

# Hedin equations, GW, GW+DMFT, and all That

Karsten Held\* (TU Wien)

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- 1 Introduction
- 2 Hedin Equations
- 3 GW
- 4 GW+DMFT
- 5 All of That: *ab initio* D $\Gamma$ A

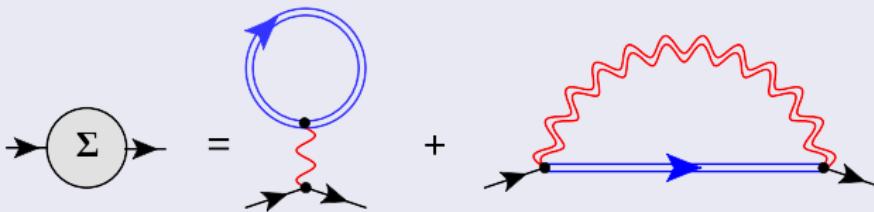
\* with C. Taranto, G. Rohringer, and A. Toschi

# 1) Introduction

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## Idea

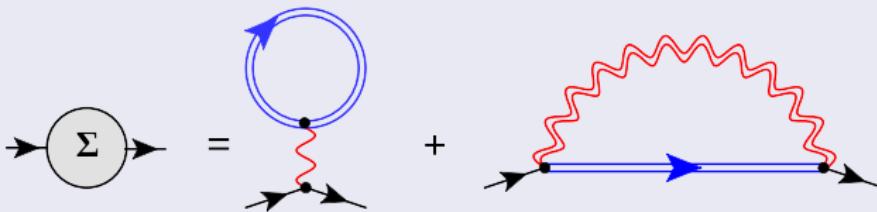
*Alternative to DFT: Hedin's (1965) GW*



# 1) Introduction

## Idea

Alternative to DFT: Hedin's (1965) GW



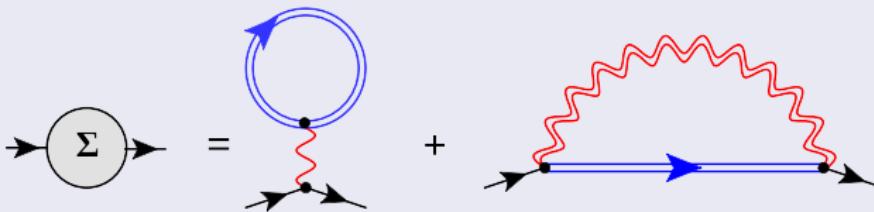
$$\Sigma^{\text{GW}}(\mathbf{r}, \mathbf{r}'; \omega) = i \int \frac{d\omega'}{2\pi} G(\mathbf{r}, \mathbf{r}'; \omega + \omega') W(\mathbf{r}, \mathbf{r}'; \omega') .$$

Self energy: Fock-like term but with screened interaction  $W$

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Alternative to DFT: Hedin's (1965) GW



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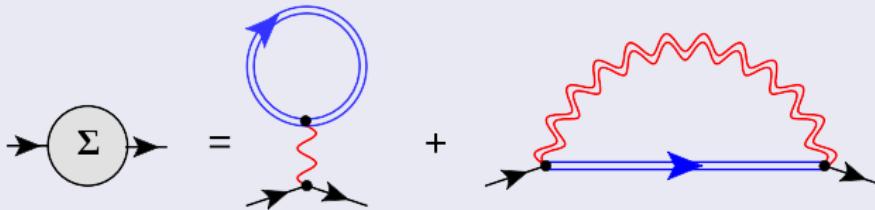
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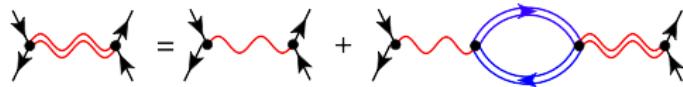
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*Self energy: Fock-like term but with screened interaction  $W$*

- often similar as LDA
- better if non-local exchange is important  
→ semiconductor band gaps

## What kind of diagrams?

Screening within random phase approx (RPA)

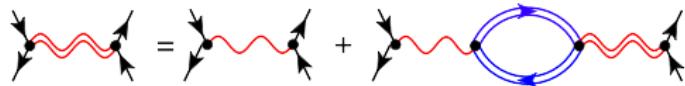


s. F. Aryasetiawan's lecture

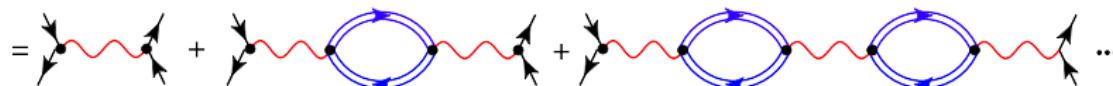


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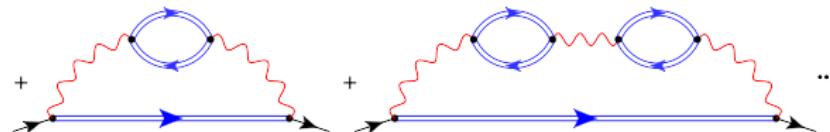
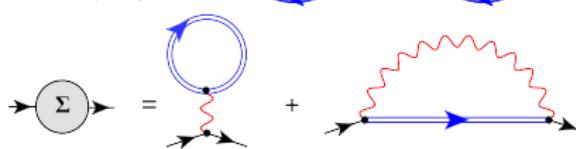
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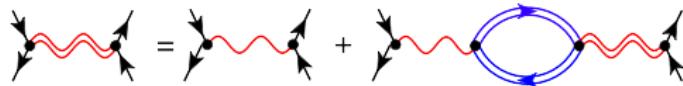


This generates:



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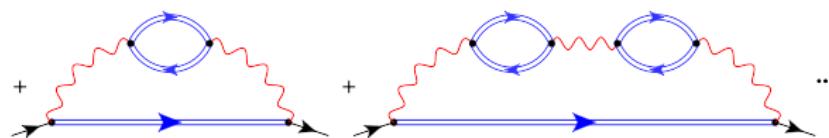
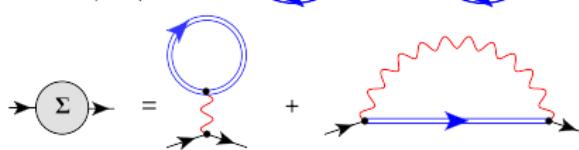
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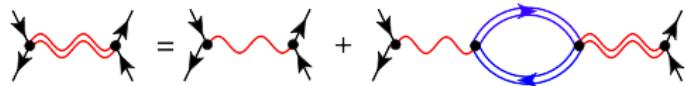
This generates:



- GW exchange “in between” LDA and bare Hartree

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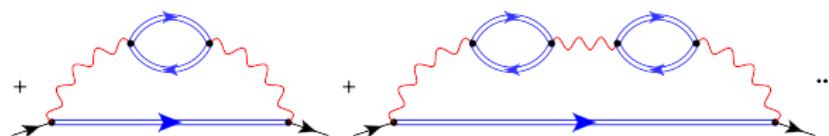
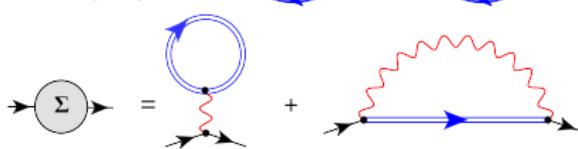
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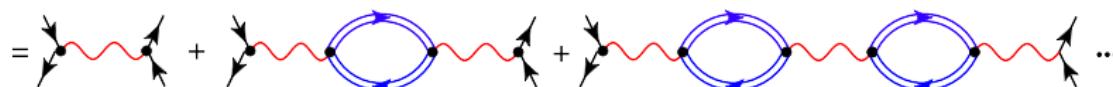
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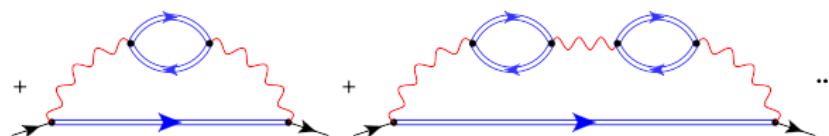
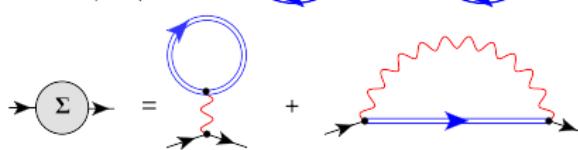
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This generates:



- GW exchange “in between” LDA and bare Hartree
- good for exchange cf. hybrid functionals
- no strong correlations (Hubbard bands) → DMFT or similar

## 2) Hedin Equations

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### Quote

Hedin (1965): "The results [i.e., the Hedin equations] are well known to the Green function people"

## 1st Hedin equation: Dyson equation $G \leftrightarrow \Sigma$

$$\overbrace{\quad\quad\quad}^{1'} \quad = \quad \overrightarrow{1'} \quad + \quad \overrightarrow{1'} \quad \overrightarrow{2'} \quad \text{Σ} \quad \overrightarrow{2} \quad \overrightarrow{1}$$

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$$\begin{aligned} G(11') &= G^0(11') + G(12)\beta\Sigma(22')G^0(2'1') \\ &= G^0(11') + G^0(12)\beta\Sigma(22')G^0(2'1') + \\ &\quad G^0(13)\beta\Sigma(33')G^0(3'2)\beta\Sigma(22')G^0(2'1') + \dots \end{aligned}$$

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Short hand notation 1 for  $(\mathbf{r}_1, \tau_1, \sigma)$  or  $(\mathbf{i}, \mathbf{m}, \omega, \sigma)$  (Bickers'04)

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Short hand notation 1 for  $(\mathbf{r}_1, \tau_1, \sigma)$  or  $(\mathbf{i}, \mathbf{m}, \omega, \sigma)$  (Bickers'04)  
 All Feynman diagrams generated by connecting

1-p irreducible building blocks  $\Sigma$  by GF lines

1-p irreducible: cutting one GF  $\Rightarrow$  diagram still connected

**2nd Hedin equation:** screened interaction  $W \leftrightarrow$  polarization op.  $P$

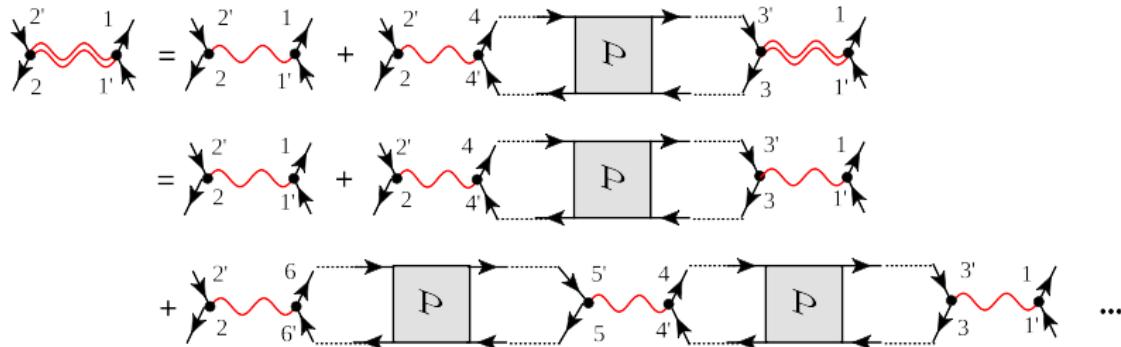
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Analogon to Dyson eq.

$W$ : all Feynman diagrams connecting left and right by  $V$ 's

$G$ : all Feynman diagrams connecting left and right by  $G^0$ 's



$$W(11'; 22') = V(11'; 22') + W(11'; 33')P(3'3; 4'4)V(44'; 22') .$$

$P$ : all diagrams irreducible w.r.t. cutting one  $V$

### 3rd Hedin equation: $P \leftrightarrow$ polarization op. $\Gamma^*$

standard relation between 2-p GF (response fct.) and vertex

$$\begin{array}{c} 2' \\ \nearrow \quad \searrow \\ \text{---} \quad \text{---} \\ \text{P} \\ \text{---} \quad \text{---} \\ \searrow \quad \nearrow \\ 2 \quad 1' \end{array} = \begin{array}{c} 2' \\ \nearrow \quad \searrow \\ \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ 2 \quad 1' \end{array} + \begin{array}{c} 2' \\ \nearrow \quad \searrow \\ \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ \Gamma^* \\ \text{---} \quad \text{---} \\ \searrow \quad \nearrow \\ 2 \quad 1' \end{array}$$

$$\left( \begin{array}{c} = \\ \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ \Gamma^* \\ \text{---} \quad \text{---} \\ \searrow \quad \nearrow \\ 2 \quad 1 \end{array} + \begin{array}{c} \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ \Gamma^* \\ \text{---} \quad \text{---} \\ \searrow \quad \nearrow \\ 2 \quad 1 \end{array} \right) \text{ in real space}$$

$$P(11'; 22') = \beta G(12')G(21') + \beta G(13)G(3'1')\Gamma^*(33'; 44')\beta G(4'2')G(24)$$

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- Note,  $P$  is irreducible w.r.t. cutting one  $V$   
 $\Rightarrow \Gamma^*$  is irreducible w.r.t. cutting one  $V$

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$$\begin{array}{ccc} \text{Diagram 1: } & \text{Diagram 2: } & \text{Diagram 3: } \\ \begin{array}{c} \text{A rectangle labeled } P \text{ with two horizontal arrows entering from the left and exiting to the right. Vertices are labeled } 2' \text{ and } 1' \text{ on the left, and } 1 \text{ and } 2 \text{ on the right.} \end{array} & = & \begin{array}{c} \text{Two parallel horizontal lines with arrows pointing right, labeled } 2' \text{ and } 1' \text{ at the top, and } 2 \text{ and } 1' \text{ at the bottom.} \end{array} \\ + & & \begin{array}{c} \text{Two parallel horizontal lines with arrows pointing right, labeled } 2' \text{ and } 1' \text{ at the top, and } 2 \text{ and } 1' \text{ at the bottom. A gray rectangle labeled } \Gamma^* \text{ is placed between them.} \end{array} \end{array}$$

$$\left( \begin{array}{ccc} \text{Diagram 4: } & \text{Diagram 5: } & \text{in real space} \\ \begin{array}{c} \text{A loop with two vertices labeled } 2 \text{ and } 1 \text{ connected by a double line. Arrows point clockwise.} \end{array} & + & \begin{array}{c} \text{A loop with two vertices labeled } 2 \text{ and } 1 \text{ connected by a double line. A gray rectangle labeled } \Gamma^* \text{ is placed inside the loop. Arrows point clockwise.} \end{array} \end{array} \right)$$

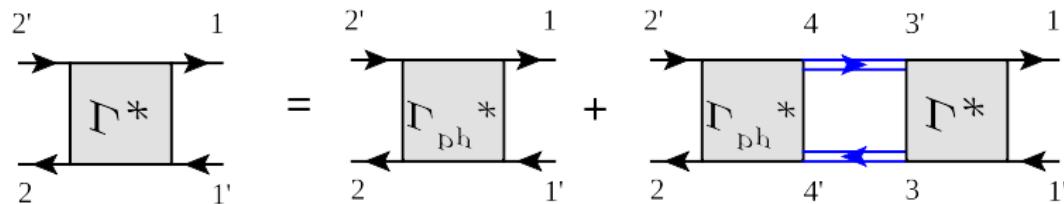
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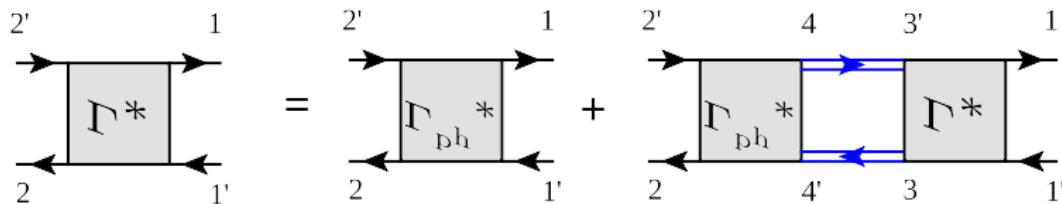
2nd line: simplifications in real space  $2=2'$ ,  $1=1'$

**4th Hedin equation:** Bethe-Salpeter eq. red.  $\Gamma^* \leftrightarrow$  irred.  $\Gamma_{ph}^*$   
 Analogon to Dyson eq. but for two-particles



$$\Gamma^*(11'; 22') = \Gamma_{ph}^*(11'; 22') + \Gamma^*(11'; 33')\beta G(3'4)G(4'3)\Gamma_{ph}^*(44'; 22')$$

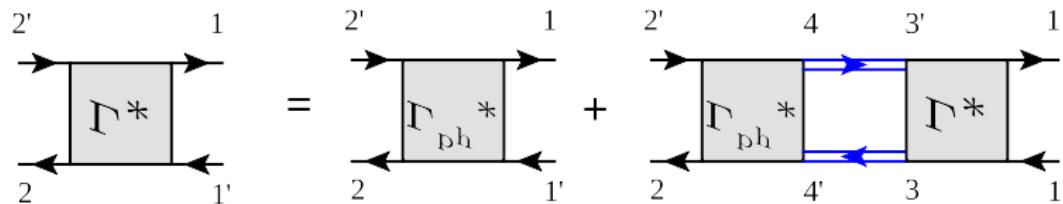
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$\Gamma_{ph}(44'; 22')$  irred. particle-hole vertex:  
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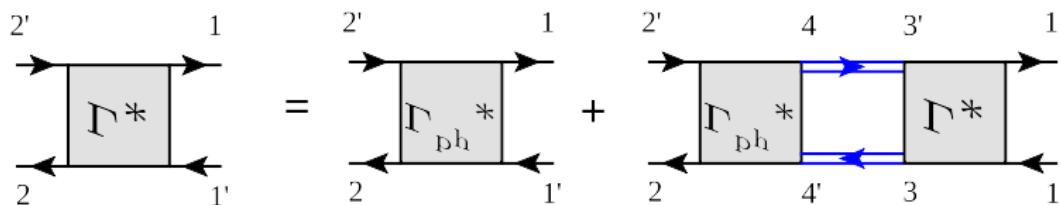
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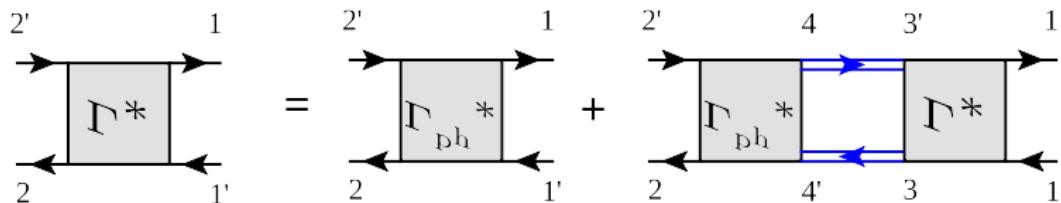
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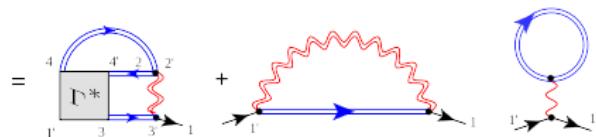
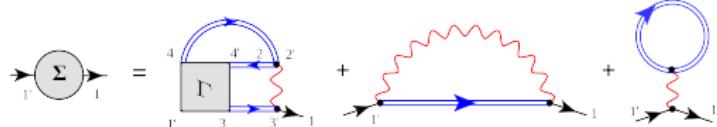
Hedin:  $\Gamma_{ph}^*(11'; 22') = \frac{\delta[\Sigma(11') - \Sigma_{\text{Hartree}}(11')]}{\delta G(2'2')}$

## 5th Hedin equation: (Heisenbeg) equation of motion

$$G(\mathbf{r}, \tau; \mathbf{r}', \tau') \equiv -\langle \mathcal{T}c(\mathbf{r}, \tau)c(\mathbf{r}, \tau')^\dagger \rangle \quad -\frac{\partial c(\mathbf{r}_1, \tau_1)}{\partial \tau_1} = [c(\mathbf{r}_1, \tau_1), H]$$

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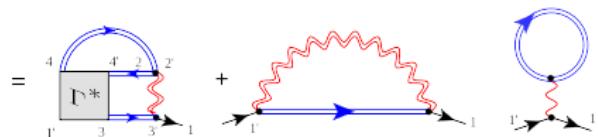
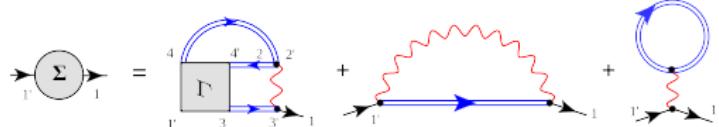


$$\begin{aligned} \Sigma(11') &= -V(13'; 22')\beta G(4'2)G(2'4)G(3'3)\Gamma(31'; 44') - V(12'; 21')G(2'2) \\ &= -W(13'; 22')\beta G(4'2)G(2'4)G(3'3)\Gamma^*(31'; 44') - W(12'; 21')G(2'2) \end{aligned}$$

for details s. Appendix of Lecture Notes

## 5th Hedin equation: (Heisenberg) equation of motion

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### 3) $GW$

**From Hedin's eq. to  $GW$**

Complicated part is irreducible vertex

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Equation of motion yields  $\Sigma^{GW}(11') = -W(12'; 21')G(2'2)$

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Bethe-Salpeter eq.  $\Rightarrow \Gamma^* = 0$

$\Rightarrow$  polarization op. is bubble  $P^{GW}(11'; 22') = \beta G(12')G(21')$

screened interaction calculated with this  $P^{GW}$  (RPA)

$$W(11'; 22') = V(11'; 22') + W(11'; 33')P^{GW}(3'3; 4'4)V(44'; 22')$$

Equation of motion yields  $\Sigma^{GW}(11') = -W(12'; 21')G(2'2)$

Dyson equation yields

$$G(11') = G^0(11') + G(12)\beta\Sigma^{GW}(22')G^0(2'1')$$

Often not self-consistent  $G_0 W_0$  – with LDA as starting point

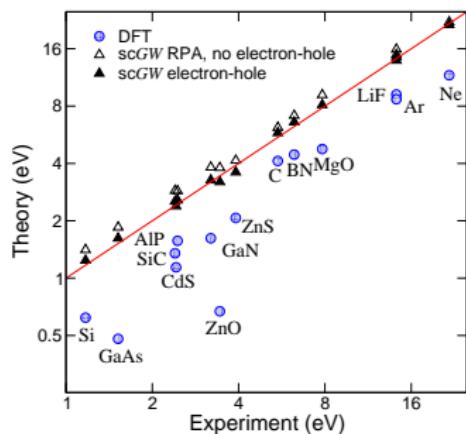
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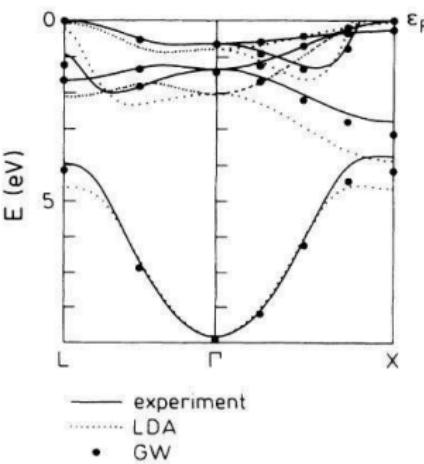
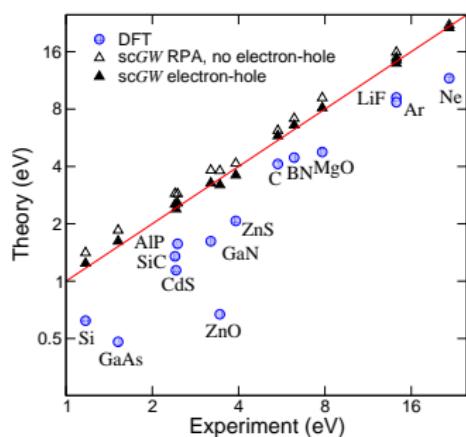
**band gaps** Shishkin et al.'07



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## bandstructure of Ni

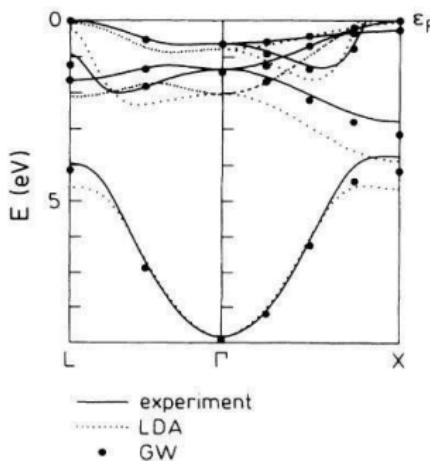
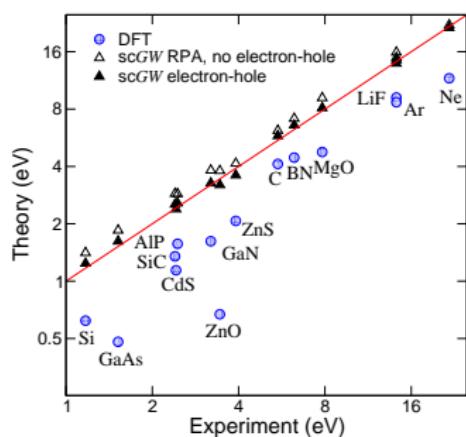
Aryasetiawan'82

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  - no -6eV satellite → DMFT

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### bandstructure of Ni

Aryasetiawan'82

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- also finite life times e.g. Ag

## 4) $GW+DMFT$

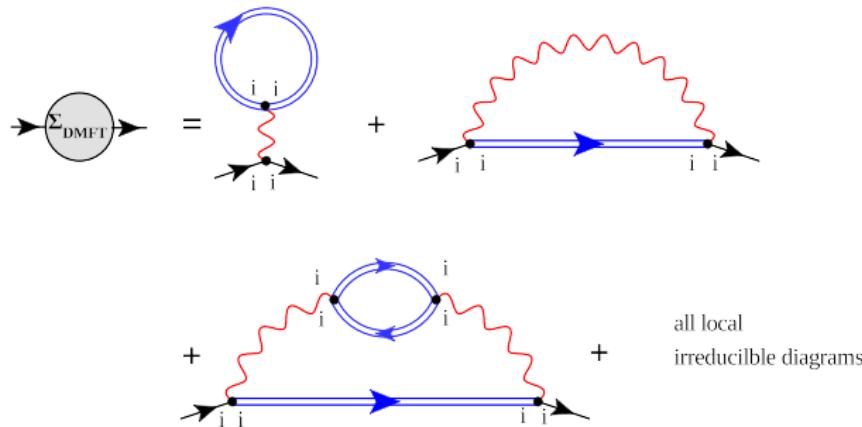
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Biermann et al.'03

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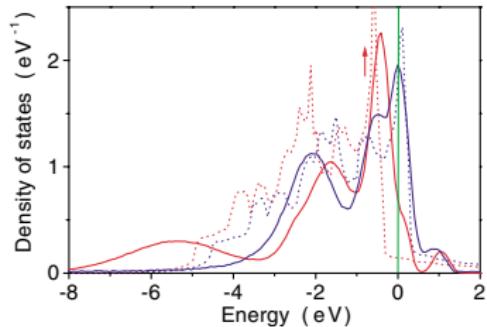


# Algorithm

reproduced from  
Held Adv. Phys. '07

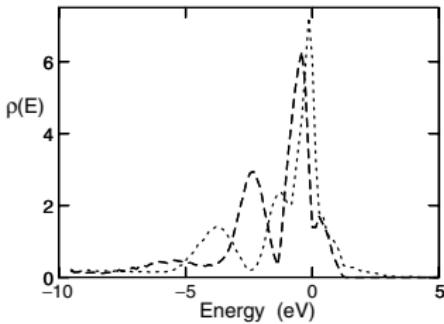
|  |  |
|--|--|
| Do LDA calculation, yielding $G_k(\omega) = [\omega\mathbf{1} + \mu\mathbf{1} - \epsilon^{\text{LDA}}(\mathbf{k})]^{-1}$ . |  |
|  | Calculate $GW$ polarization $\mathbf{P}^{GW}(\omega) = -2i \int \frac{d\omega'}{2\pi} \mathbf{G}(\omega + \omega') \mathbf{G}(\omega')$ .  |
|  | If DMFT polarization $\mathbf{P}^{\text{DMFT}}$ is known (after the 1st iteration), include it   |
|  | $\mathbf{P}^{\text{GW+DMFT}}(\mathbf{k}, \omega) = \mathbf{P}^{GW}(\mathbf{k}, \omega) - \frac{1}{V_{\text{BZ}}} \int d^3k \mathbf{P}^{GW}(\mathbf{k}, \omega) + \mathbf{P}^{\text{DMFT}}(\omega).$  |
|  | With this polarization, calculate the screened interaction:  |
|  | $\mathbf{W}(\mathbf{k}; \omega) = \mathbf{V}_{ee}(\mathbf{k}) [\mathbf{1} - \mathbf{V}_{ee}(\mathbf{k}) \mathbf{P}(\mathbf{k}; \omega)]^{-1}.$   |
|  | Calculate $\Sigma_k^{\text{Hartree}} = \int \frac{d^3q}{V_{\text{BZ}}} \mathbf{G}_q(\tau=0^-) \mathbf{W}(\mathbf{k}-\mathbf{q}, 0)$ and $\Sigma_{dc}^{\text{Hartree}}$ .   |
|  | Calculate $\Sigma^{GW}(\mathbf{r}, \mathbf{r}'; \omega) = i \int \frac{d\omega'}{2\pi} \mathbf{G}(\mathbf{r}, \mathbf{r}'; \omega + \omega') \mathbf{W}(\mathbf{r}, \mathbf{r}'; \omega')$ .   |
|  | Calculate the DMFT self-energy $\Sigma^{\text{DMFT}}$ and polarization $\mathbf{P}^{\text{DMFT}}$ as follows:  |
|  | From the local Green function $\mathbf{G}$ and old self-energy $\Sigma^{\text{DMFT}}$ calculate $(\mathbf{G}^0)^{-1}(\omega) = \mathbf{G}^{-1}(\omega) + \Sigma^{\text{DMFT}}(\omega)$ . $\Sigma^{\text{DMFT}} = 0$ in 1st iteration.  |
|  | Extract the local screening contributions from $\mathbf{W}$ :<br>$\mathbf{U}(\omega) = [\mathbf{W}^{-1}(\omega) - \mathbf{P}^{\text{DMFT}}(\omega)]^{-1}.$   |
|  | With $\mathbf{U}$ and $\mathbf{G}^0$ , solve impurity problem with effective action  |
|  | $\mathcal{A} = \sum_{\nu\sigma} \psi_{\nu m}^{\sigma*} (\mathcal{G}_{\nu mn}^{\sigma 0})^{-1} \psi_{\nu n}^{\sigma} + \sum_{lm\sigma\sigma'} \int d\tau \psi_l^{\sigma*}(\tau) \psi_l^{\sigma}(\tau) U_{lm}(\tau-\tau') \psi_m^{\sigma'*}(\tau') \psi_m^{\sigma'}(\tau'),$ resulting in $\mathbf{G}$ and susceptibility $\chi$ . |
|  | From $\mathbf{G}$ and $\chi$ , calculate $\Sigma^{\text{DMFT}}(\omega) = (\mathbf{G}^0)^{-1}(\omega) - \mathbf{G}^{-1}(\omega)$ ,<br>$\mathbf{P}^{\text{DMFT}}(\omega) = \mathbf{U}^{-1}(\omega) - [\mathbf{U} - \mathbf{U}\chi\mathbf{U}]^{-1}(\omega).$  |
|  | Combine this to the total $GW$ self-energy:<br>$\Sigma^{\text{GW+DMFT}}(\mathbf{k}, \omega) = \Sigma^{GW}(\mathbf{k}, \omega) - \int d^3k \Sigma^{GW}(\mathbf{k}, \omega) + \Sigma^{\text{Hartree}}(\mathbf{k}) - \Sigma_{dc}^{\text{Hartree}} + \Sigma^{\text{DMFT}}(\omega).$  |
|  | From this and $\mathbf{G}^0$ , calculate $\mathbf{G}_k^{\text{new}}(\omega)^{-1} = \mathbf{G}_k^0(\omega)^{-1} - \Sigma_k(\omega).$  |
|  | Iterate with $\mathbf{G}_k = \mathbf{G}_k^{\text{new}}$ until convergence, i.e. $\ \mathbf{G}_k - \mathbf{G}_k^{\text{new}}\  < \epsilon$ .  |

## Results for Ni



LDA+DMFT

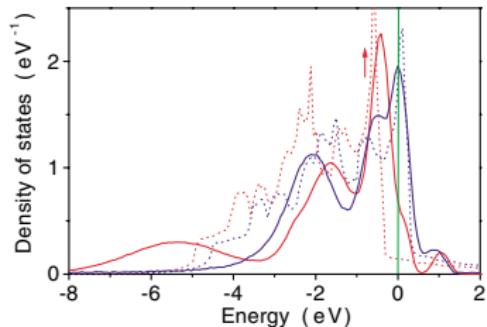
Lichtenstein et al.'01



GW+DMFT

Biermann et al.'03

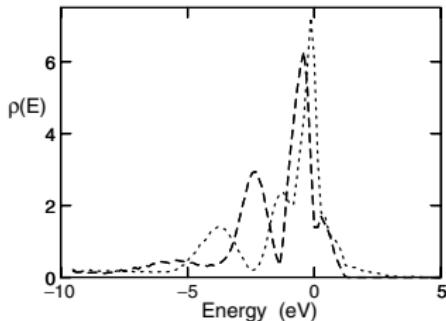
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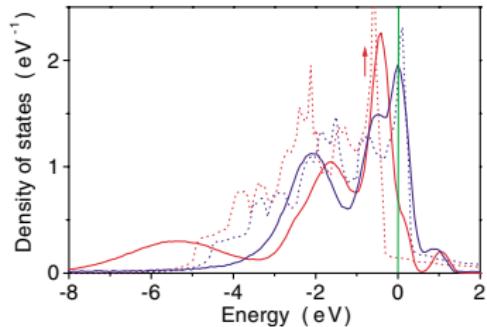
- satellite at -6eV; both similar



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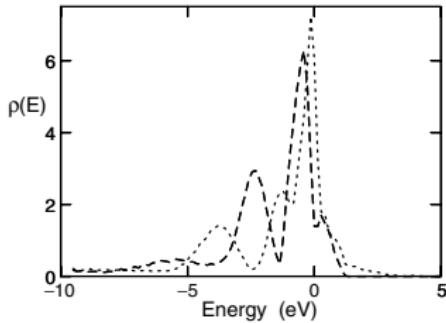
## Results for Ni



LDA+DMFT

Lichtenstein et al.'01

- satellite at -6eV; both similar
- no self consistency,  $P^{\text{DMFT}}$  ...
- $W(\omega) \rightarrow W$



GW+DMFT

Biermann et al.'03

## 4) All of That: *ab initio* D $\Gamma$ A

Hedin's eq.: combine  $GW$  exchange and correlations on 2p level  
instead of 1p  $GW+DMFT$

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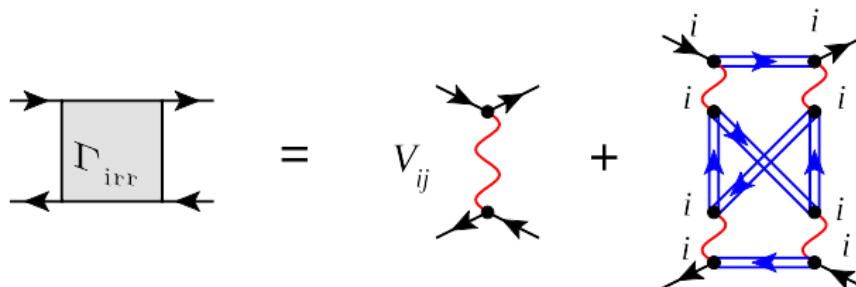
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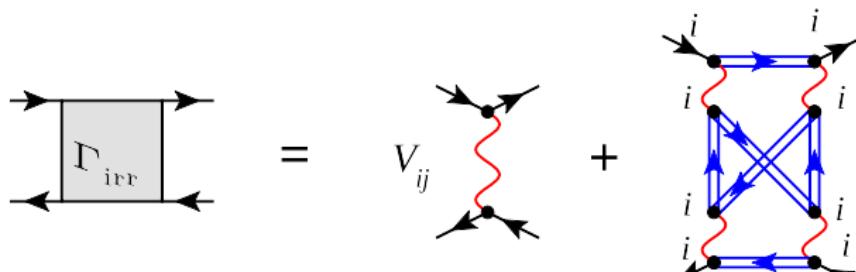
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→ starting point of Hedin eq.