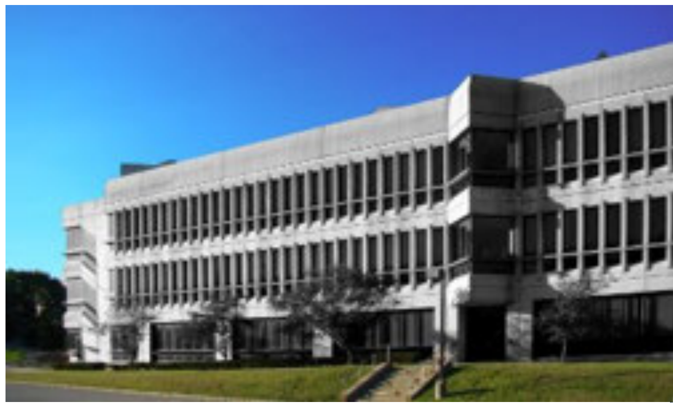
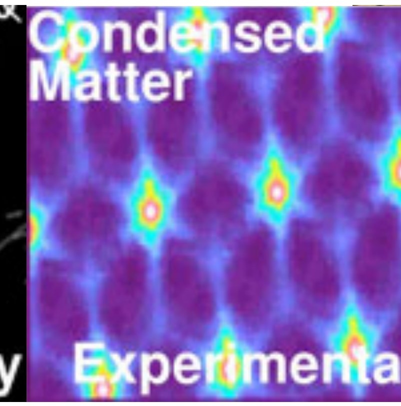
Heavy Fermion Physics: a 21st Century perspective

Julich,
21 Sept, 2015

Piers Coleman: Rutgers Center for Materials Theory, USA

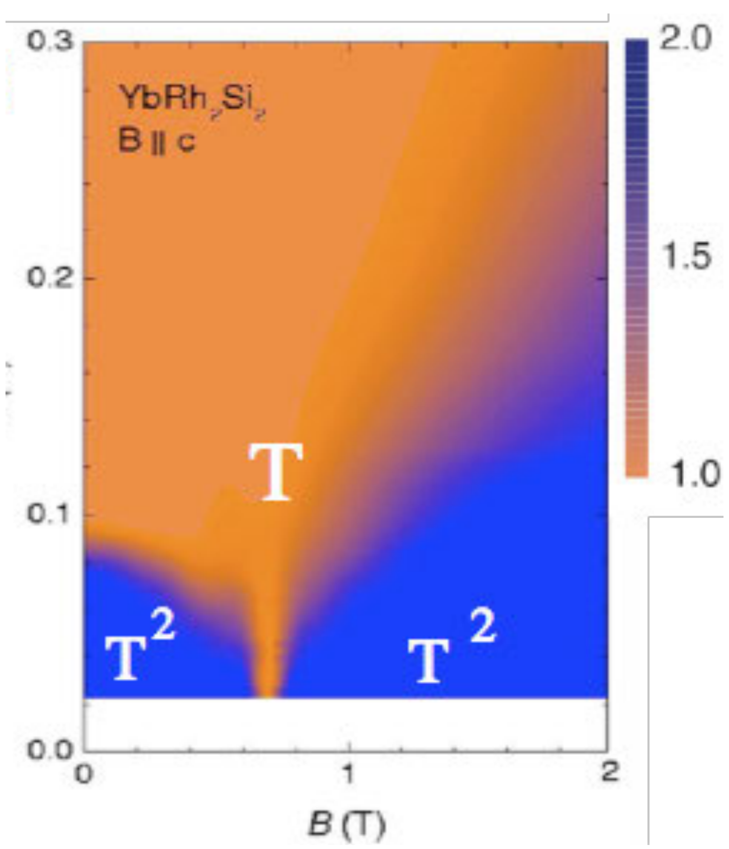


Heavy Fermion Physics: a 21st Century perspective

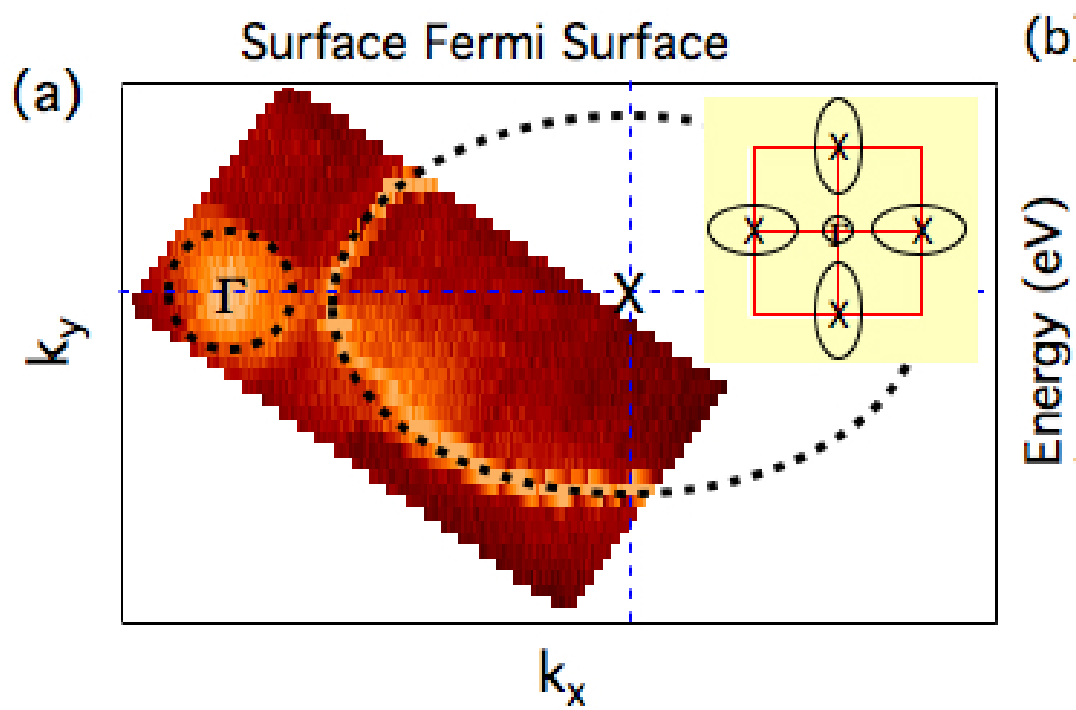
Piers Coleman: Rutgers Center for Materials Theory, USA

Quantum Criticality & Strange Metals



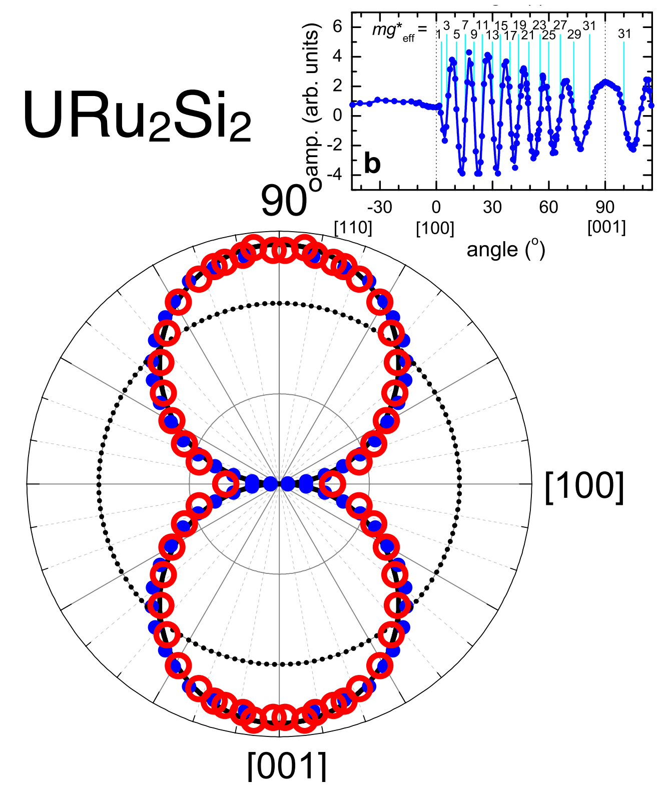
Heavy Fermion Superconductivity

Topological Kondo Insulators



Hidden Order

URu$_2$Si$_2$



Collaborators.

Q. Si	Rice
R. Ramazashvili	Toulouse
C. Pepin	CEA, Saclay
Aline Ramires	ETH
Rebecca Flint	Iowa State
Premi Chandra	Rutgers
Andriy Nevidomskyy	Rice
Alexei Tsvelik	Brookhaven NL
Hai Young Kee	U. Toronto
Natan Andrei	Rutgers
Onur Erten	Rutgers
Maxim Dzero	Kent State
Victor Galitski	U. Maryland
Kai Sun	U. Michigan

Experimentalists:

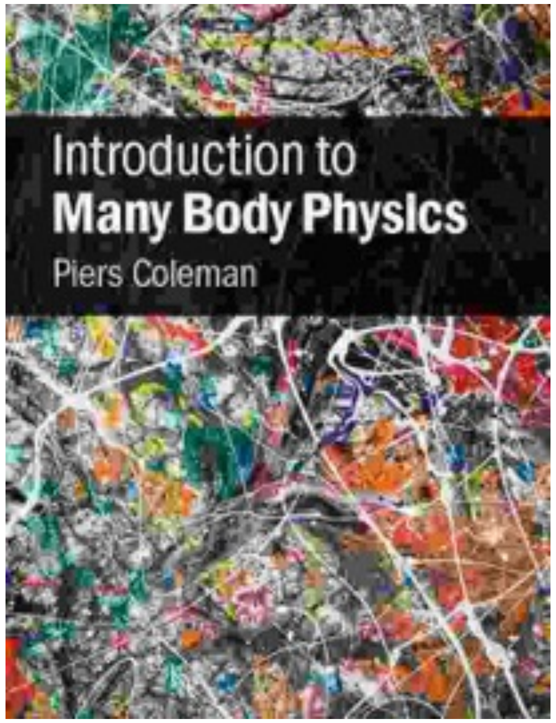
H. von Lohneysen	Karlsruhe
G. Aeppli	ETH, Zurich
A. Schröder	Kent State
S. Nakatsuji	ISSP
G. Lonzarich	Cambridge
S. Paschen	Vienna
J. Thompson	Los Alamos
J. Allen	U. Michigan
Z. Fisk	UC Irvine
F Steglich	Dresden/Zhejiang

U.S. DEPARTMENT OF ENERGY

NSF

Notes:

"Introduction to Many Body Physics", Ch 8,15-16", PC, CUP to be published (2015).



"Heavy Fermions: electrons at the edge of magnetism." Wiley encyclopedia of magnetism. PC. cond-mat/0612006.

"I2CAM-FAPERJ Lectures on Heavy Fermion Physics", (X=I, II, III)
http://physics.rutgers.edu/~coleman/talks/RIO13_X.pdf

General reading:

A. Hewson, "Kondo effect to heavy fermions", CUP, (1993).
"The Theory of Quantum Liquids", Nozieres and Pines (Perseus 1999).

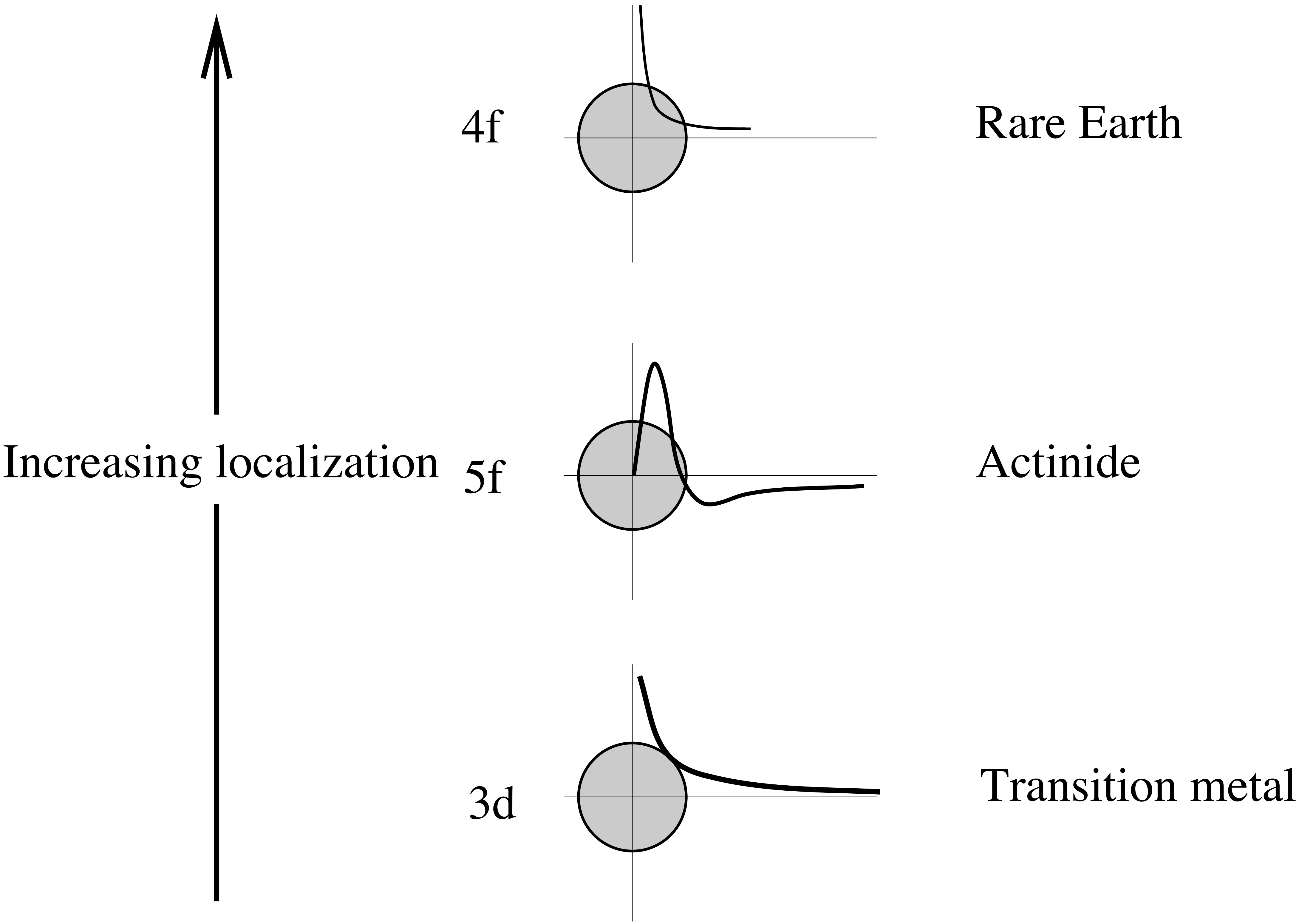
Rutgers
Center for Materials Theory

Outline of the Topics

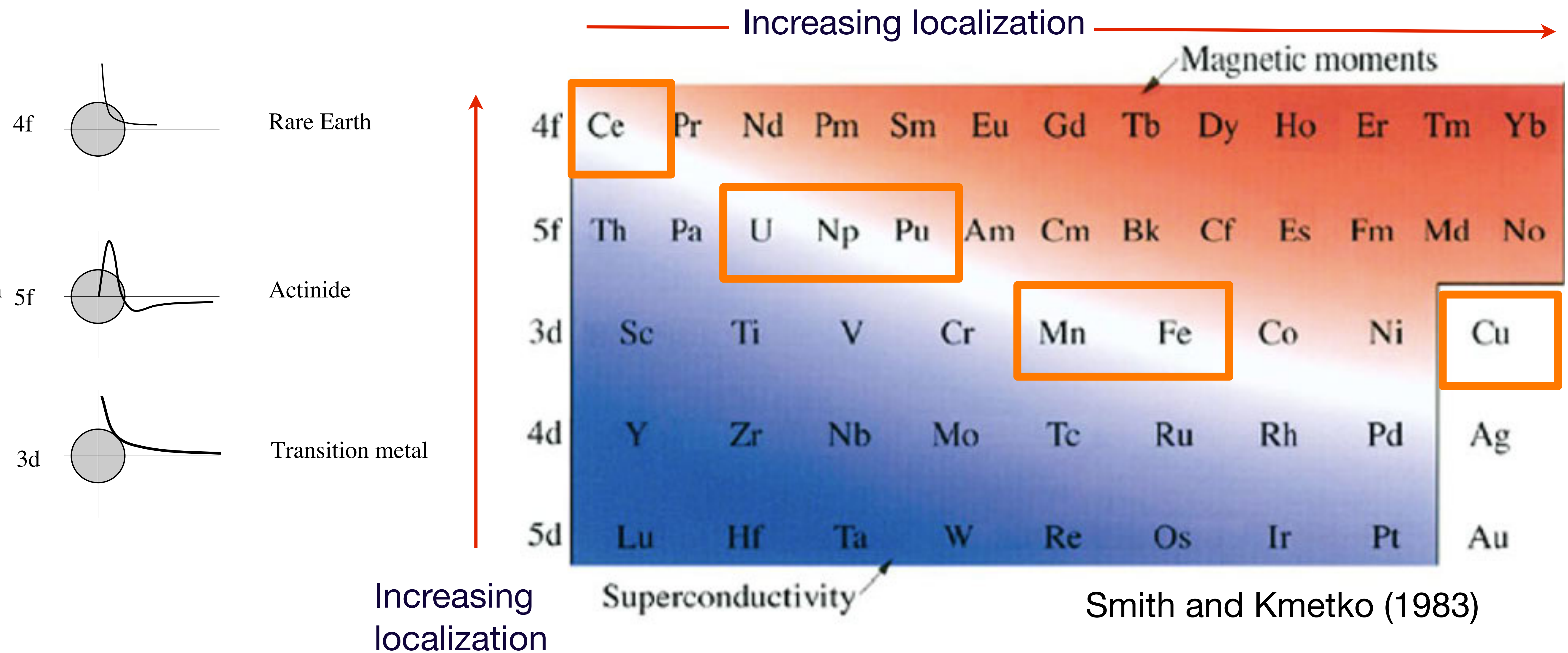
1. **Trends in the periodic table.**
2. Introduction: Heavy Fermions and the Kondo Lattice.
3. Kondo Insulators: the simplest heavy fermions.
4. Large N expansion for the Kondo Lattice
5. Heavy Fermion Superconductivity
6. Topological Kondo Insulators
7. Co-existing magnetism and the Kondo Effect.

Please ask questions!

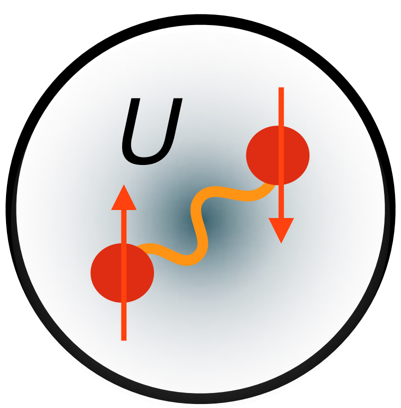
Rutgers
Center for Materials Theory



Increasing localization
4f
Rare Earth
5f
Actinide
3d
Transition metal



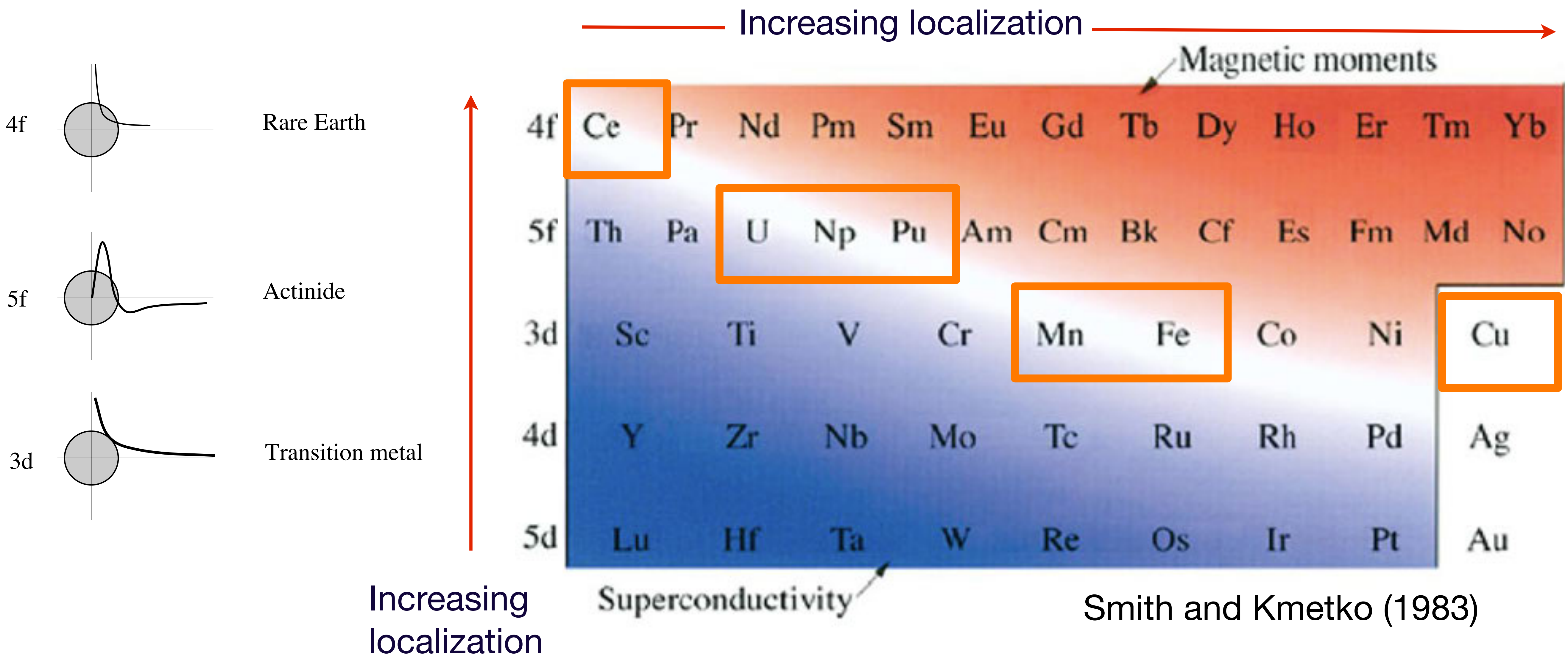
Smith and Kmetko (1983)



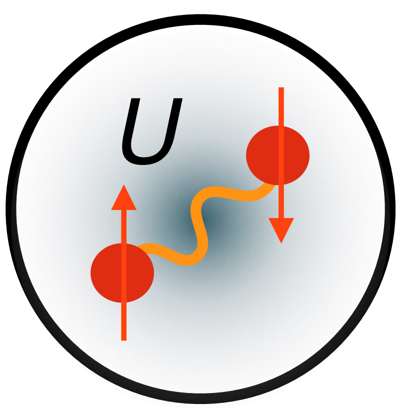
- No double occupancy: strongly correlated
- Residual valence fluctuations induce AFM Superexchange.

Mott Mechanism.

Anderson U (Anderson 1959)



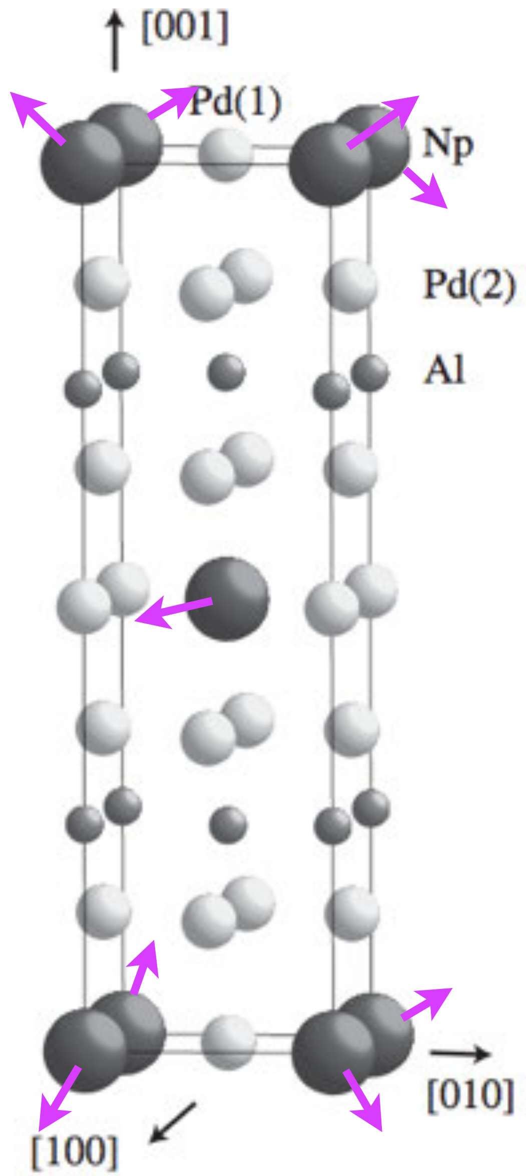
Smith and Kmetko (1983)



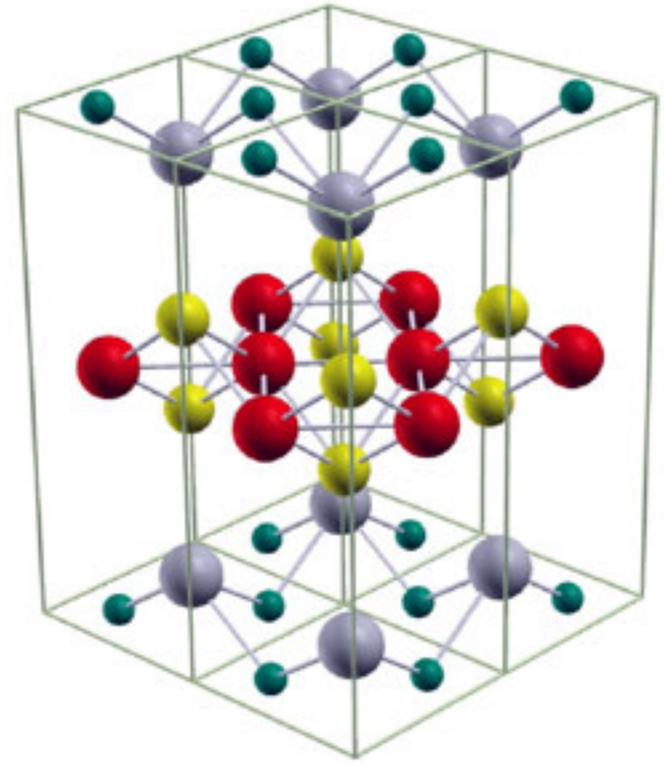
Many things are possible at the brink of magnetism.

Mott Mechanism.

Anderson U (Anderson 1959)

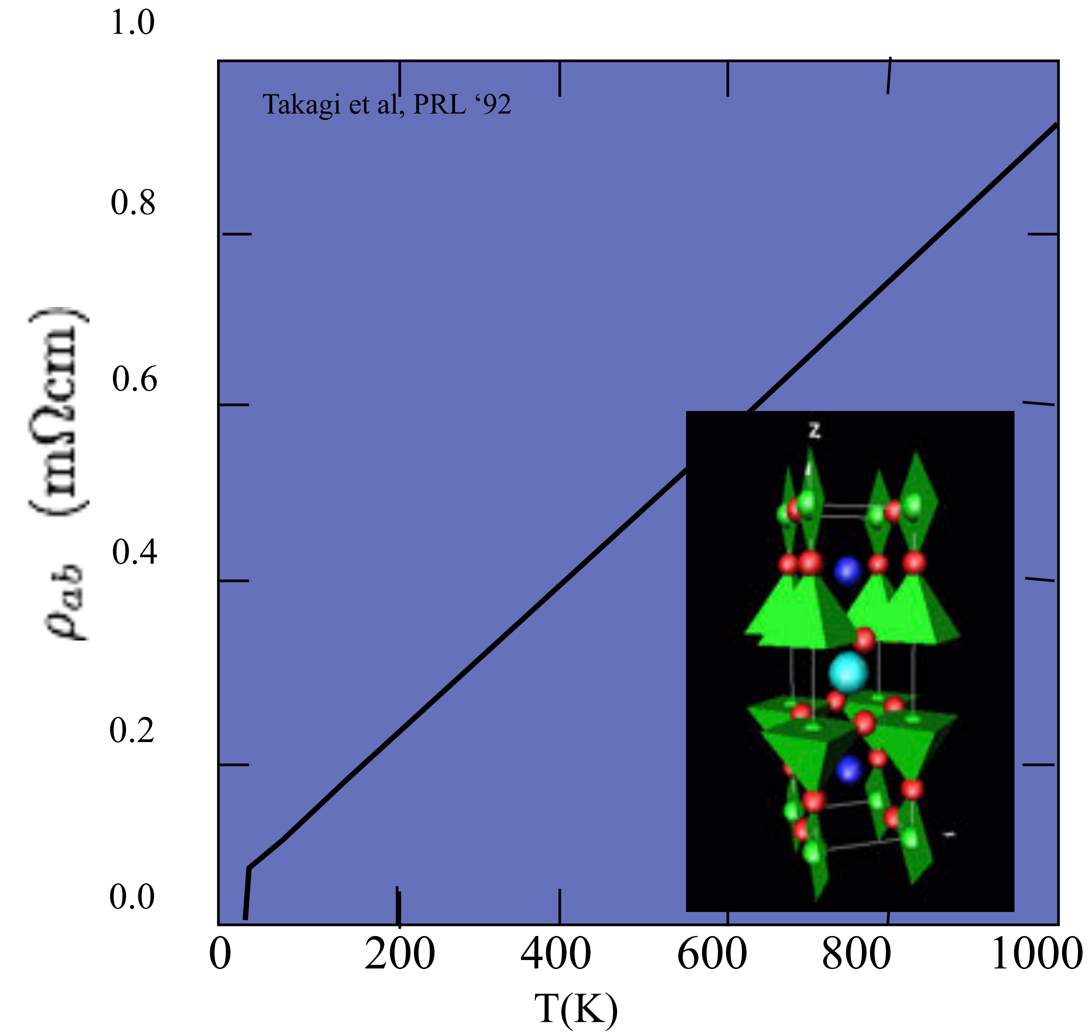


$NpAl_2Pd_5$



$PrF_xO_{1-x}FeAs$

Z.A. Ren et.al, Beijing, (08)



HF 115s
T_c=0.2 -18.5 K

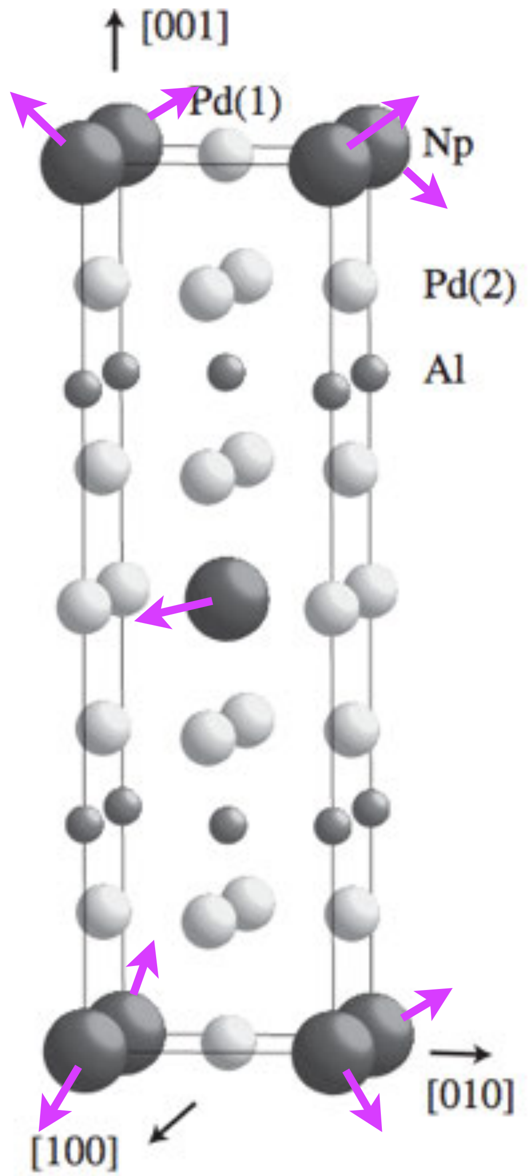
Iron based sc
T_c= 6 - 53 ++ ? K

Cuprates T_c=11-92K

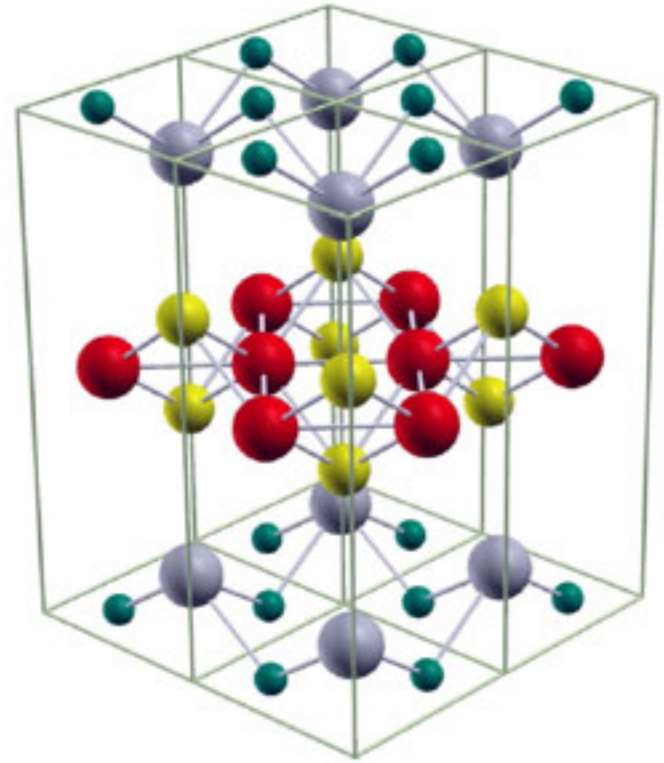
Diversity of new ground-states on the brink of localization.

f-electron systems: 4f Ce, Yb systems 5f U, Np, Pu systems.

d-electron systems: e.g Pnictides, Cuprate SC.

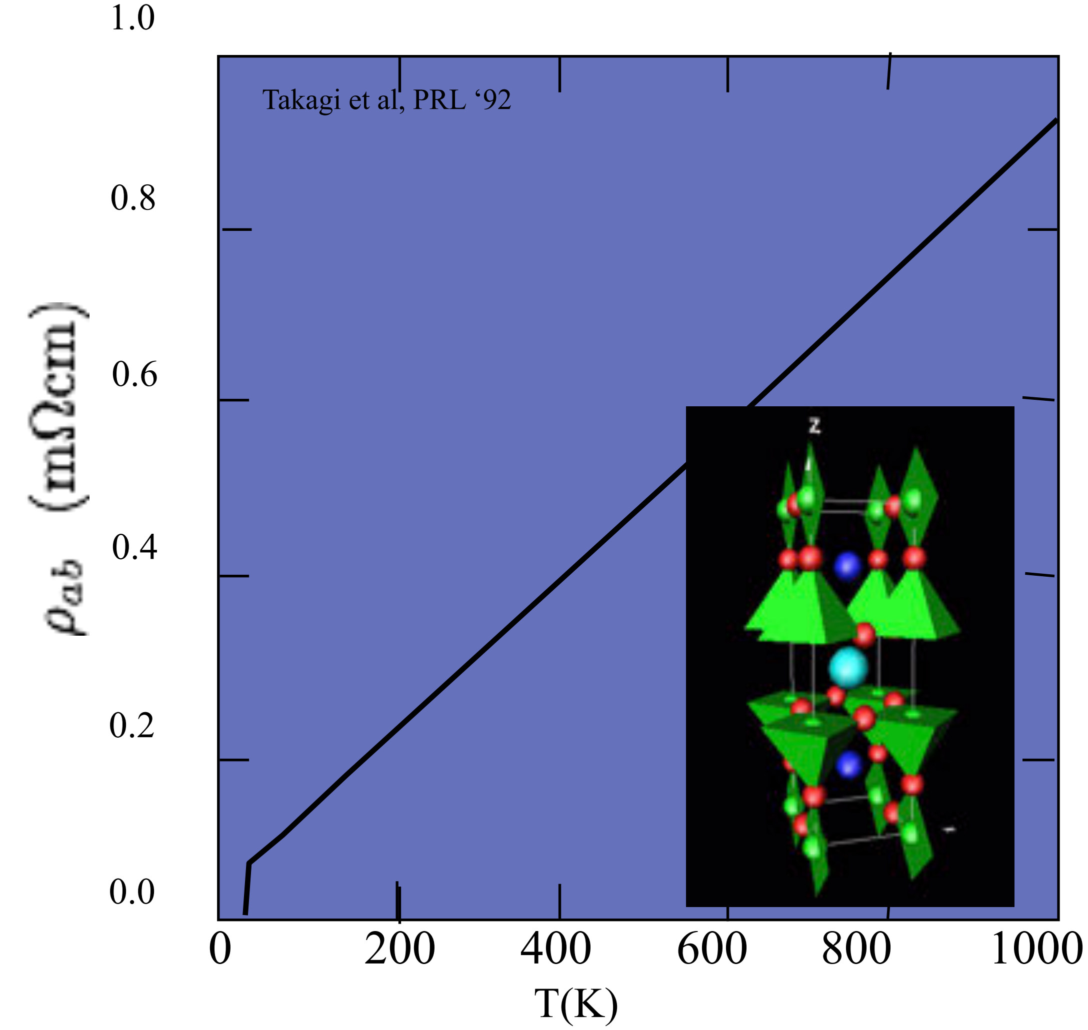


$NpAl_2Pd_5$



$PrF_xO_{1-x}FeAs$

Z.A. Ren et.al, Beijing, (08)



HF 115s

T_c=0.2 -18.5 K

Iron based sc

T_c= 6 - 53 ++ ? K

Cuprates T_c=11-92K

A new era of mysteries

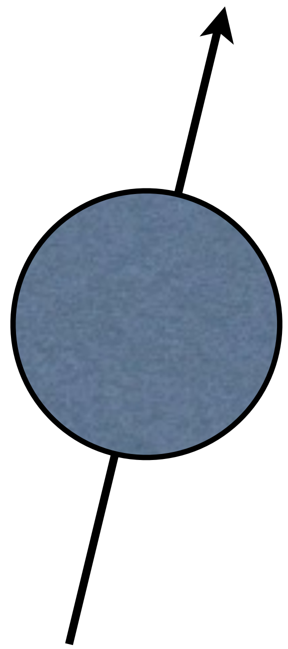
Outline of the Topics

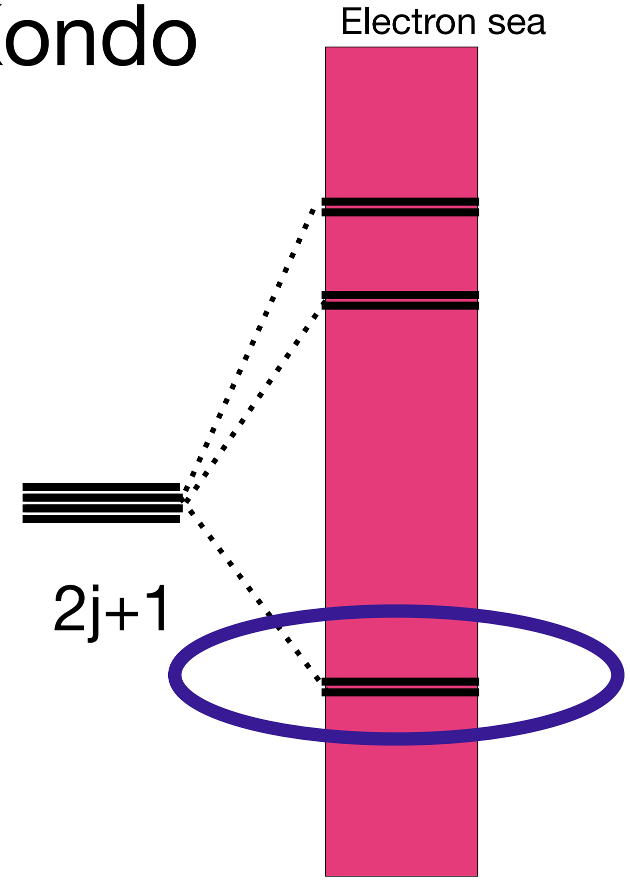
1. Trends in the periodic table.
2. **<u>Introduction: Heavy Fermions and the Kondo Lattice.</u>**
3. Kondo Insulators: the simplest heavy fermions.
4. Large N expansion for the Kondo Lattice
5. Heavy Fermion Superconductivity
6. Topological Kondo Insulators
7. Co-existing magnetism and the Kondo Effect.

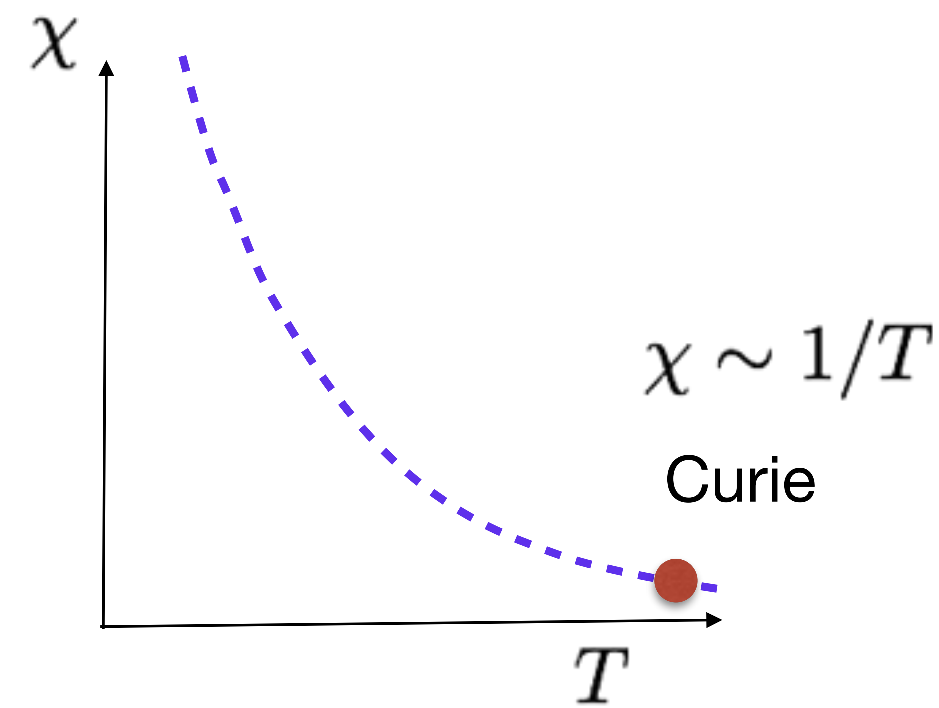
Please ask questions!

Rutgers
Center for Materials Theory

Heavy Fermions + Kondo



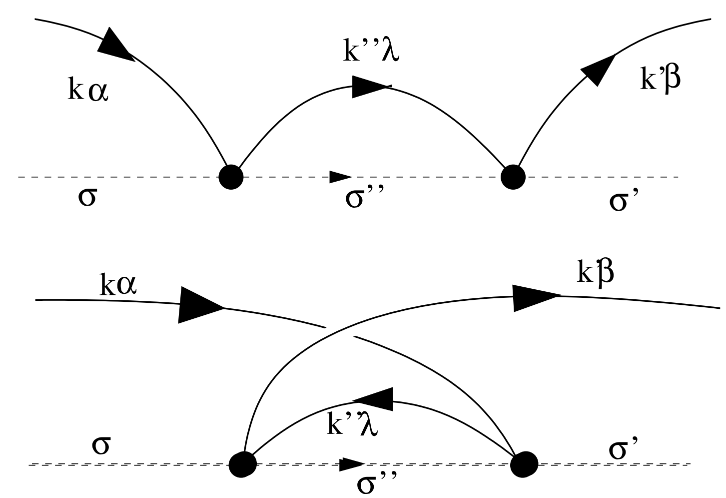




Spin (4f,5f):
“quark” of heavy
electron physics.

$$J \to J(T) = J + 2J^2\rho \ln \frac{D}{T}$$

“Scales to
Strong Coupling”



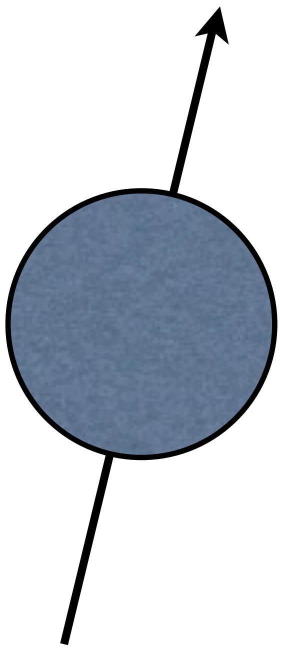
$$\approx -\frac{J^2\rho\delta D}{D}(\sigma^b\sigma^a)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$

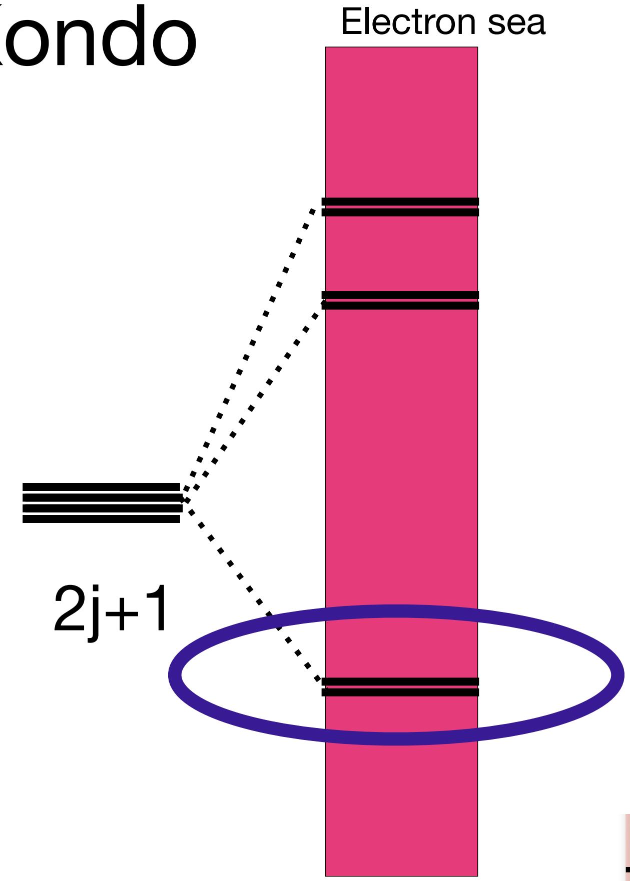
$$\approx \frac{J^2\rho\delta D}{D}(\sigma^a\sigma^b)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$

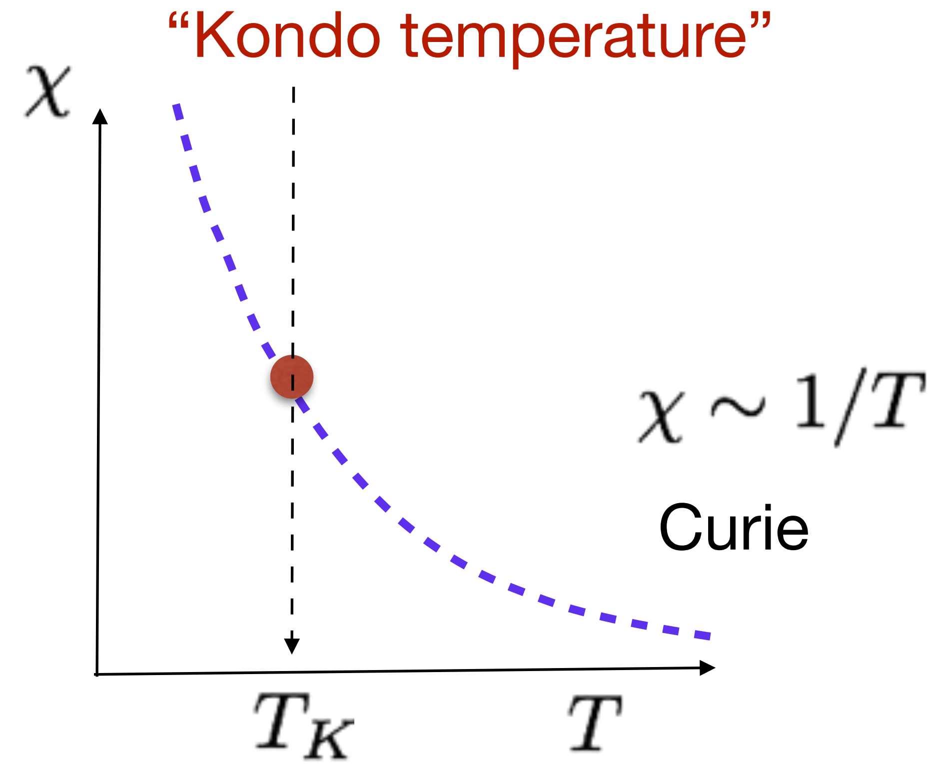
$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \vec{S}\cdot\vec{\sigma}(0)$$

J. Kondo, 1962

Heavy Fermions + Kondo







$$T_K = W\sqrt{J\rho}\,e^{-\frac{1}{2J\rho}}$$

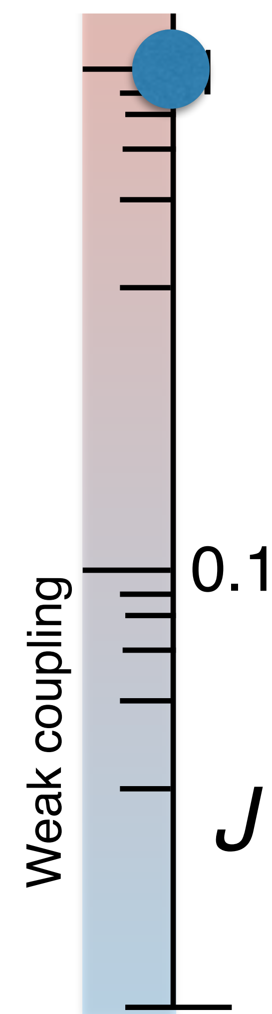
Spin (4f,5f):
"quark" of heavy
electron physics.

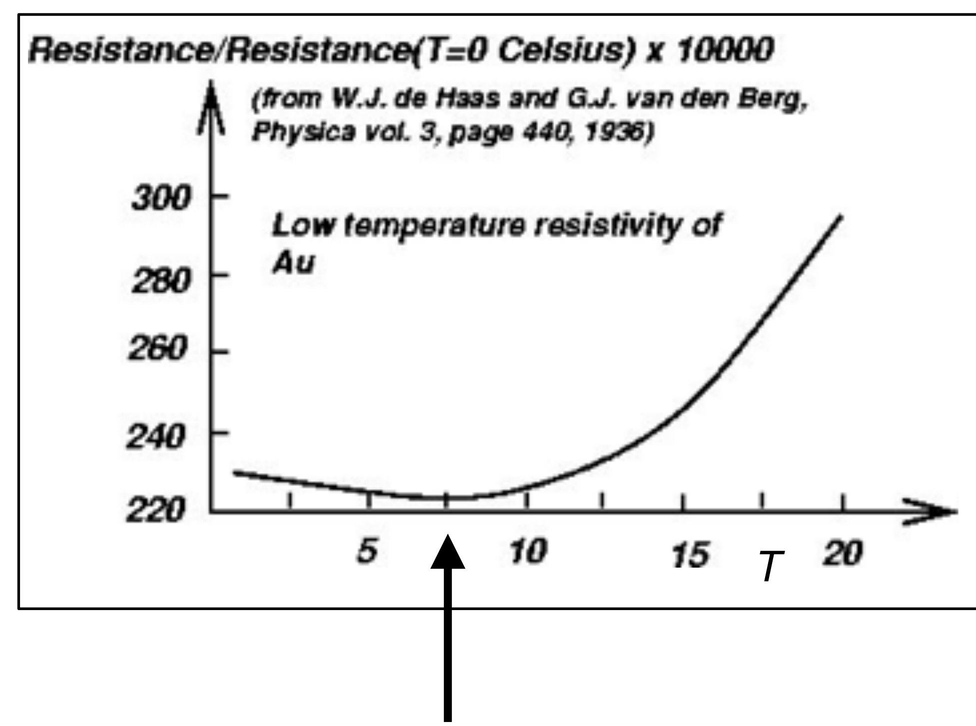
$$J \rightarrow J(T) = J + 2J^2\rho \ln \frac{D}{T}$$

"Scales to
Strong Coupling"

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J\,\vec{S}\cdot\vec{\sigma}(0)$$

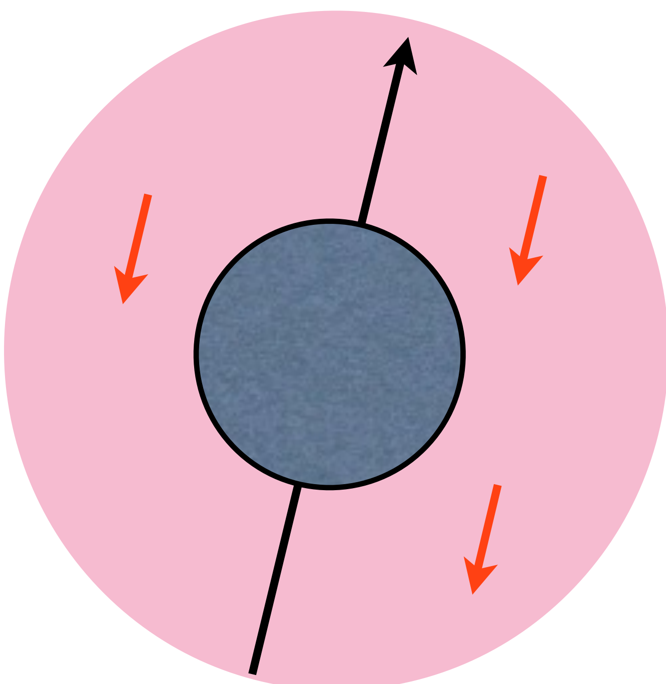
J. Kondo, 1962

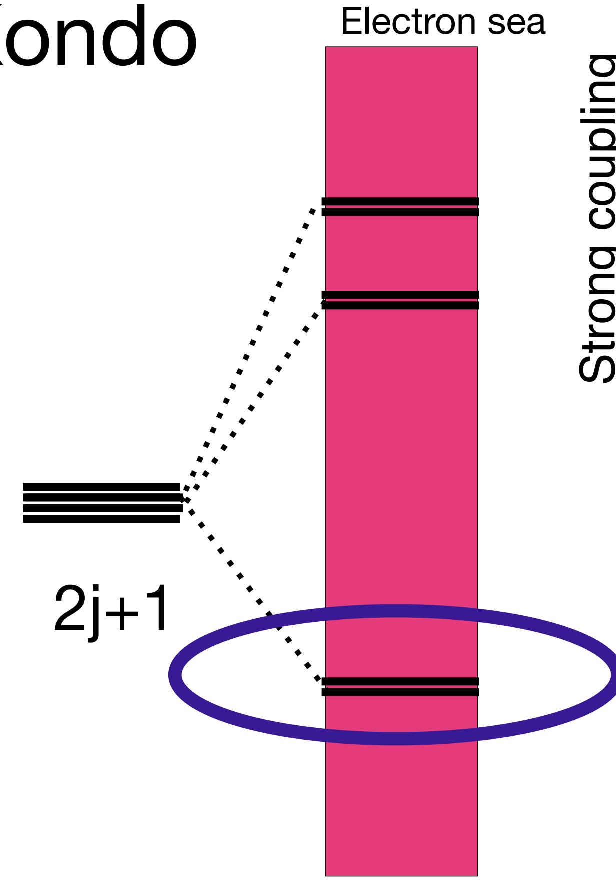


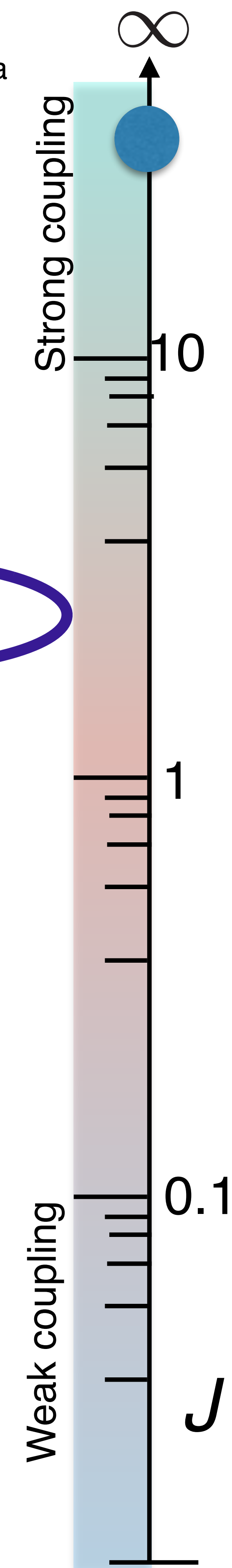


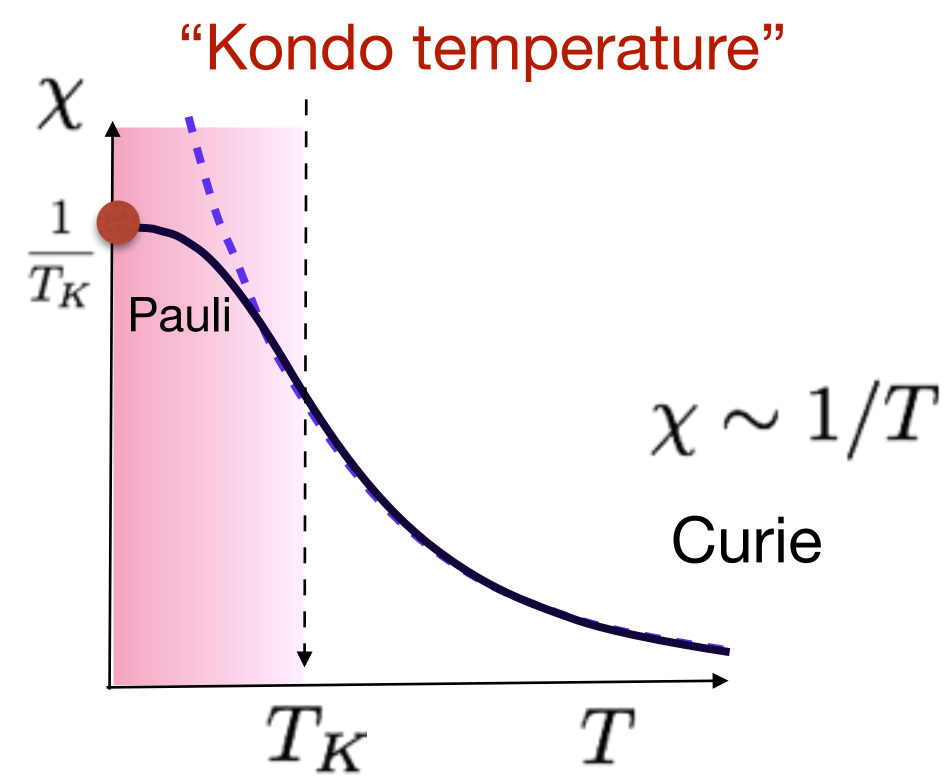
"Kondo Resistance Minimum"

Heavy Fermions + Kondo









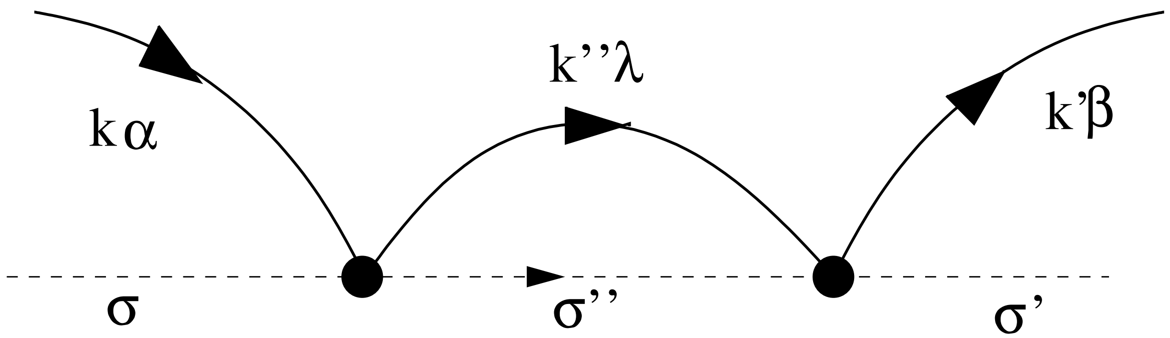
$$T_K = W\sqrt{J\rho}\,e^{-\frac{1}{2J\rho}}$$

Spin screened by conduction electrons: entangled

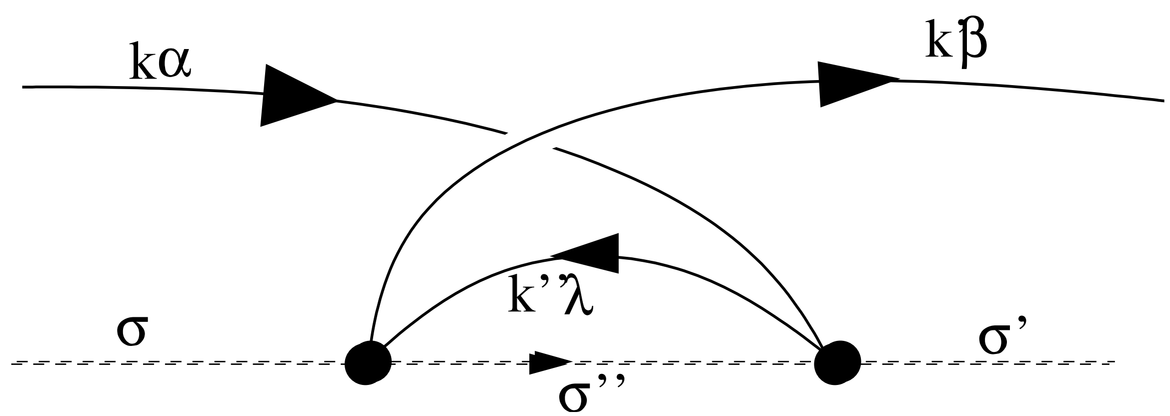
$$\boxed{|GS\rangle = \frac{1}{\sqrt{2}}\left(|\Uparrow\downarrow\rangle - |\Downarrow\uparrow\rangle\right)}$$

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \quad \vec{S}\cdot\vec{\sigma}(0)$$

J. Kondo, 1962



$$\hat{T}^I \approx -\frac{J^2\rho\delta D}{D}(\sigma^b\sigma^a)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$



$$\hat{T}^{II} \approx \frac{J^2\rho\delta D}{D}(\sigma^a\sigma^b)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$

$$\delta H^{int}_{k'\beta\sigma';k\alpha\sigma} = \hat{T}^I + T^{II} = -\frac{J^2\rho\delta D}{D}\overbrace{[\sigma^a,\sigma^b]}^{2i\epsilon^{abc}\sigma^c_{\beta\alpha}}{}_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$

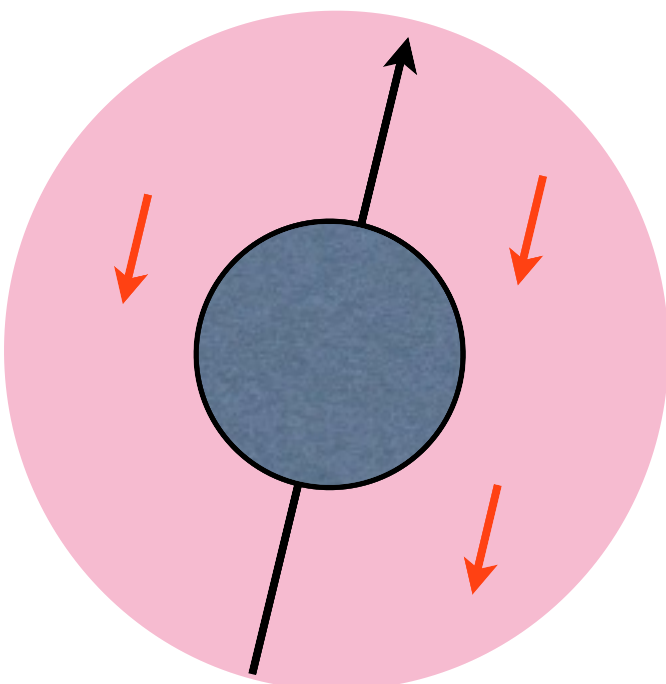
.

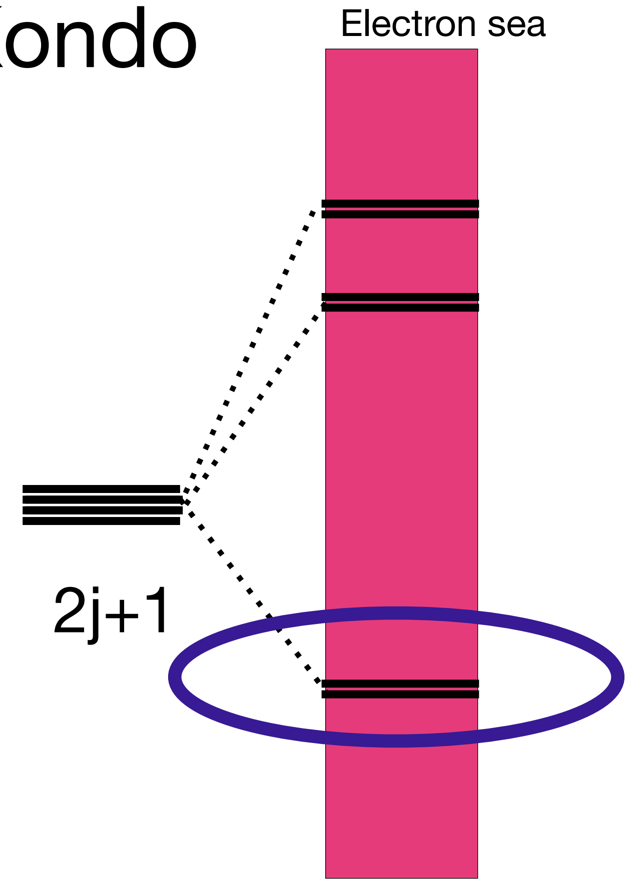
$$J(D') = J(D) + 2J^2\rho\frac{\delta D}{D}$$

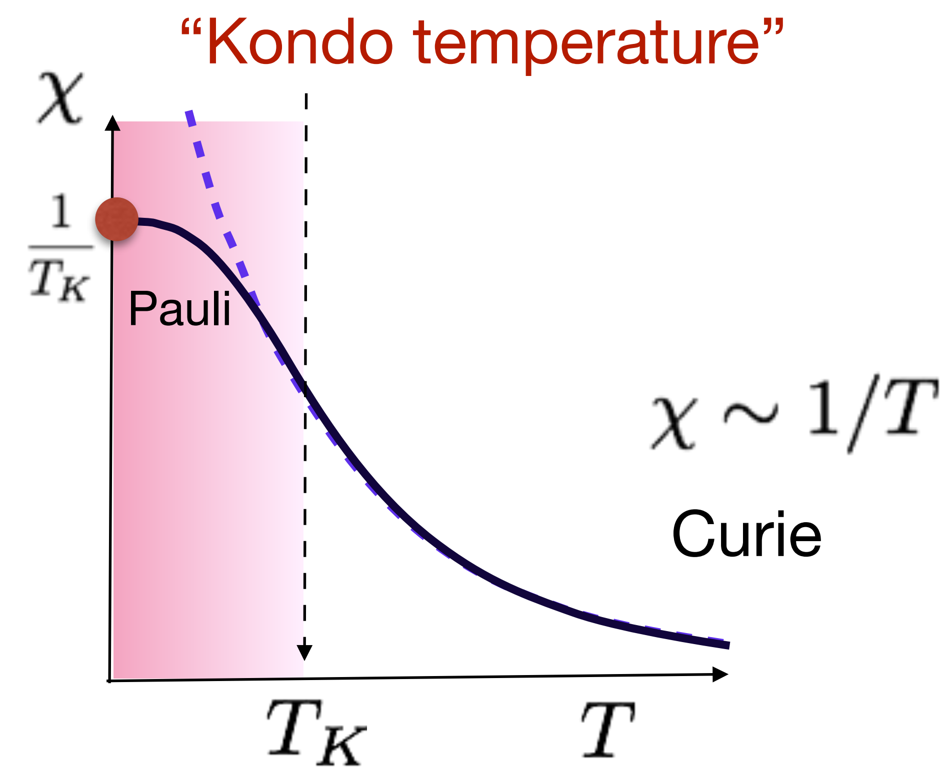
$$\boxed{\frac{\partial g}{\partial \ln D} = \beta(g) = -2g^2 + O(g^3).}$$

$(g = \rho J)$

Heavy Fermions + Kondo







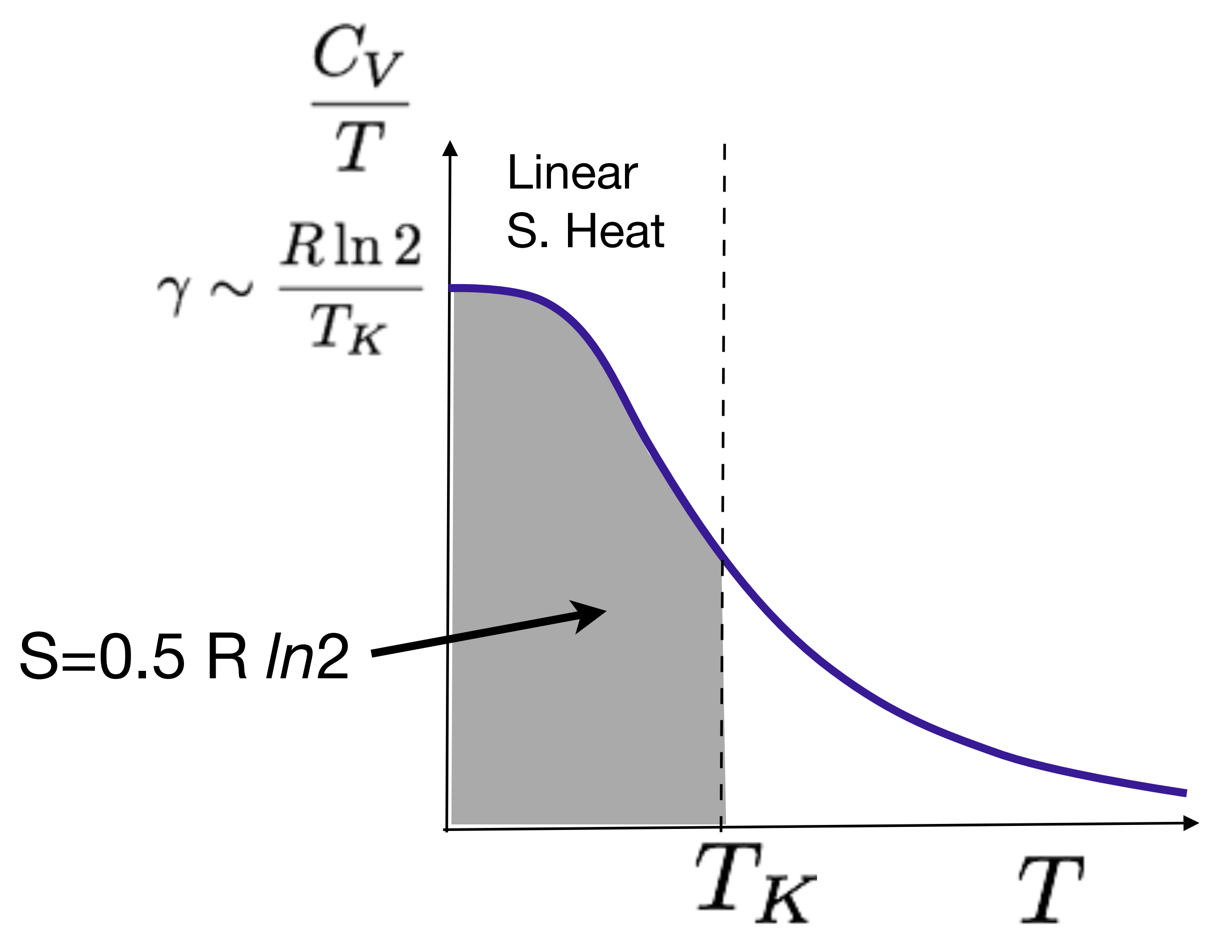
$$T_K = W\sqrt{J\rho}\, e^{-\frac{1}{2J\rho}}$$

Spin screened by conduction electrons: <u>entangled</u>

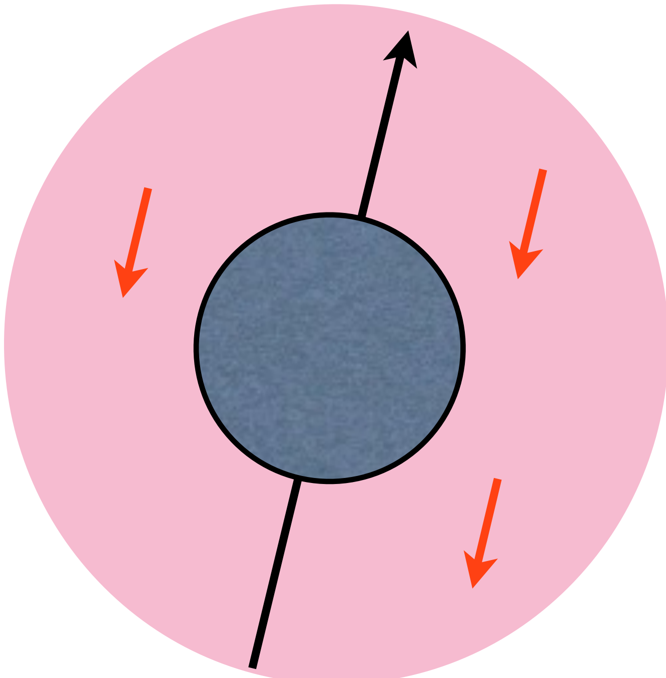
$$\boxed{|GS\rangle = \frac{1}{\sqrt{2}}(|\Uparrow\downarrow\rangle - |\Downarrow\uparrow\rangle)}$$

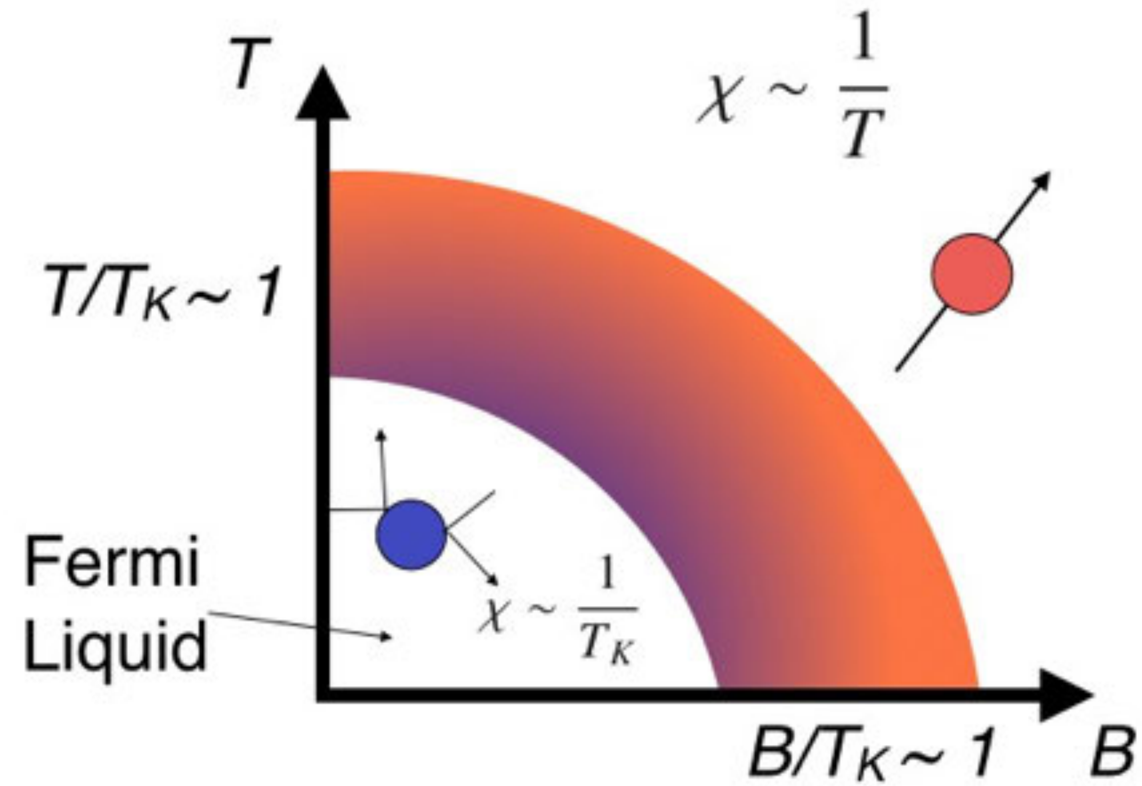
$$S(T) = \int_0^T \frac{C_V}{T'}dT'$$

Spin entanglement entropy



Heavy Fermion Primer



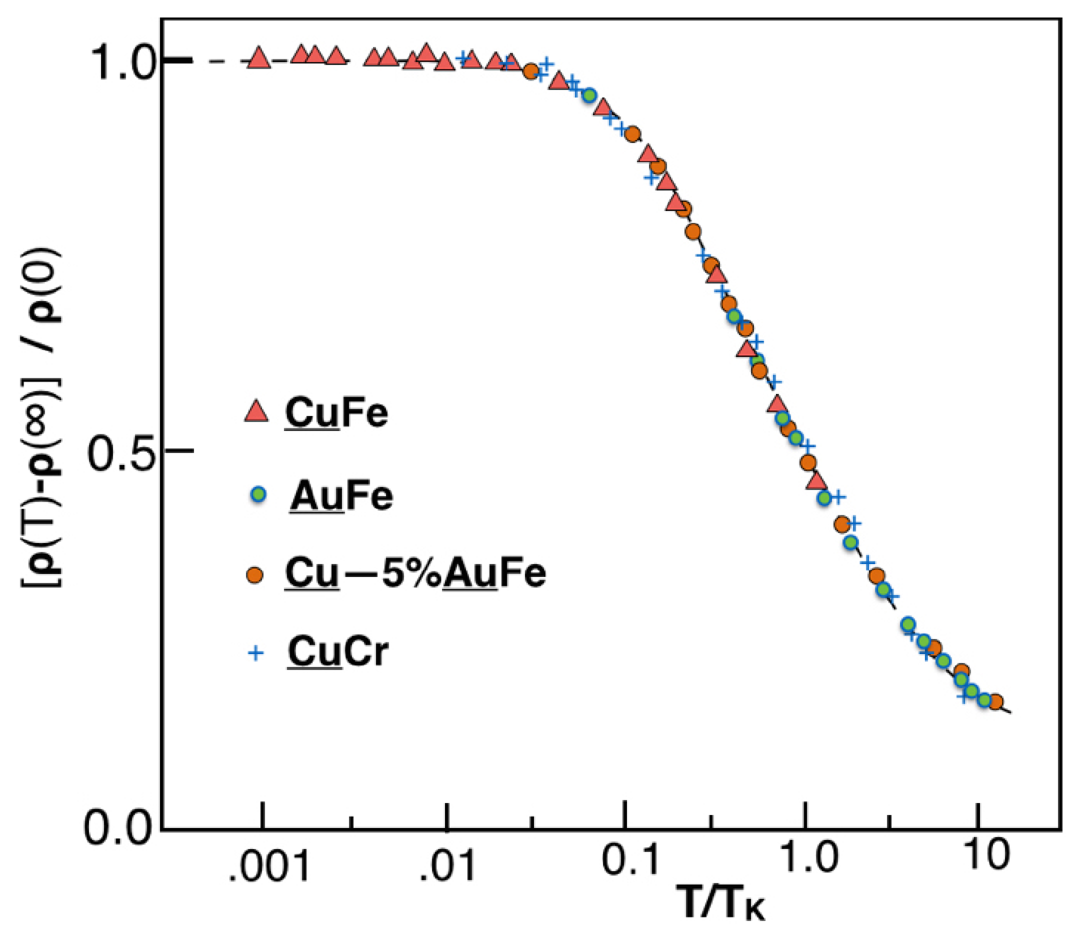


$$\frac{R(T)}{R_U} = n_i \Phi\left(\frac{T}{T_K}\right)$$

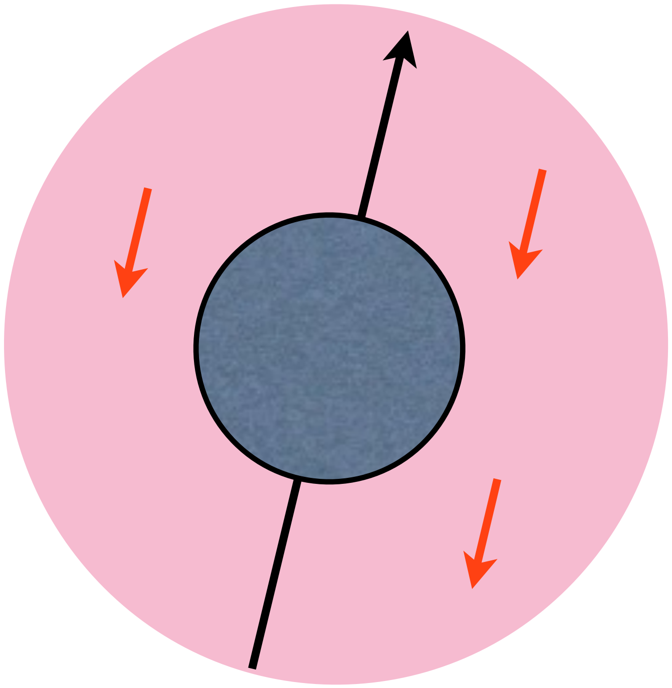
Universality

$$S(T) = \int_0^T \frac{C_V}{T'} dT'$$

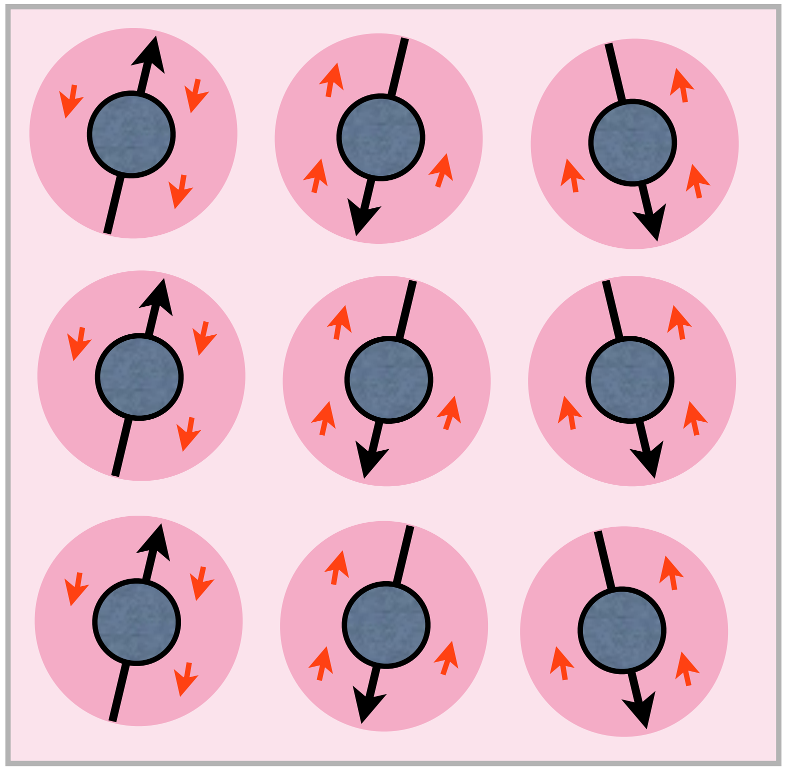
Spin entanglement entropy



Heavy Fermion Primer



$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \sum_{j} \vec{S}_j \cdot \vec{\sigma}(j)$$



“Kondo Lattice”

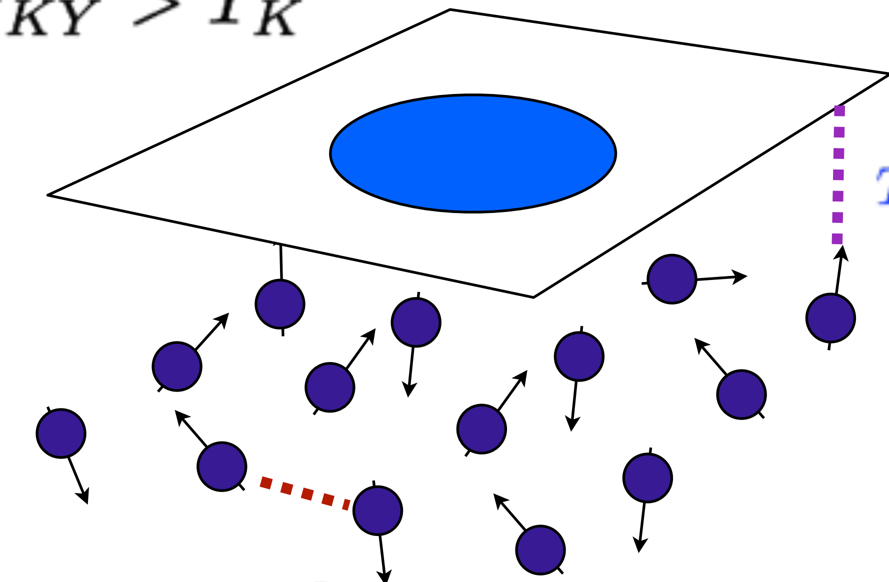
DONIACH'S Hypothesis.

Doniach (1977)

$$H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum_j (\psi^{\dagger}_j \vec{\sigma} \psi_j) \cdot \vec{S}_j$$

Kondo Lattice Model
(Kasuya, 1951)

$T_{RKKY} > T_K$

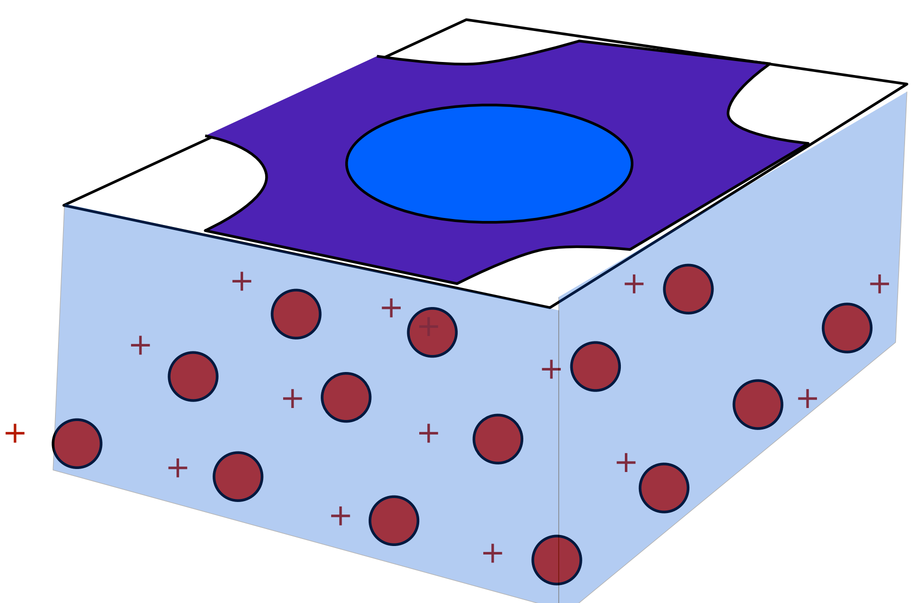


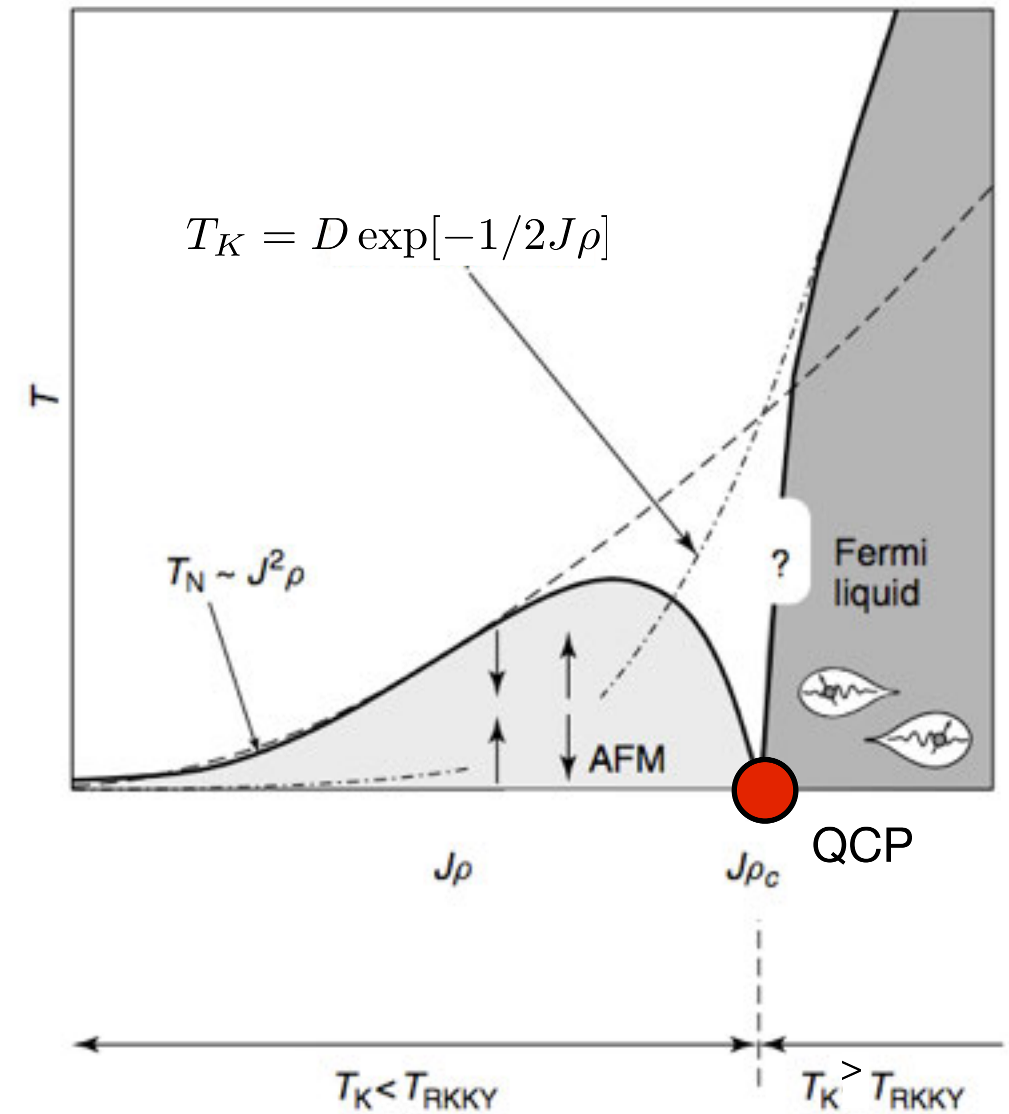
$T_K \sim D \exp\left[-\frac{1}{2J\rho}\right]$

$T_{RKKY} \sim J^2 \rho$

$T_{RKKY} < T_K$

Large Fermi surface of composite Fermions



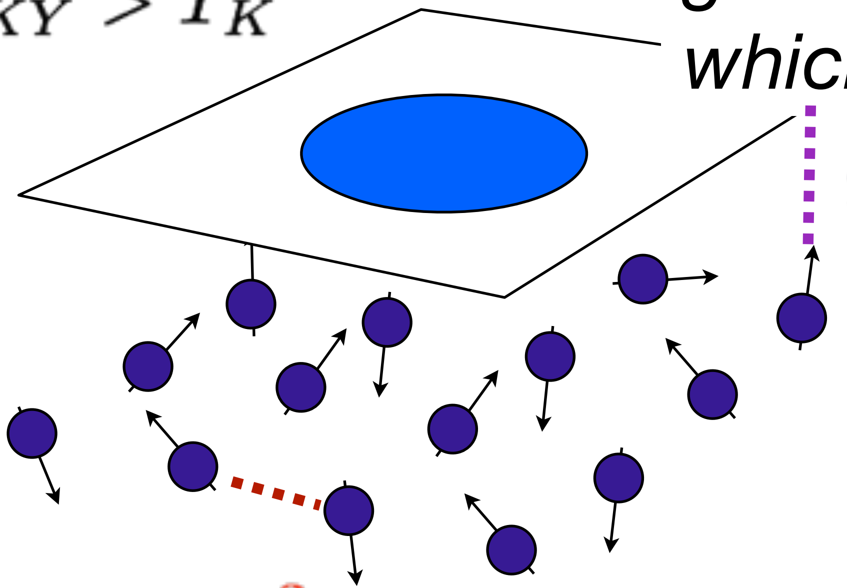


DONIACH'S Hypothesis.

Doniach (1977)

The main result ... is that there should be a second-order transition at zero temperature, as the exchange is varied, between an antiferromagnetic ground state for weak J and a Kondo-like state in which the local moments are quenched.

$$T_{RKKY} > T_K$$

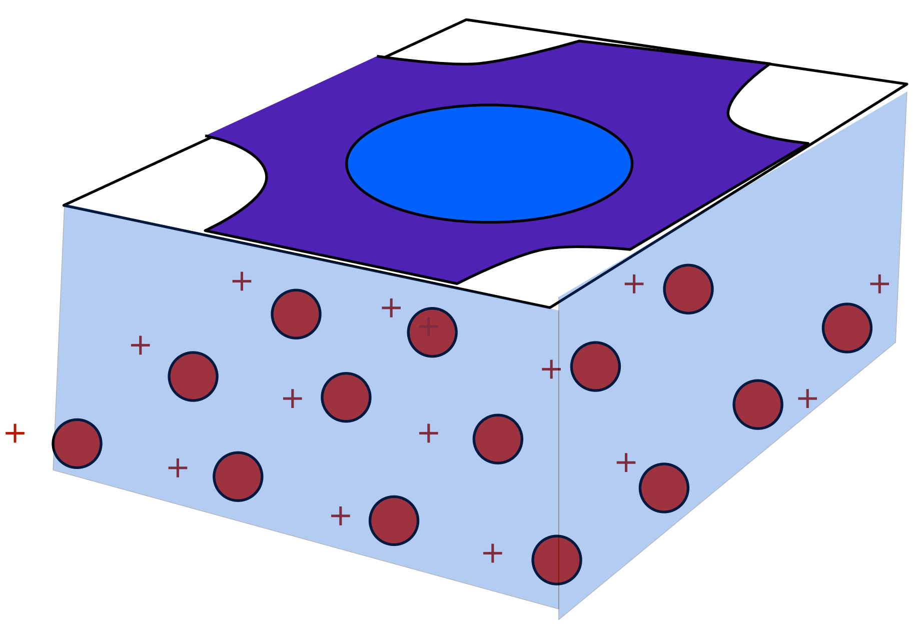


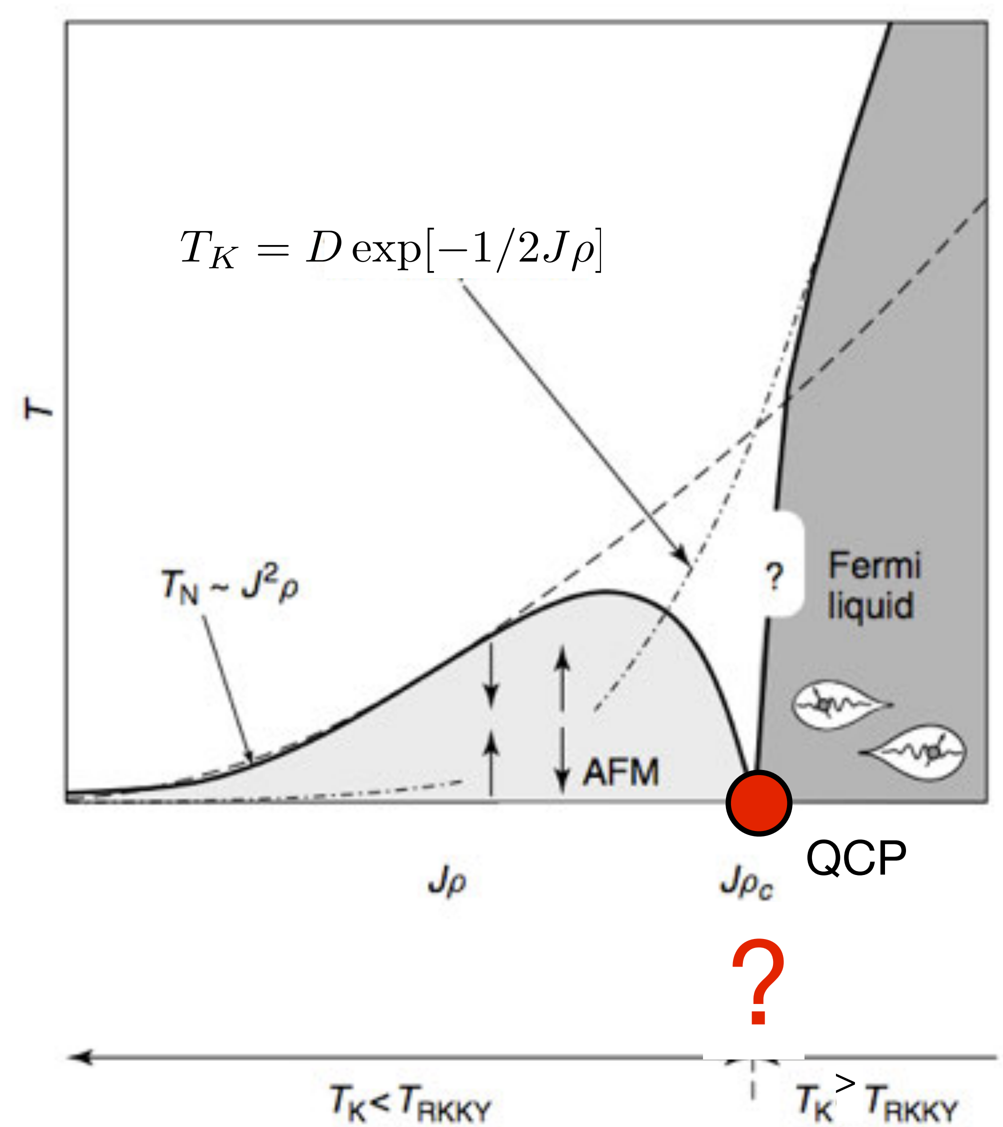
$$T_K \sim D \exp\left[-\frac{1}{2J\rho}\right]$$

$$T_{RKKY} \sim J^2 \rho$$

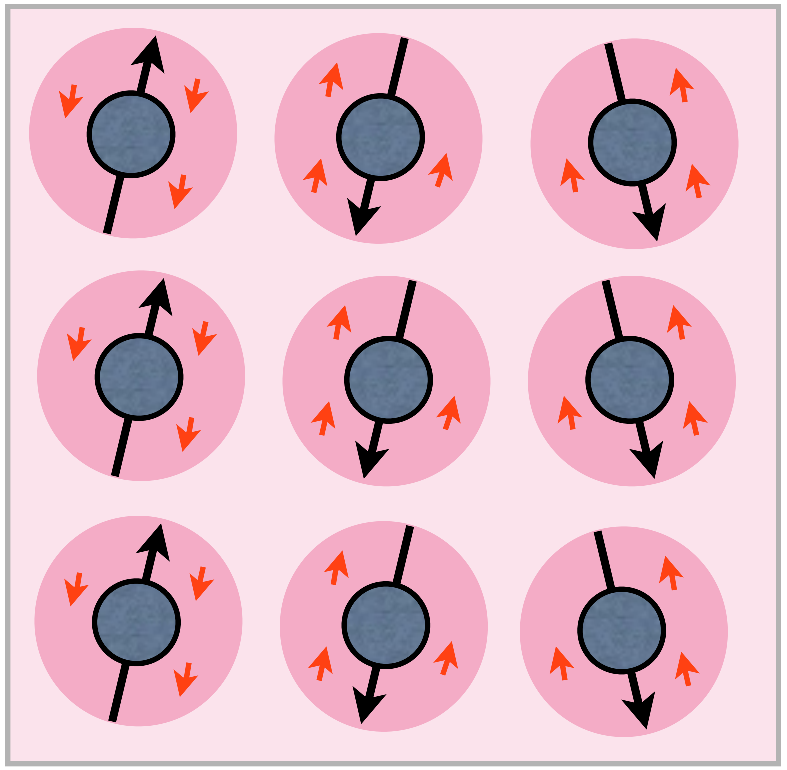
$$T_{RKKY} < T_K$$

Large Fermi surface of composite Fermions





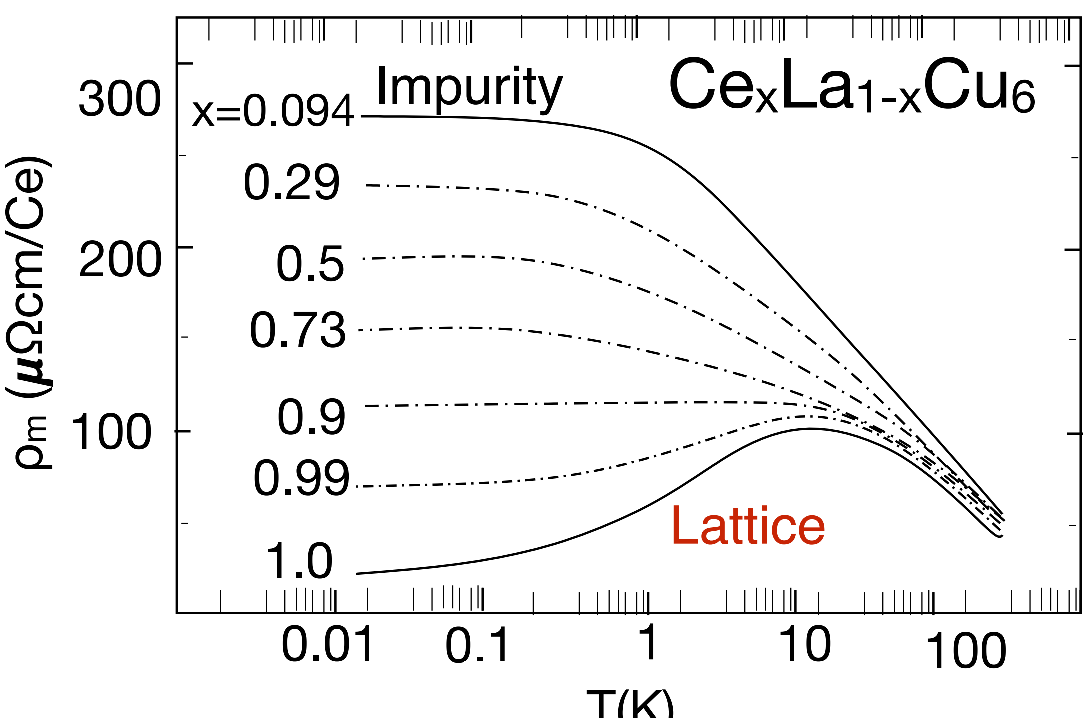
$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \sum_{j} \vec{S}_j \cdot \vec{\sigma}(j)$$

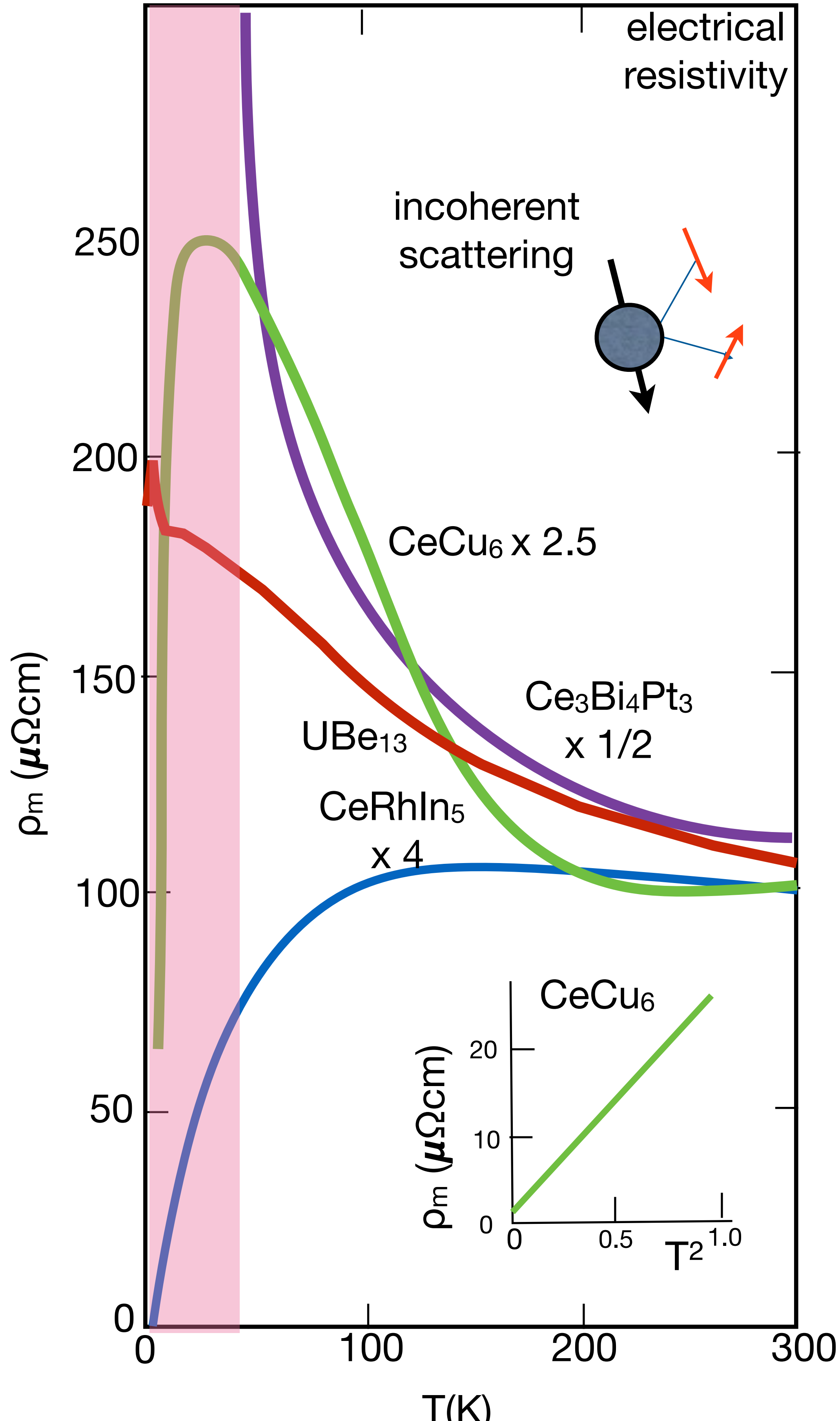


“Kondo Lattice”

Entangled spins and electrons

→ **<u>Heavy Fermion Metals</u>**





$$\rho(T) = \rho_0 + AT^2$$

Coherent Heavy Fermions

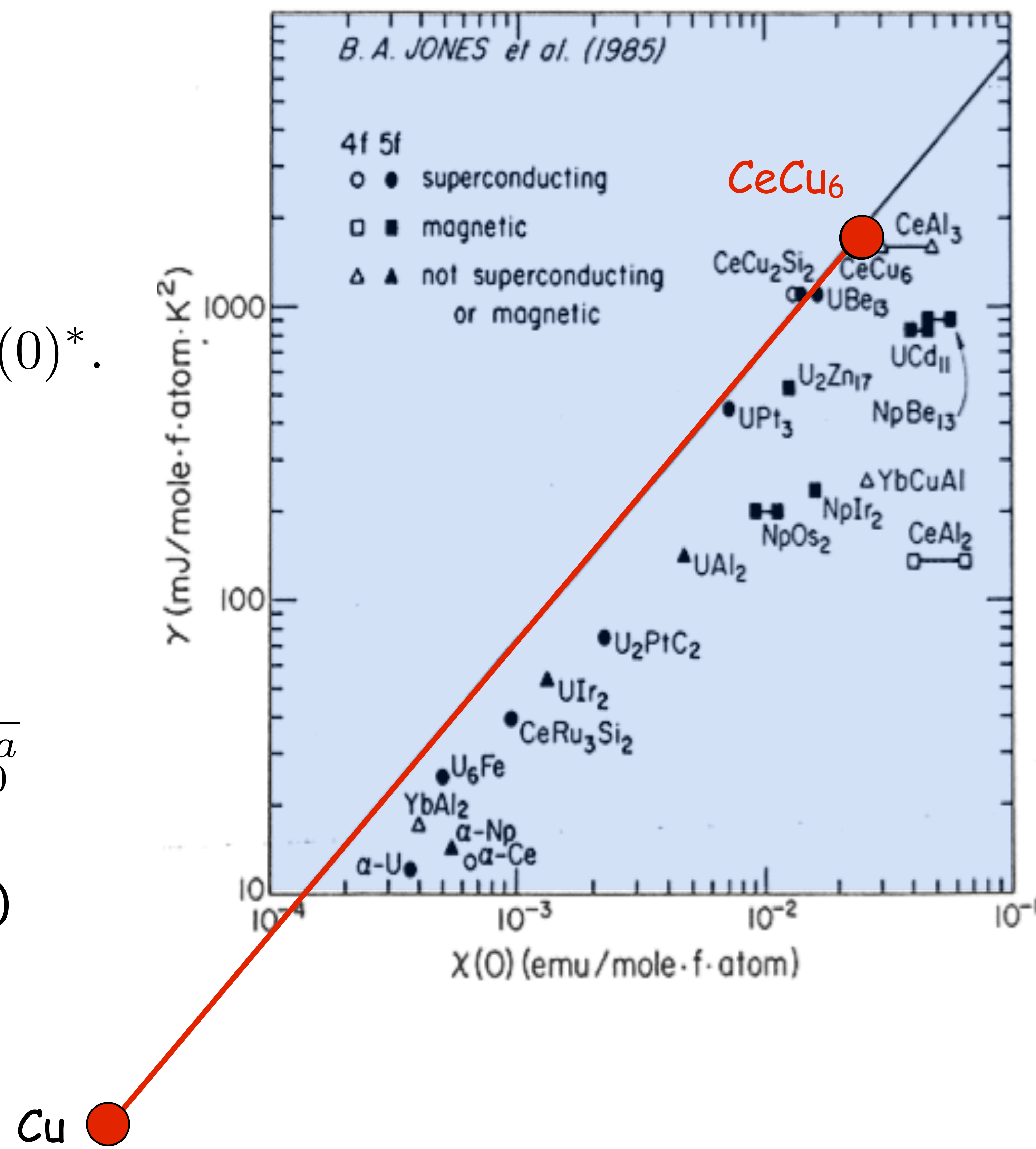
Heavy Fermions: magnetically polarizable Landau Fermi liquids.

$$E_{\mathbf{p}} = \frac{p^2}{2m^*}, \qquad N^*(0) = \frac{m^* p_F}{\pi^2 \hbar^3}$$

$$\gamma = \mathrm{Lim}_{T\to 0}\left(\frac{C_V}{T}\right) = \frac{\pi^2 k_B^2}{3} N(0)^*.$$

$$\chi = \frac{\mu_B^2 N^*(0)}{1 + F_0^a}$$

$$W = \frac{\chi}{\gamma} = 3\left(\frac{\mu_{\mathrm{B}}}{2\pi k_B}\right)^2 \frac{1}{1 + F_0^a}$$



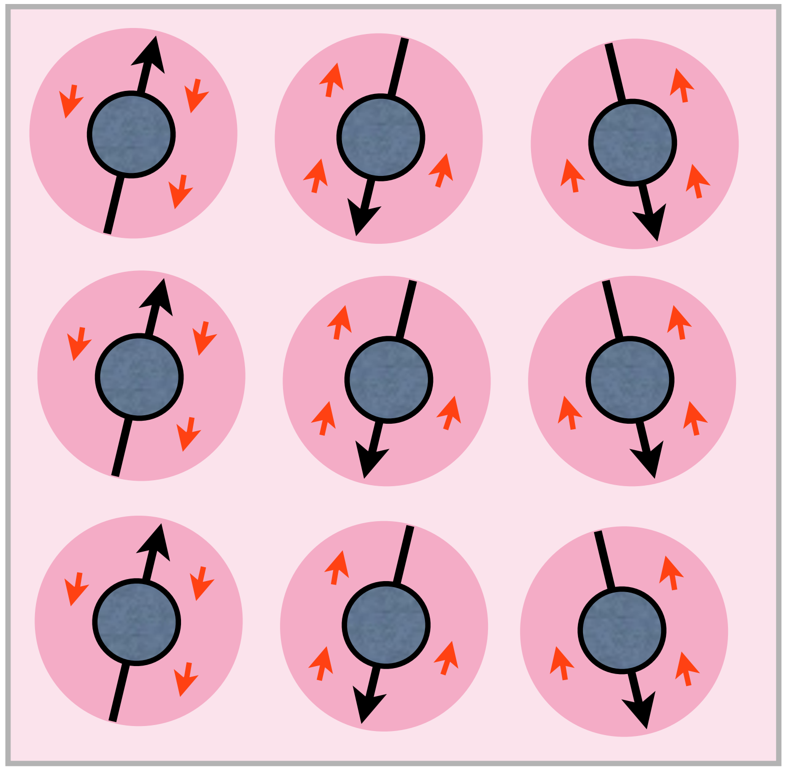
eg Cu vs $CeCu_6$ (copper, spin doped)

γCu ~ 1 mJ/mol/K^2,

$\Upsilon[CeCu_6]$ ~ 1000 mJ/mol/K^2,

$m^*/m_e \sim 1000$

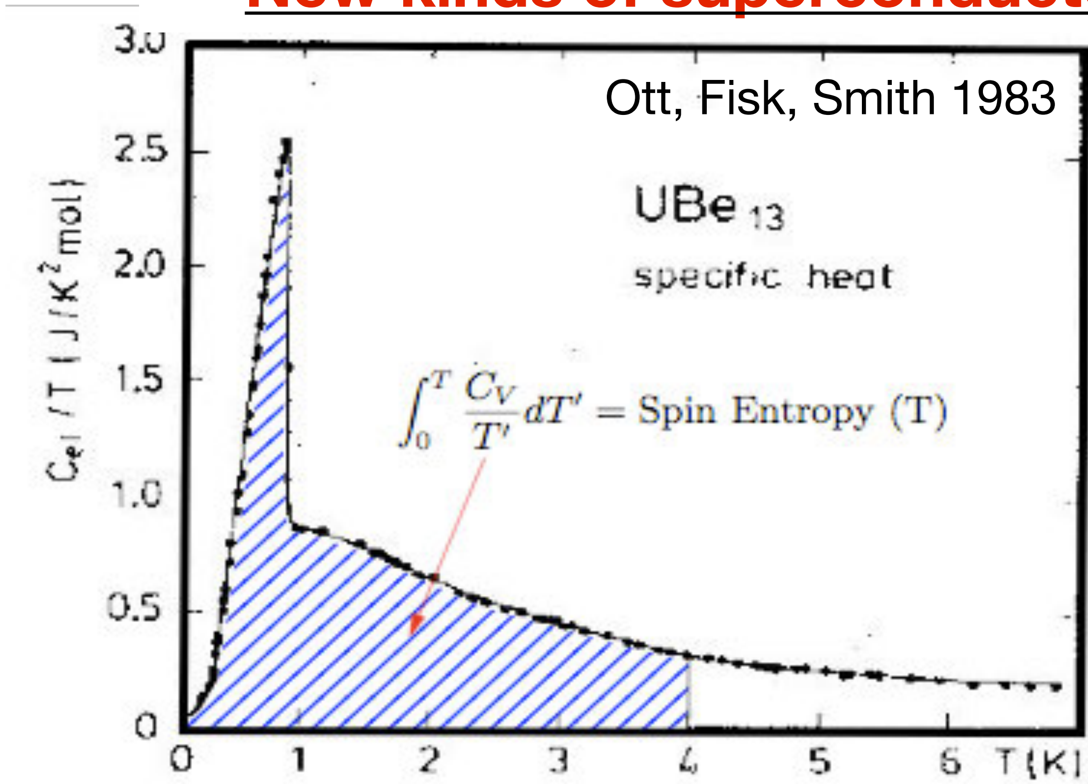
$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \sum_{j} \vec{S}_j \cdot \vec{\sigma}(j)$$

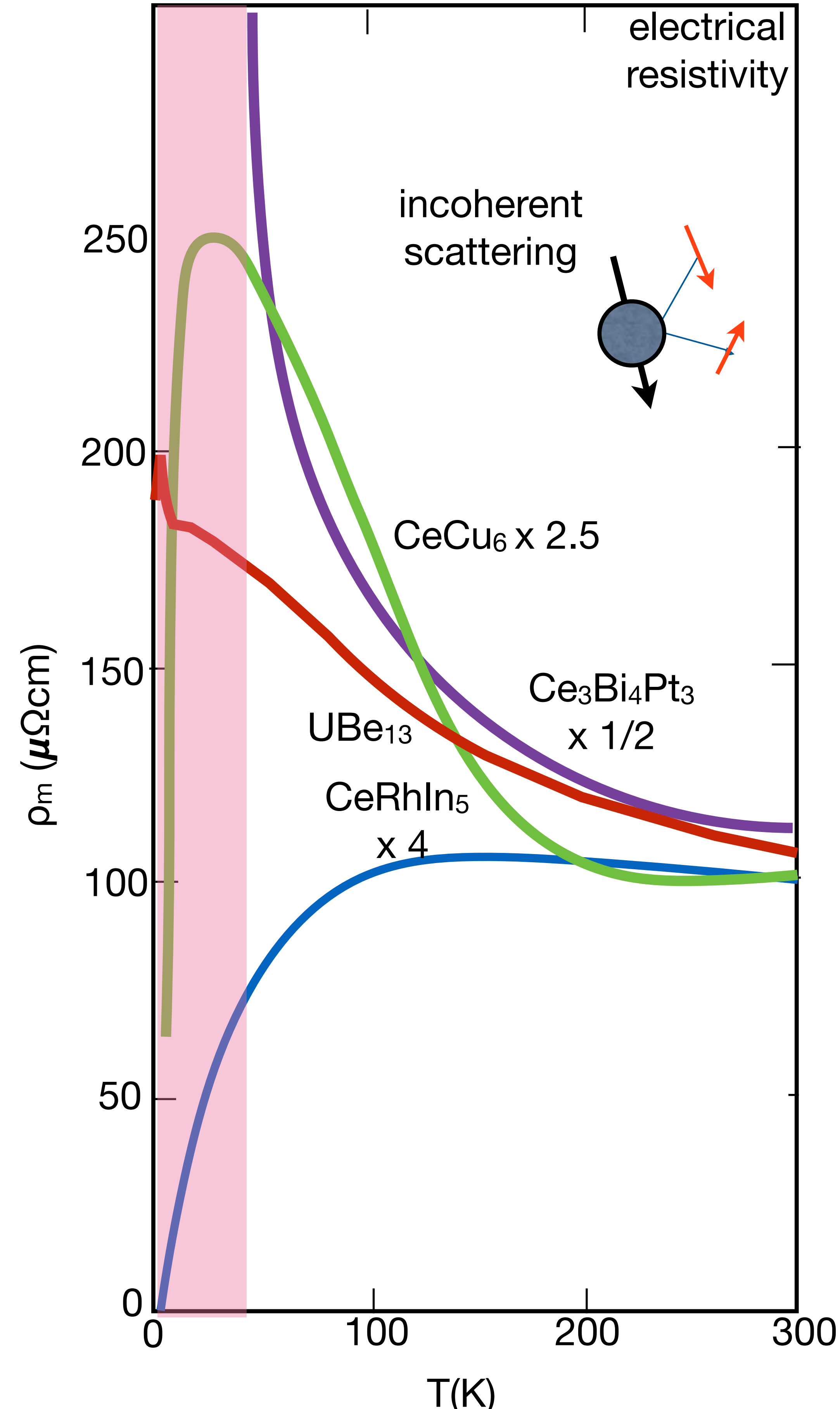


"Kondo Lattice"

Entangled spins and electrons

→ **New kinds of superconductor**

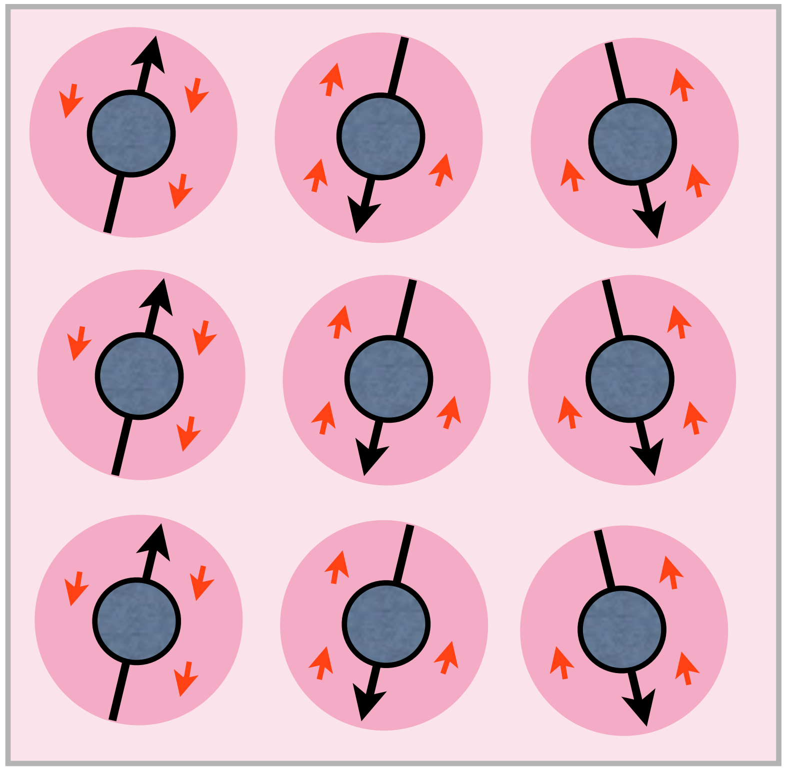




$$\rho(T) = \rho_0 + AT^2$$

Coherent Heavy Fermions

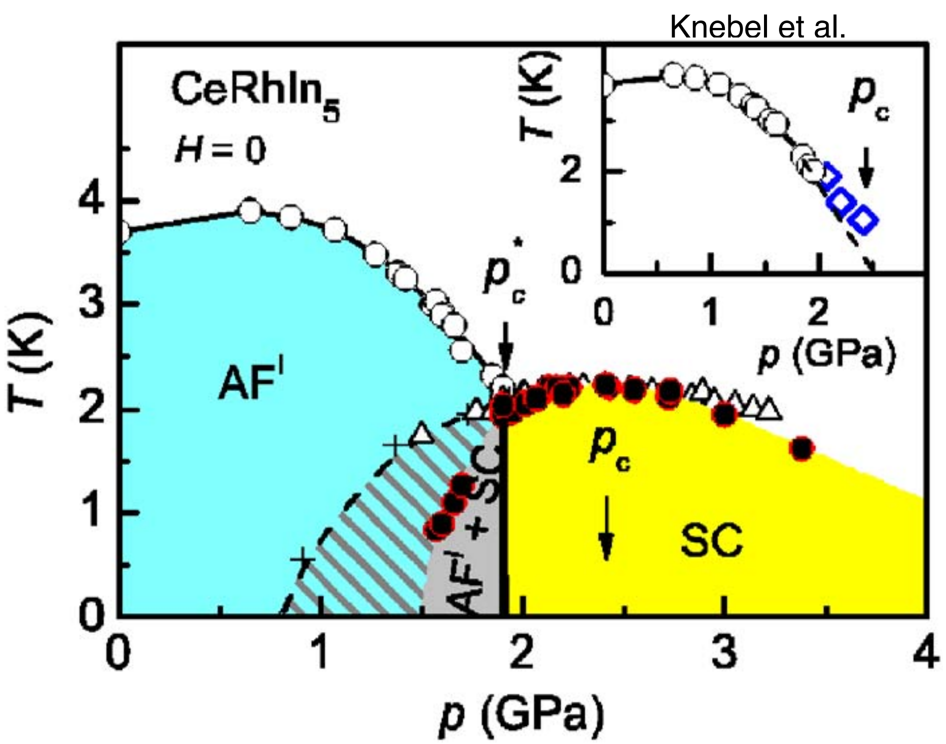
$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \sum_{j} \vec{S}_j \cdot \vec{\sigma}(j)$$

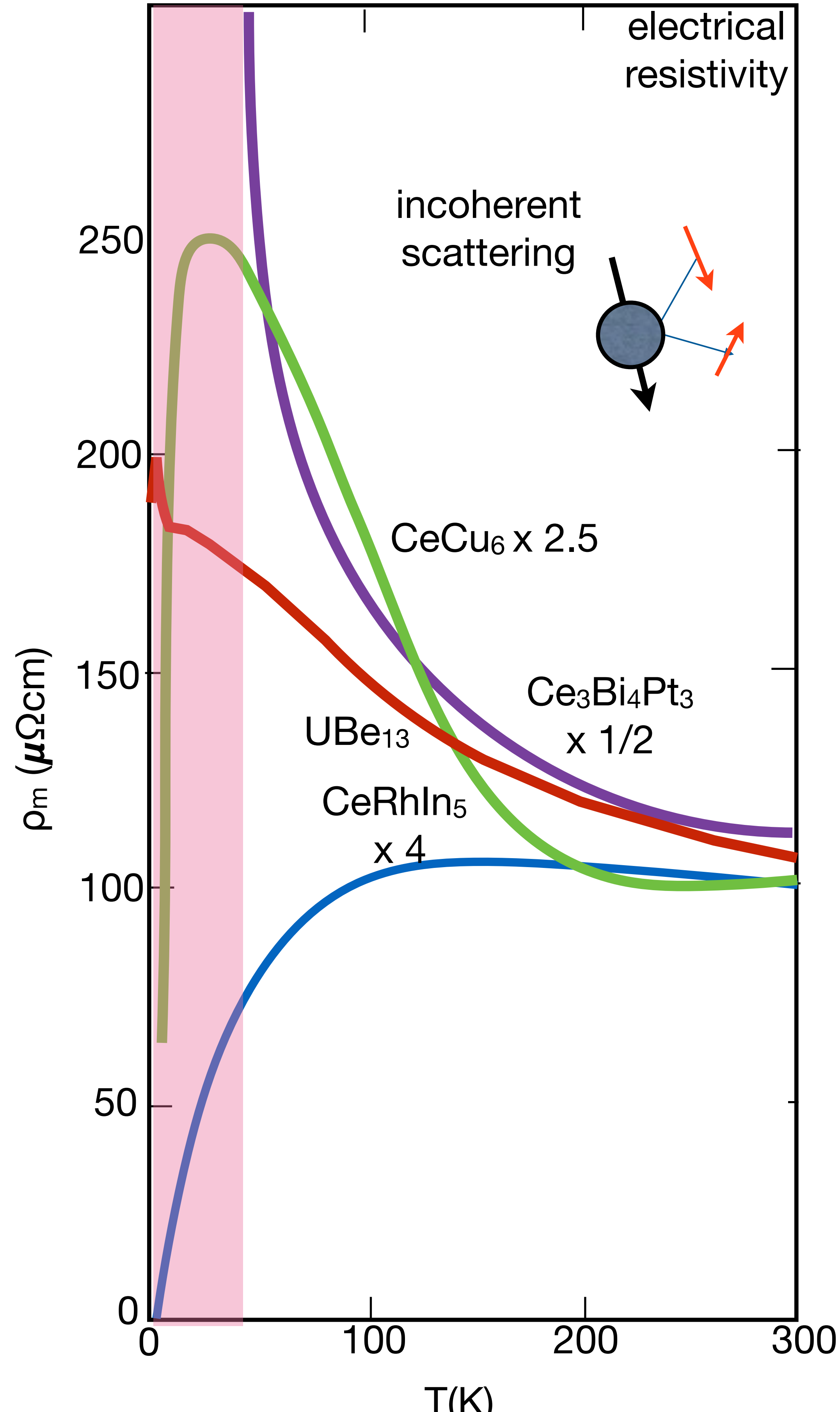


“Kondo Lattice”

Entangled spins and electrons

→ **<u>AFM/Superconductivity</u>**



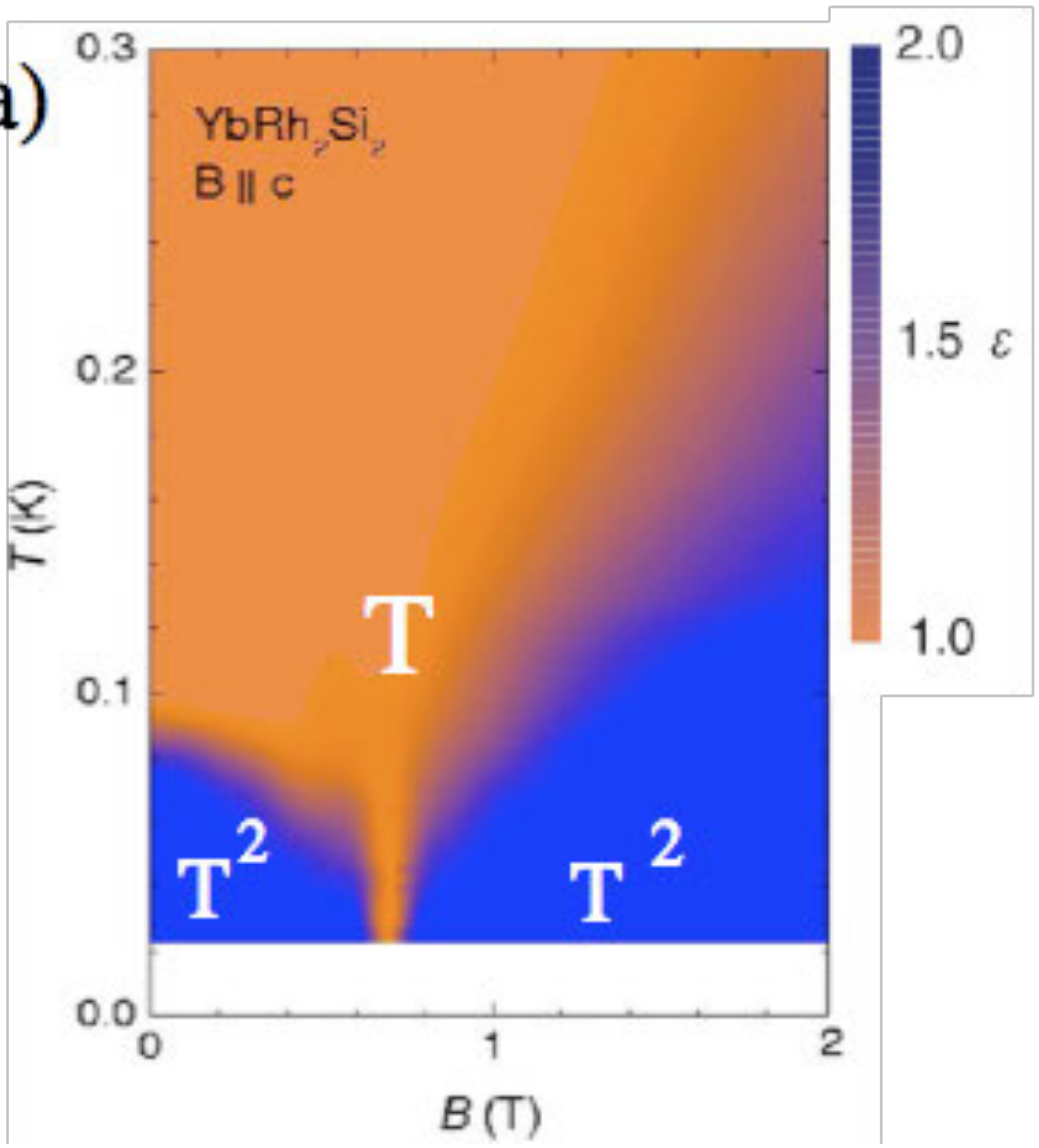


$$\rho(T) = \rho_0 + AT^2$$

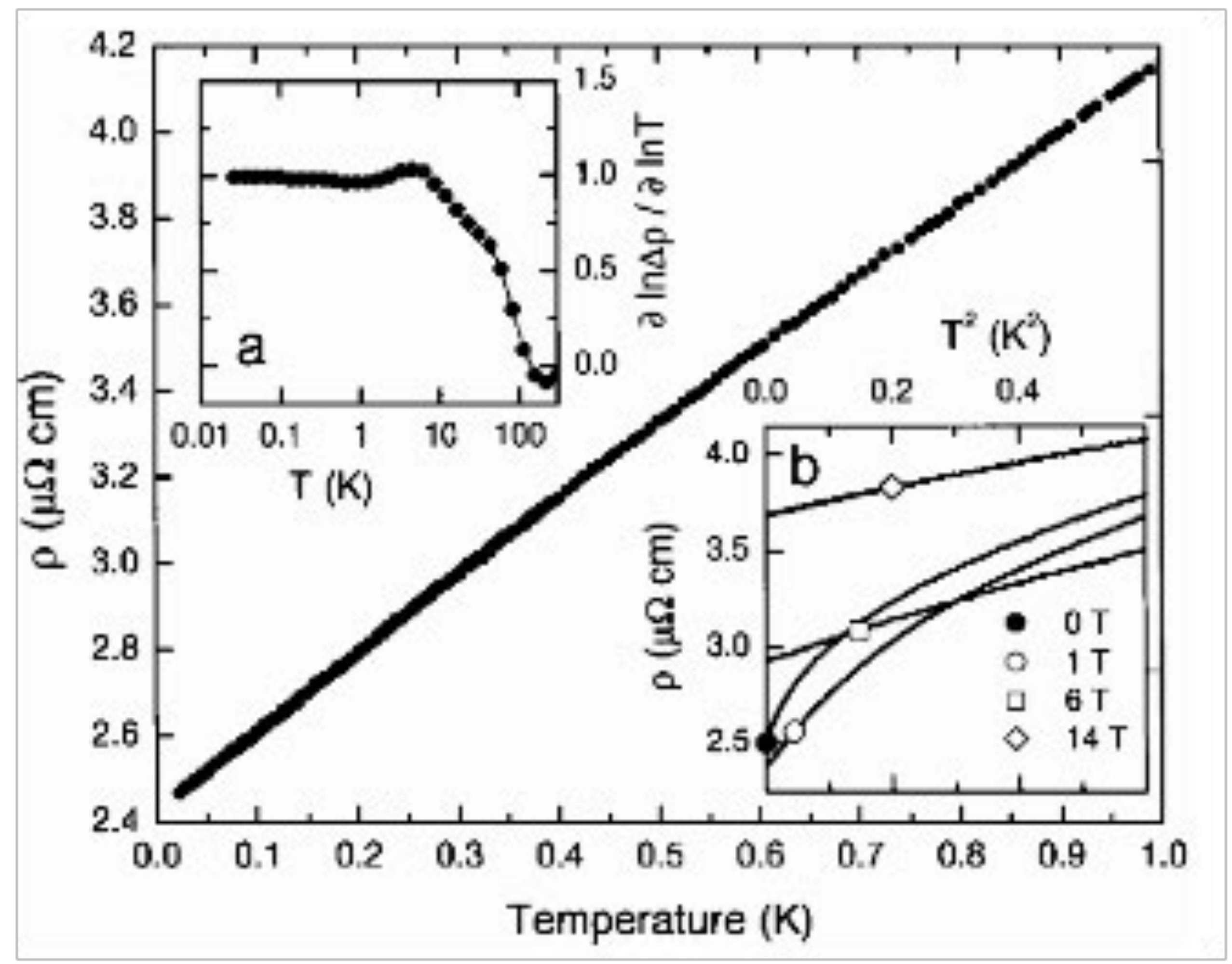
Coherent Heavy Fermions

$YbRh_2Si_2$: Field tuned quantum criticality.

(a)



(b)



Custers et al, (2003)

Outline of the Topics

1. Trends in the periodic table.
2. Introduction: Heavy Fermions and the Kondo Lattice.
3. **Kondo Insulators: the simplest heavy fermions.**
4. Large N expansion for the Kondo Lattice
5. Heavy Fermion Superconductivity
6. Topological Kondo Insulators
7. Co-existing magnetism and the Kondo Effect.

Please ask questions!

Kondo insulators: History

Menth, Buehler and Geballe (PRL 22,295, 1969)

Aeppli and Fisk (Comments CMP 16, 155, 1992)

MAGNETIC AND SEMICONDUCTING PROPERTIES OF SmB_6†

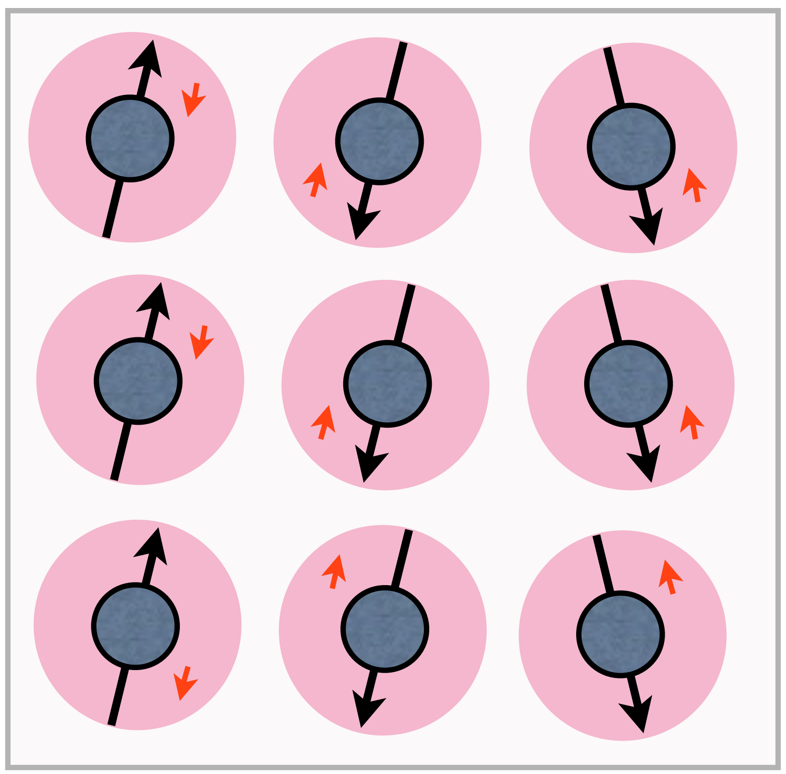
A. Menth and E. Buehler
Bell Telephone Laboratories, Murray Hill, New Jersey

and

T. H. Geballe
Department of Applied Physics, Stanford University, Stanford, California, and Bell Telephone Laboratories, Murray Hill, New Jersey

(Received 21 November 1968)

Simplest Kondo Lattice



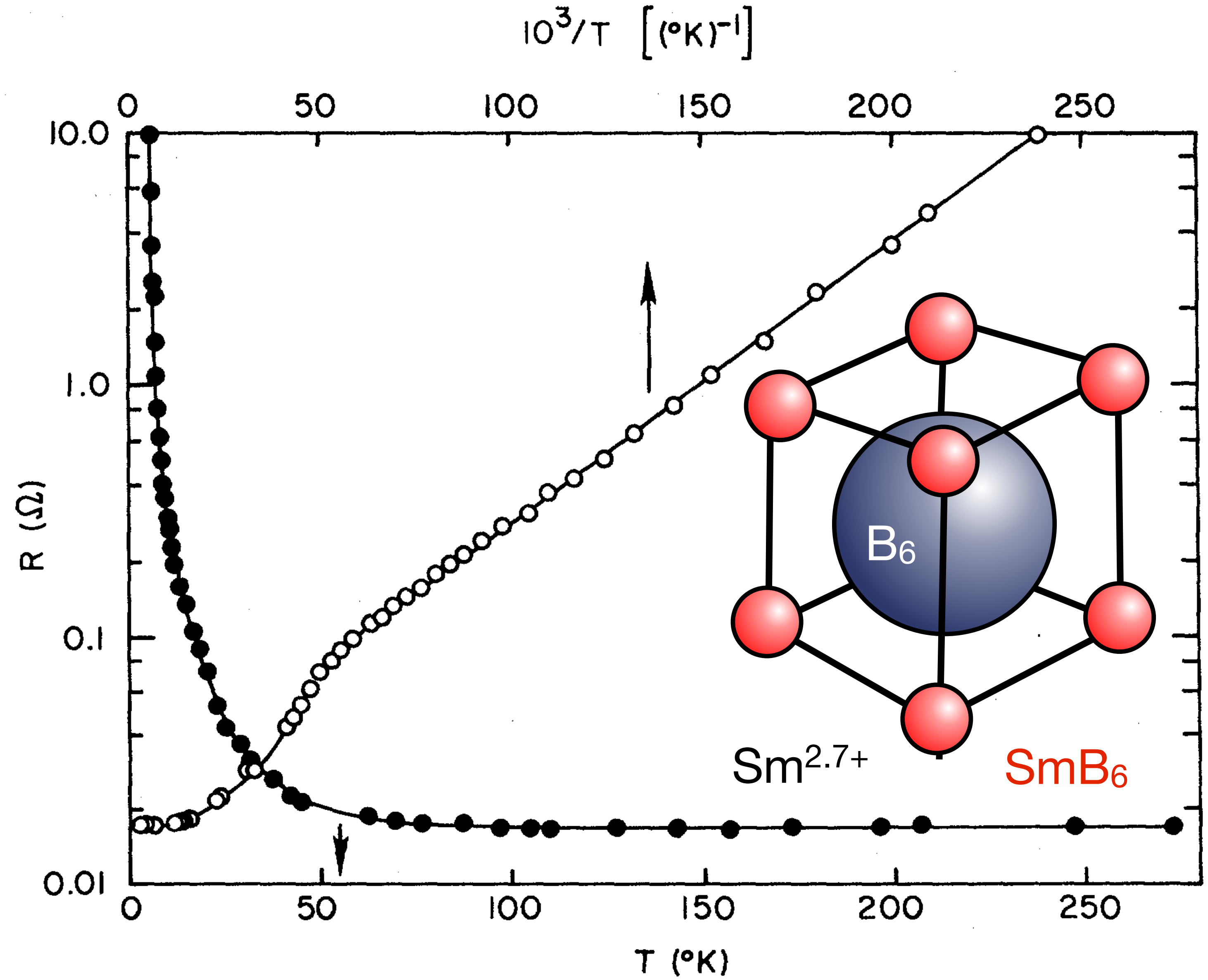


FIG. 1. Resistance of SmB_6 as a function of temperature. Closed circles: resistance versus T; open circles: resistance versus $10^3/T$.

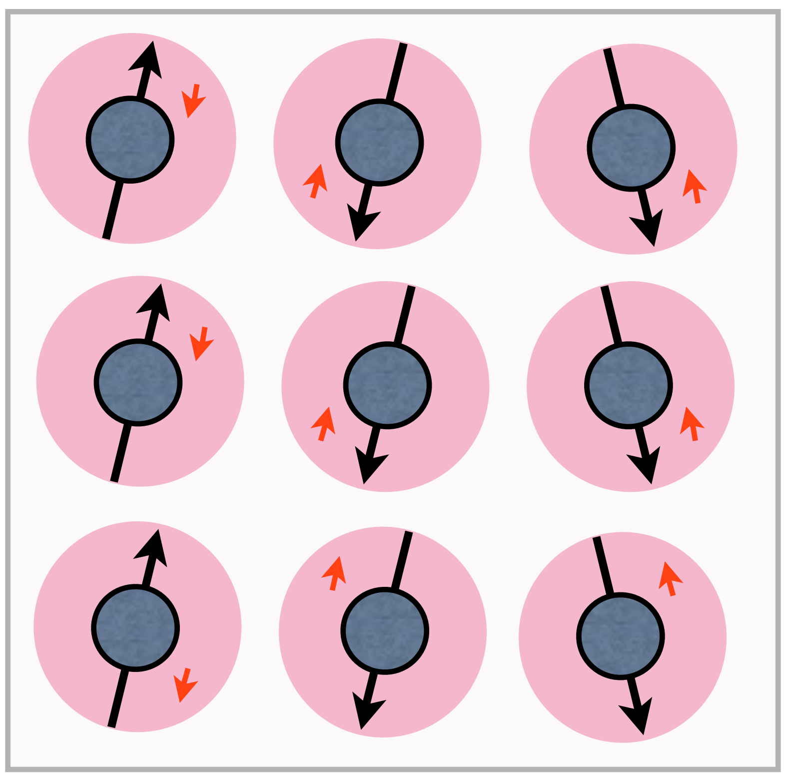
Kondo insulators: History

Menth, Buehler and Geballe (PRL 22,295, 1969)
Aeppli and Fisk (Comments CMP 16, 155, 1992)

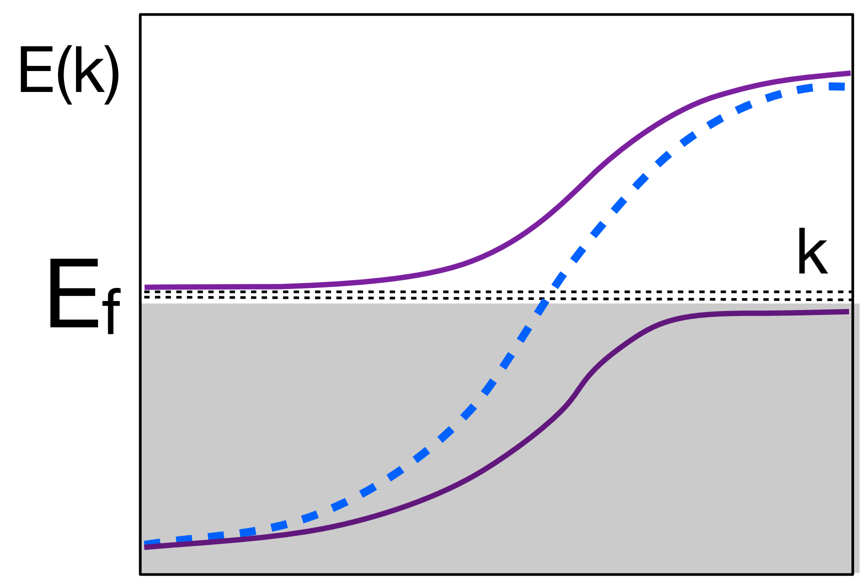
Hybridization picture.

Maple + Wohlleben, 1972
Mott Phil Mag, 30,403,1974
Allen and Martin, 1979

Simplest Kondo Lattice



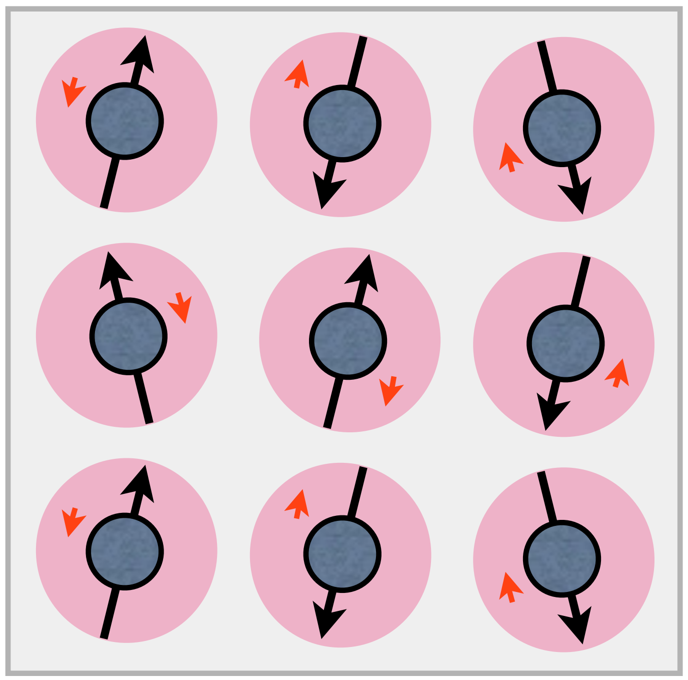
$$\mathcal{H} = (|\mathbf{k}\sigma\rangle V_{\sigma\alpha}(\mathbf{k})\langle\alpha| + \mathrm{H.c})$$



"In SmB6 and high-pressure SmS a very small gap separates occupied from unoccupied states, this in our view being due to hybridization of 4f and 4d bands." Mott 1974

Strong coupling Kondo Lattice $J >> t$

$$H = J\sum \vec{\sigma}(j)\cdot\vec{S}_j - t\sum_{(i,j)}(c^\dagger_{i\sigma}c_{j\sigma} + \mathrm{H.c})$$

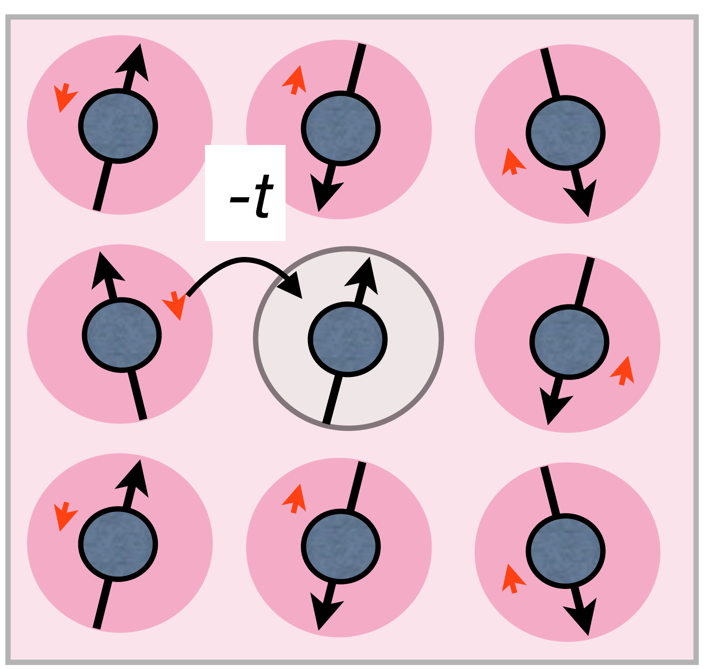


$$n_e = n_{\mathrm{spins}}$$

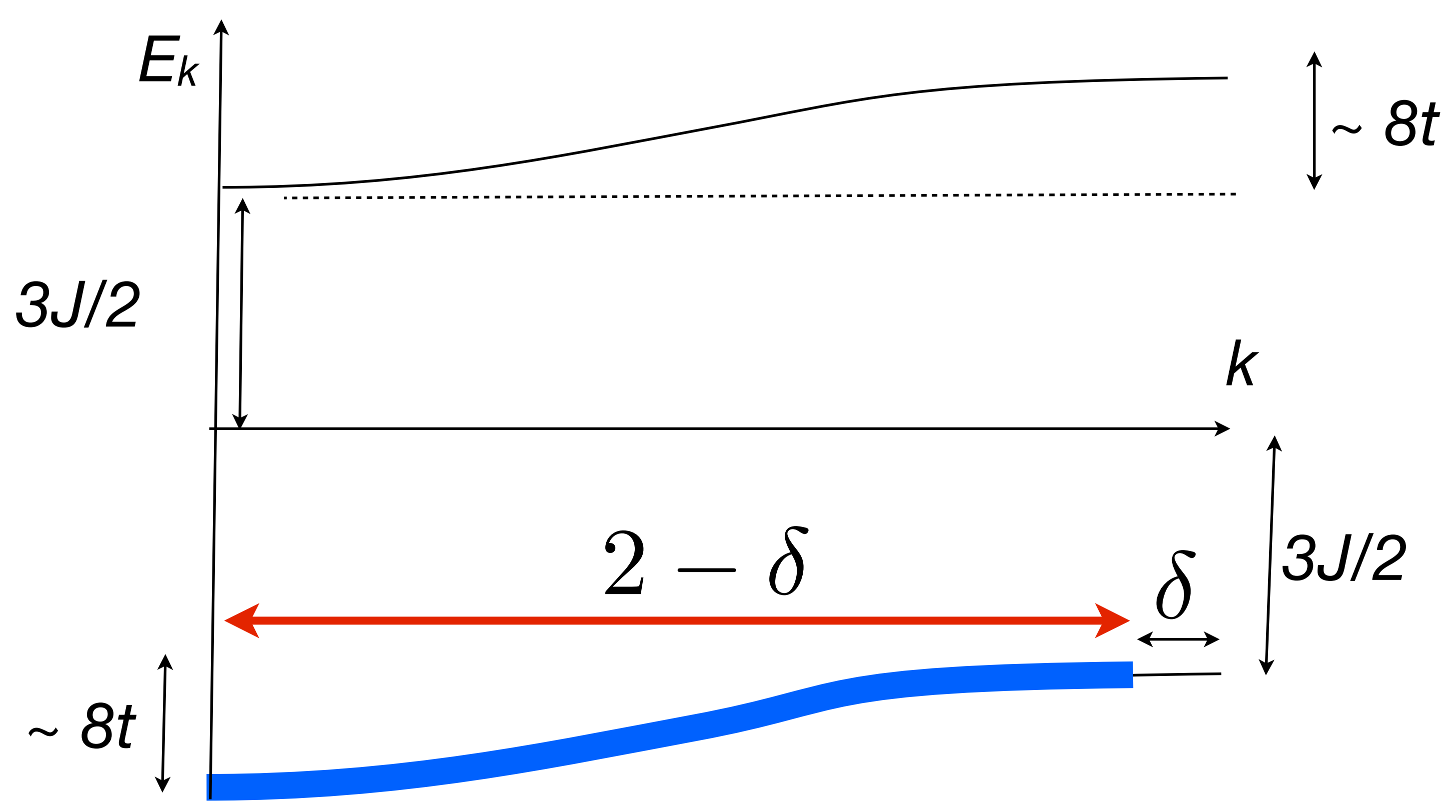
Kondo insulator

$$2\left(\frac{v_{\mathrm{FS}}}{(2\pi)^D}\right) = 2-\delta = n_{\mathrm{spins}} + n_e$$

FS sum rule counts spins as charged qp.



Hole doping: mobile heavy holes $n_e = n_{\mathrm{spins}} - \delta$



But what about weak coupling: J << t?

Outline of the Topics

1. Trends in the periodic table.
2. Introduction: Heavy Fermions and the Kondo Lattice.
3. Kondo Insulators: the simplest heavy fermions.
4. **<u>Large N expansion for the Kondo Lattice</u>**
5. Heavy Fermion Superconductivity
6. Topological Kondo Insulators
7. Co-existing magnetism and the Kondo Effect.

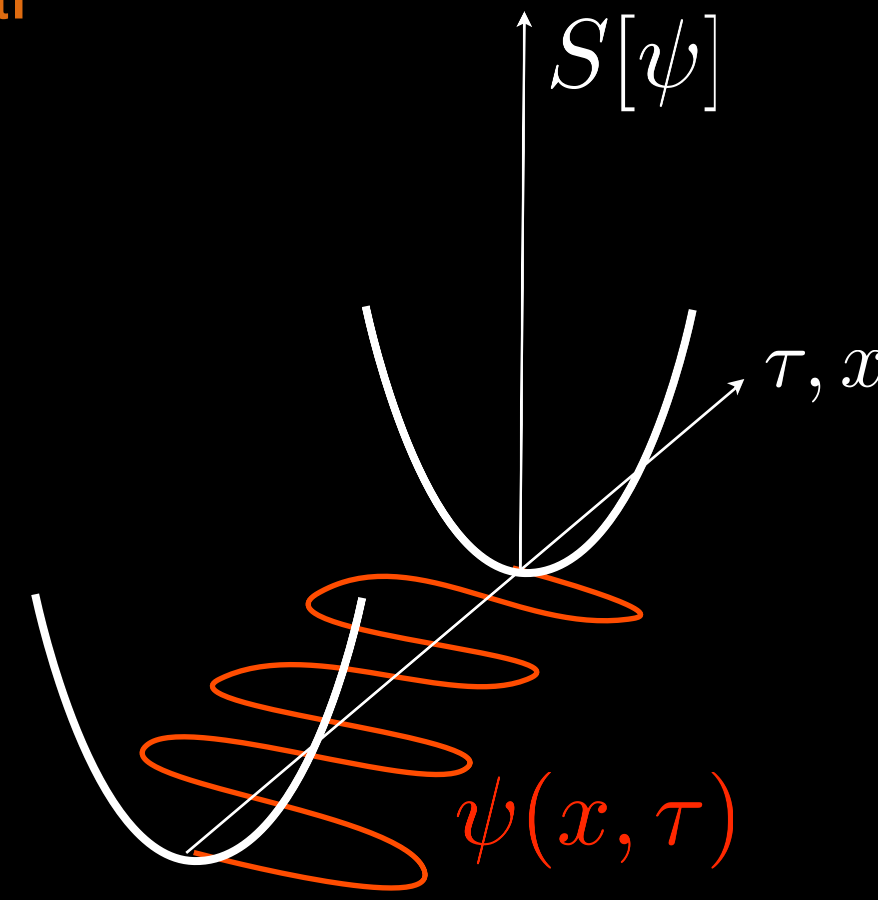
Please ask questions!

Rutgers
Center for Materials Theory

How can we tame the wild Quantum fluctuations?

Path Integral

$$Z = \int_{\text{Fields}} e^{-S[\psi]}$$



$$H = \sum_k \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{J}{N}\sum_j \psi^\dagger_a(j)\psi_a(j) S^{ba}(j)$$

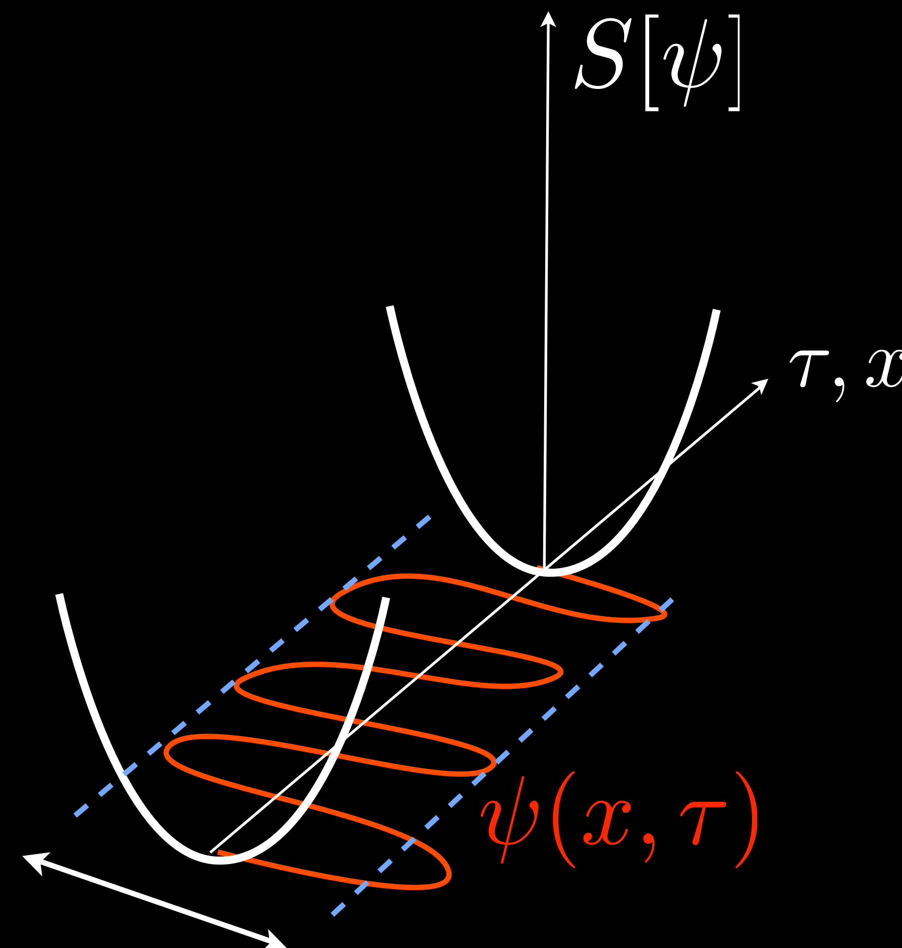
Single FS, two channels.

$$c^\dagger_\sigma(j) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} c_{\mathbf{k}\sigma} e^{i\mathbf{k}\cdot\mathbf{R}_j}$$

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Large *N* expansion.

$$Z = \int_{\mathrm{Fields}} e^{-\frac{S[\psi]}{1/N}}$$



$$\frac{1}{N} \sim \hbar_{eff}$$

$$\sigma \in \left(-\frac{1}{2}, \frac{1}{2}\right) \longrightarrow \left(-\frac{N}{2}, \frac{N}{2}\right)$$

$$H = \sum_k \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{J}{N} \sum_j \psi^\dagger_a(j)\psi_a(j) S^{ba}(j)$$

Single FS, two channels.

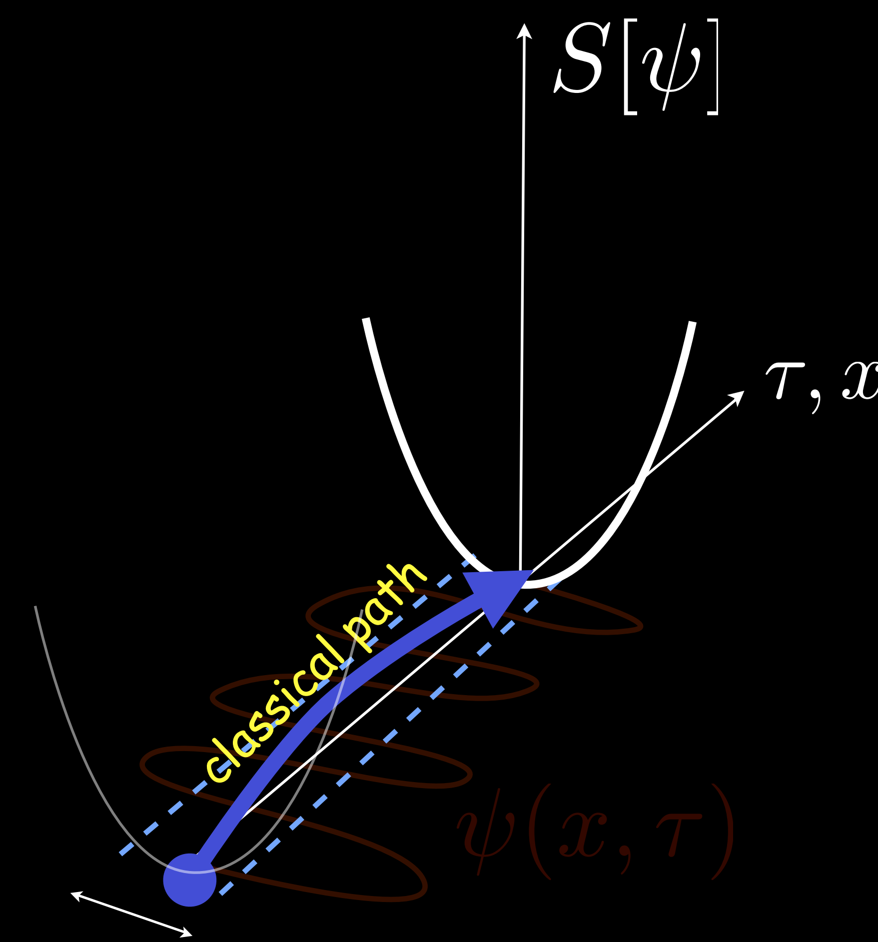
$$c^\dagger_\sigma(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} c_{\mathbf{k}\sigma} e^{i\mathbf{k}\cdot\mathbf{R}_j}$$

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Large N expansion.

$$N \longrightarrow \infty$$

$$Z = \int_{\text{Fields}} e^{-\frac{S[\psi]}{1/N}}$$



$$\frac{1}{N} \sim \hbar_{eff}$$

$$S^{ba} = f_b^\dagger f_a - \frac{n_f}{N}\delta_{ab}$$

$$H_I \rightarrow \frac{J}{N}\left(f^\dagger_{j\alpha} c_{j\alpha}\right)\left(c^\dagger_{j\beta} f_\beta\right)$$

$$\rightarrow \bar{V}\left(c^\dagger_{j\beta} f_\beta\right) + \left(f^\dagger_{j\alpha} c_{j\alpha}\right) V + N\frac{\bar{V}V}{J}$$

$$H = \sum_k \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{J}{N}\sum_j \psi_a^\dagger(j)\psi_a(j) S^{ba}(j)$$

Single FS, two channels.

$$c^\dagger_\sigma(j) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} c_{\mathbf{k}\sigma} e^{i\mathbf{k}\cdot\mathbf{R}_j}$$

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Large N Approach.

Read and Newns '83.

$$c^\dagger_{j\alpha} = \frac{1}{\sqrt{N_s}} \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} e^{-i\mathbf{k}\cdot\vec{R}_j}$$

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_j H_I(j)$$

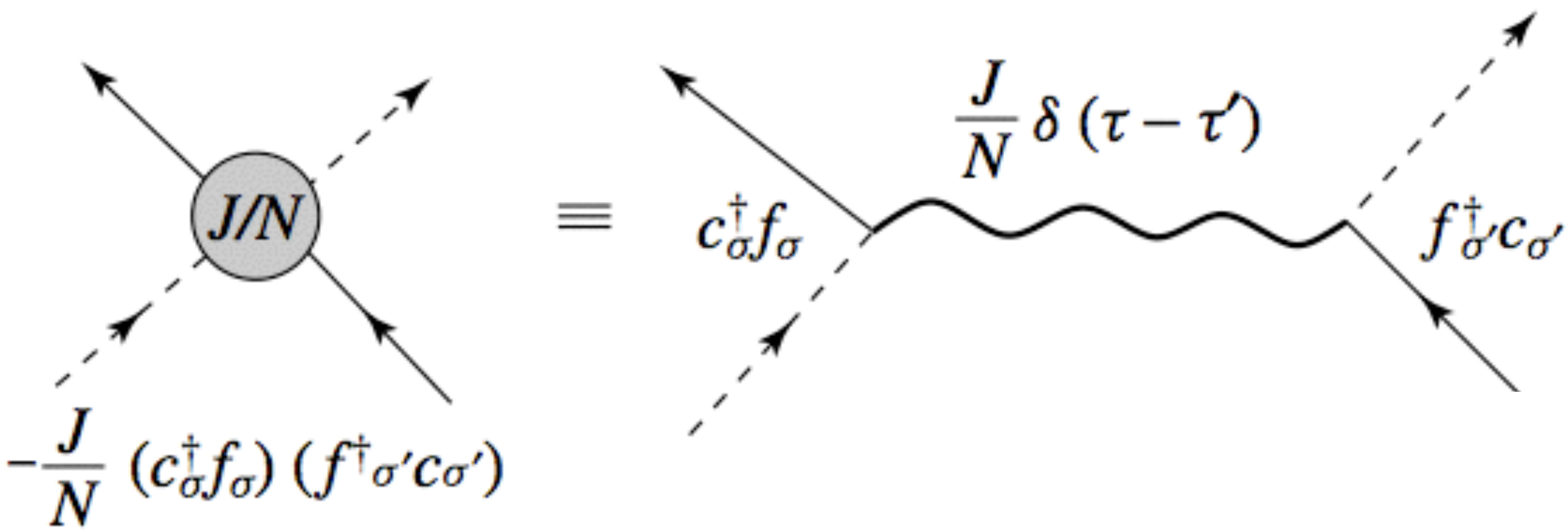
$$H_I(j) = -\frac{J}{N}\left(c^\dagger_{j\beta} f_{j\beta}\right)\left(f^\dagger_{j\alpha} c_{j\alpha}\right)$$

Constraint n_f = Q=qN

all terms extensive in N

$$-gA^\dagger A \rightarrow A^\dagger V + \bar{V} A + \frac{\bar{V}V}{g}$$

$$H_I(j) \rightarrow H_I[V,j] = \bar{V}_j\left(c^\dagger_{j\alpha} f_{j\alpha}\right) + \left(f^\dagger_{j\alpha} c_{j\alpha}\right) V_j + N\frac{\bar{V}_j V_j}{J}.$$



Large N Approach

Read and Newns '83.

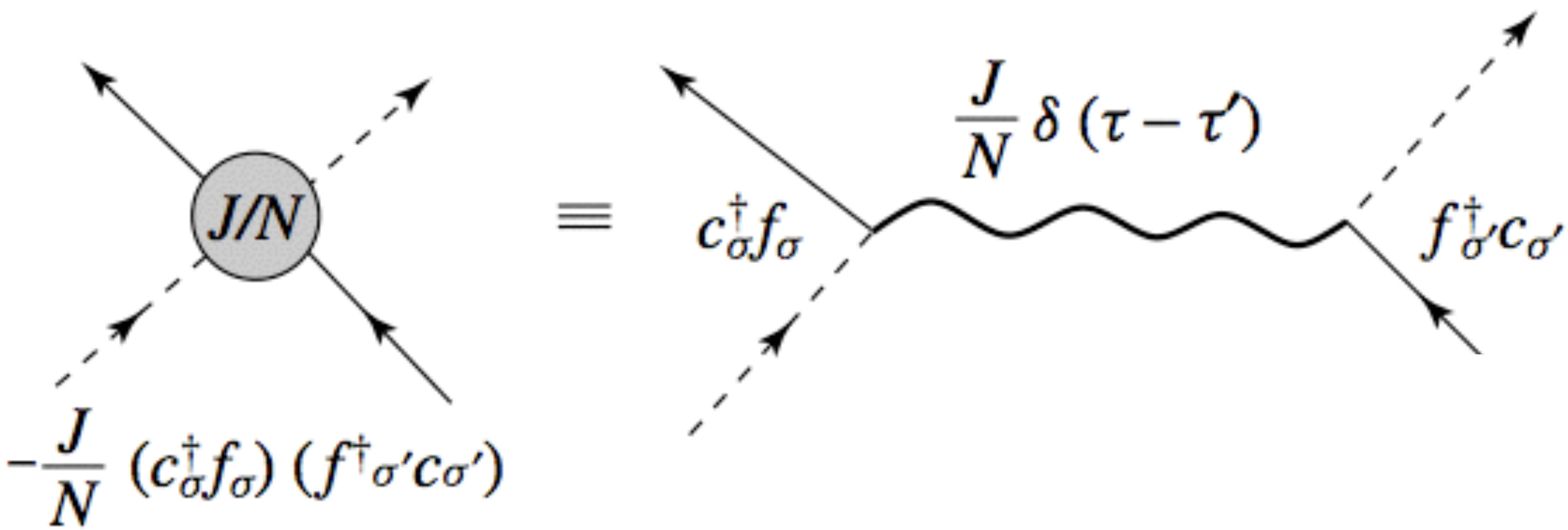
Extensive in N

$$Z = \int \mathcal{D}[V,\lambda] \overbrace{\int \mathcal{D}[c,f] \exp\left[- \int_0^\beta \left(\sum_{k\sigma} c^\dagger_{\mathbf{k}\sigma} \partial_\tau c_{\mathbf{k}\sigma} + \sum_{j\sigma} f^\dagger_{j\sigma} \partial_\tau f_{j\sigma} + H[V,\lambda] \right) \right]}^{= \mathrm{Tr}\left[T exp\left(- \int_0^\beta H[V,\lambda] d\tau \right) \right]}$$

$$H[V,\lambda] = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_j \left(H_I[V_j, j] + \lambda_j [n_f(j) - Q] \right),$$

$$H_I[V,j] = \bar{V}_j \left(c^\dagger_{j\alpha} f_{j\alpha} \right) + \left(f^\dagger_{j\alpha} c_{j\alpha} \right) V_j + N \frac{\bar{V}_j V_j}{J}.$$

U(1) constraint: note $n_f = Q = (qN)$



Large N Approach.

Read and Newns '83.

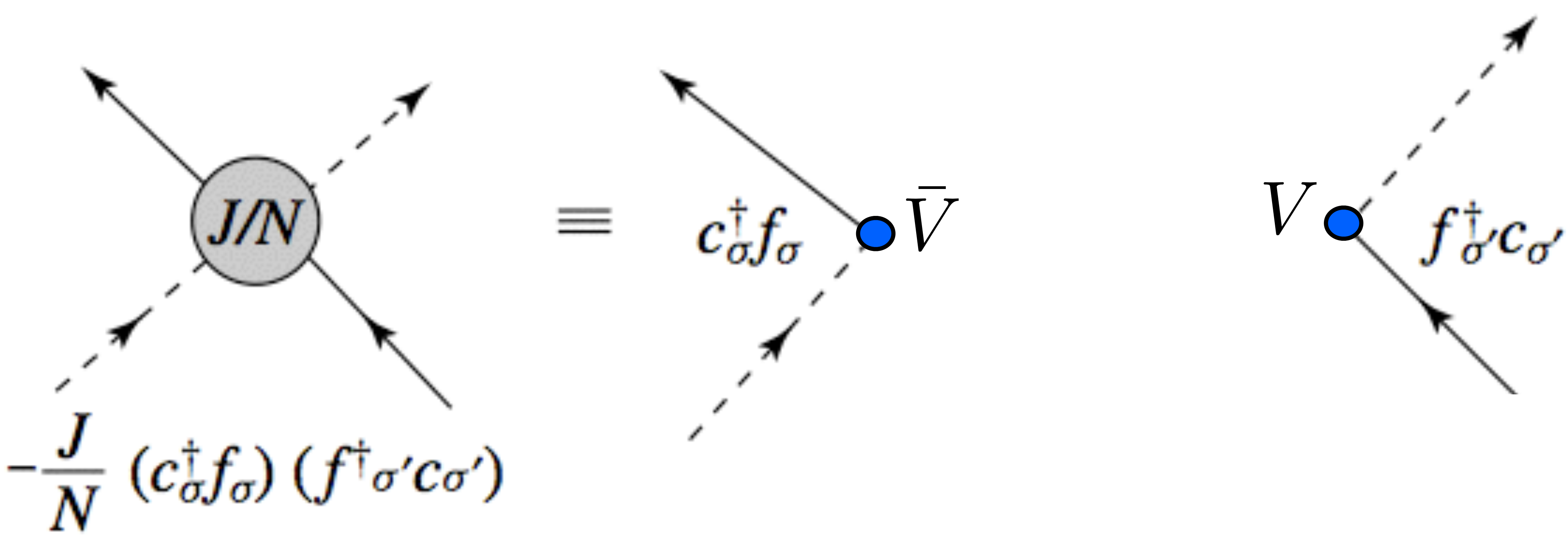
$$Z = \mathrm{Tr} e^{-\beta H_{MFT}}, \qquad (N \to \infty)$$

$V_j = V$

at each site

$$H[V,\lambda] = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_j \left(H_I[V_j, j] + \lambda_j [n_f(j) - Q] \right),$$

$$H_I[V,j] = \bar{V}_j \left(c^\dagger_{j\alpha} f_{j\alpha} \right) + \left(f^\dagger_{j\alpha} c_{j\alpha} \right) V_j + N \frac{\bar{V}_j V_j}{J}.$$



Detailed calcn.

$$H_{MFT} = \sum_{\mathbf{k}\sigma} \left(c^{\dagger}{}_{\mathbf{k}\sigma}, f^{\dagger}{}_{\mathbf{k}\sigma} \right) \overbrace{\begin{pmatrix} \epsilon_{\mathbf{k}} & V \\ \bar{V} & \lambda \end{pmatrix}}^{\underline{h}(\mathbf{k})} \begin{pmatrix} c_{\mathbf{k}\sigma} \\ f_{\mathbf{k}\sigma} \end{pmatrix} + N\mathcal{N}_s \left(\frac{|V|^2}{J} - \lambda q \right)$$

$$= \sum_{\mathbf{k}\sigma} \psi^{\dagger}{}_{\mathbf{k}\sigma} \, \underline{h}(\mathbf{k}) \, \psi_{\mathbf{k}\sigma} + N\mathcal{N}_s \left(\frac{|V|^2}{J} - \lambda q \right).$$

$$f^{\dagger}{}_{\vec{k}\sigma} = \frac{1}{\sqrt{n}} \sum_j f^{\dagger}{}_{j\sigma} e^{i\vec{k}\cdot\vec{R}_j}$$

$$H_{MFT} = \sum_{\mathbf{k}\sigma} \left(a^{\dagger}{}_{\mathbf{k}\sigma}, b^{\dagger}{}_{\mathbf{k}\sigma} \right) \begin{pmatrix} E_{\mathbf{k}^+} & 0 \\ 0 & E_{\mathbf{k}^-} \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}\sigma} \\ b_{\mathbf{k}\sigma} \end{pmatrix} + Nn \left(\frac{\bar{V}V}{J} - \lambda q \right).$$

$$\mathrm{Det}\left[E^{\pm}_{\mathbf{k}} \underline{1} - \begin{pmatrix} \epsilon_{\mathbf{k}} & V \\ \bar{V} & \lambda \end{pmatrix} \right] = (E_{\mathbf{k}\pm} - \epsilon_{\mathbf{k}})(E_{\mathbf{k}\pm} - \lambda) - |V|^2 = 0,$$

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2}\right)^2 + |V|^2\right]^{\frac{1}{2}},$$

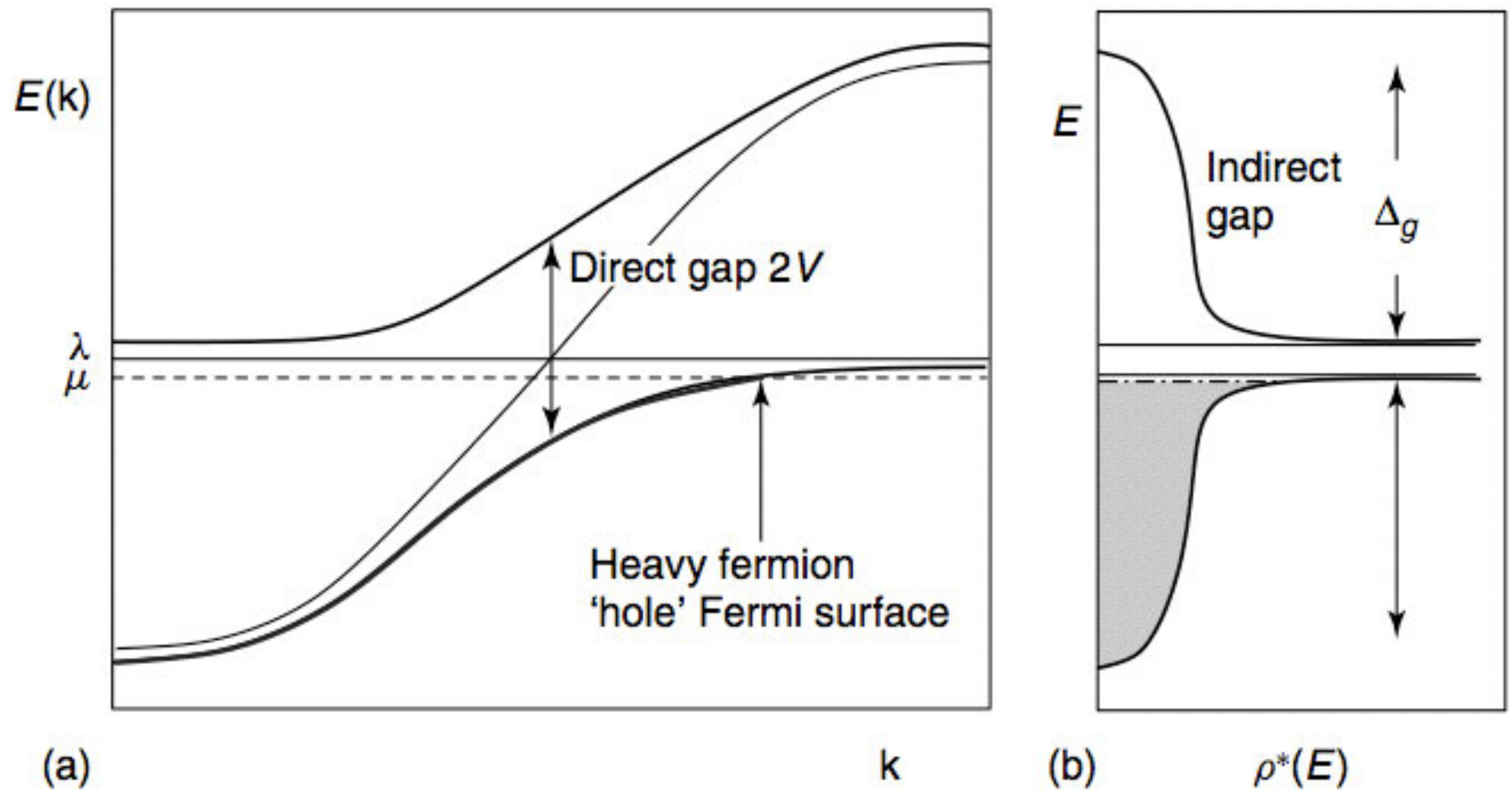
$$|MF\rangle = \prod_{|\mathbf{k}|<k_F\sigma} b^{\dagger}{}_{\mathbf{k}\sigma}|0\rangle = \prod_{|\mathbf{k}|<k_F\sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f^{\dagger}{}_{\mathbf{k}\sigma})|0\rangle.$$

$$a^{\dagger}{}_{\mathbf{k}\sigma} = u_{\mathbf{k}} c^{\dagger}{}_{\mathbf{k}\sigma} + v_{\mathbf{k}} f^{\dagger}{}_{\mathbf{k}\sigma}$$
$$b^{\dagger}{}_{\mathbf{k}\sigma} = -v_{\mathbf{k}} c^{\dagger}{}_{\mathbf{k}\sigma} + u_{\mathbf{k}} f^{\dagger}{}_{\mathbf{k}\sigma}$$

$$\left\{ \begin{array}{c} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{array} \right\} = \left[\frac{1}{2} \pm \frac{(\epsilon_{\mathbf{k}} - \lambda)/2}{2\sqrt{\left(\frac{\epsilon_{\mathbf{k}}-\lambda}{2}\right)^2 + |V|^2}}\right]^{\frac{1}{2}}$$

$$\boxed{|GW\rangle = P_Q \prod_{|\mathbf{k}|<k_F\sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f^{\dagger}{}_{\mathbf{k}\sigma})|0\rangle,}$$

"Gutzwiller" wavefunction



Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2}\right)^2 + |V|^2\right]^{\frac{1}{2}},$$

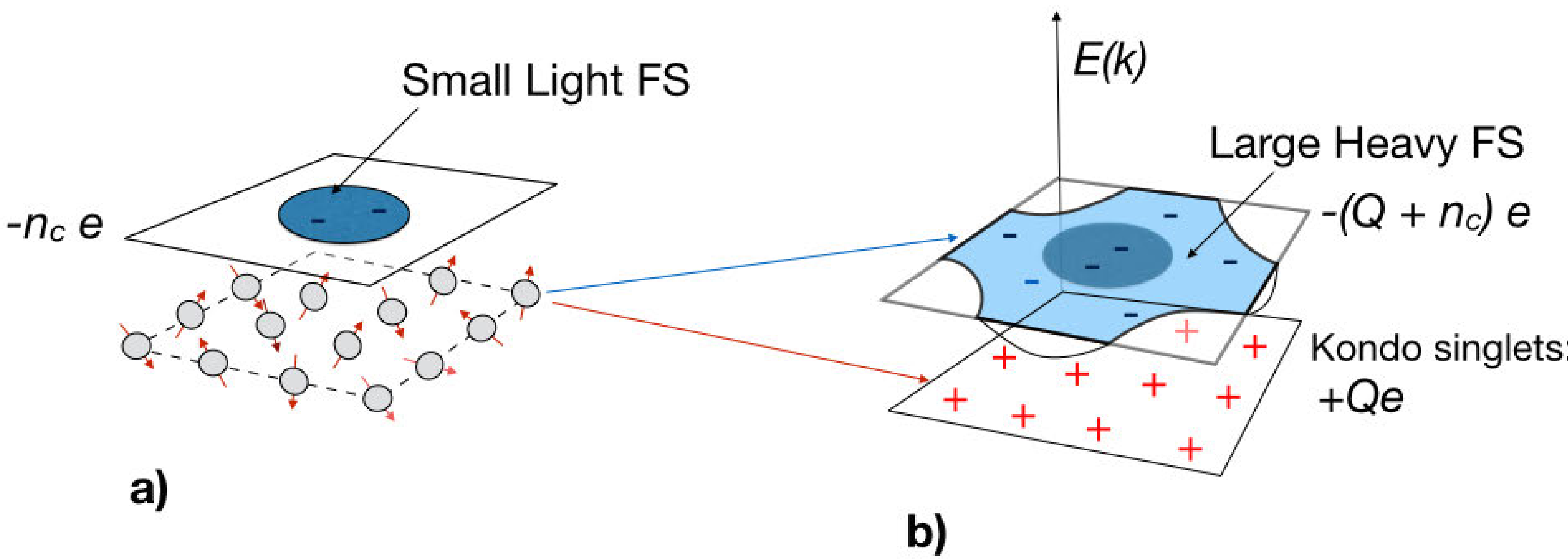
$$|MF\rangle = \prod_{|\mathbf{k}|<k_F\sigma} b^{\dagger}{}_{\mathbf{k}\sigma}|0\rangle = \prod_{|\mathbf{k}|<k_F\sigma} (-v_{\mathbf{k}}c_{\mathbf{k}\sigma} + u_{\mathbf{k}}f^{\dagger}{}_{\mathbf{k}\sigma})|0\rangle.$$

$$a^{\dagger}{}_{\mathbf{k}\sigma} = u_{\mathbf{k}}c^{\dagger}{}_{\mathbf{k}\sigma} + v_{\mathbf{k}}f^{\dagger}{}_{\mathbf{k}\sigma}$$
$$b^{\dagger}{}_{\mathbf{k}\sigma} = -v_{\mathbf{k}}c^{\dagger}{}_{\mathbf{k}\sigma} + u_{\mathbf{k}}f^{\dagger}{}_{\mathbf{k}\sigma}$$

$$\left\{\begin{array}{c} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{array}\right\} = \left[\frac{1}{2} \pm \frac{(\epsilon_{\mathbf{k}} - \lambda)/2}{2\sqrt{\left(\frac{\epsilon_{\mathbf{k}}-\lambda}{2}\right)^2 + |V|^2}}\right]^{\frac{1}{2}}$$

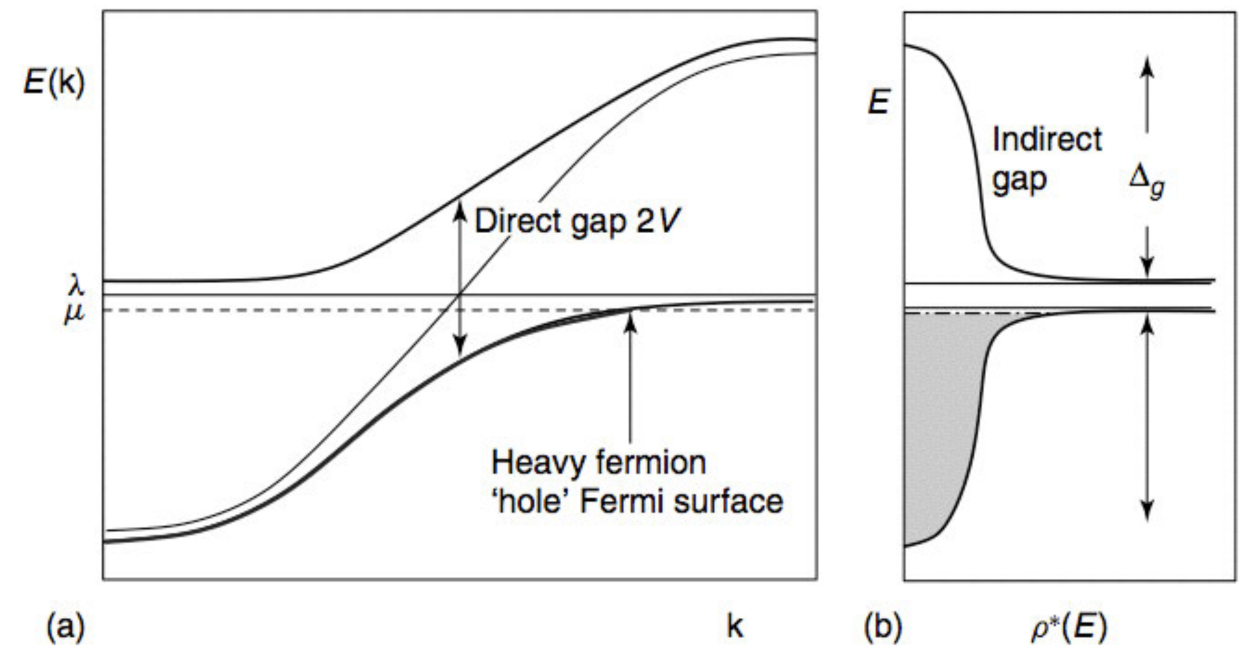
$$\boxed{|GW\rangle = P_Q \prod_{|\mathbf{k}|<k_F\sigma} (-v_{\mathbf{k}}c_{\mathbf{k}\sigma} + u_{\mathbf{k}}f^{\dagger}{}_{\mathbf{k}\sigma})|0\rangle,}$$

"Gutzwiller" wavefunction



Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2}\right)^2 + |V|^2\right]^{\frac{1}{2}},$$



$$\frac{F}{N} = -T\sum_{\mathbf{k},\pm} \ln\left[1 + e^{-\beta E_{\mathbf{k}\pm}}\right] + \mathcal{N}_s\left(\frac{V^2}{J} - \lambda q\right).$$

$$\frac{E_o}{N\mathcal{N}_s} = \int_{-\infty}^{0} dE \rho^*(E) E + \left(\frac{V^2}{J} - \lambda q\right)$$

$$E = \epsilon + \frac{V^2}{E-\lambda} \qquad \rho^*(E) = \rho\frac{d\epsilon}{dE} = \rho\left(1 + \frac{V^2}{(E-\lambda)^2}\right)$$

$$\frac{E_o}{N\mathcal{N}_s} = \rho\int_{-D-V^2/D}^{0} dE E\left(1 + \frac{V^2}{(E-\lambda)^2}\right) + \left(\frac{V^2}{J} - \lambda q\right)$$

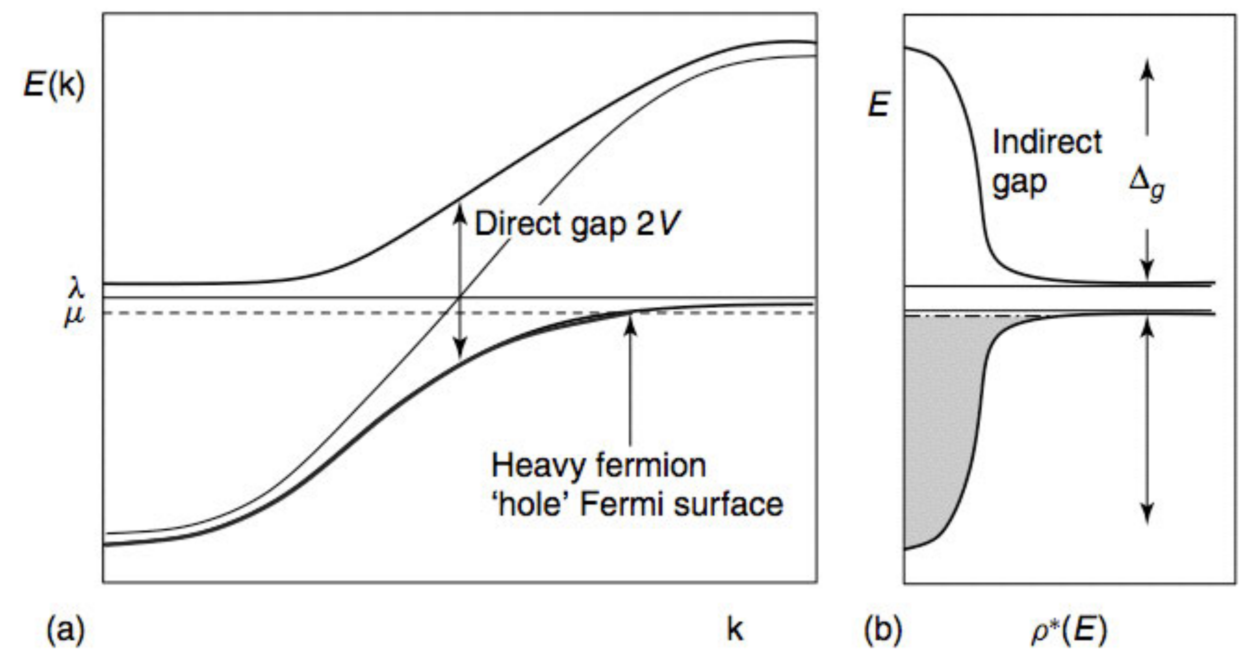
$$\begin{aligned}\frac{E_o}{N\mathcal{N}_s} &= -\frac{\rho}{2}\left(D + \frac{V^2}{D}\right)^2 + \frac{\Delta}{\pi}\int_{-D}^{0} dE\left(\frac{1}{E-\lambda} + \frac{\lambda}{(E-\lambda)^2}\right) + \left(\frac{V^2}{J} - \lambda q\right)\\ &= -\frac{D^2\rho}{2} + \frac{\Delta}{\pi}\ln\left(\frac{\lambda}{D}\right) + \left(\frac{V^2}{J} - \lambda q\right)\end{aligned}$$

$$(\Delta = \pi\rho|V|^2)$$

Detailed calcn.

$$(\Delta = \pi\rho|V|^2)$$

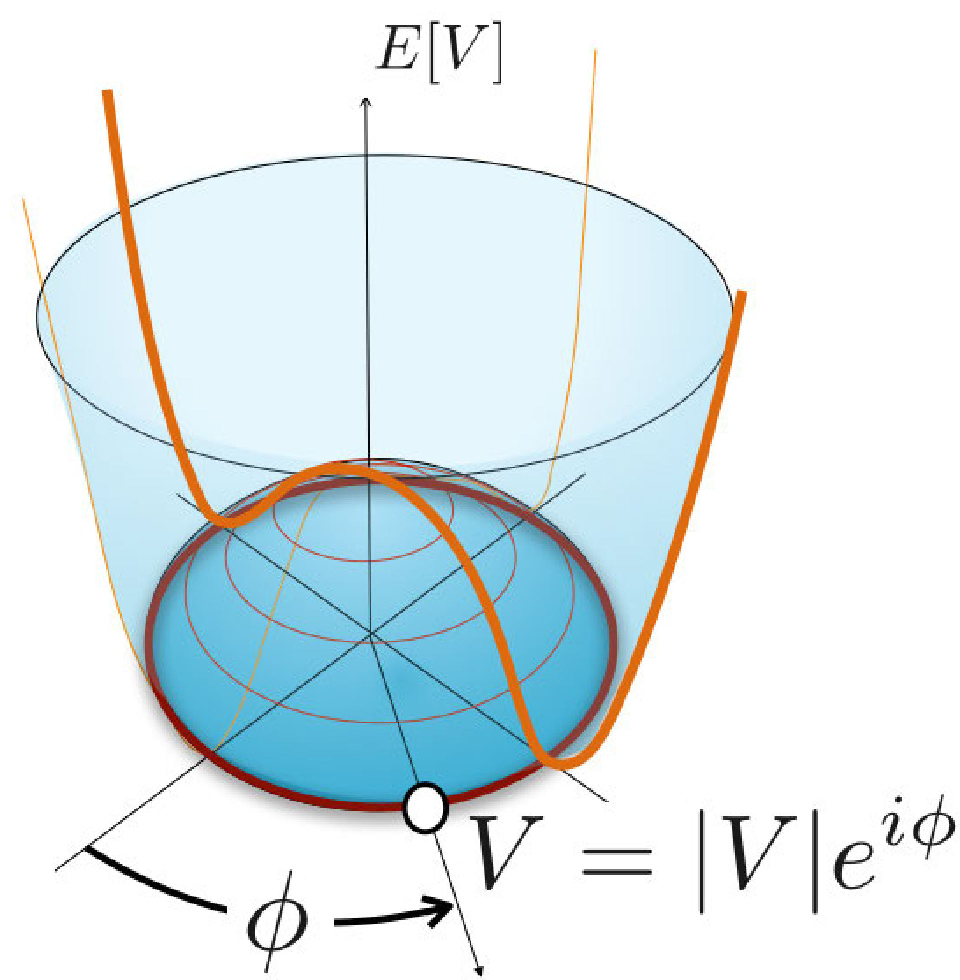
$$\begin{aligned}\frac{E_0}{N\mathcal{N}_s} &= -\frac{D^2\rho}{2} + \frac{\Delta}{\pi}\ln\left(\frac{\lambda}{D}\right) + \left(\frac{\pi\rho V^2}{\pi\rho J} - \lambda q\right)\\ &= -\frac{D^2\rho}{2} + \frac{\Delta}{\pi}\ln\left(\frac{\lambda}{D}\right) + \left(\frac{\Delta}{\pi\rho J} - \lambda q\right)\\ &= -\frac{D^2\rho}{2} + \frac{\Delta}{\pi}\ln\left(\frac{\lambda}{De^{-\frac{1}{J\rho}}}\right) - \lambda q\\ &= -\frac{D^2\rho}{2} + \frac{\Delta}{\pi}\ln\left(\frac{\lambda}{T_K}\right) - \lambda q.\end{aligned}$$



$$T_K = De^{-\frac{1}{J\rho}}$$

$$\frac{\partial E_0}{\partial \lambda} = \langle n_f\rangle - Q = 0 \qquad \frac{\Delta}{\pi\lambda} - q = 0$$

$$\frac{E_o(V)}{N\mathcal{N}_s} = \frac{\Delta}{\pi}\ln\left(\frac{\Delta}{\pi q e T_K}\right) - \frac{D^2\rho}{2},$$

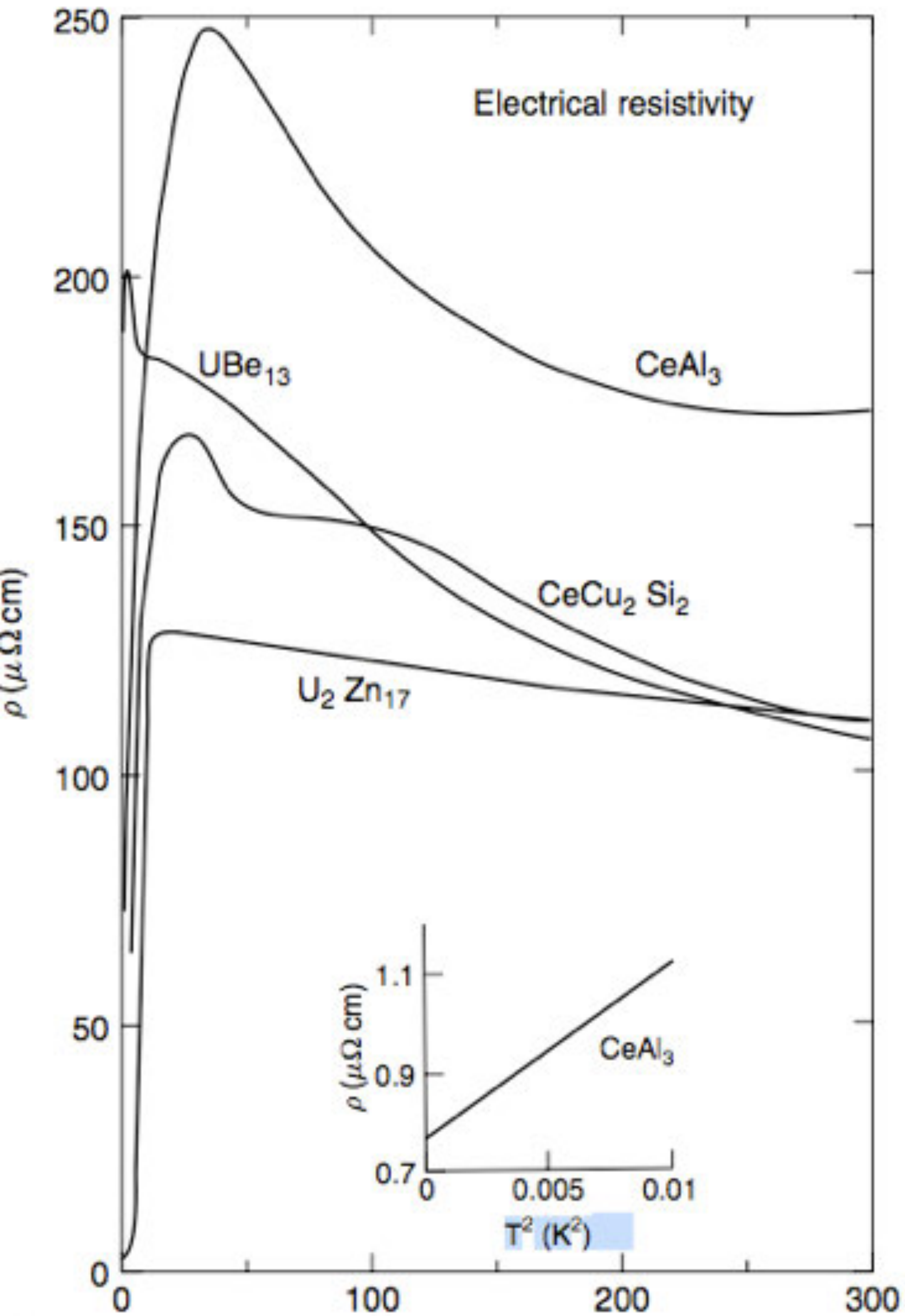


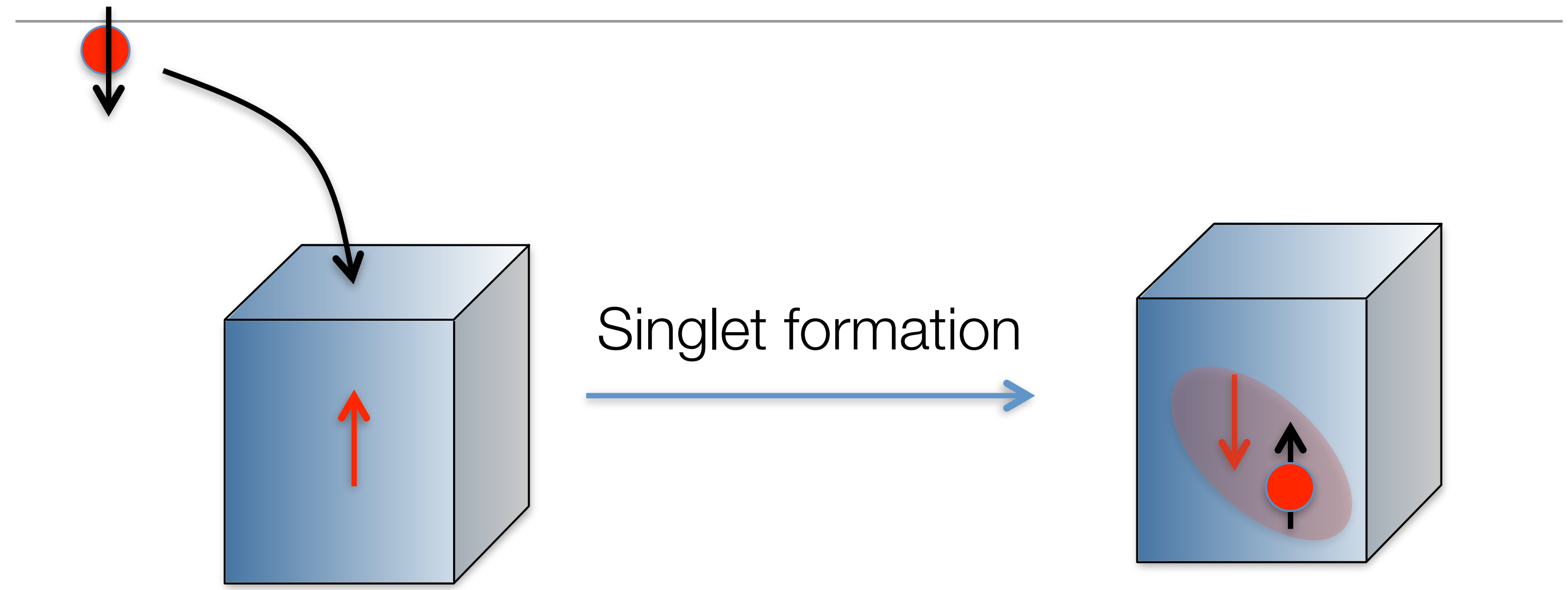
$$\frac{\partial E_0}{\partial \Delta} = 0$$

$$0 = \frac{1}{\pi}\ln\left(\frac{\Delta e^2}{\pi q T_K}\right)$$

$$\Delta = \frac{\pi q}{e^2}T_K$$

Coherence and composite fermions





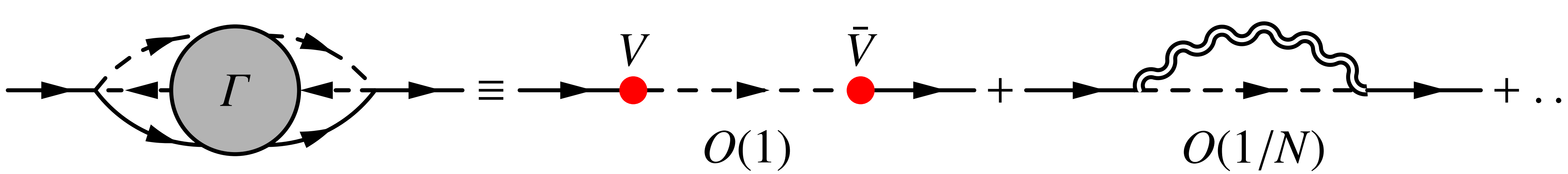
Heavy electron = (electron x spinflip)

- The large N approach to the Kondo lattice.
  Spin x conduction = composite fermion

$$\frac{J}{N} c^{\dagger}{}_{\beta} S_{\alpha\beta} c_{\alpha} \longrightarrow \bar{V}\left(c^{\dagger}{}_{\alpha} f_{\alpha}\right) + \left(f^{\dagger}{}_{\alpha} c_{\alpha}\right) V + N\frac{\bar{V}V}{J},$$

Composite Fermion

$$\frac{J}{N} c^{\dagger}_{j\alpha} S_{\alpha\beta} \equiv \bar{V} f^{\dagger}_{j\beta}$$



Outline of the Topics

1. Trends in the periodic table.
2. Introduction: Heavy Fermions and the Kondo Lattice.
3. Kondo Insulators: the simplest heavy fermions.
4. Large N expansion for the Kondo Lattice
5. **Heavy Fermion Superconductivity**
6. Topological Kondo Insulators
7. Co-existing magnetism and the Kondo Effect.

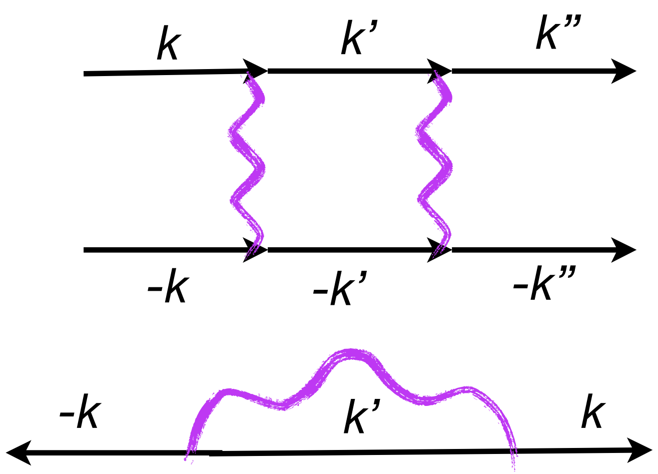
Please ask questions!

Rutgers
Center for Materials Theory

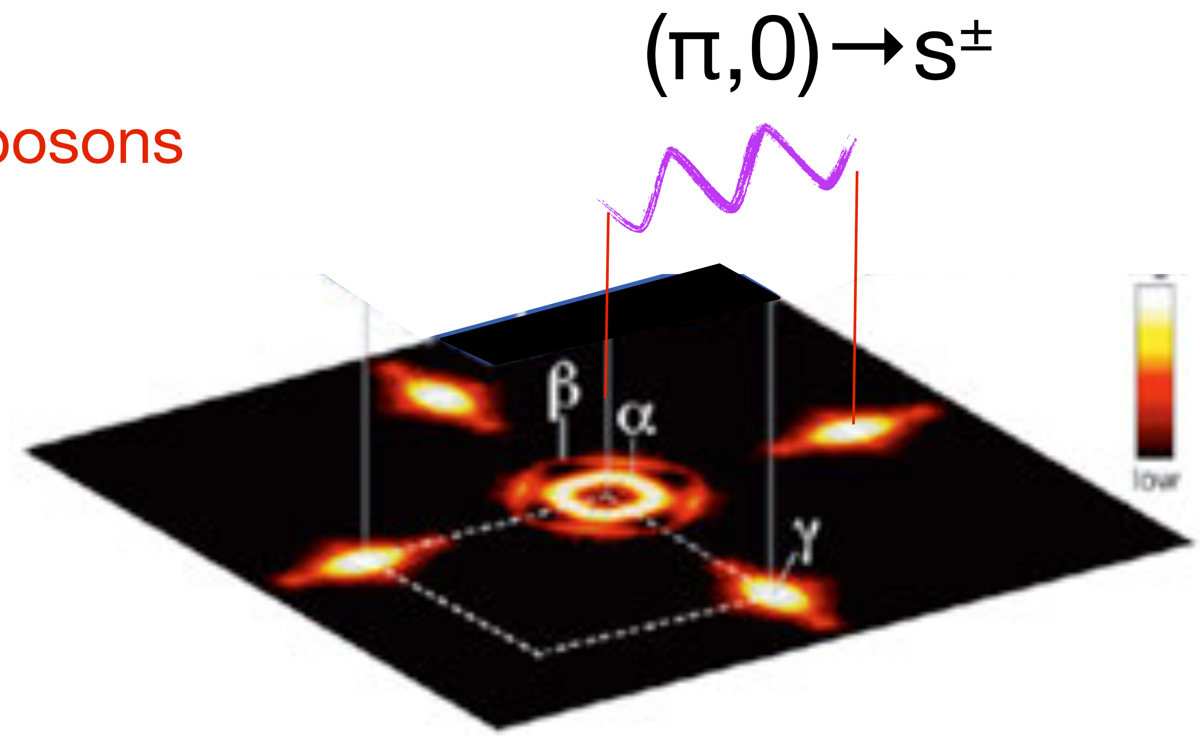
Glue vs Fabric.

Glue

Spin fluctuations = pairing bosons



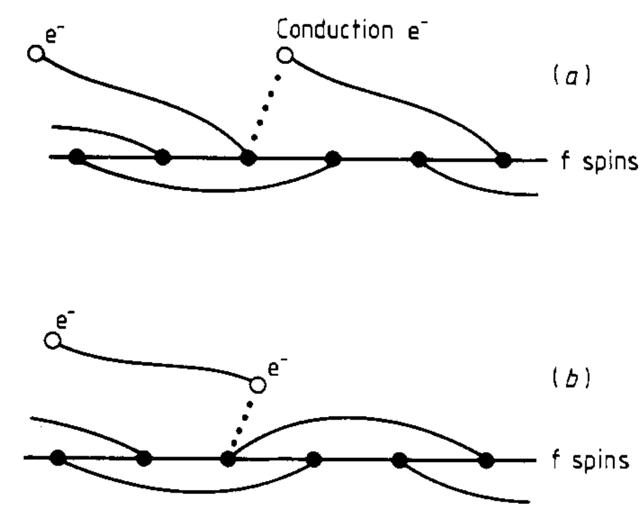
Eliashberg Approach (cf B. Keimer et al)

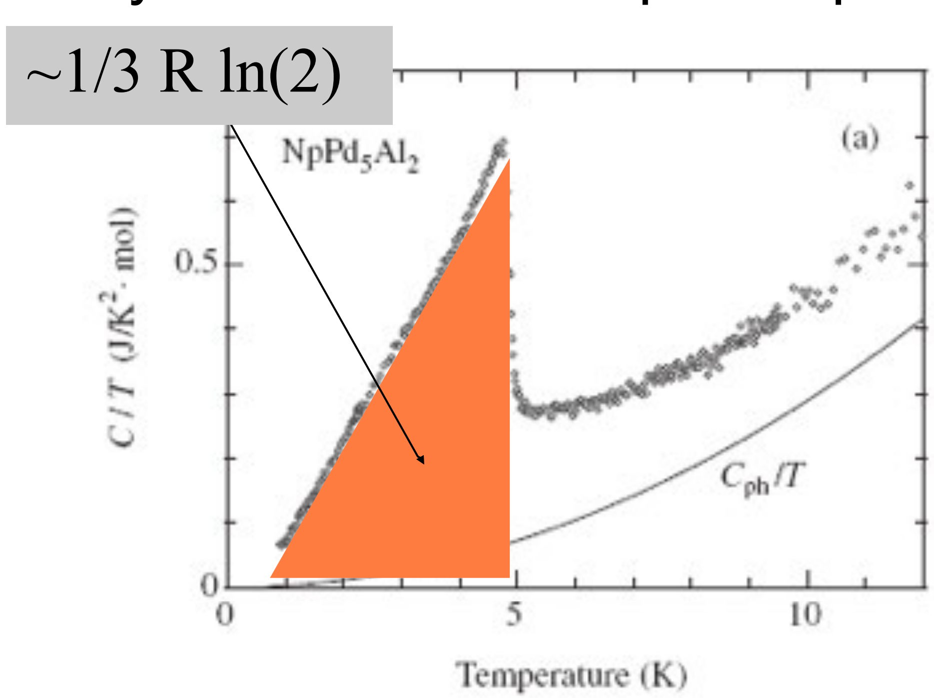


Fabric: spins *make* the pairs

Anderson: RVB (1987); Coleman Andrei (1989)

Emery & Kivelson: composite pairs (1993)





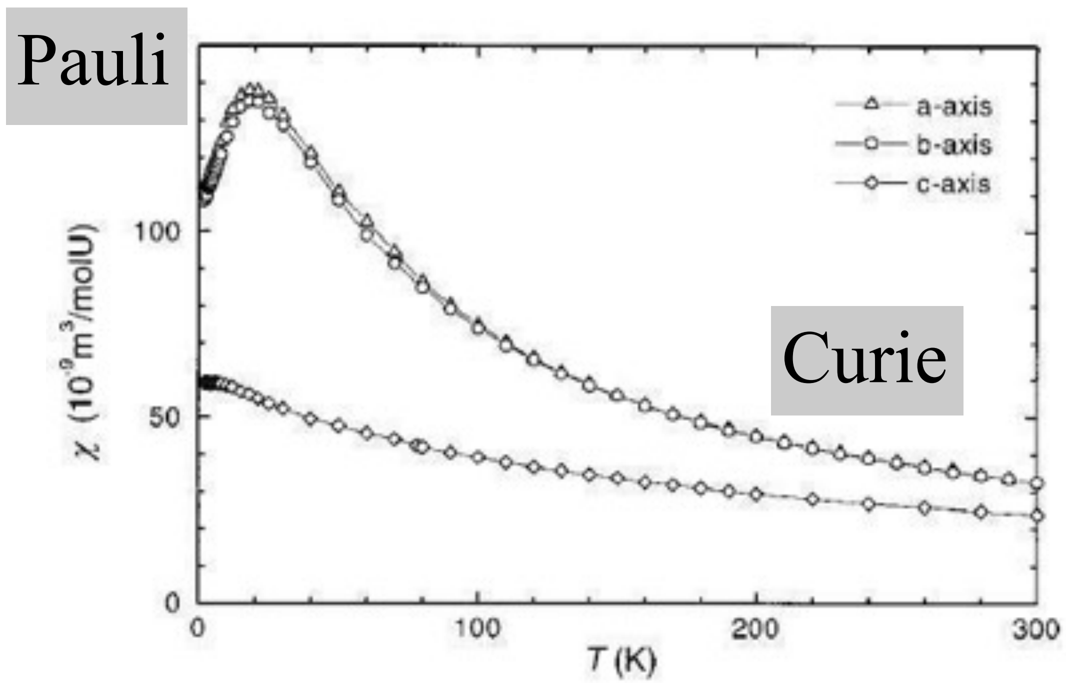
$$R\ln W = \int_0^T dT' \frac{C'}{T'}$$

"Hilbert Space Spectroscopy"

SPIN Hilbert space BUILDS the pairs.

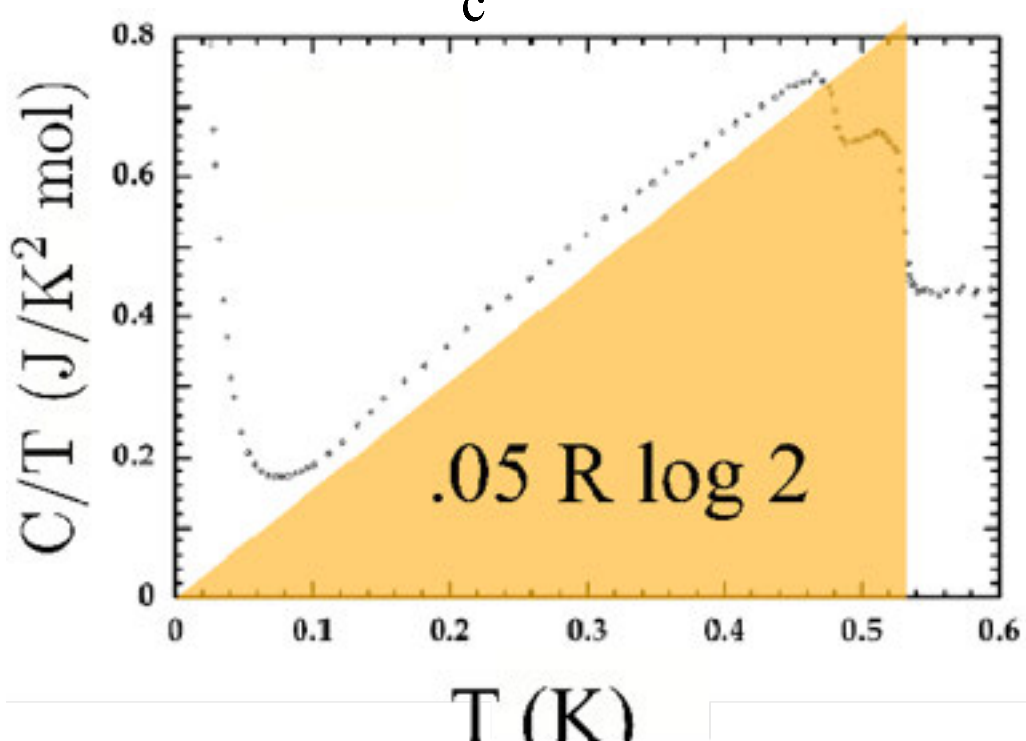
How?

UPt_3



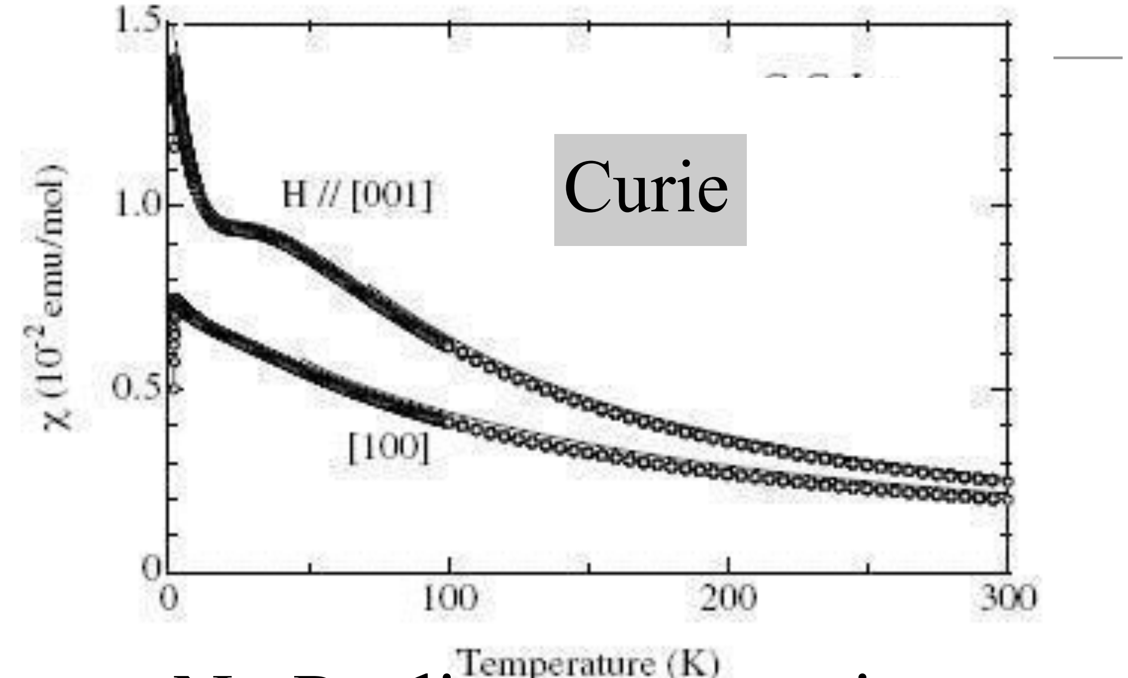
Pauli paramagnetic by 30K

$T_c = 0.5K$



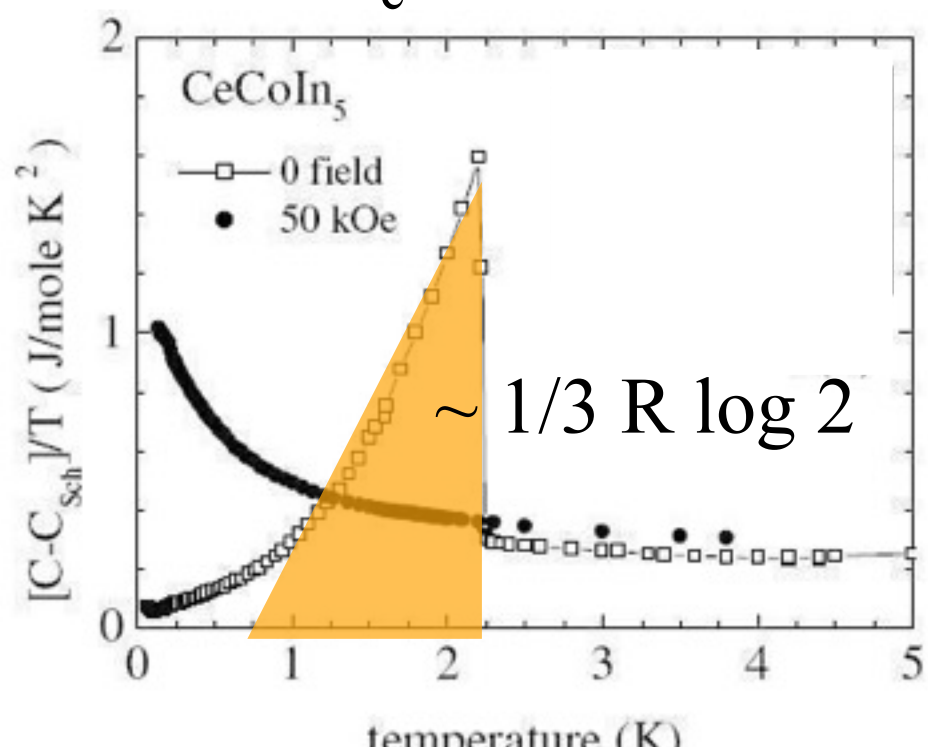
Frings *et al.* J. Magn. Magn. Mater. **31**, 240(1983)
Brison *et al.* J. Low Temp. Phys. **95**, 145(1994)

$CeCoIn_5$



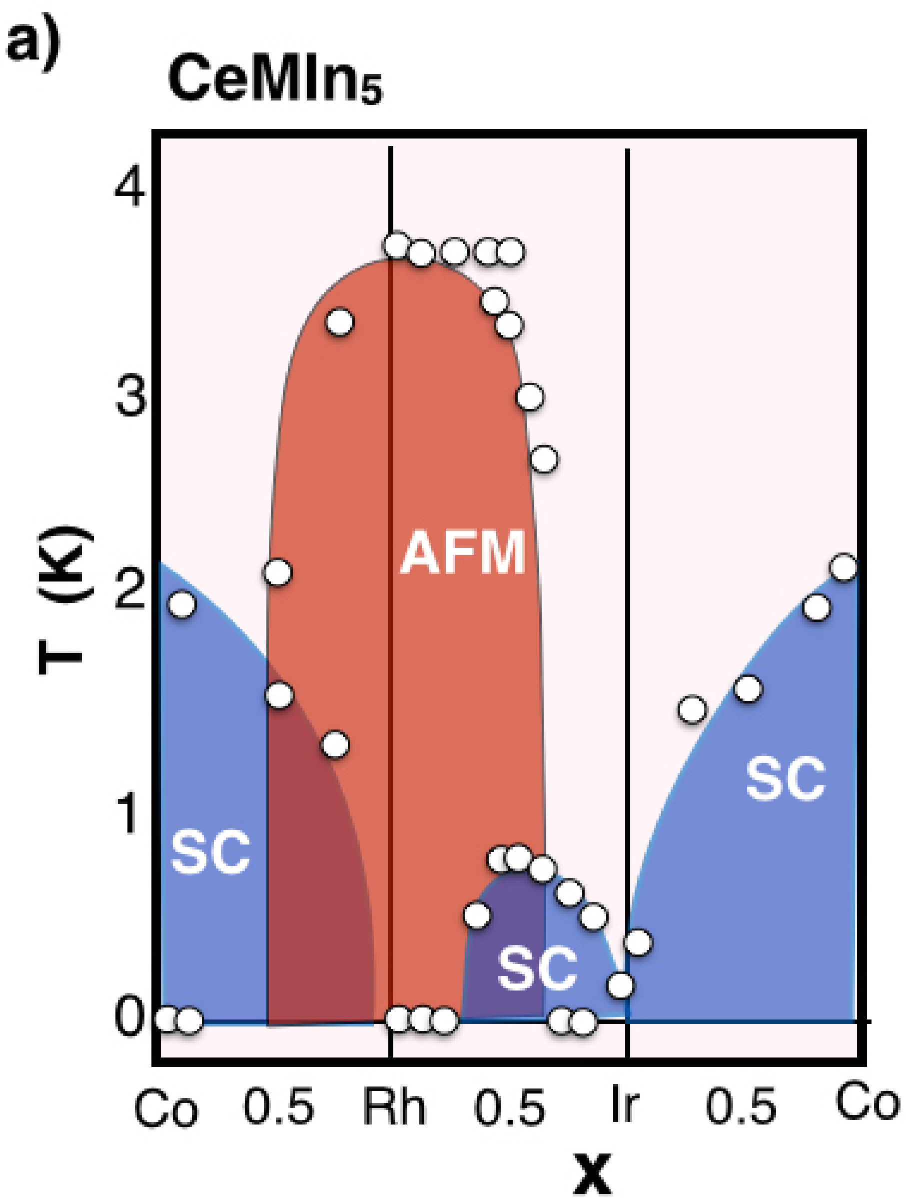
No Pauli paramagnetism

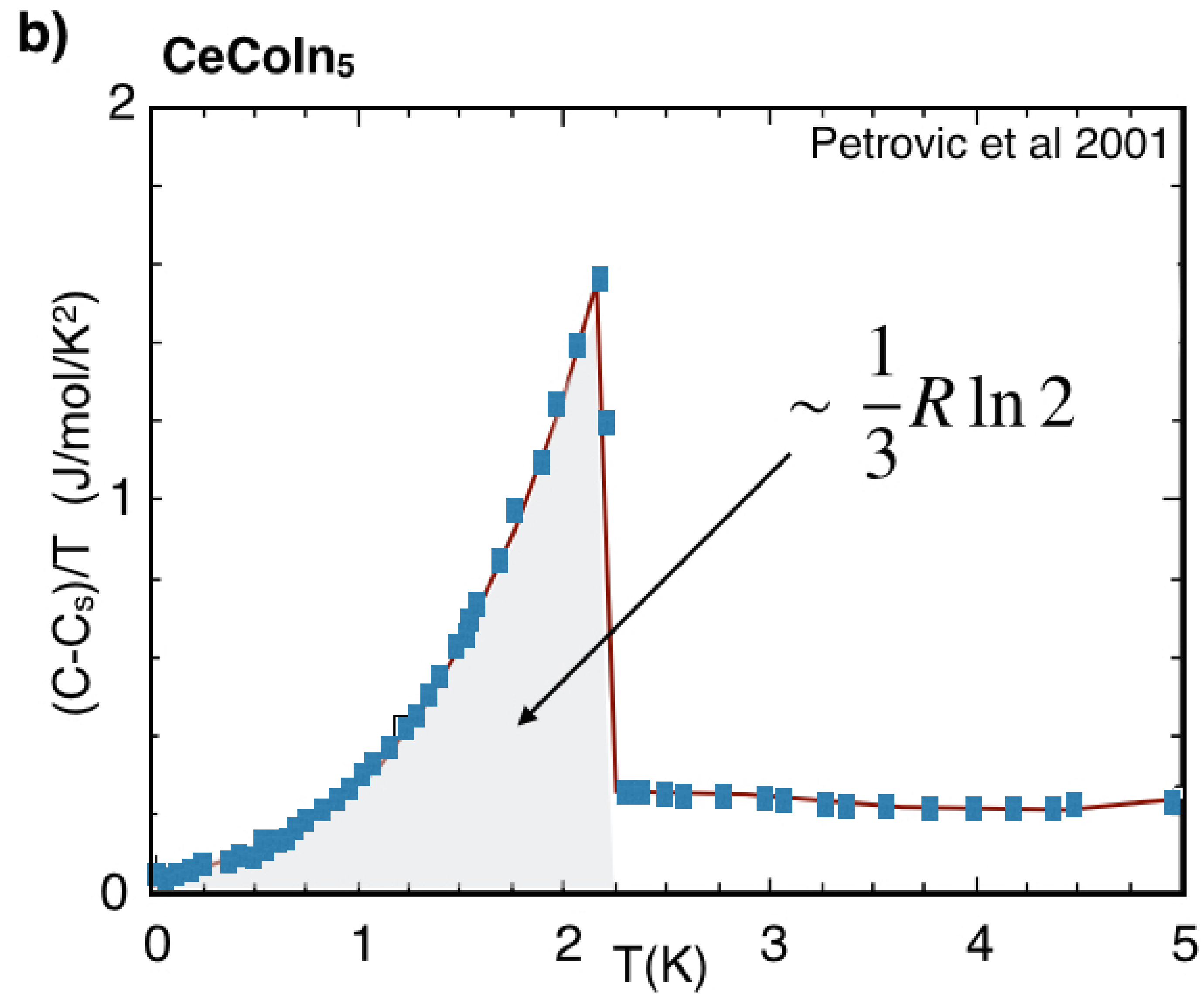
$T_c = 2.3K$



Shishido *et al.* JPSJ **71**, 162 (2002)
Petrovic *et al.* J.Phys Condens. Matter **13** 337 (2001)

115 Materials.





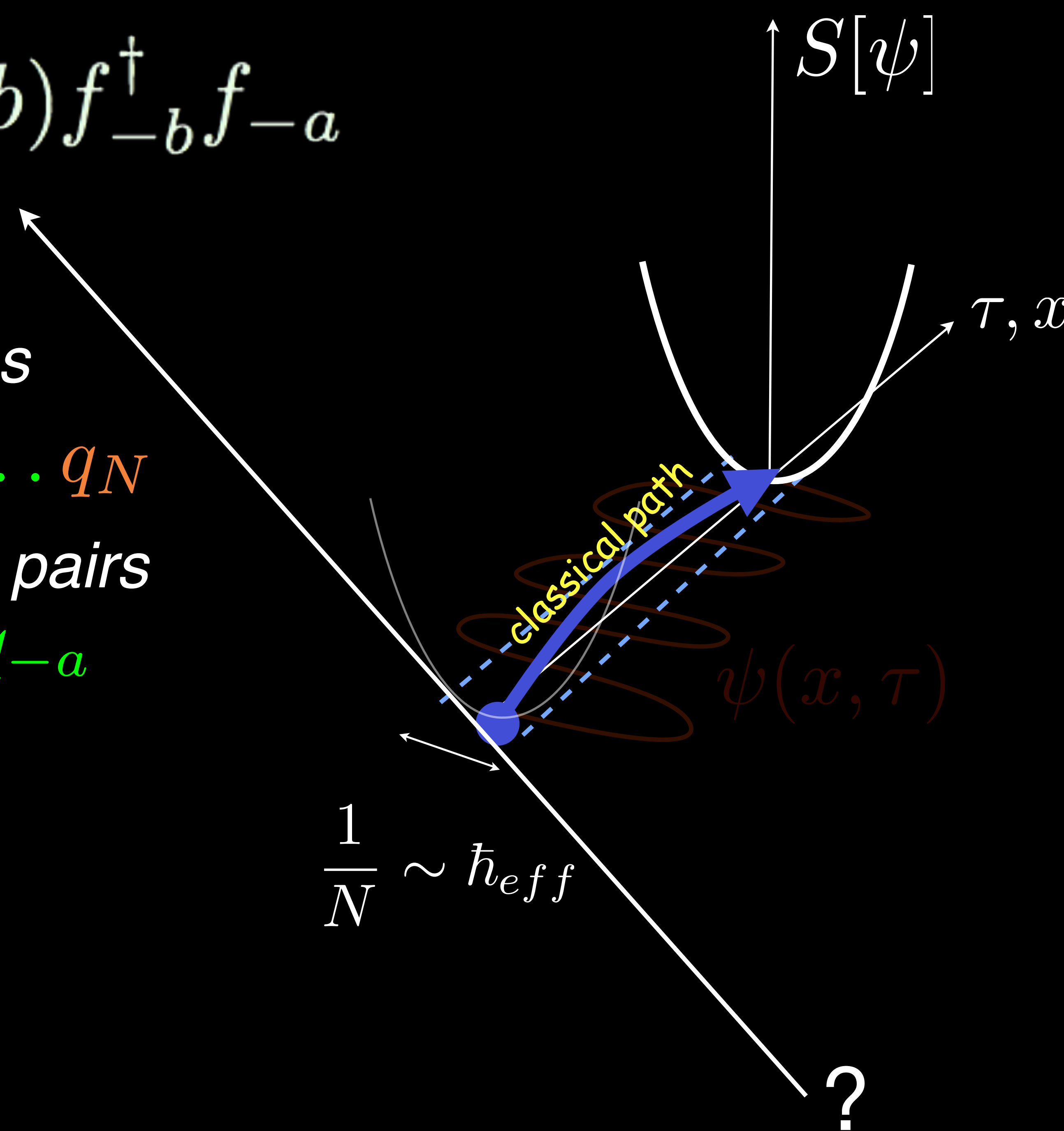
$N \rightarrow \infty$

"Symplectic Large N"

R. Flint and PC '08

$$S^{ba} = f_b^\dagger f_a - \mathrm{sgn}(a)\mathrm{sgn}(b) f^\dagger_{-b} f_{-a}$$

	Mesons	*Baryons*
SU(N):	$\bar{q}q$	$q_1 q_2 \cdots q_N$
No Pairs !		
		Cooper pairs
SP(N):	$\bar{q}q$	$q_a q_{-a}$



$$H = \sum_{\mathbf{k}\alpha} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \frac{J_K}{N} \sum_j c^\dagger_{j\alpha} c_{j\beta} S_{\beta\alpha}(j) + \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(i) S_{\beta\alpha}(j)$$

SP(N) Large N Approach.

$$H = H_c + H_K + H_{RKKY}$$

$$\begin{aligned} H_K &= \frac{J_K}{N}\sum_j c^\dagger{}_{j\alpha} c_{j\beta} S_{\beta\alpha}(j) \rightarrow -\frac{J_K}{N}\sum_{i,j}\Big((c^\dagger{}_{j\alpha} f_{j\alpha})(f^\dagger{}_{j\beta} c_{j\beta}) + \tilde{\alpha}\tilde{\beta}(c^\dagger{}_{j\alpha} f^\dagger{}_{j-\alpha})(f_{j-\beta} c_{j\beta})\Big) \\ H_M &= \frac{J_H}{2N}\sum_{(i,j)} S_{\alpha\beta}(j) S_{\beta\alpha}(j) \rightarrow -\frac{J_H}{N}\sum_j \Big[(f^\dagger{}_{i\alpha} f_{j\alpha})(f^\dagger{}_{j\beta} f_{i\beta}) + \tilde{\alpha}\tilde{\beta}(f^\dagger{}_{i\alpha} f^\dagger{}_{j-\alpha})(f_{j-\beta} f_{i\beta})\Big] \end{aligned}$$

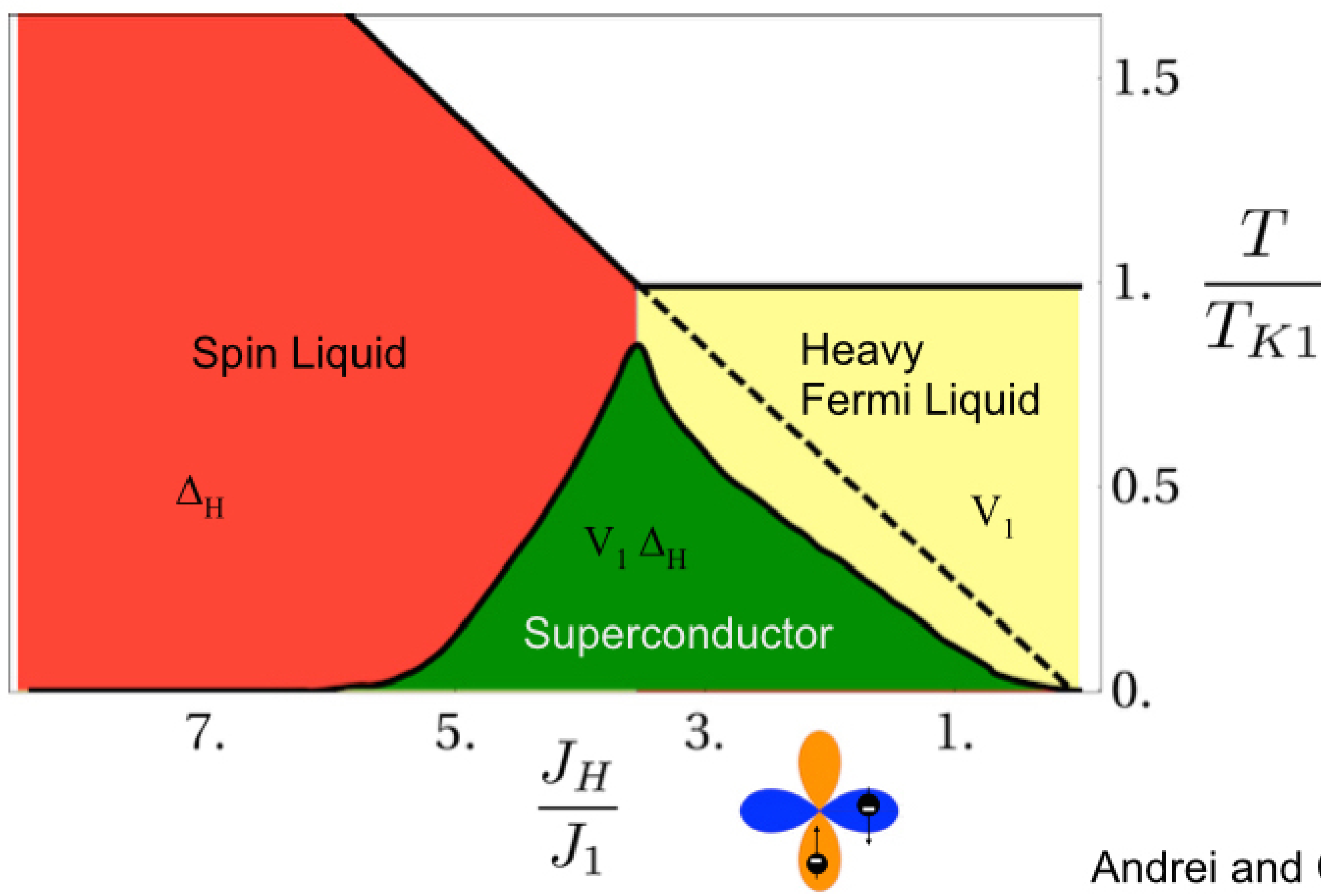
$$\begin{aligned} H_K &\rightarrow \sum_j \Big[c^\dagger{}_{j\alpha}\Big(V_j f_{j\alpha} + \tilde{\alpha}\Delta^K_j f^\dagger{}_{j-\alpha}\Big) + \text{H.c}\Big] + N\left(\frac{|V_j|^2 + |\Delta^K_j|^2}{J_K}\right) \\ H_H &\rightarrow \sum_{(i,j)} \Big[t_{ij} f^\dagger{}_{i\alpha} f_{j\alpha} + \Delta_{ij}\tilde{\alpha} f^\dagger{}_{i\alpha} f^\dagger{}_{j-\alpha} + \text{H.c}\Big] + N\left[\frac{|t_{ij}|^2 + |\Delta_{ij}|^2}{J_H}\right] \end{aligned}$$

Uniform solution:

$$H = \sum_{\mathbf{k},\alpha>0} (\tilde{c}^\dagger{}_{\mathbf{k}\alpha}, \tilde{f}^\dagger{}_{\mathbf{k}\alpha}) \begin{bmatrix} \epsilon_{\mathbf{k}}\tau_3 & V\tau_1 \\ V\tau_1 & \vec{w}\cdot\vec{\tau} + \Delta_{H\mathbf{k}}\tau_1 \end{bmatrix} \begin{pmatrix} \tilde{c}_{\mathbf{k}\alpha} \\ \tilde{f}_{\mathbf{k}\alpha} \end{pmatrix} + \mathcal{N}_s N\left(\frac{|V|^2}{J_K} + 2\frac{|\Delta_H|^2}{J_H}\right)$$

SP(N) Large N Approach.

$$H = H_c + H_K + H_{RKKY}$$



Andrei and Coleman 1989

Uniform solution:

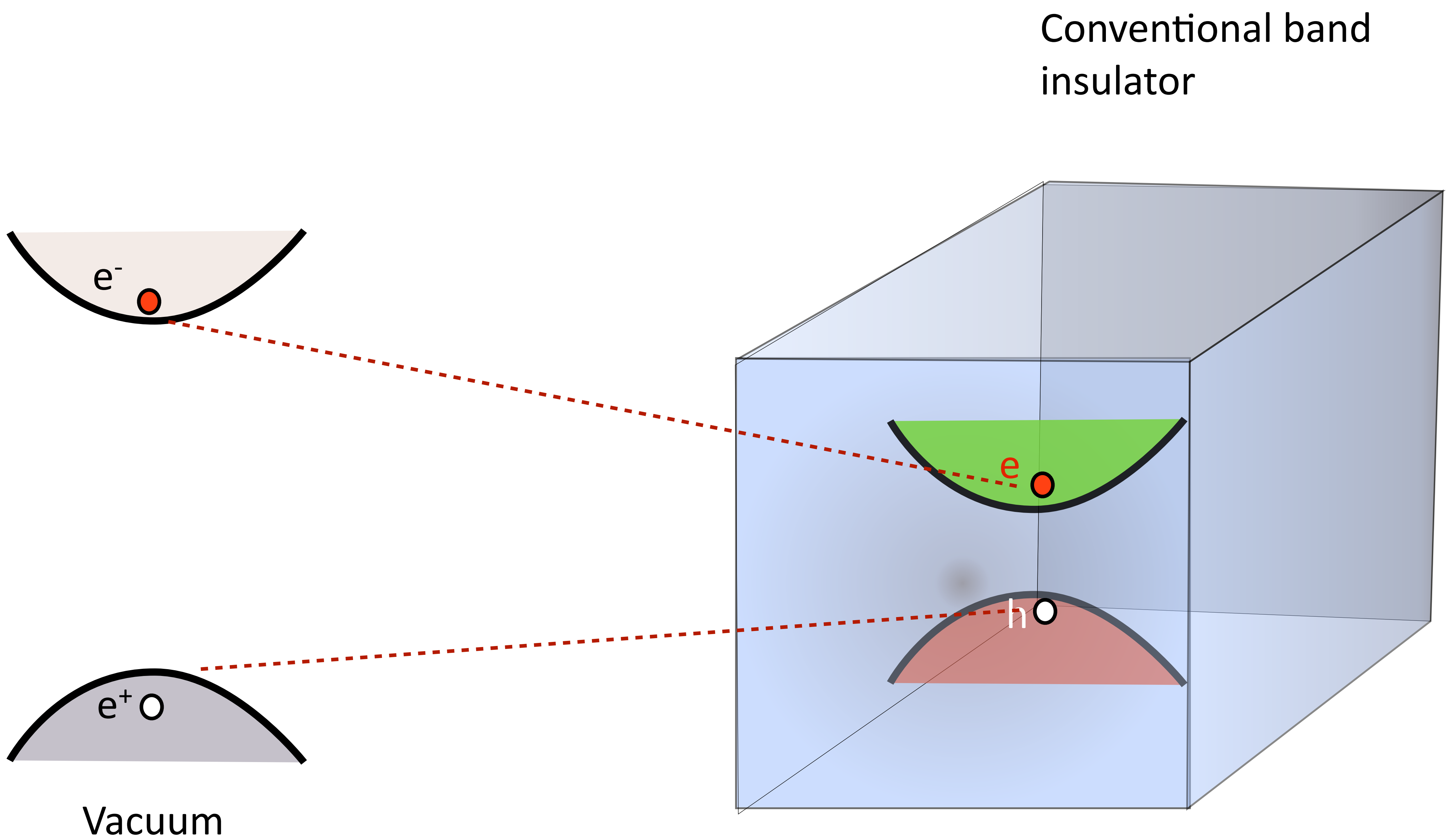
$$H = \sum_{\mathbf{k},\alpha>0} (\tilde{c}^{\dagger}{}_{\mathbf{k}\alpha}, \tilde{f}^{\dagger}{}_{\mathbf{k}\alpha}) \begin{bmatrix} \epsilon_{\mathbf{k}}\tau_3 & V\tau_1 \\ V\tau_1 & \vec{w}\cdot\vec{\tau} + \Delta_{H\mathbf{k}}\tau_1 \end{bmatrix} \begin{pmatrix} \tilde{c}_{\mathbf{k}\alpha} \\ \tilde{f}_{\mathbf{k}\alpha} \end{pmatrix} + \mathcal{N}_s N \left(\frac{|V|^2}{J_K} + 2\frac{|\Delta_H|^2}{J_H} \right)$$

Outline of the Topics

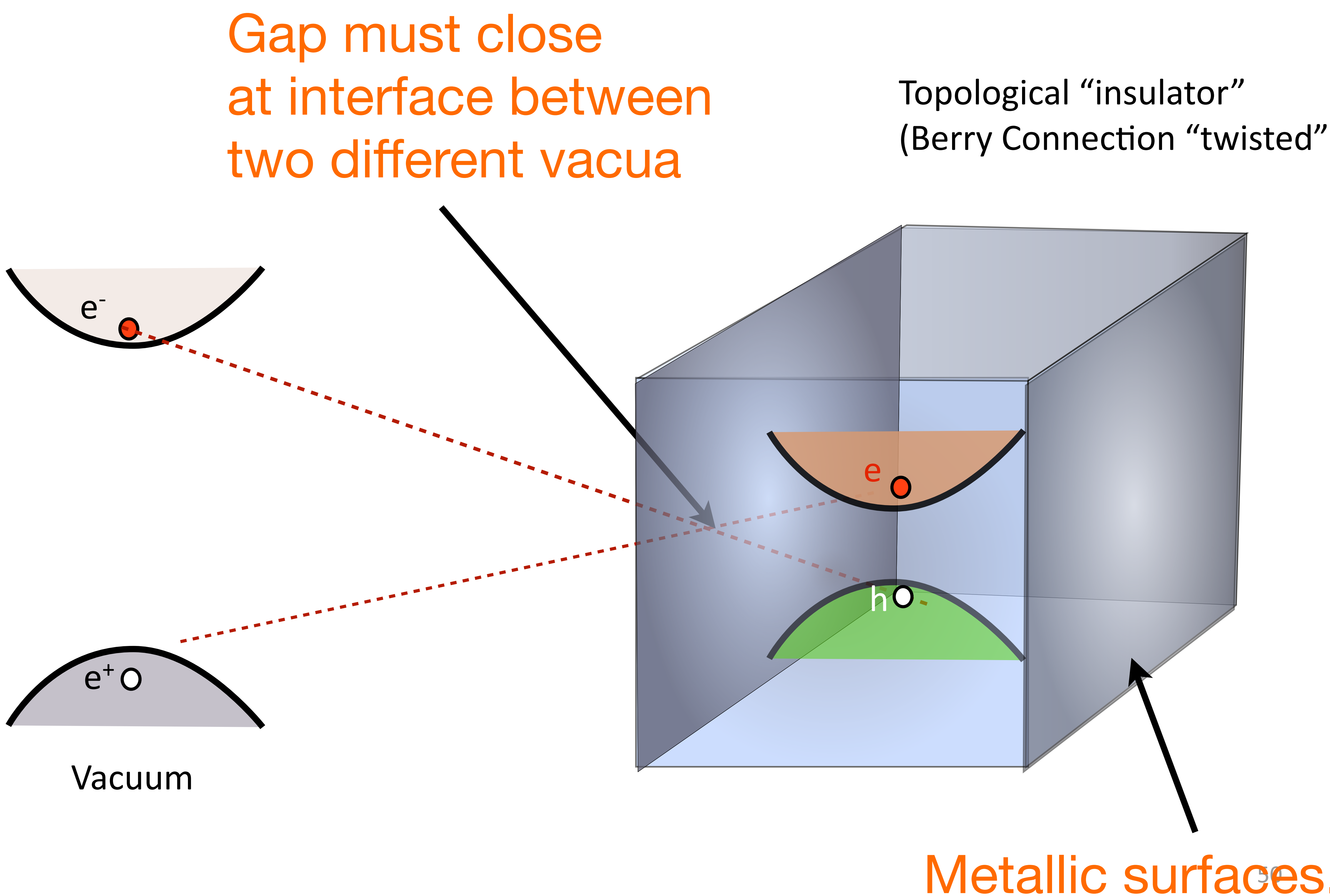
1. Trends in the periodic table.
2. Introduction: Heavy Fermions and the Kondo Lattice.
3. Kondo Insulators: the simplest heavy fermions.
4. Large N expansion for the Kondo Lattice
5. Heavy Fermion Superconductivity
6. **<u>Topological Kondo Insulators</u>**
7. Co-existing magnetism and the Kondo Effect.

Please ask questions!

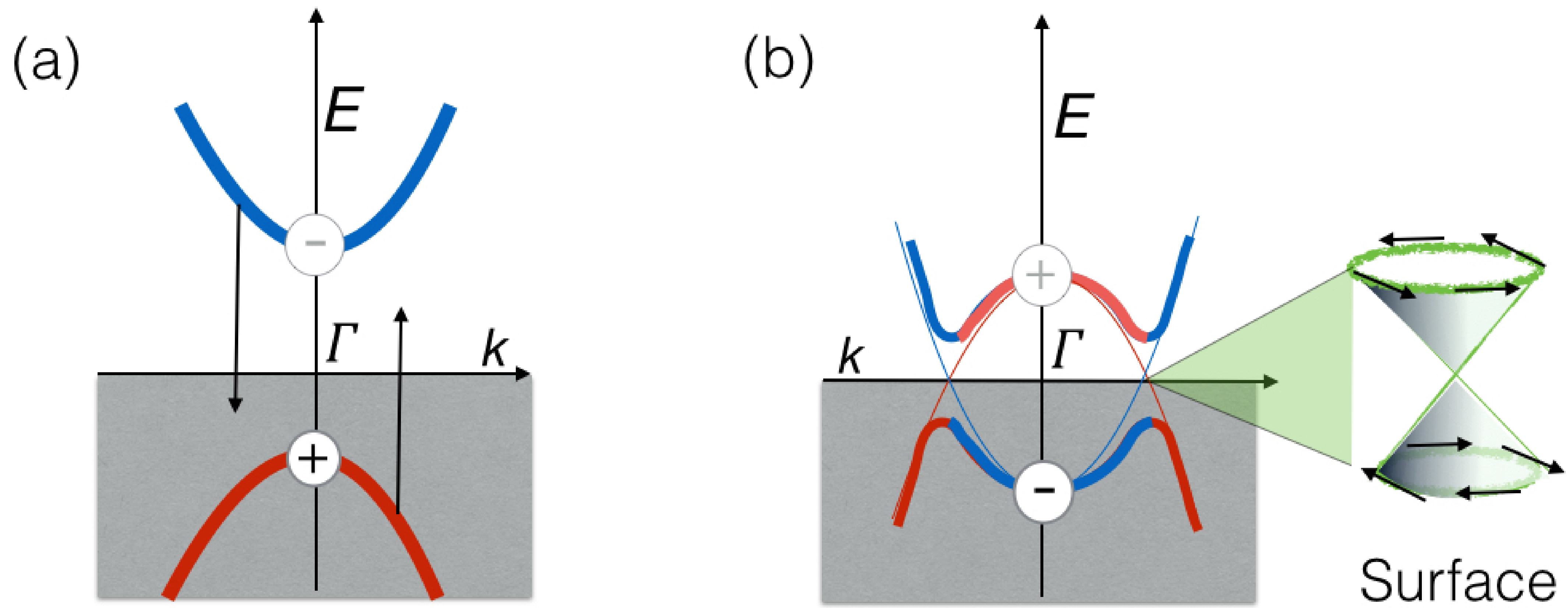
Conventional band insulator: adiabatic continuation of the vacuum.

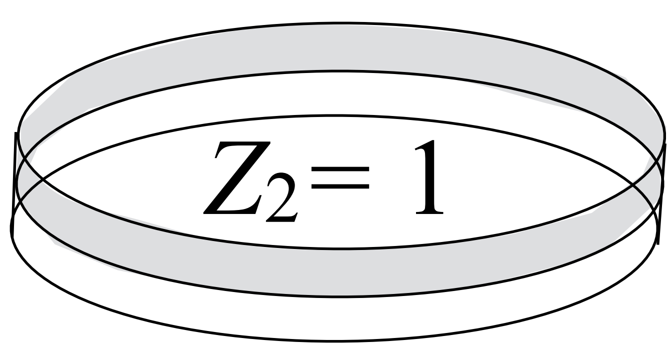


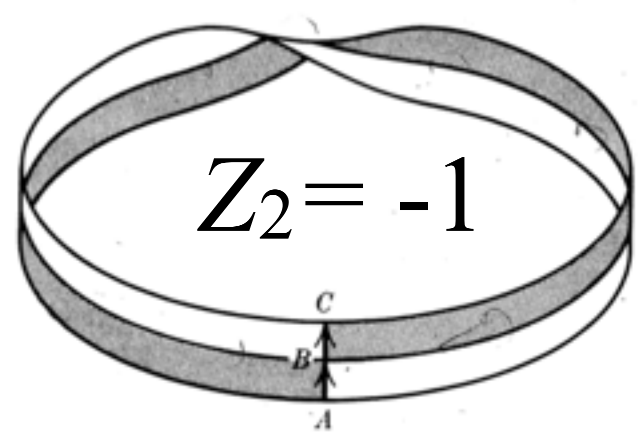
Topological insulator : adiabatically disconnected from the vacuum.



Band Crossing of odd and even parity states Yields a Z_2 Topological Insulator (Fu, Kane, Mele, 2007)





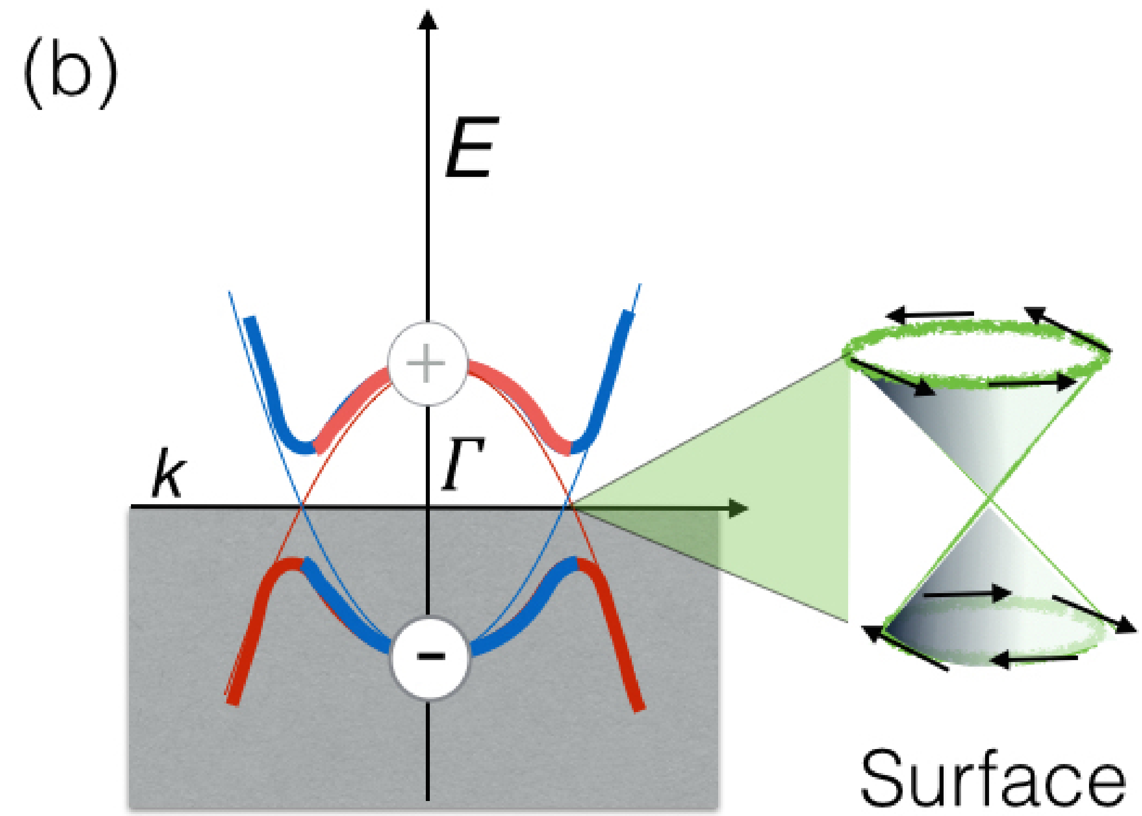


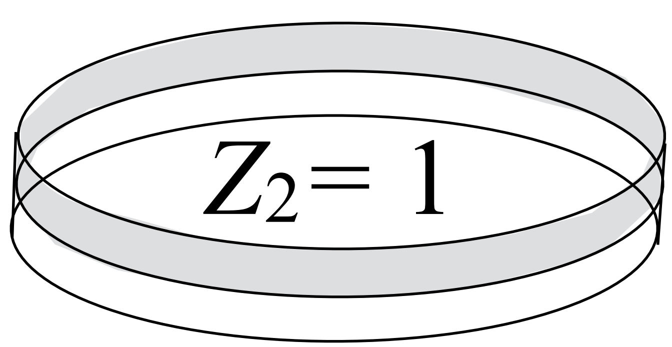
Topological Texture of Berry Connection

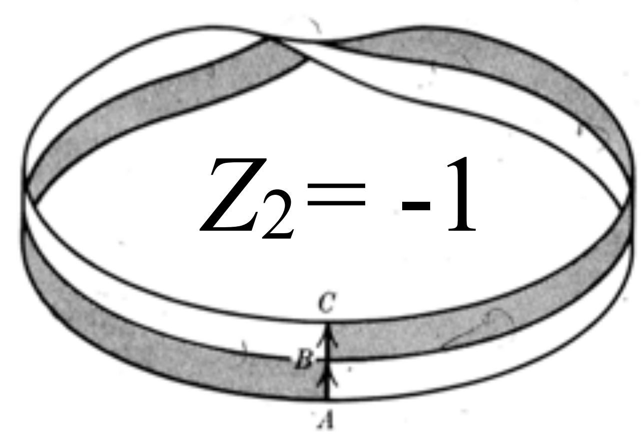
Band Crossing of odd and even parity states Yields a Z_2 Topological Insulator (Fu, Kane, Mele, 2007)

$$Z_2 = \prod_i \delta(\Gamma_i)$$

Fu and Kane (2008) :Parity equation.



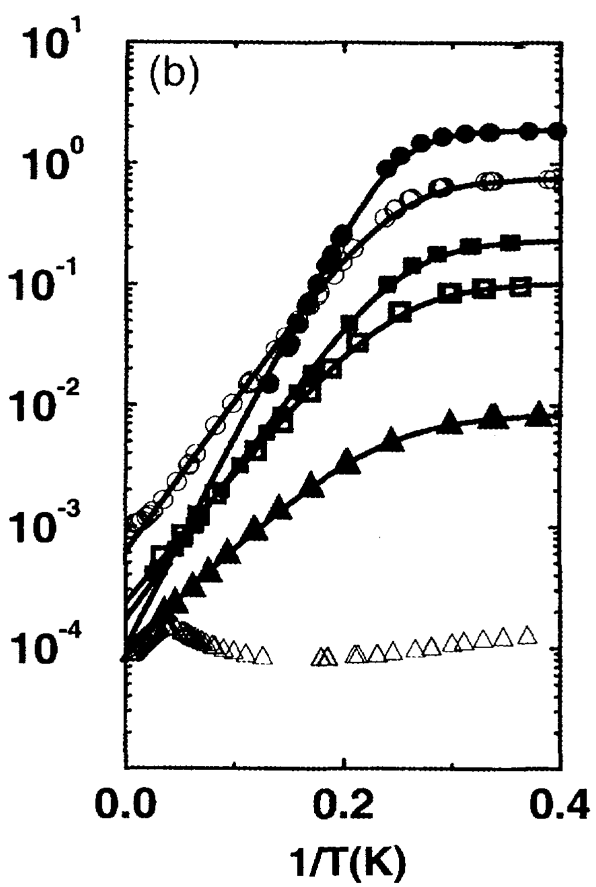


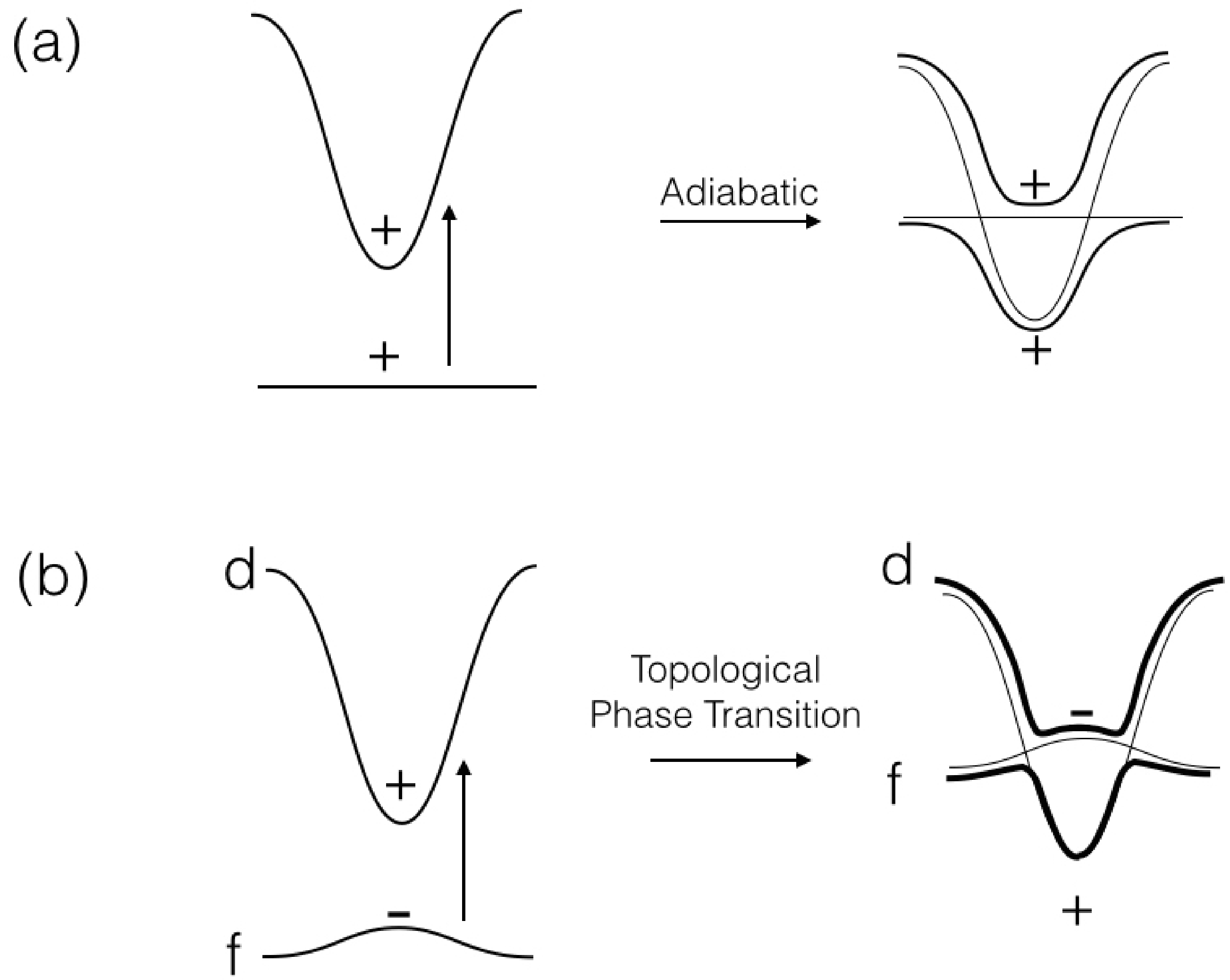


Topological Texture of Berry Connection

Are Kondo insulators topological?

Persistent conductivity Plateau





R. Martin & J. Allen, J. Applied Physics, **50,**7561 (1979)

Dzero, Sun, Galitski, PC Phys. Rev. Lett. **104**, 106408 (2010)

Many Body Localized

Many Body Delocalization

Band Theory SmB_6: T. Takimoto, J. Phys. Soc. Jpn. 80, 123710 (2011).

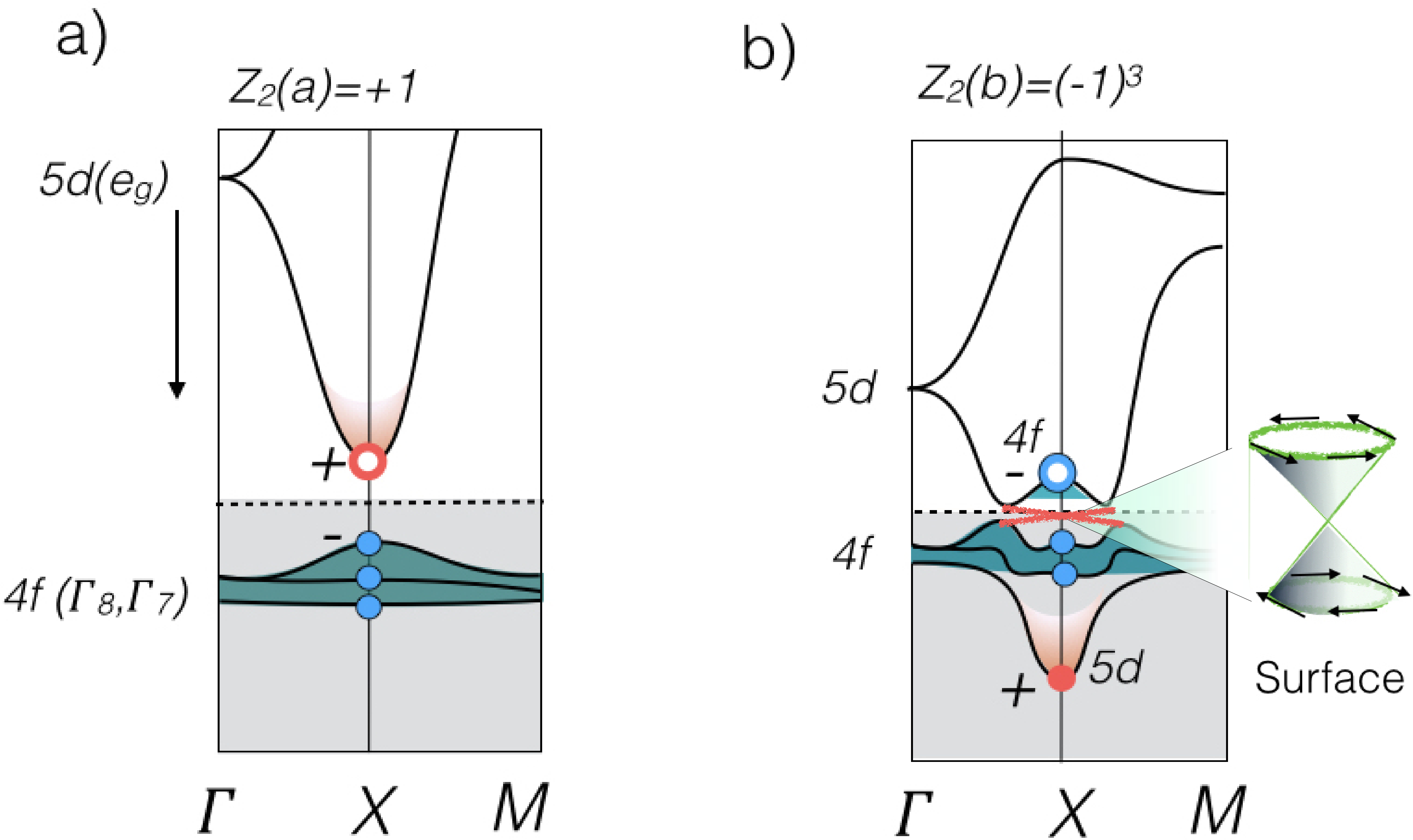
Maxim Dzero, Kai Sun, Piers Coleman and Victor Galitski, Phys. Rev. B 85 , 045130-045140 (2012).

Gutzwiller + Band Theory F. Lu, J. Zhao, H. Weng, Z. Fang and X. Dai, Phys. Rev. Lett. 110, 096401 (2013).

Victor Alexandrov, Maxim Dzero and Piers Coleman PRL (2013).

Features of the new model

Dzero et al, Annual Reviews of Condensed Matter Physics (2016), arXiv 1506.05635



Three crossings: THREE DIRAC CONES ON SURFACE.

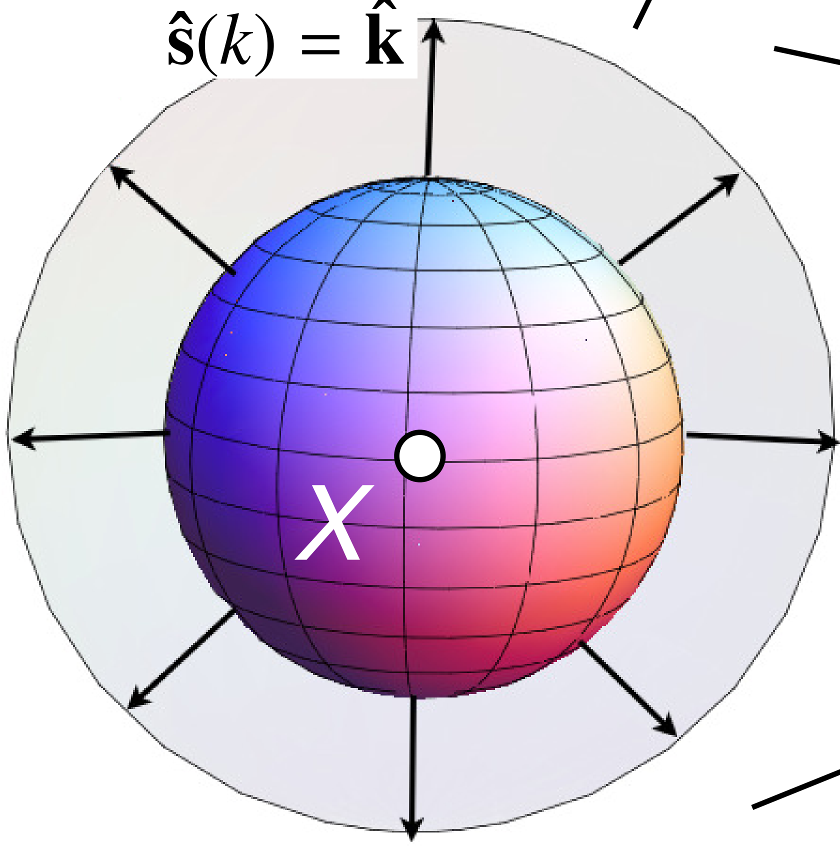
Features of the new model

Dzero et al, Annual Reviews of Condensed Matter Physics (2016), arXiv 1506.05635

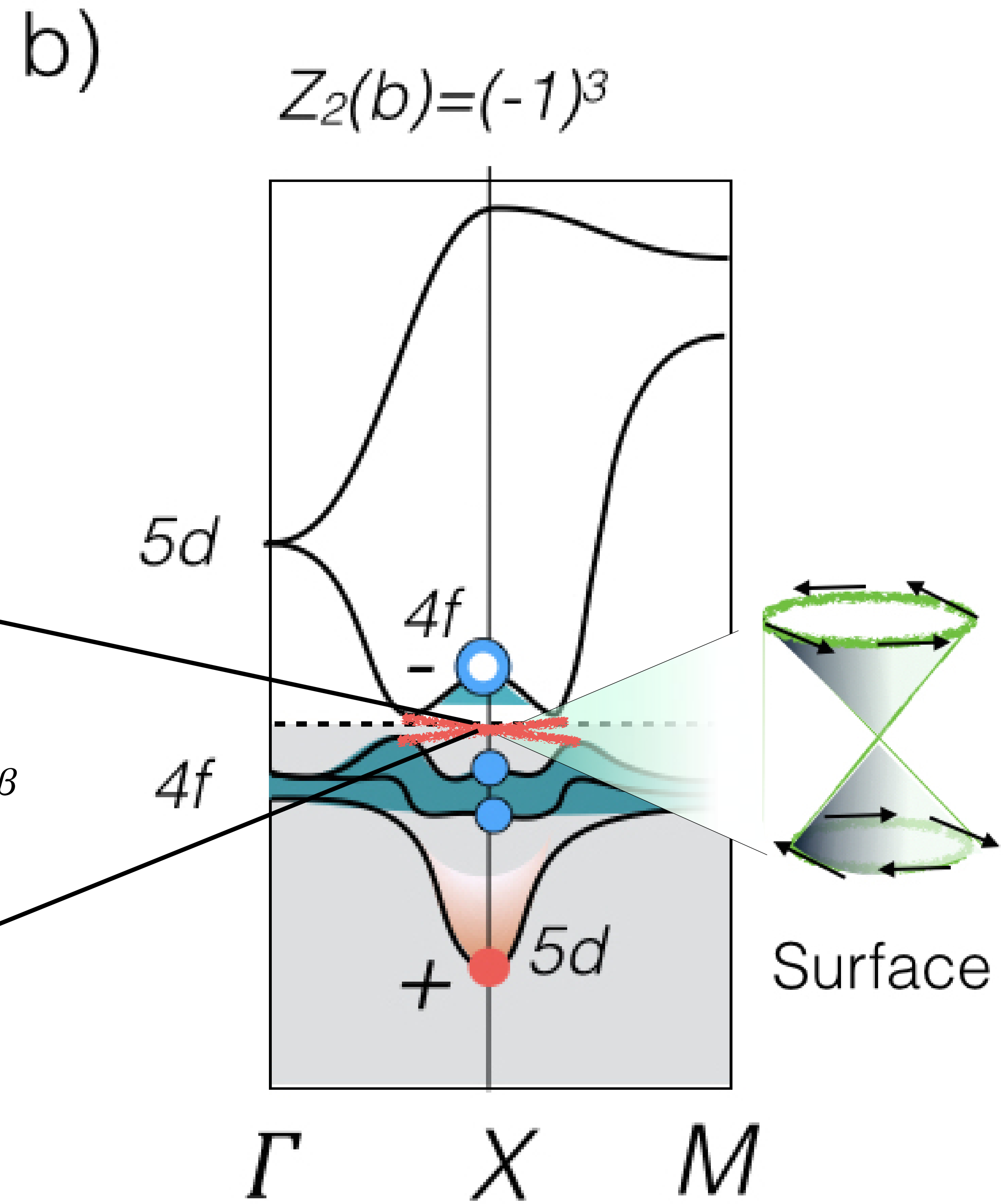
Like He-3B: an adaptive insulator.

$$\mathcal{H}(\mathbf{k}) = \left(\begin{array}{c|c} \epsilon_{\mathbf{k}} & V\,\mathbf{s}_{\mathbf{k}}\cdot\vec{\sigma} \\ \hline V\,\mathbf{s}_{\mathbf{k}}\cdot\vec{\sigma} & \epsilon_{f\mathbf{k}} \end{array}\right)$$

$$\hat{\mathbf{s}}(k) = \hat{\mathbf{k}}$$



$$V_{\alpha\beta}(k) = V\mathbf{s}_{\mathbf{k}}\cdot\vec{\sigma}_{\alpha\beta}$$



$$\mathbf{s}_{\mathbf{k}} = (\sin k_x, \sin k_y, \sin k_z) \sim \hat{\mathbf{k}}$$

Hybridization of **f (P=+)** and **d** (P=-) vanishes at X point.

Three crossings: THREE DIRAC CONES ON SURFACE.

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} \psi^{\dagger}{}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J_K \sum_j \psi^{\dagger}{}_{j\alpha} \psi_{j\beta} S_{\beta\alpha}(j) + J_H \sum_{i,j} S_{\alpha\beta}(i) S_{\beta\alpha}(j)$$

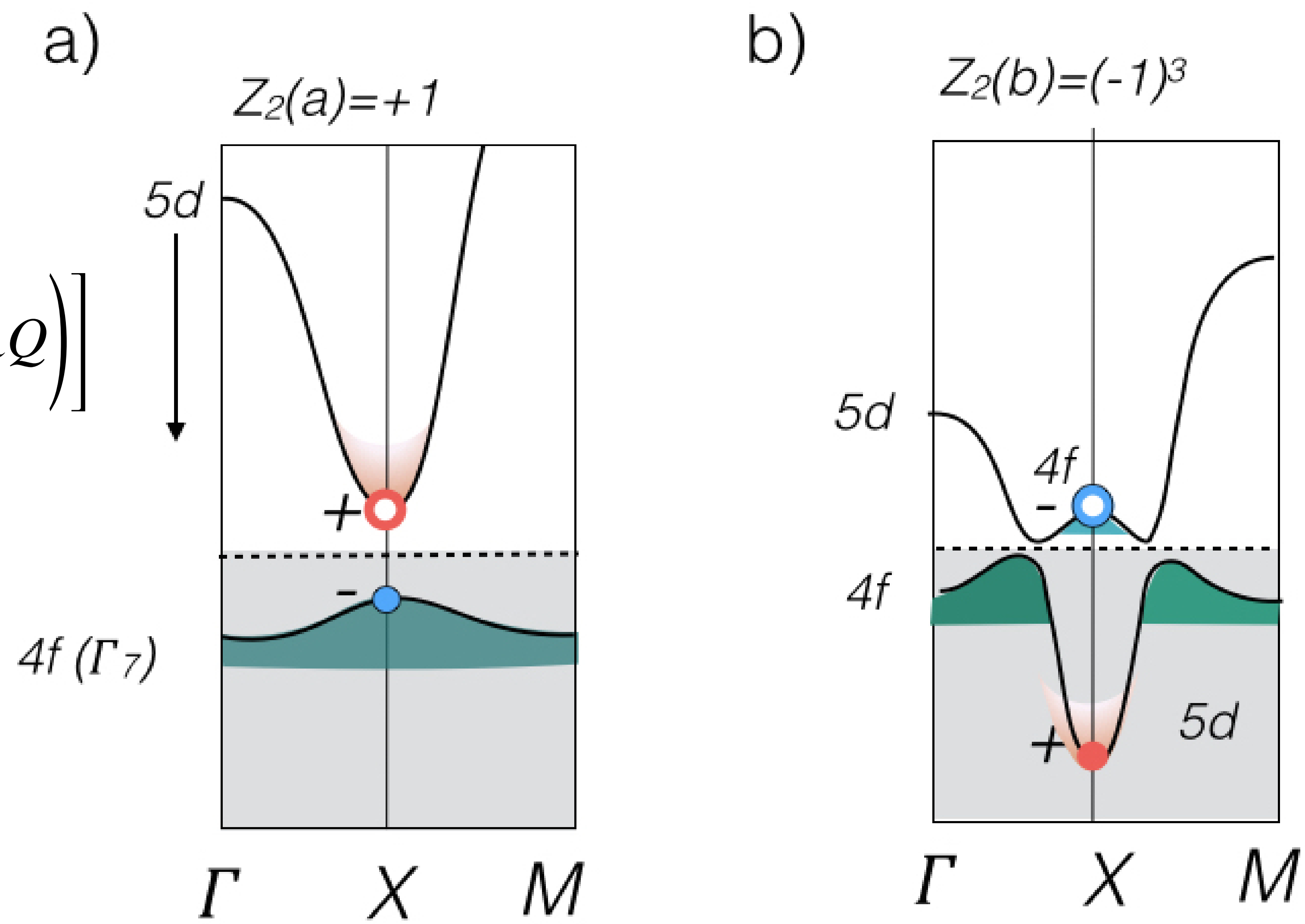
$$\psi^{\dagger}{}_{j\alpha} = \sum_{i,\sigma} c^{\dagger}{}_{i\sigma} \Phi_{\sigma\alpha}(\mathbf{R}_i - \mathbf{R}_j)$$

$$\Phi(\mathbf{R}) = \begin{cases} -i\hat{R} \cdot \frac{\vec{\sigma}}{2}, & \mathbf{R} \in \text{n.n} \\ 0 & \text{otherwise} \end{cases}$$

$$\Phi(\mathbf{k}) = \vec{s}_{\mathbf{k}} \cdot \vec{\sigma}$$

$$H_{TKI} = \sum_{\mathbf{k}} \psi^{\dagger}{}_{\mathbf{k}} h(\mathbf{k}) \psi_{\mathbf{k}} + \mathcal{N}_s \left[\left(\frac{V^2}{J_K} + \frac{3t^2}{J_H} - \lambda Q \right) \right]$$

$$h(\mathbf{k}) = \begin{pmatrix} \epsilon_{\mathbf{k}} & V\vec{\sigma} \cdot \vec{s}_{\mathbf{k}} \\ V\vec{\sigma} \cdot \vec{s}_{\mathbf{k}} & \epsilon_{f\mathbf{k}} \end{pmatrix}$$



Outline of the Topics

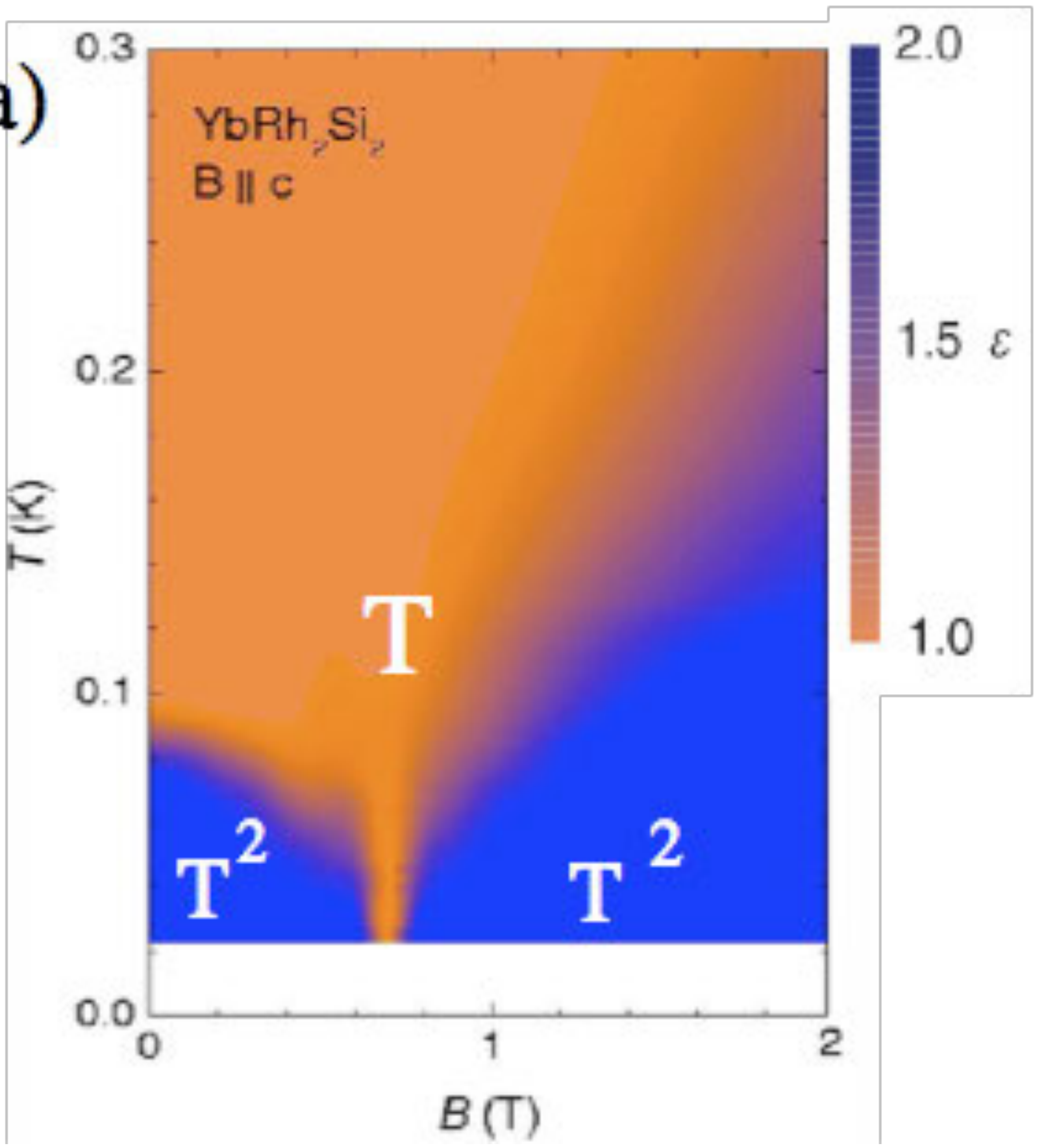
1. Trends in the periodic table.
2. Introduction: Heavy Fermions and the Kondo Lattice.
3. Kondo Insulators: the simplest heavy fermions.
4. Large N expansion for the Kondo Lattice
5. Heavy Fermion Superconductivity
6. Topological Kondo Insulators
7. **AFM meets the Kondo Effect**.

Please ask questions!

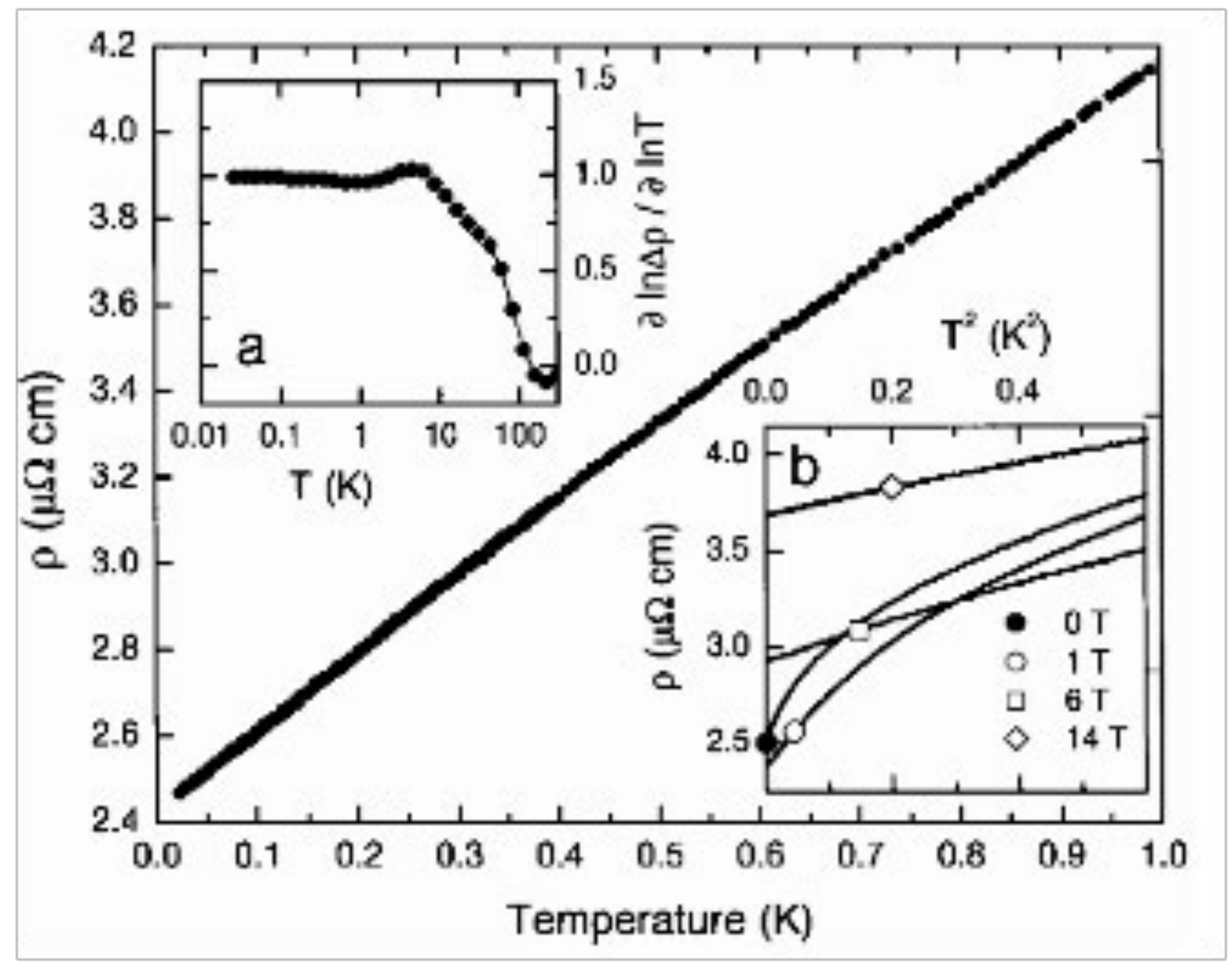
Rutgers
Center for Materials Theory

$YbRh_2Si_2$: Field tuned quantum criticality.

(a)



(b)

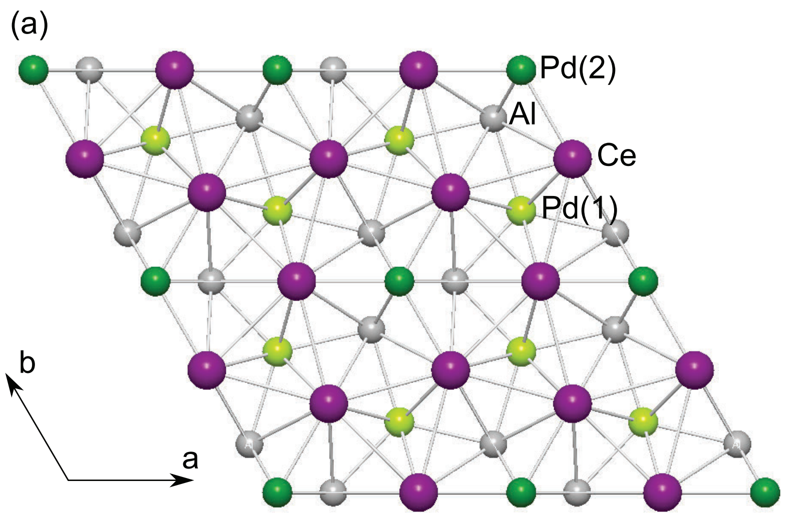


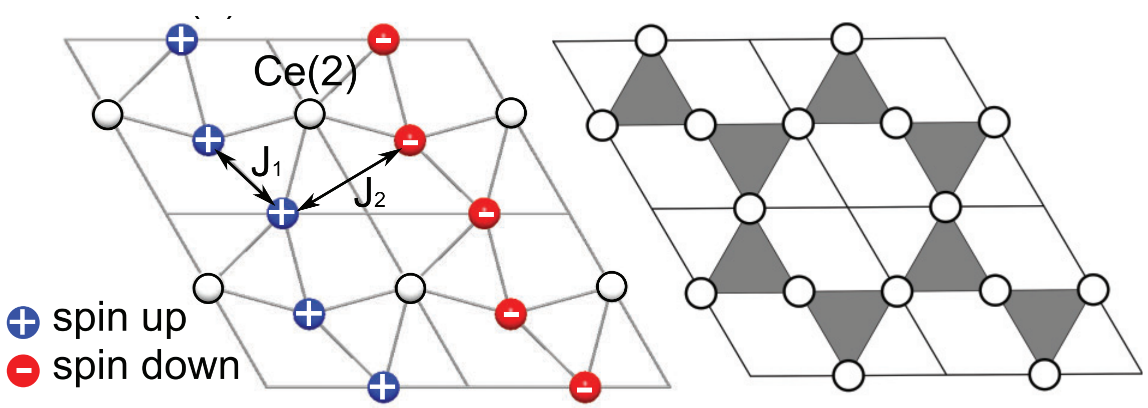
Custers et al, (2003)

Magnetism meets Kondo

1. CePdAl

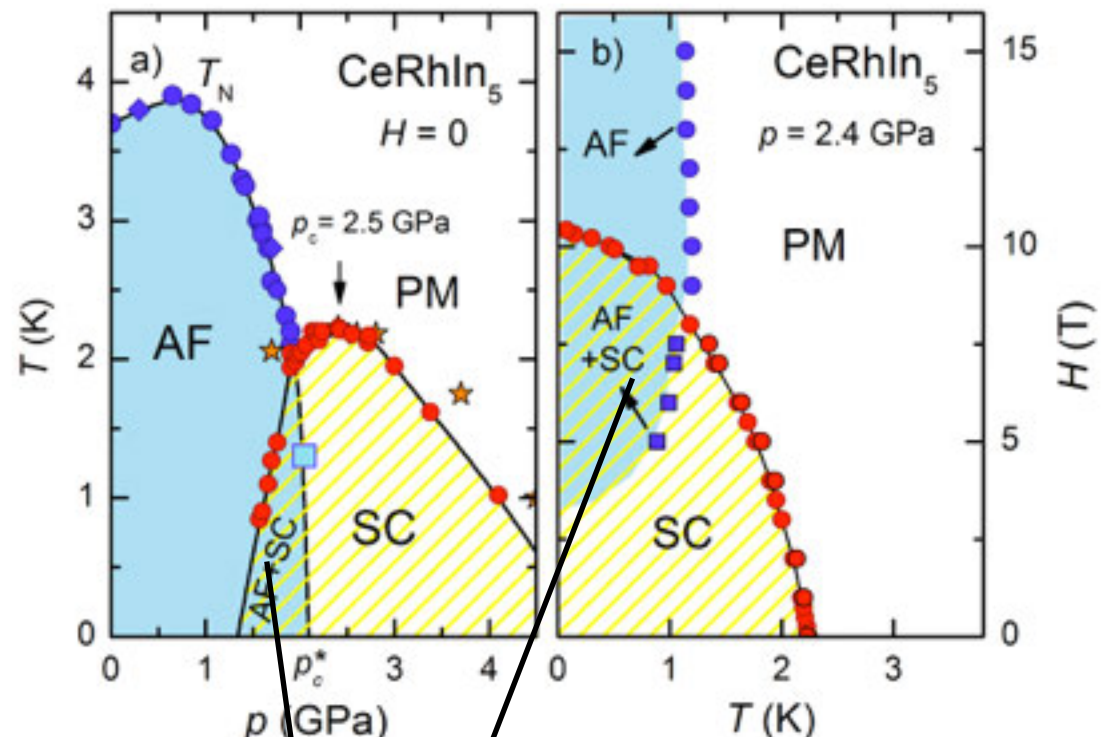
Inhomogenious Kondo/AFM





V. Frisch et al,
PRB 89, 054416 (2014),

2. $CeRh_2In_5$



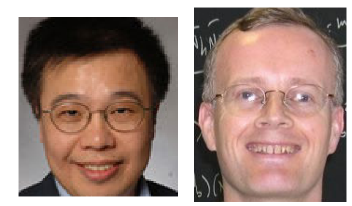
Magnetism LM + Kondo Pure Kondo

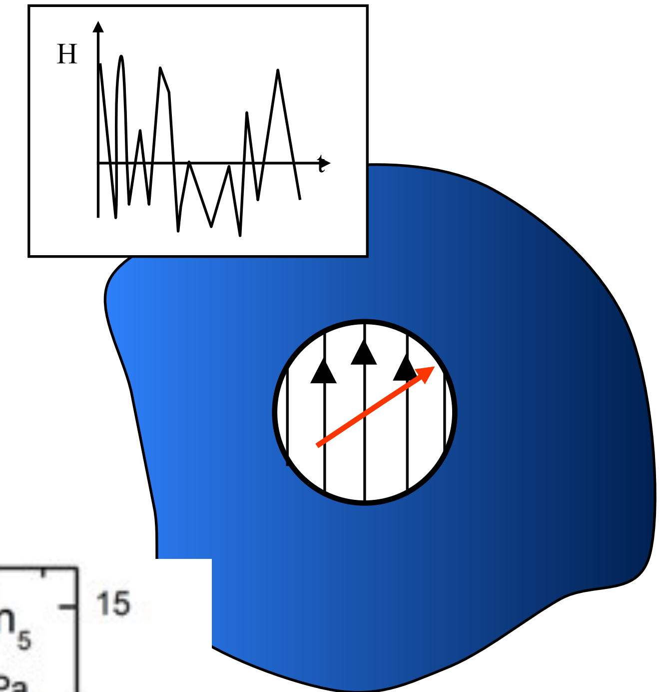
New Ideas

- **Local quantum criticality**

(Si, Ingersent, Smith, Rabello, Nature 2001):
Spin is the critical mode,
Fluctuations critical in time.

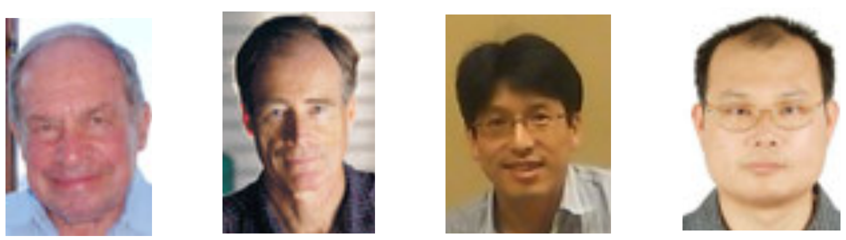
Si, Ingersent





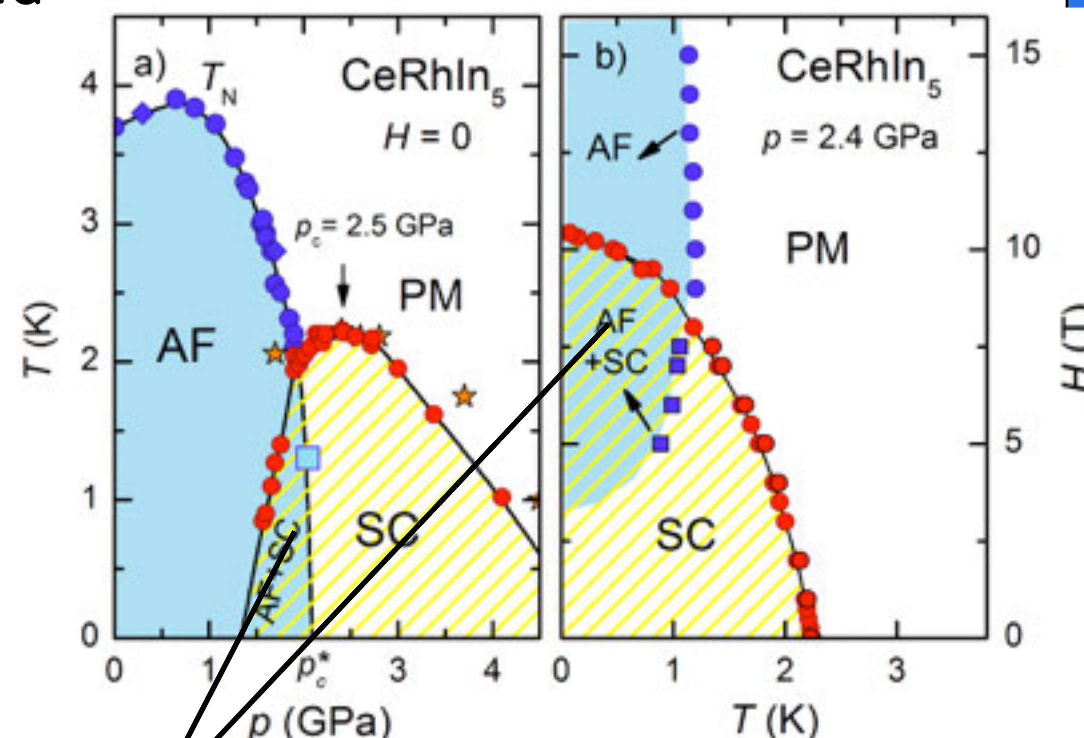
Requires a two dimensional spin fluid

- **Two fluid scenario.**



D. Pines Z. Fisk S. Nakatsuji Y. Yang

Nature (2008), PRL (2004)



Magnetism LM + Kondo Pure Kondo

Spin = B Spin = B,F Spin = F

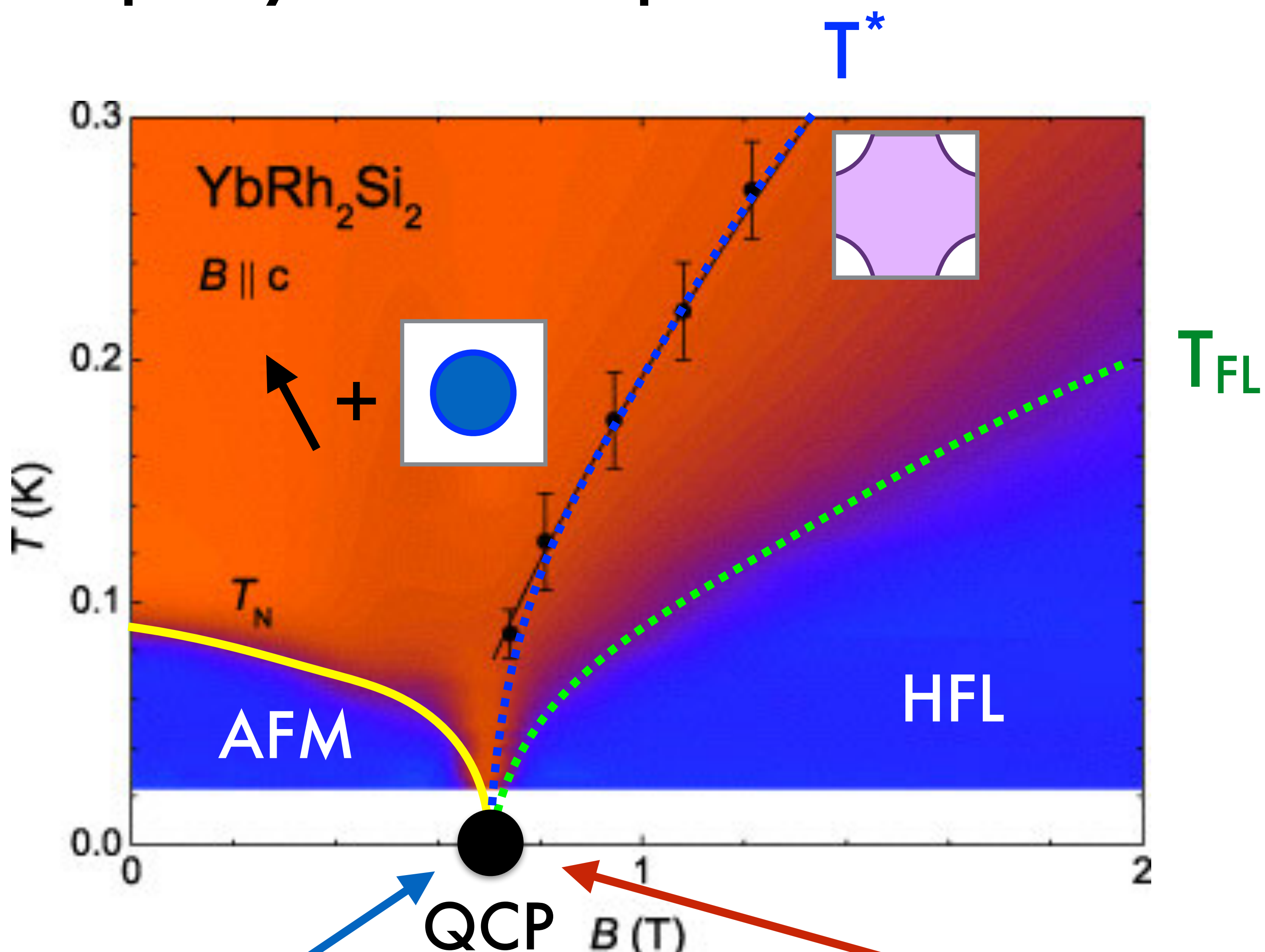
- **Supersymmetry?**

Coleman, Pepin, Tsvelik (1999)
Ramires Coleman (2014)

Description of unconventional QCP requires new formalism.

Strange Metal = Unbroken Susy?

Why Supersymmetric Spins?



Schwinger Boson

Abrikosov Fermion

SUSY

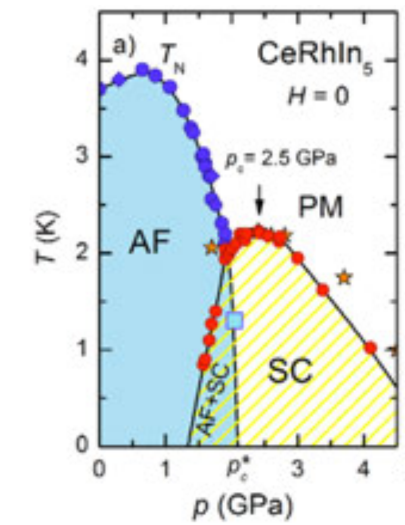
Gan, Coleman and Andrei, PRL (1992)
Coleman, Pepin, Tsvelik, PRB (2000)

Unusual QCP:

$$C/T \sim LogT$$

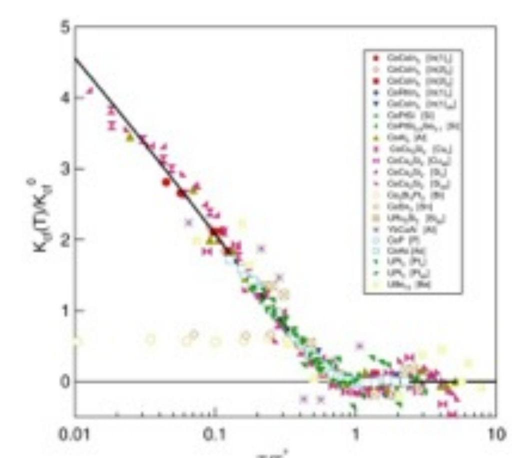
$$\rho \sim T^{\alpha}, \quad \alpha < 2$$

Coexistance:



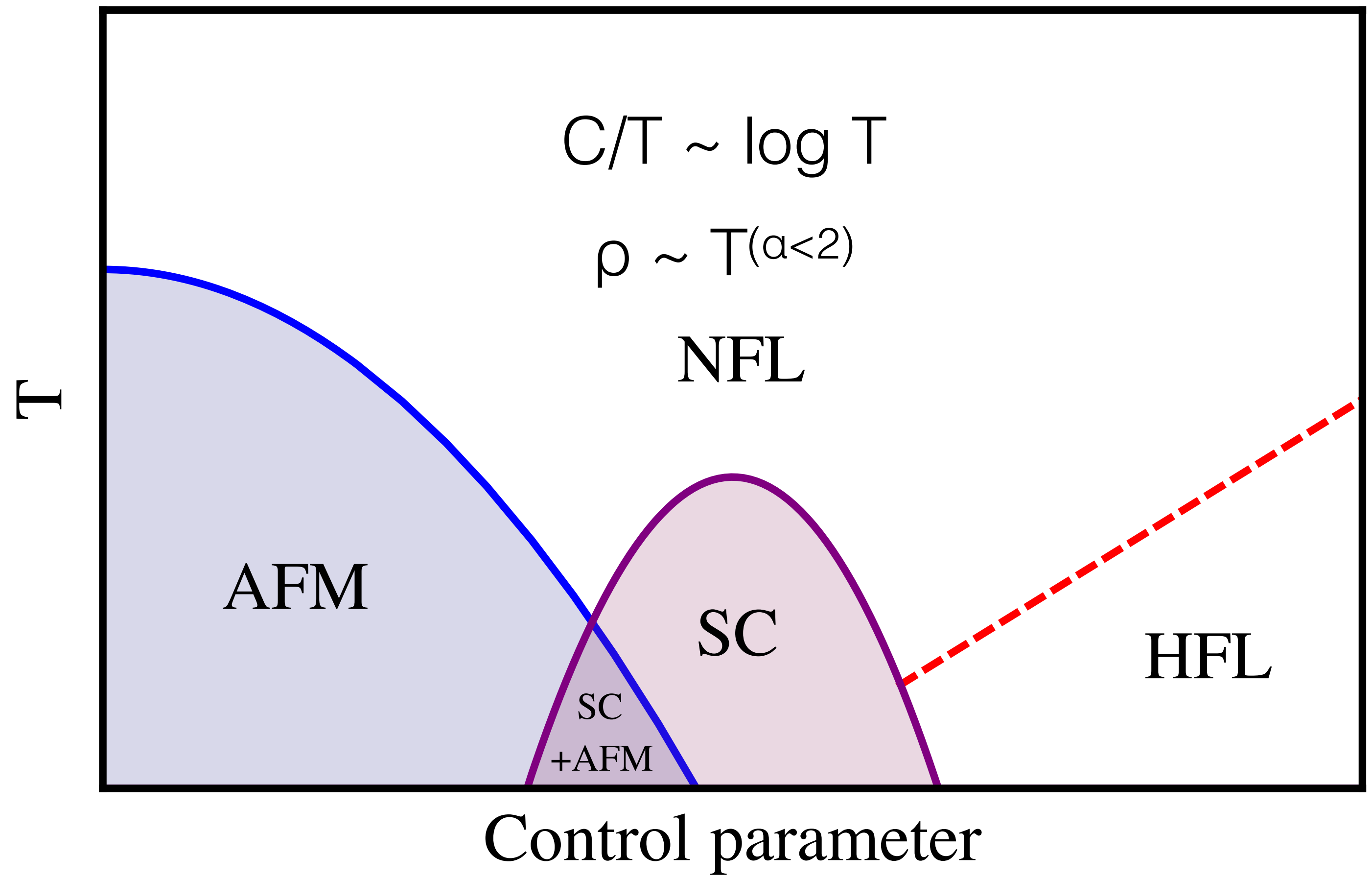
Knebel, arXiv 2009

2-fluid picture:



Nakatsuji, Pines and Fisk, PRL (2004)

Heavy Fermion Systems: Why Supersymmetric Spins?



Bosonic

$$\mathbf{S}_B = b^\dagger_\alpha \mathbf{\Gamma}_{\alpha\beta} b_\beta$$

$$|\Psi\rangle = P_G |\Psi_B\rangle$$

Fermionic

$$\mathbf{S}_F = f^\dagger_\alpha \mathbf{\Gamma}_{\alpha\beta} f_\beta$$

$$|\Psi\rangle = P_G |\Psi_F\rangle$$

How to describe the generic HF phase diagram in its entirety?

Supersymmetric Spin

$$\mathbf{S} = f^\dagger_\alpha \mathbf{\Gamma}_{\alpha\beta} f_\beta + b^\dagger_\alpha \mathbf{\Gamma}_{\alpha\beta} b_\beta$$

$$|\Psi\rangle = P_G |\Psi_B\rangle \otimes |\Psi_F\rangle$$

Gan, Coleman and Andrei, 1992
Coleman, Pepin, Tsvelik, 2000

Results

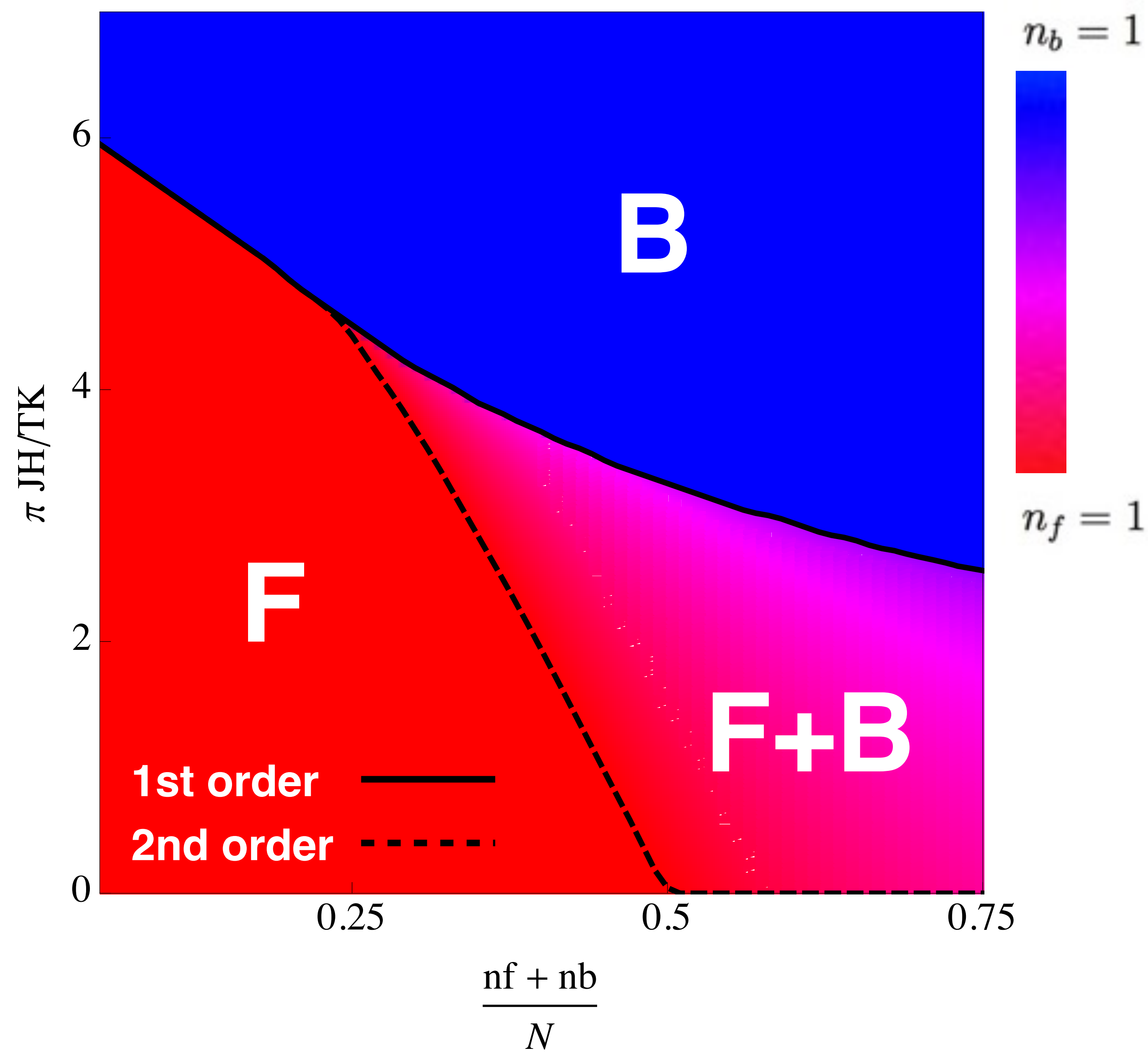
Within a static mean field solution the free energy have the following closed form:

$$F = -2\sin(\pi n_f) - \frac{\pi J_H}{T_K}(q_0 - n_f)(q_0 - n_f + 1)$$

$$n_f + n_b = q_0$$

The energy will be minimized by different representations in different areas of the phase diagram

- ✦ **F+B Phase → Coexistence;**
- ✦ **2nd order transition F → F+B;**
- ✦ **Fermionic modes go soft;**
- ✦ **Unusual critical behavior;**



Thank You!