Exercise Sheet 9 due 9 July

1. *Clebsch-Gordan coefficients*

Prove the following recursion relation for the Clebsch-Gordan coefficients:

$$\begin{aligned}&\sqrt{j(j{+}1)-m(m{\pm}1)}\,\langle j_1,m_1;\,j_2,m_2|j,m{\pm}1;\,j_1;j_2\rangle\\&=\sqrt{j_1(j_1{+}1)-m_1(m_1{\mp}1)}\,\langle j_1,m_1{\mp}1;\,j_2,m_2|j,m;\,j_1;j_2\rangle\\&+\sqrt{j_2(j_2{+}1)-m_2(m_2{\mp}1)}\,\langle j_1,m_1;\,j_2,m_2{\mp}1|j,m;\,j_1;j_2\rangle\end{aligned}$$

2. *Clebsch-Gordan coefficients*

Write a program that takes two angular momentum quantum numbers j_a and j_b as input and produces a matrix for transforming from the product states $|j_a,m_a;\,j_b,m_b\rangle$ to the total angular momentum states $|j,m\rangle$. Example:

$j_a=1$	$j_b=1/2$	$j=3/2$				$j=1/2$	
m_a	m_b	$m=3/2$	$m=1/2$	$m=-1/2$	$m=-3/2$	$m=1/2$	$m=-1/2$
1	$1/2$	1					
1	$-1/2$		$\sqrt{1/3}$			$\sqrt{2/3}$	
0	$1/2$		$\sqrt{2/3}$			$-\sqrt{1/3}$	
0	$-1/2$			$\sqrt{2/3}$			$\sqrt{1/3}$
-1	$1/2$			$\sqrt{1/3}$			$-\sqrt{2/3}$
-1	$-1/2$				1		

Hint: Expand the square of the coefficients into a continued fraction.

3. *Addition of three angular momenta*

Consider three independent spin 1/2 systems with spin operators $\vec{S}_a$, $\vec{S}_b$, and $\vec{S}_c$. Add the spins in two different ways:

i. $(\vec{S}_a+\vec{S}_b)+\vec{S}_c$

ii. $\vec{S}_a+(\vec{S}_b+\vec{S}_c)$

Compare the results.