The Kondo Effect

Frithjof B. Anders

Lehrstuhl für Theoretische Physik II - Technische Universität Dortmund

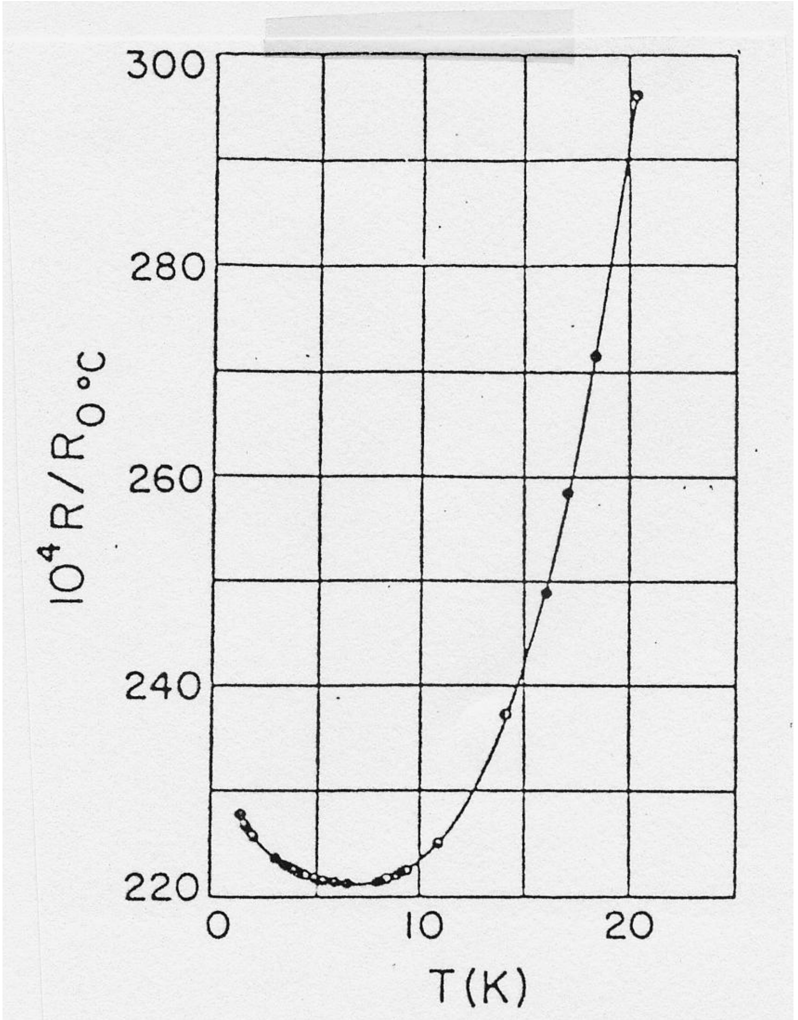
Autumn-School on Correlated Electrons 2012

technische universität
dortmund

Outline

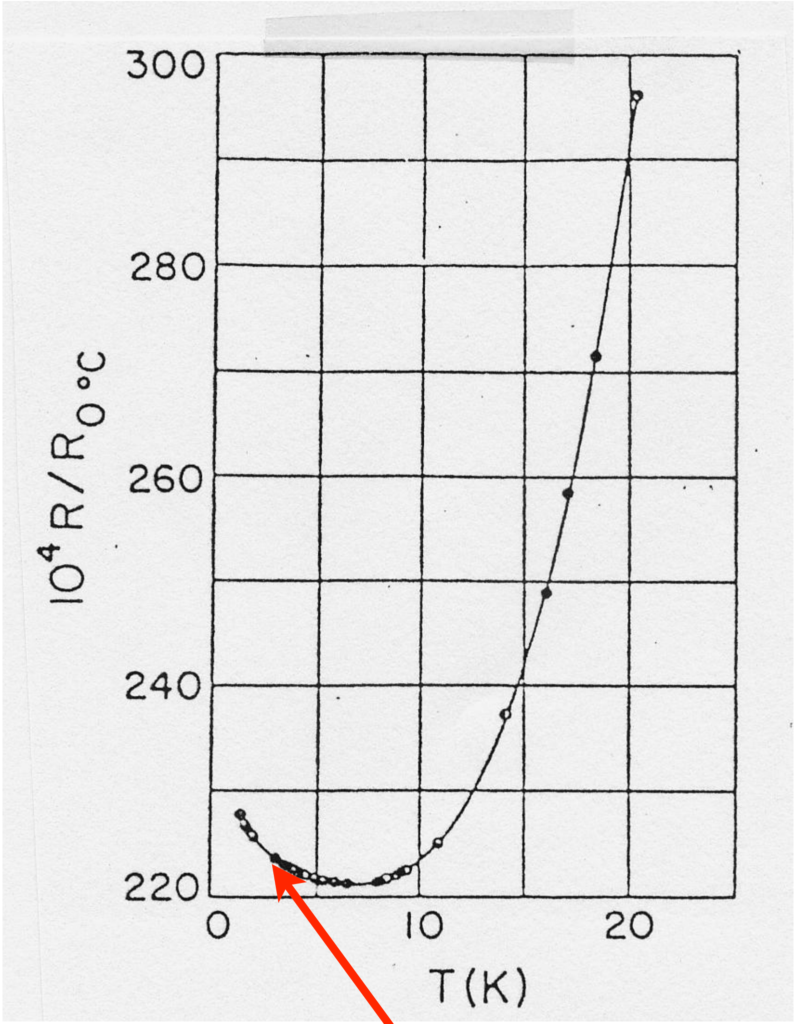
1. Introduction
   a. History: resistance minimum
   b. Anderson model
2. Renormalization Group
   a. Poor man’s scaling
   b. NRG-> see Ralf Bulla'
   c. exotic Kondo effects in metal
3. Kondo effect in Lattice systems
   a. Heavy Fermion materials
   b. Dynamical mean field theory
   c. impurity solver
4. Kondo effect in nano-device
   a. Kondo effect in single-electron transistors
   b. Charge Kondo effect

Resistance minimum



de Haas, van der Berg 1936

Resistance minimum

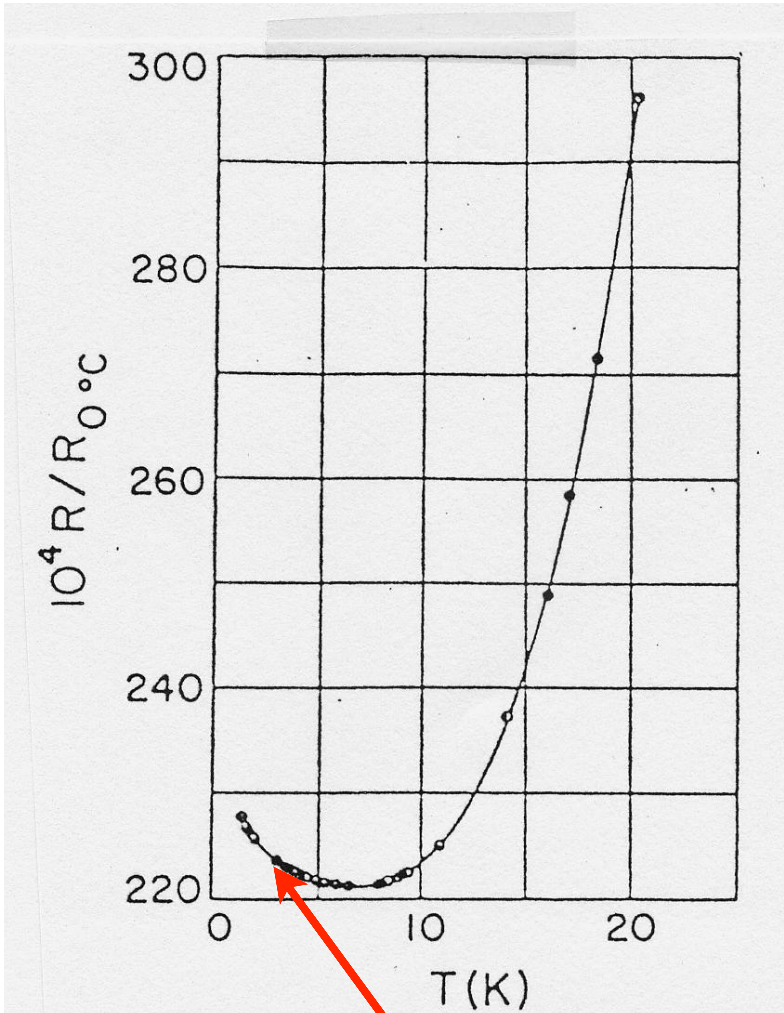


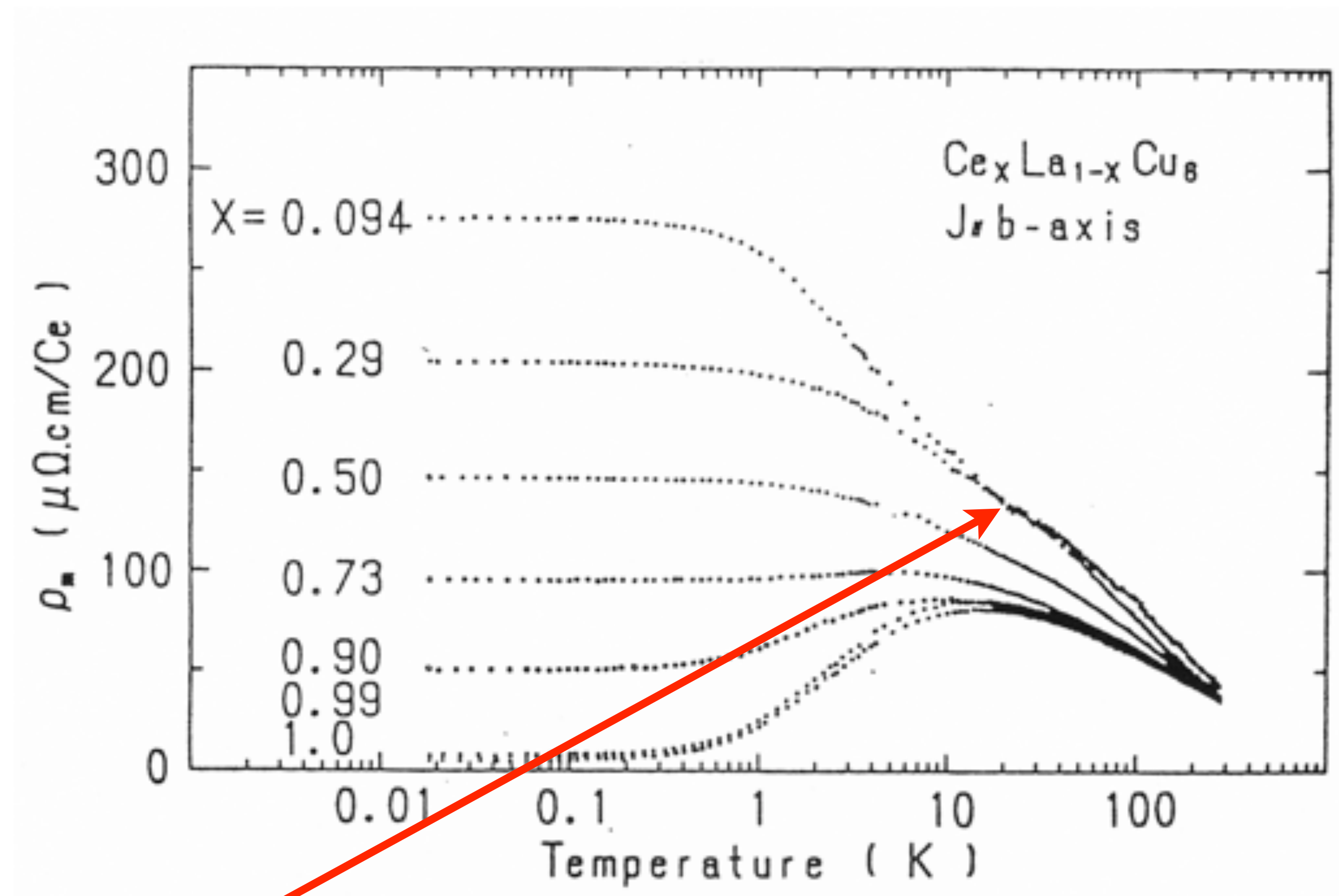
de Haas, van der Berg 1936

increase: ∝ log(T)

technische universität dortmund

Resistance minimum



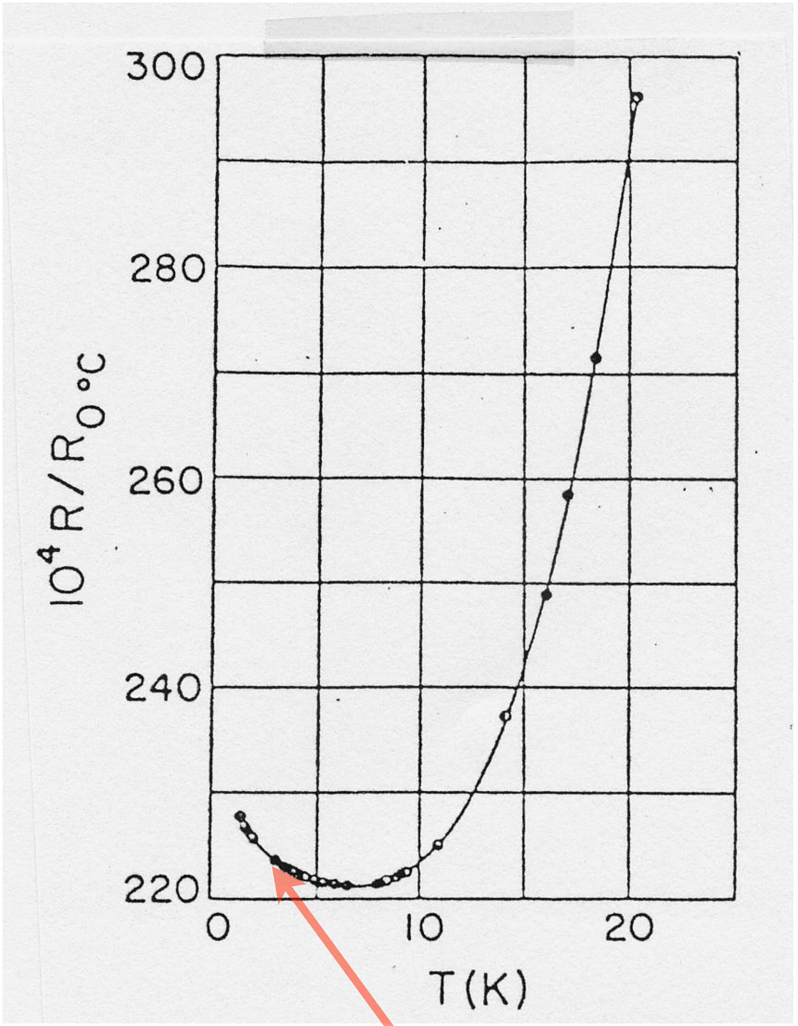


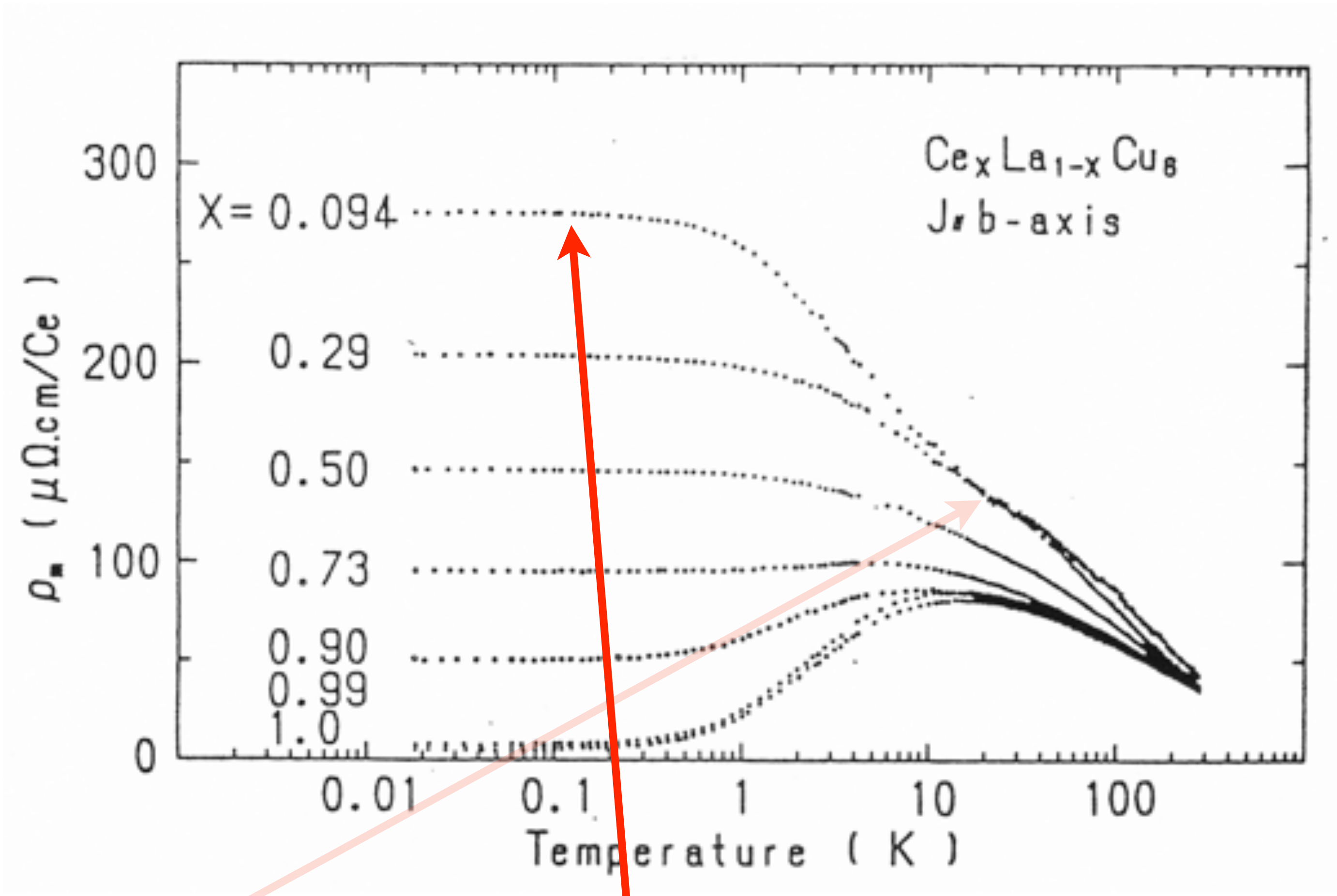
Onuki et al 1987

de Haas, van der Berg 1936

increase: $\propto$ log(T)

Resistance minimum





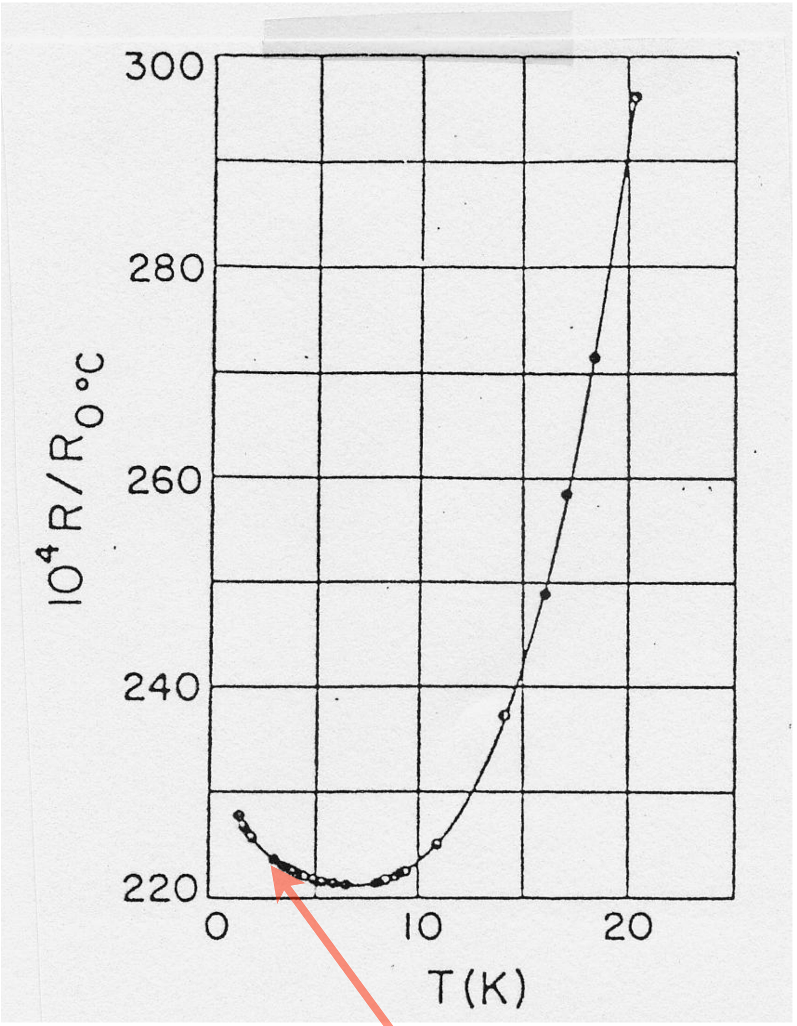
Onuki et al 1987

de Haas, van der Berg 1936

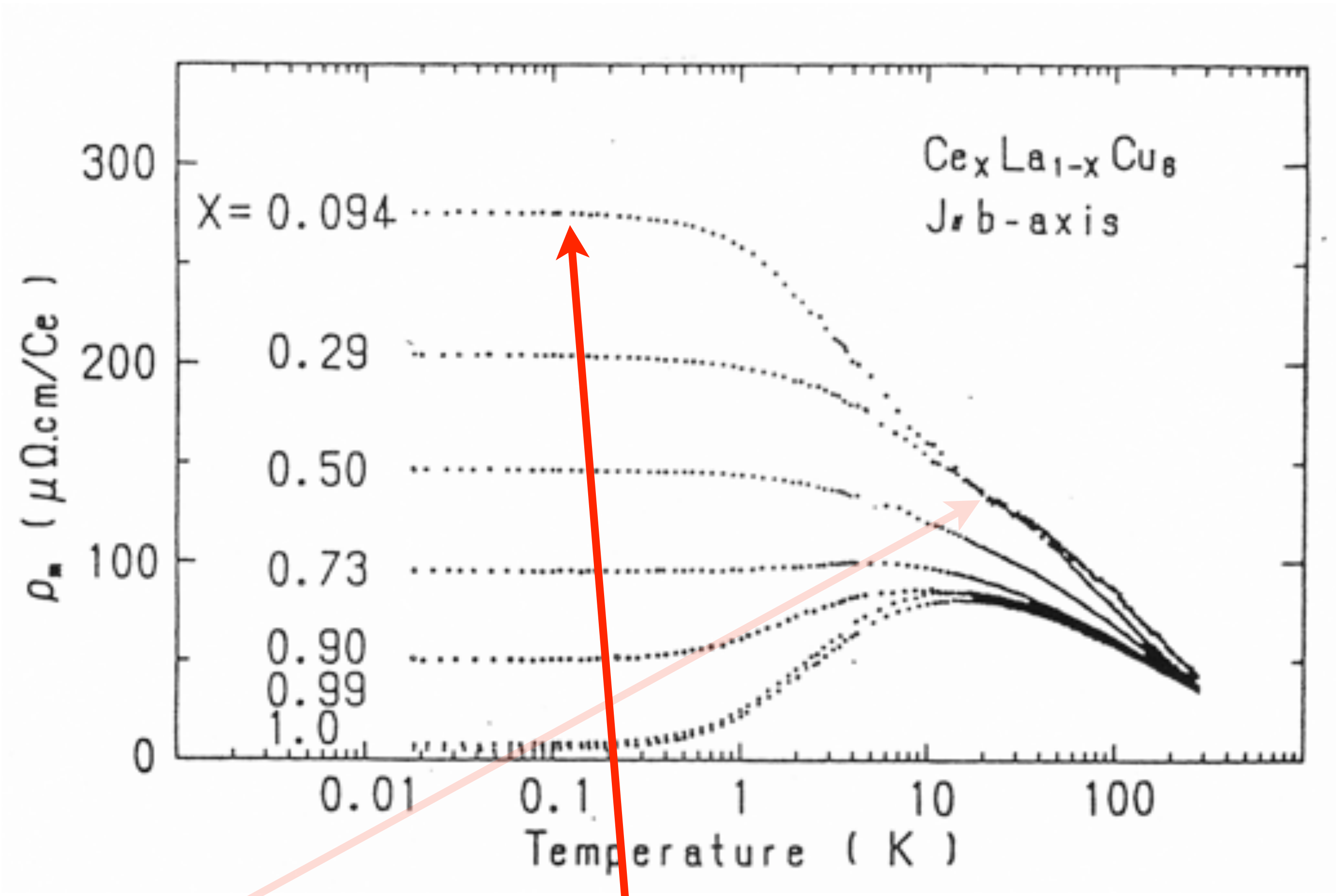
increase: ∝ log(T)   but saturation for $T<T_K$

Resistance minimum

Jun Kondo



de Haas, van der Berg 1936



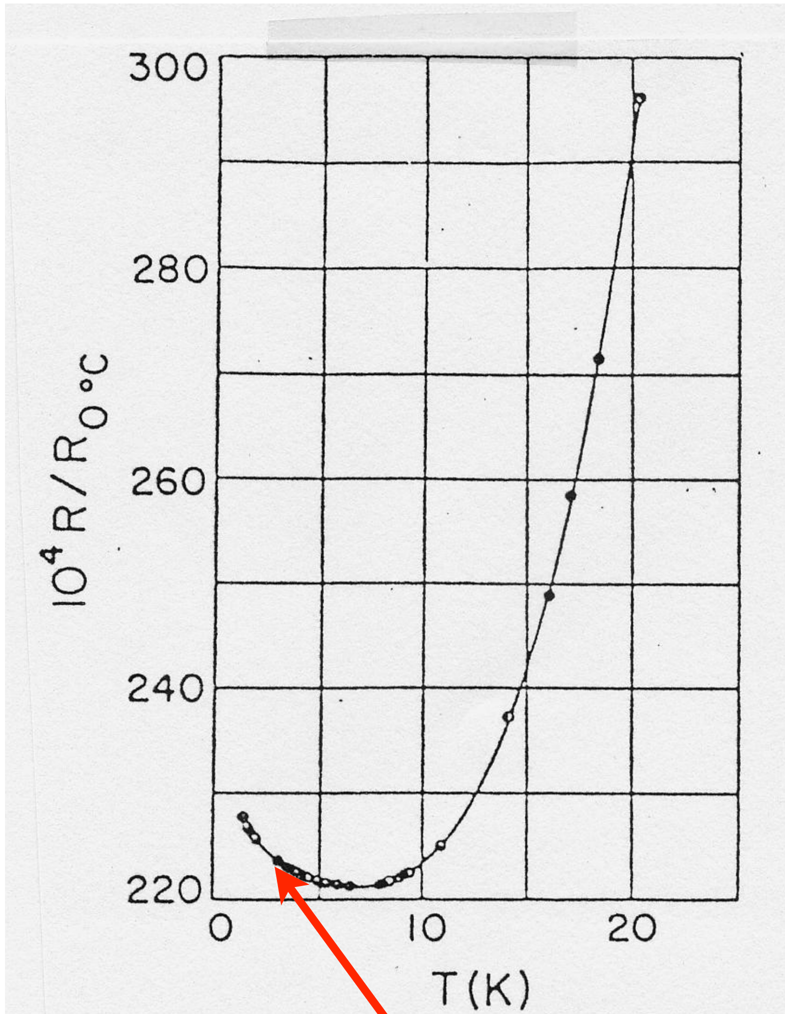
Onuki et al 1987

increase: ∝ log(T)   but saturation for T<T$_K$

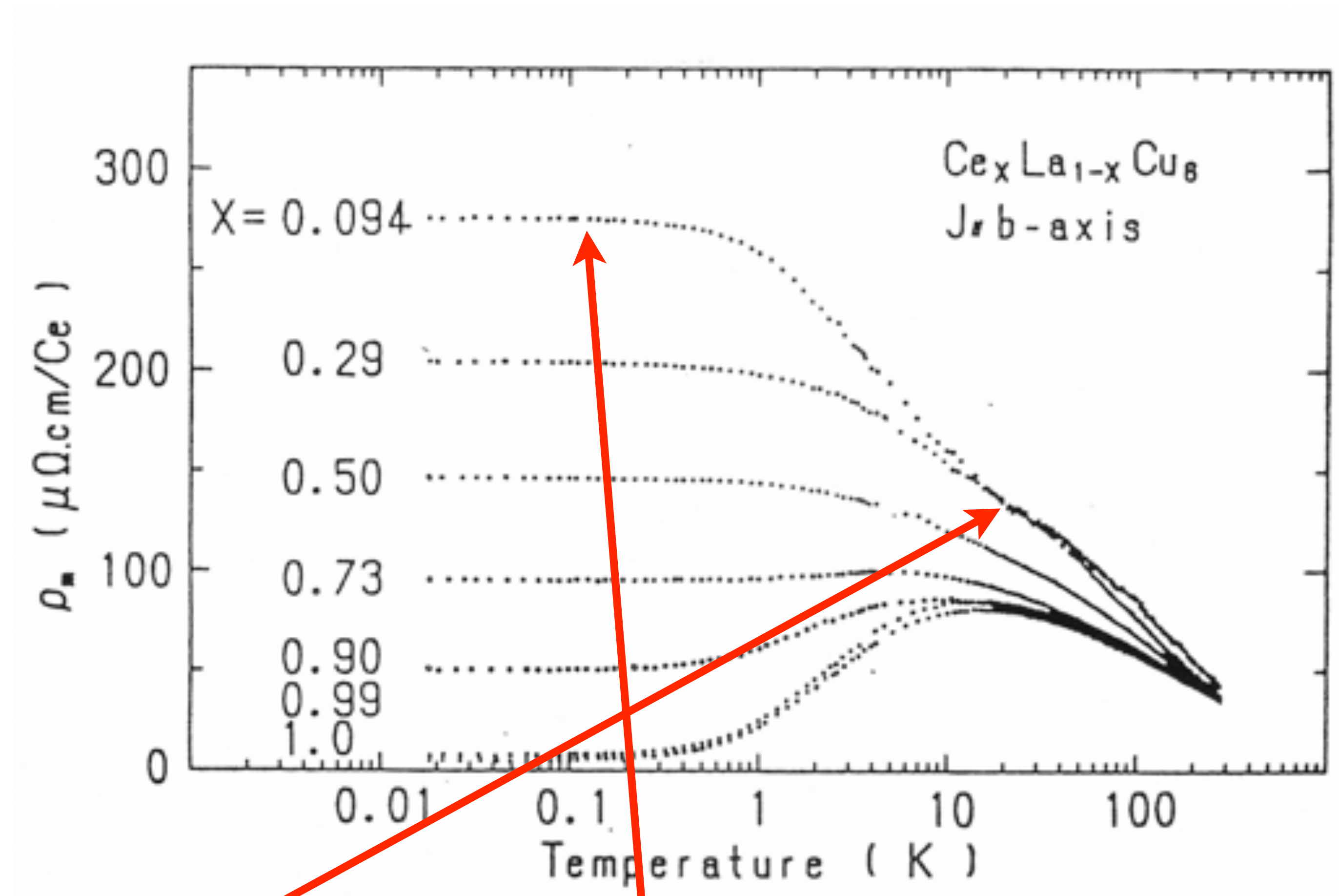
Jun Kondo(1964) magnetic scattering: $H_K = J\vec{S}_{loc}\vec{s}_{\rm band}$

Resistance minimum

Jun Kondo



de Haas, van der Berg 1936



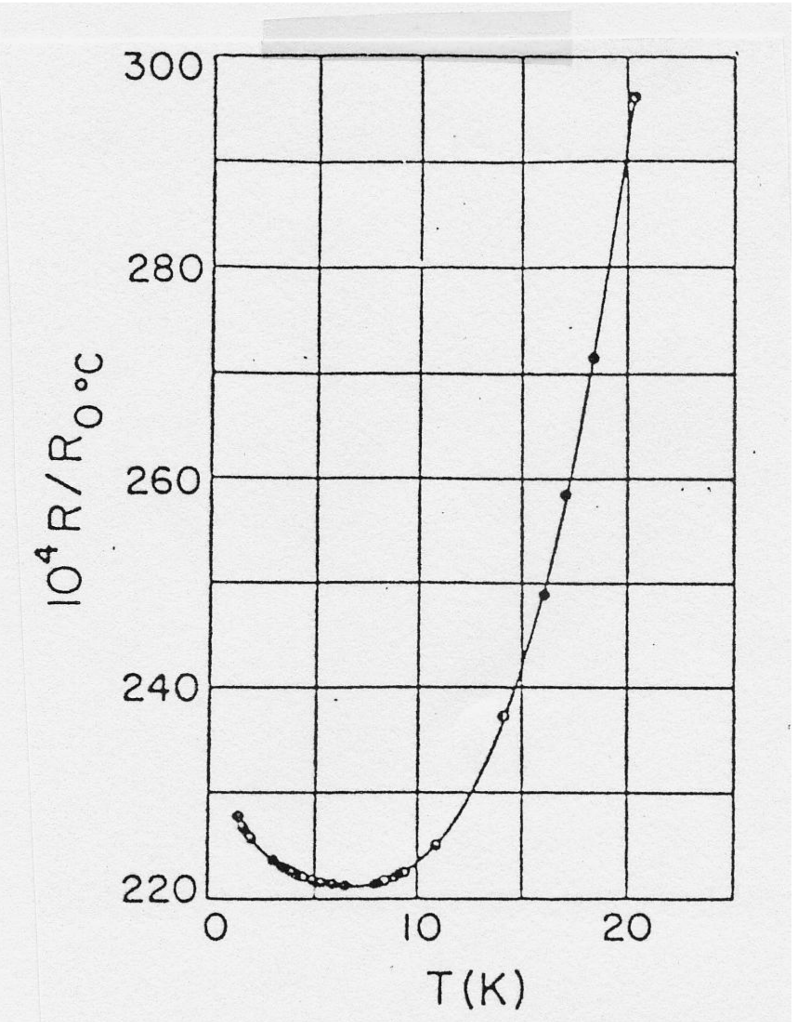
Onuki et al 1987

increase: ∝ log(T)

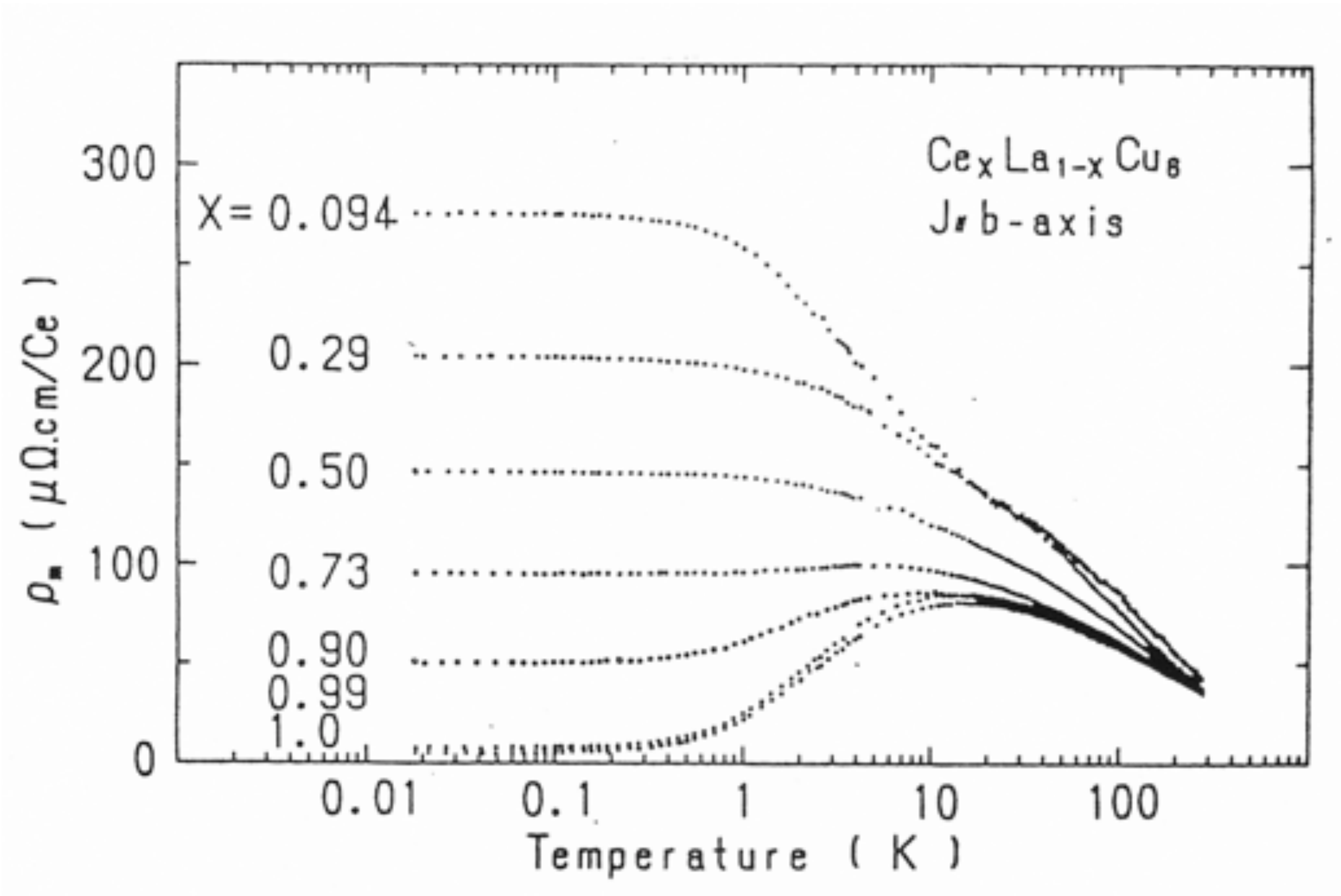
but saturation for $T<T_K$

Resistance minimum

Jun Kondo



de Haas, van der Berg 1936



Onuki et al 1987

increase: ∝ log(T)   but saturation for $T<T_K$

$$\rho_{imp} = \frac{3\pi m J^2 S(S+1)}{2e^2\hbar\varepsilon_F}\left[1 - J\rho(\varepsilon_F)\ln\left(\frac{k_B T}{D}\right) + O(J^2)\right]$$

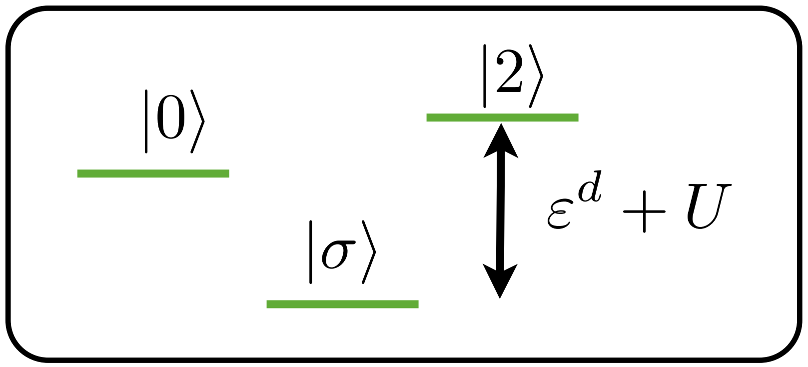
Anderson Model

Single level: $$H_{imp} = \sum_{\sigma} \varepsilon^{d} d_{\sigma}^{\dagger} d_{\sigma} + U n_{\uparrow} n_{\downarrow}$$

Anderson Model

Single level: $$H_{imp} = \sum_{\sigma} \varepsilon^d d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow$$

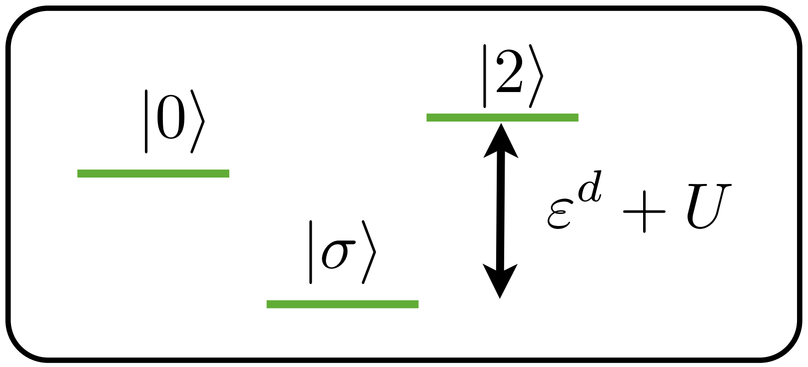
Eigenstates:



Anderson Model

Single level: $$H_{imp} = \sum_{\sigma} \varepsilon^d d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow$$

Eigenstates:

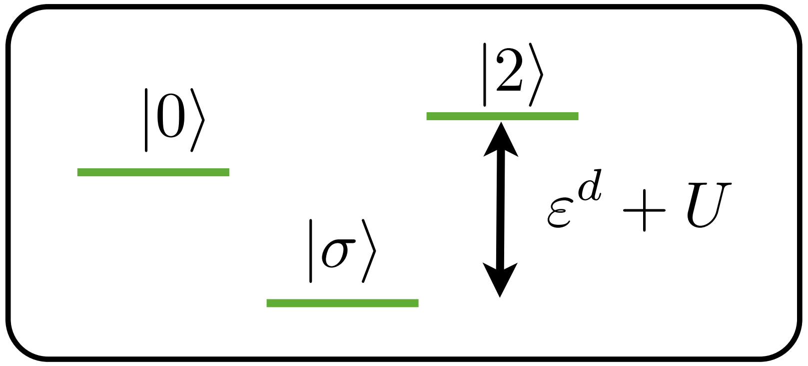


local moment formation: $T < U$

Anderson Model

Single level: $$H_{imp} = \sum_\sigma \varepsilon^d d_\sigma^\dagger d_\sigma + U n_\uparrow n_\downarrow$$

Eigenstates:



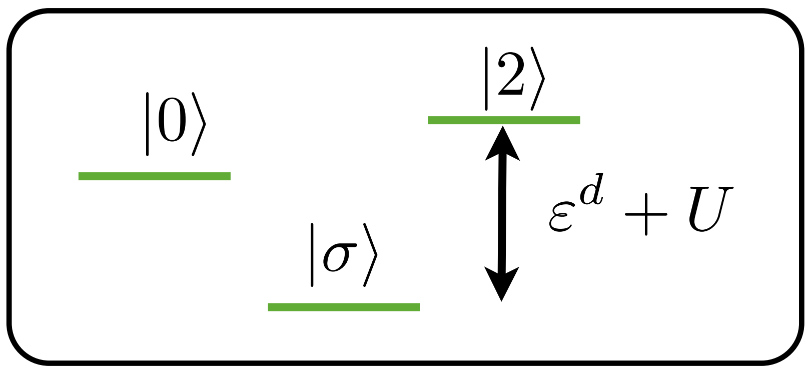
local moment formation: $T < U$

charge fluctuations:

Anderson Model

Single level: $$H_{imp} = \sum_{\sigma} \varepsilon^d d_\sigma^\dagger d_\sigma + U n_\uparrow n_\downarrow$$

Eigenstates:



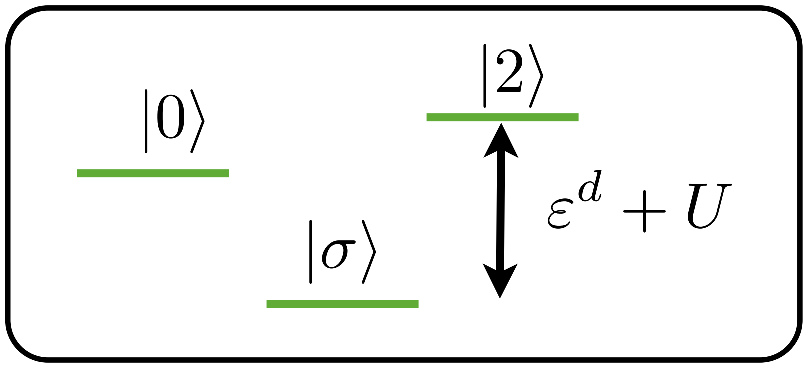
local moment formation: $T < U$

charge fluctuations:

Anderson Model

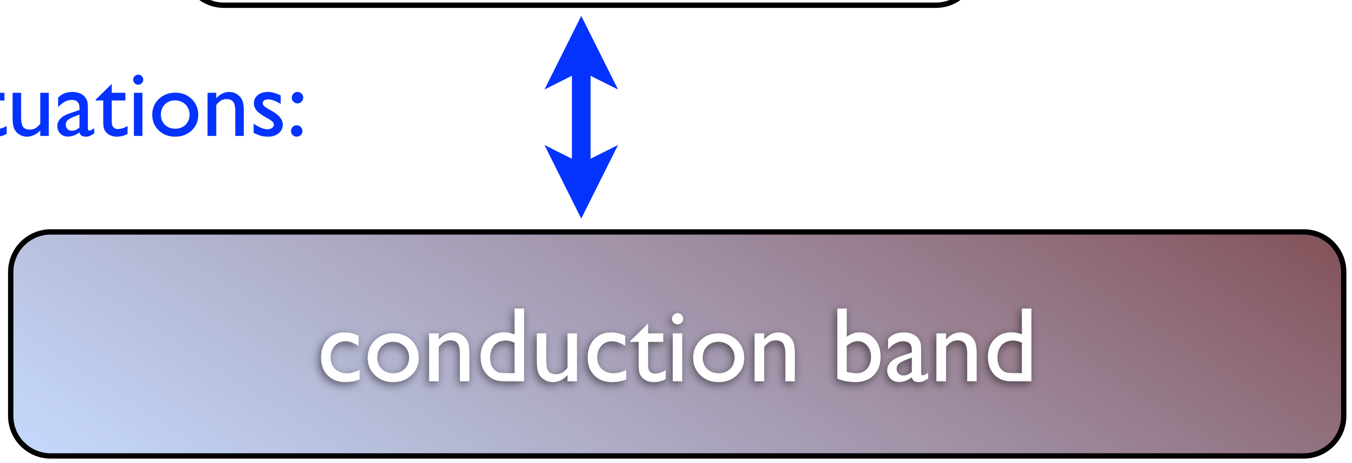
Single level: $$H_{imp} = \sum_{\sigma} \varepsilon^d d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow$$

Eigenstates:



local moment formation: $T < U$

charge fluctuations:



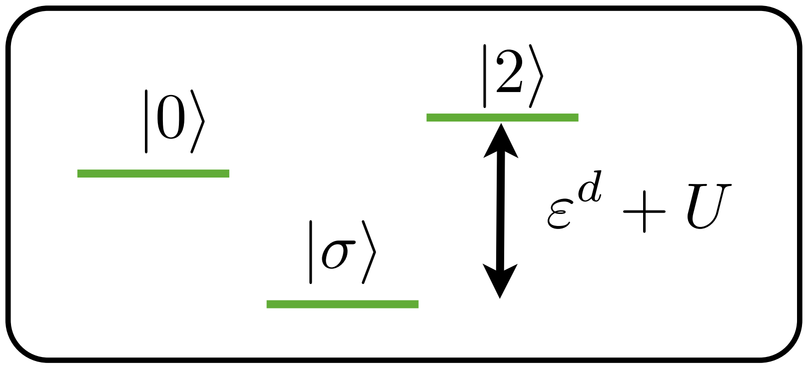
hybridization:

$$H_{hyp} = \sum_{\sigma k} V(\sigma k) \left(d^\dagger_\sigma c_{k\sigma} + c^\dagger_{k\sigma} d_\sigma \right)$$

Anderson Model

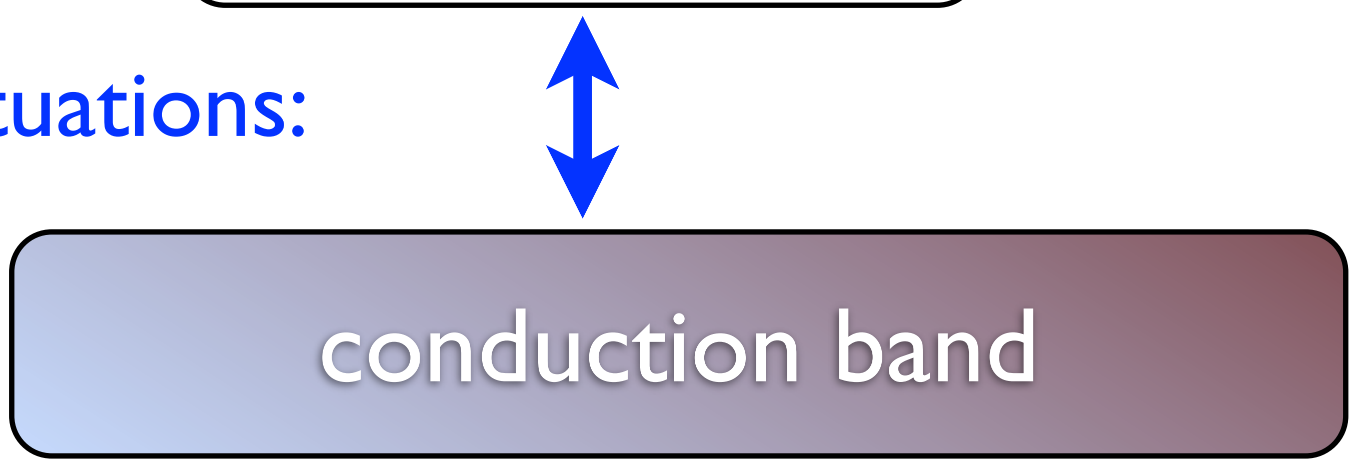
Single level: $$H_{imp} = \sum_{\sigma} \varepsilon^d d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow$$

Eigenstates:

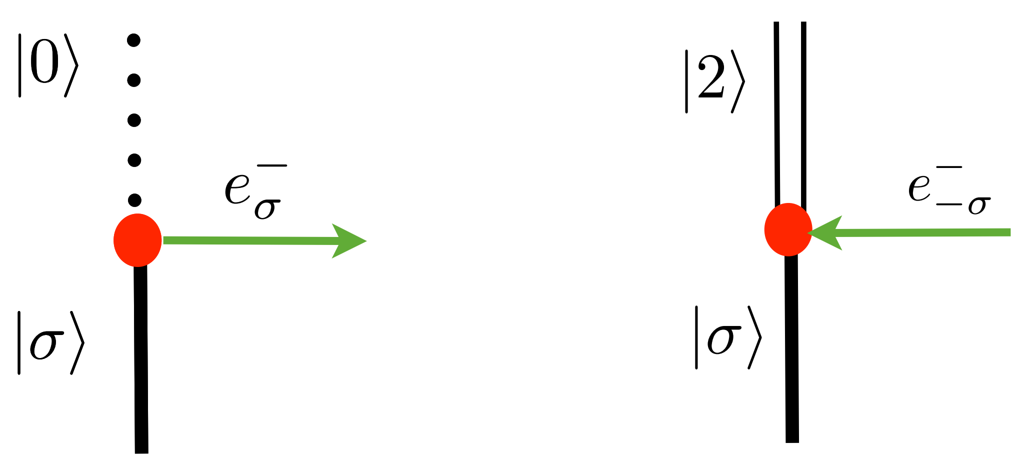


local moment formation: $T < U$
model related to the Kondo model

charge fluctuations:



hybridization:

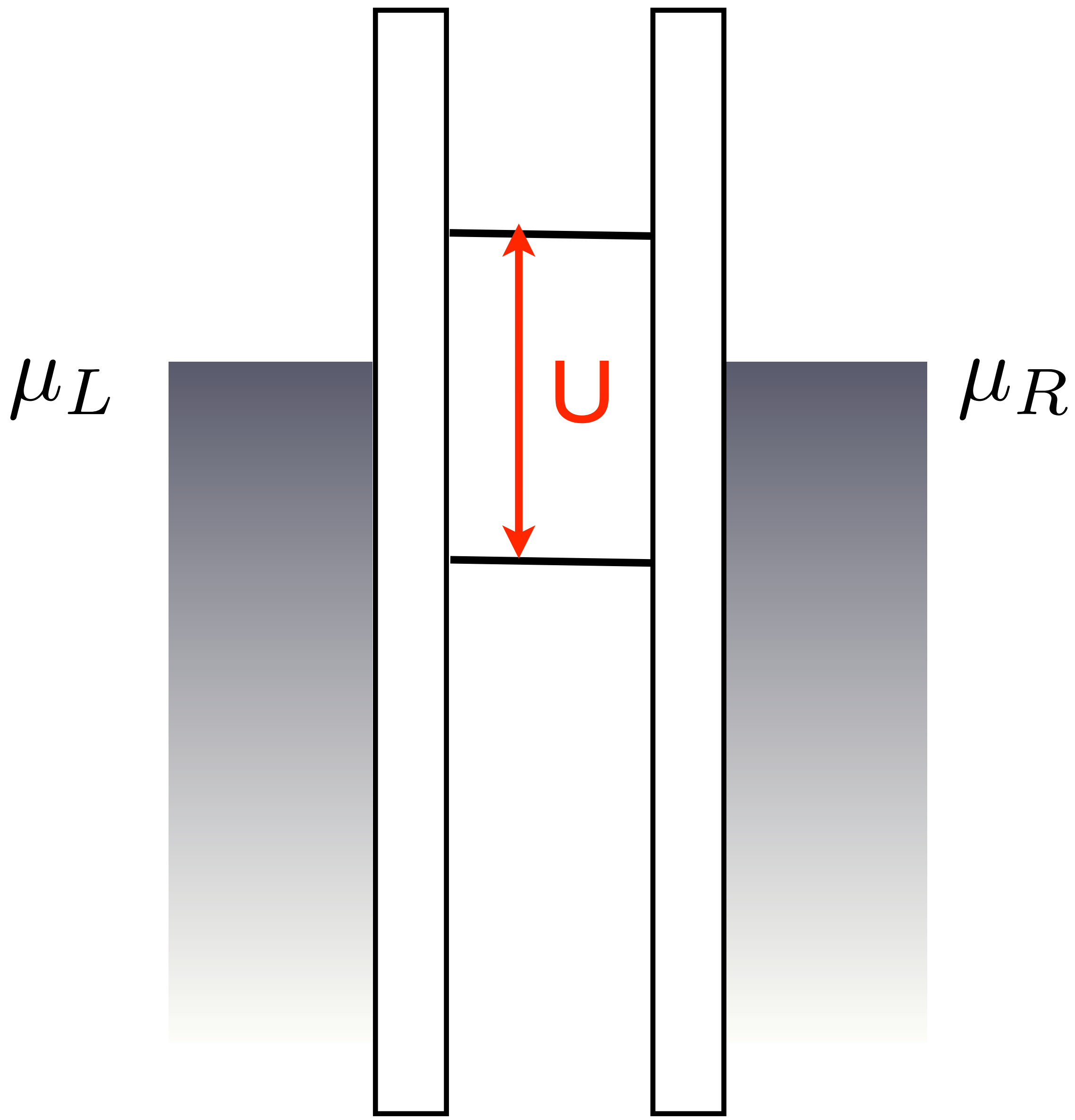


H_{imp}: artifical atom

$$H = \sum_\sigma E_\sigma d^\dagger_\sigma d_\sigma + U n^d_\uparrow n^d_\downarrow$$

$$|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |2\rangle$$

$$|0\rangle, E_\uparrow, E_\downarrow, 2E + U$$

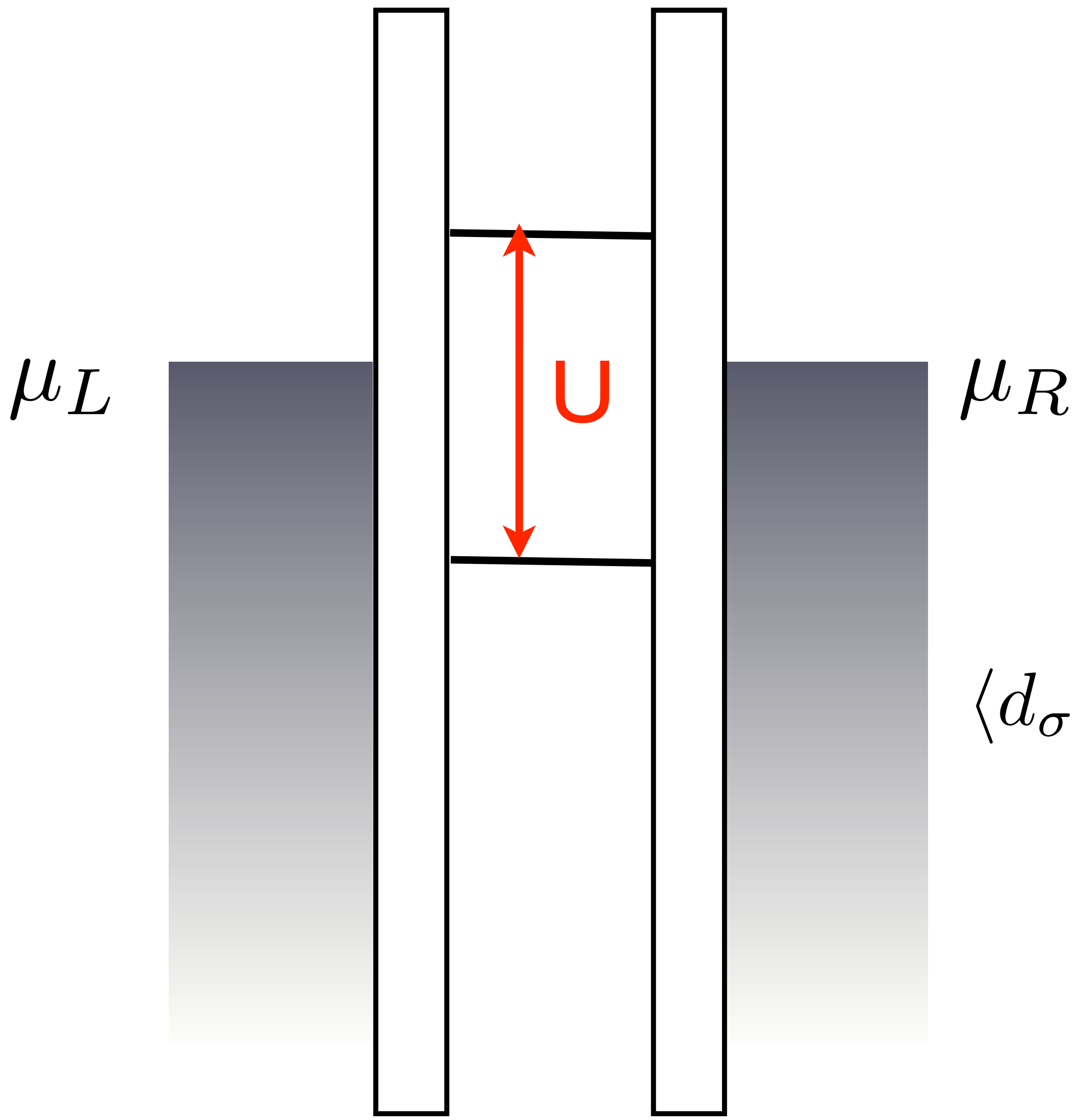


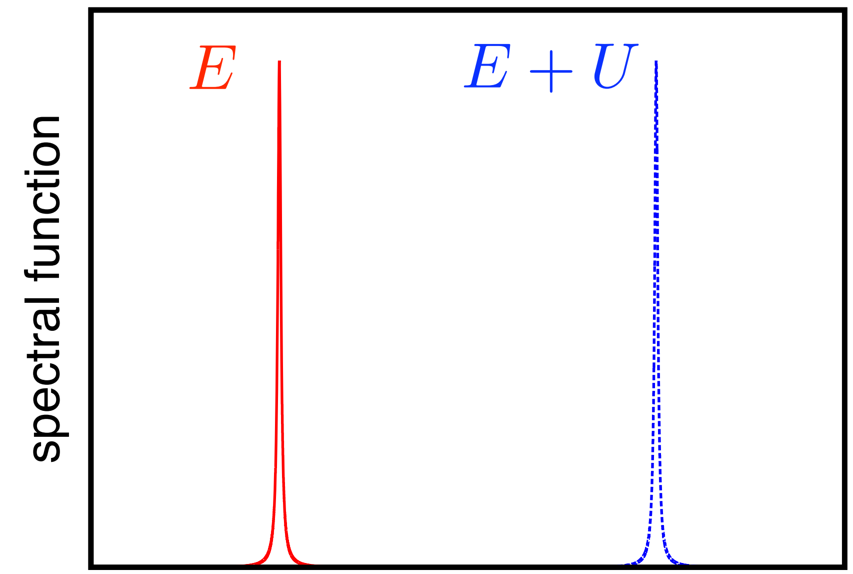
H_{imp}: artifical atom

$$H = \sum_\sigma E_\sigma d^\dagger_\sigma d_\sigma + U n^d_\uparrow n^d_\downarrow$$

$$|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |2\rangle$$

$$|0\rangle, E_\uparrow, E_\downarrow, 2E + U$$





$$\langle d_\sigma | d^\dagger_\sigma \rangle (z) = \frac{1 - n_{-\sigma}}{z - E_\sigma} + \frac{n_{-\sigma}}{z - (E_\sigma + U)}$$

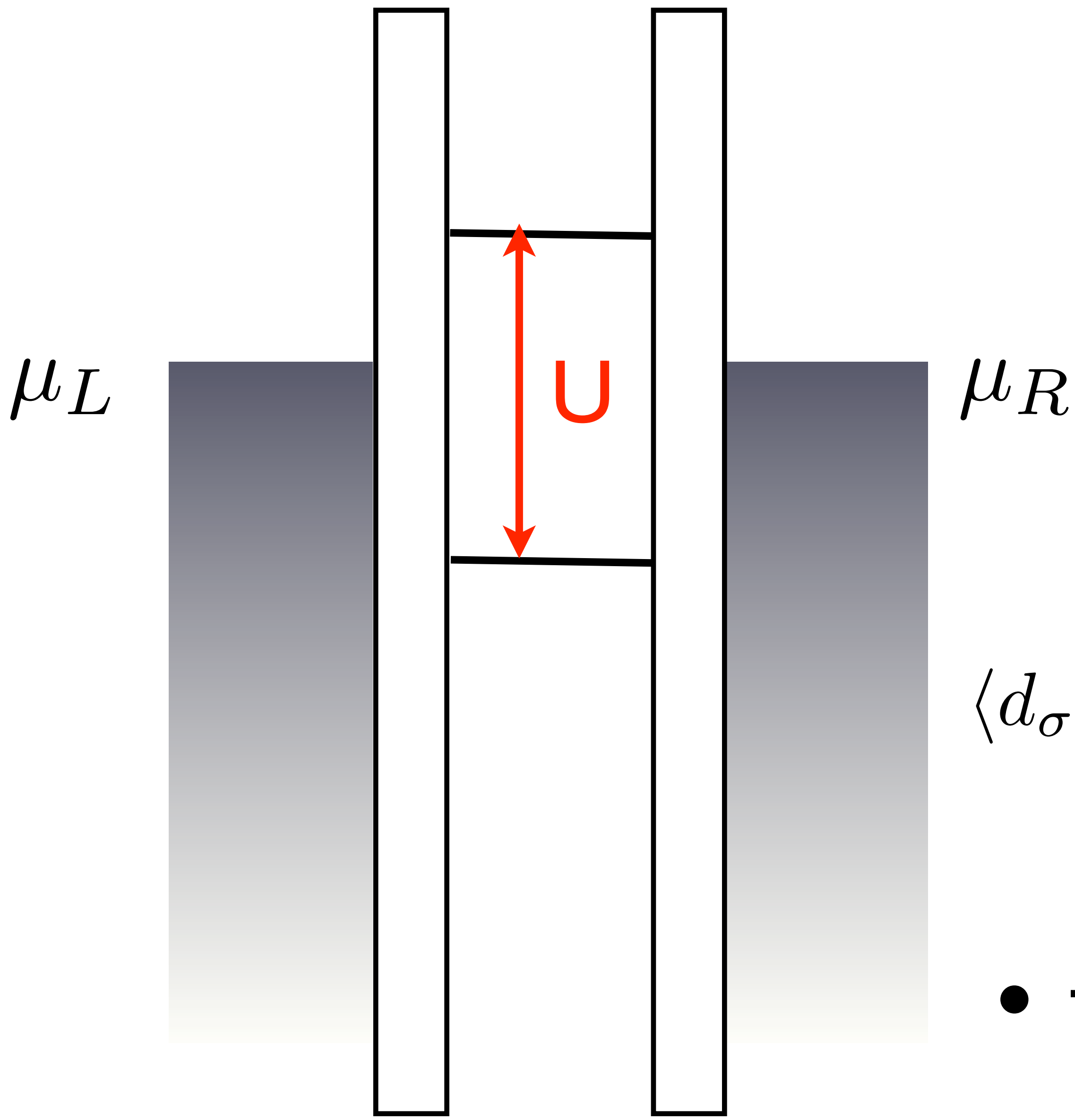
Spektralfunktion

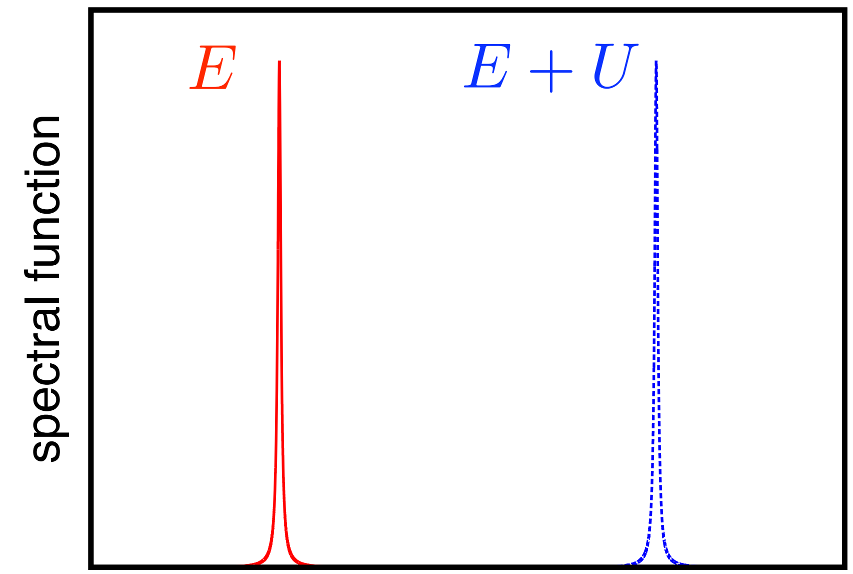
H_{imp}: artifical atom

$$H = \sum_\sigma E_\sigma d^\dagger_\sigma d_\sigma + U n^d_\uparrow n^d_\downarrow$$

$$|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |2\rangle$$

$$|0\rangle, E_\uparrow, E_\downarrow, 2E + U$$





$$\langle d_\sigma | d^\dagger_\sigma \rangle (z) = \frac{1 - n_{-\sigma}}{z - E_\sigma} + \frac{n_{-\sigma}}{z - (E_\sigma + U)}$$

Spektralfunktion

- fractal weighs for n=1

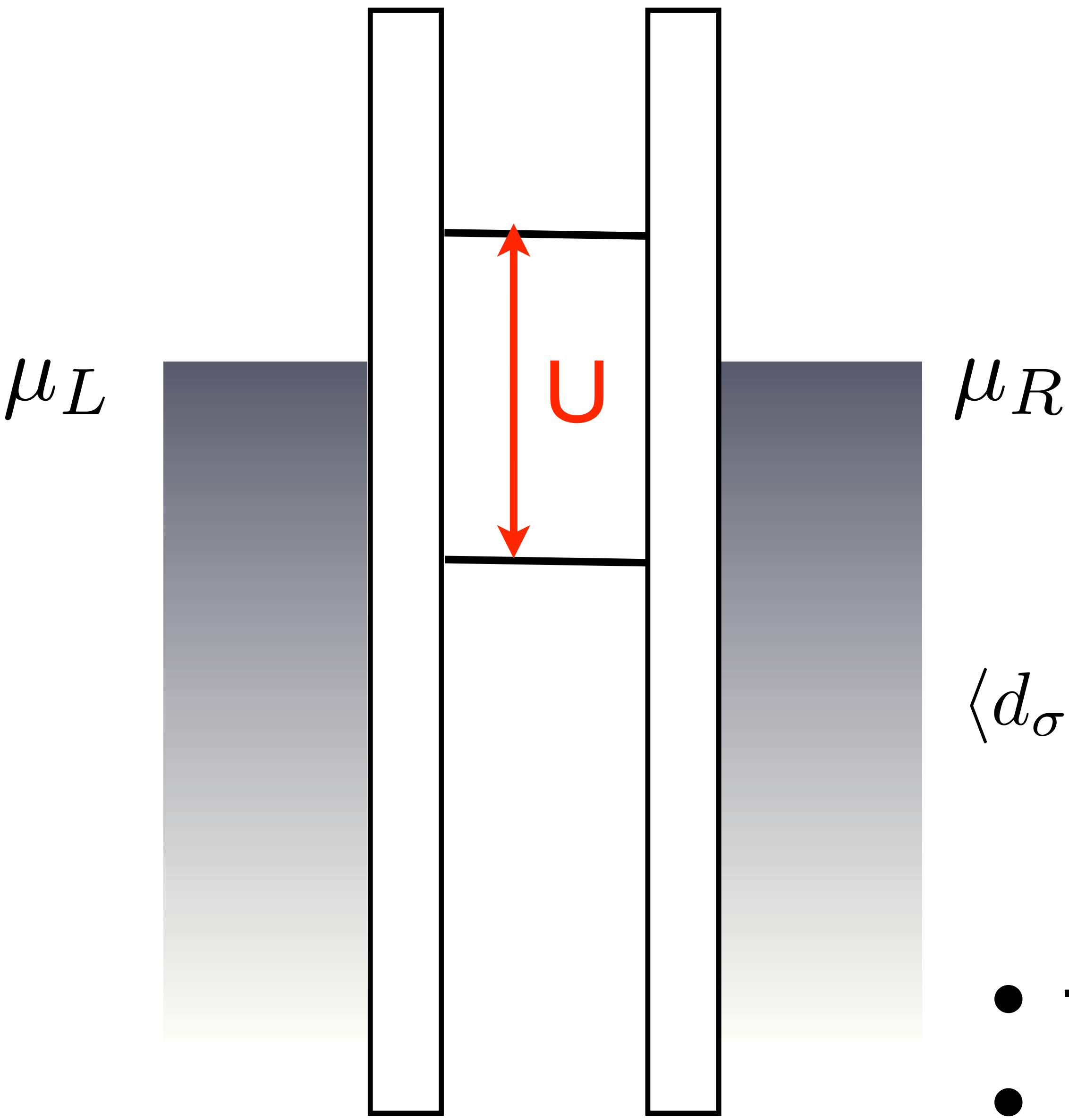
Wednesday, September 5, 2012

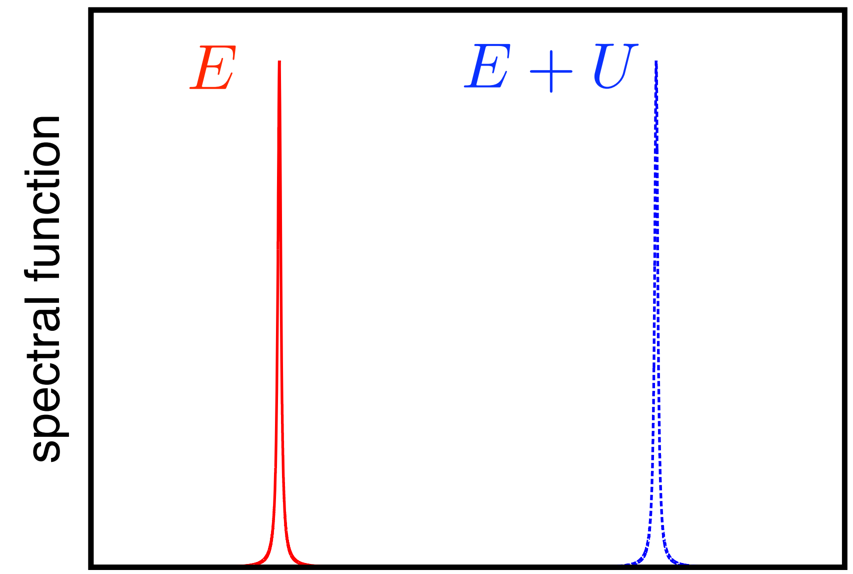
H_{imp}: artifical atom

$$H = \sum_\sigma E_\sigma d^\dagger_\sigma d_\sigma + U n^d_\uparrow n^d_\downarrow$$

$$|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |2\rangle$$

$$|0\rangle, E_\uparrow, E_\downarrow, 2E + U$$





$$\langle d_\sigma | d^\dagger_\sigma \rangle(z) = \frac{1 - n_{-\sigma}}{z - E_\sigma} + \frac{n_{-\sigma}}{z - (E_\sigma + U)}$$

Spektralfunktion

- fractal weighs for n=1
- no single-particle descriptions

multi-orbital Anderson Model

spin, charge and orbital fluctuations

$$H_{imp} = \sum_{i\sigma} \varepsilon_i^d n_{i\sigma}^d + \sum_{\substack{\sigma\sigma' \\ mnpq}} U_{mnpq} d^\dagger_{n\sigma} d^\dagger_{m\sigma'} d_{p\sigma'} d_{n\sigma}$$

see R. Eder's lecture

multi-orbital Anderson Model

spin, charge and orbital fluctuations

$$H_{imp} = \sum_{i\sigma} \varepsilon_i^d n_{i\sigma}^d + \sum_{\substack{\sigma\sigma' \\ mnpq}} U_{mnpq} d^\dagger_{n\sigma} d^\dagger_{m\sigma'} d_{p\sigma'} d_{n\sigma}$$

U includes Hund's rule couplings   see R. Eder's lecture

multi-orbital Anderson Model

spin, charge and orbital fluctuations

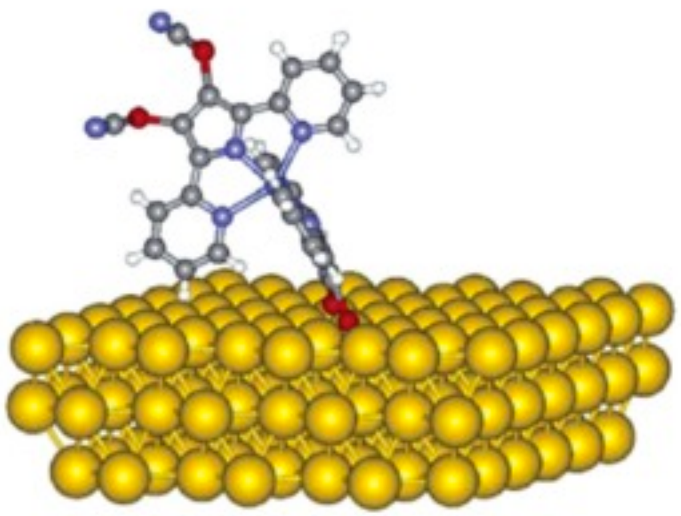
$$H_{imp} = \sum_{i\sigma} \varepsilon_i^d n_{i\sigma}^d + \sum_{\substack{\sigma\sigma' \\ mnpq}} U_{mnpq} d^\dagger_{n\sigma} d^\dagger_{m\sigma'} d_{p\sigma'} d_{n\sigma}$$

U includes Hund's rule couplings see R. Eder's lecture

$$H_{hyp} = \sum_{i\sigma k,\nu} V_{i,\nu}(\sigma k) \left(d^\dagger_{i\sigma} c_{k\nu\sigma} + c^\dagger_{k\nu\sigma} d_{i\sigma} \right)$$

example impurity

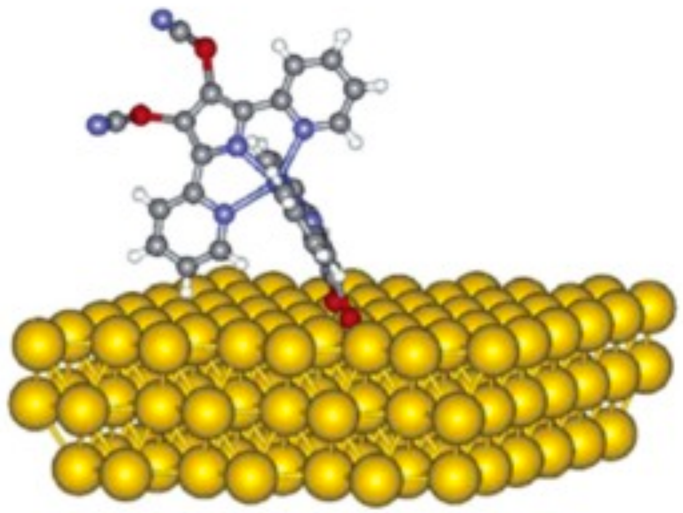
$$H_{imp} = \sum_{i\sigma} \varepsilon_i^d n_{i\sigma}^d + \sum_{\substack{\sigma\sigma' \\ mnpq}} U_{mnpq} d_{n\sigma}^\dagger d_{m\sigma'}^\dagger d_{p\sigma'} d_{n\sigma}$$



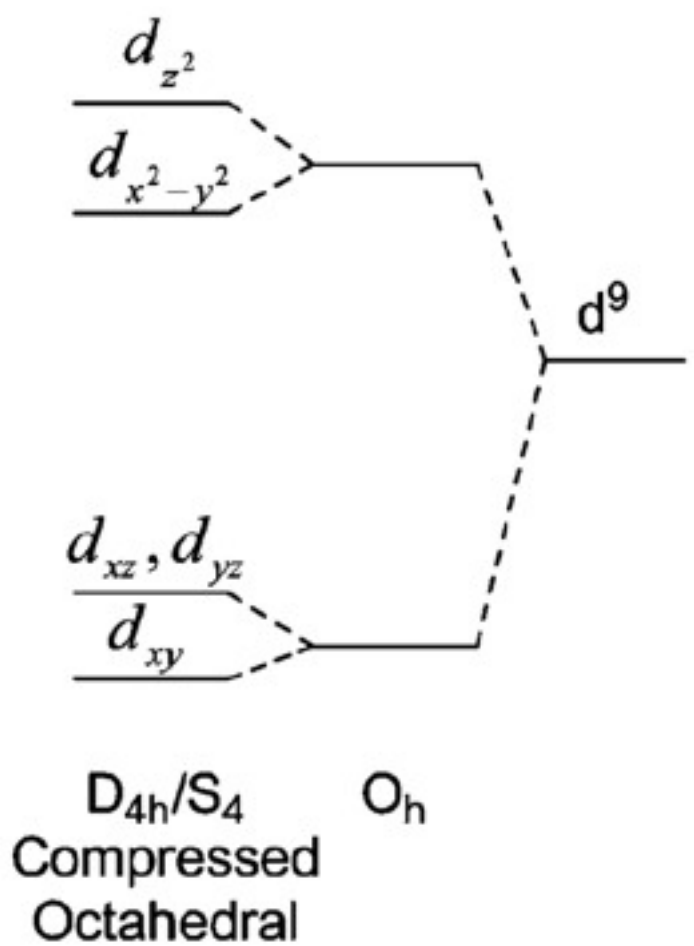
transition metal complex on a surface

example impurity

$$H_{imp} = \sum_{i\sigma} \varepsilon_i^d n_{i\sigma}^d + \sum_{\substack{\sigma\sigma' \\ mnpq}} U_{mnpq} d_{n\sigma}^\dagger d_{m\sigma'}^\dagger d_{p\sigma'} d_{n\sigma}$$

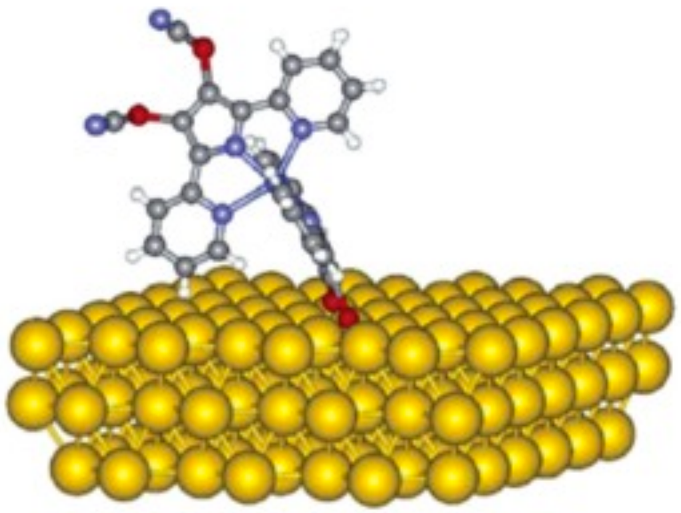


transition metal complex on a surface

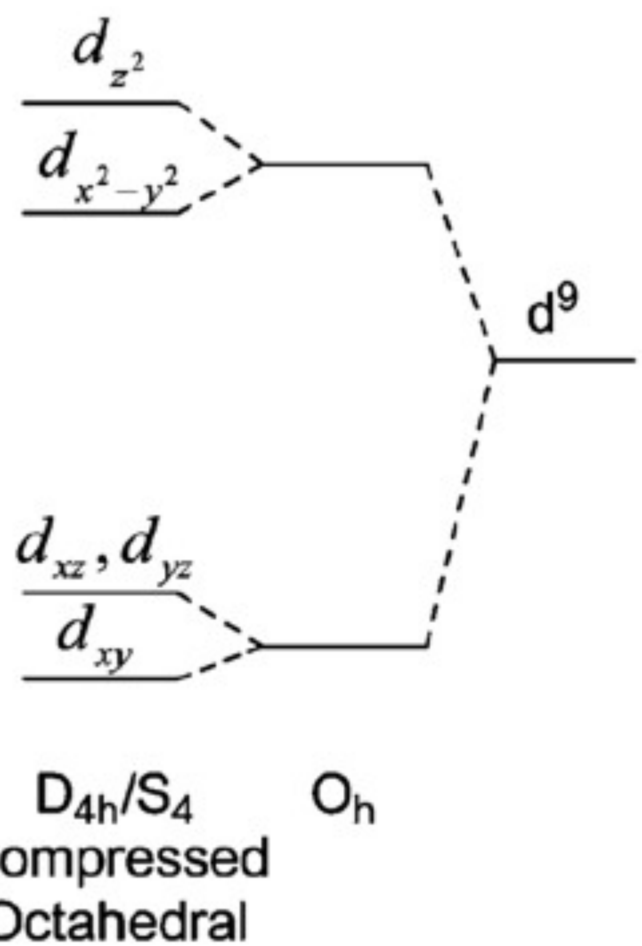


example impurity

$$H_{imp} = \sum_{i\sigma} \varepsilon_i^d n_{i\sigma}^d + \sum_{\substack{\sigma\sigma' \\ mnpq}} U_{mnpq} d^\dagger_{n\sigma} d^\dagger_{m\sigma'} d_{p\sigma'} d_{n\sigma}$$



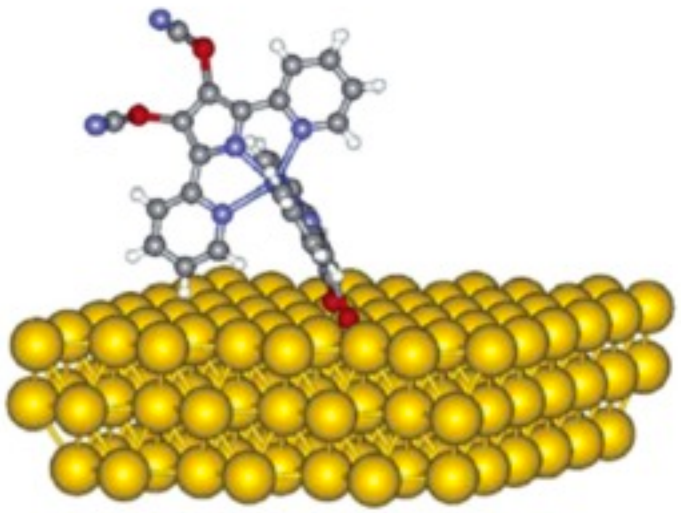
transition metal complex on a surface



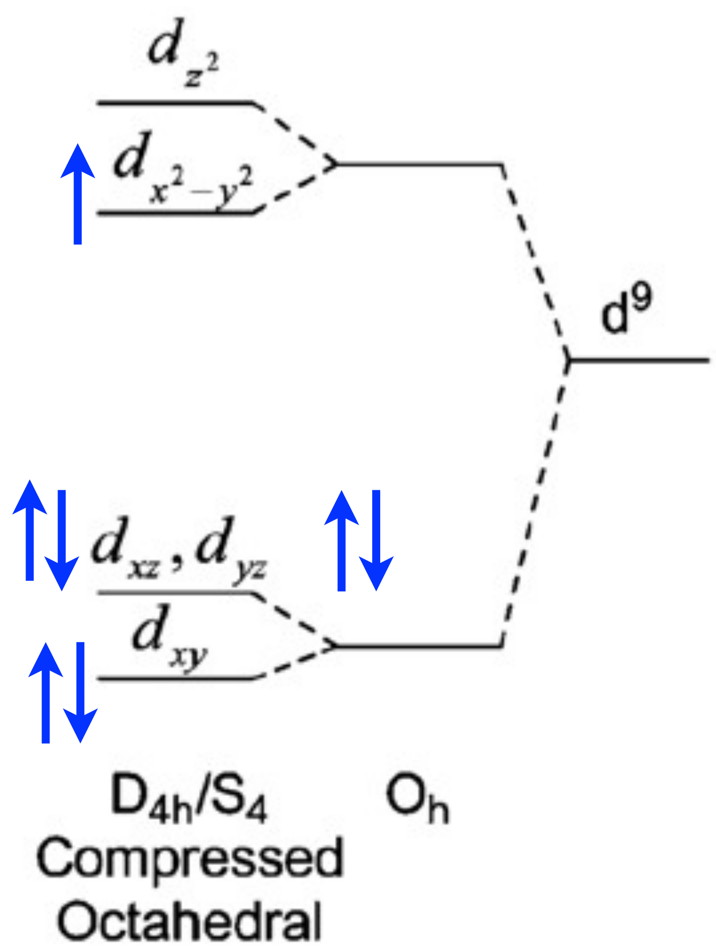
Co^{2+} low spin $S=1/2$

example impurity

$$H_{imp} = \sum_{i\sigma} \varepsilon_i^d n_{i\sigma}^d + \sum_{\substack{\sigma\sigma' \\ mnpq}} U_{mnpq} d_{n\sigma}^\dagger d_{m\sigma'}^\dagger d_{p\sigma'} d_{n\sigma}$$



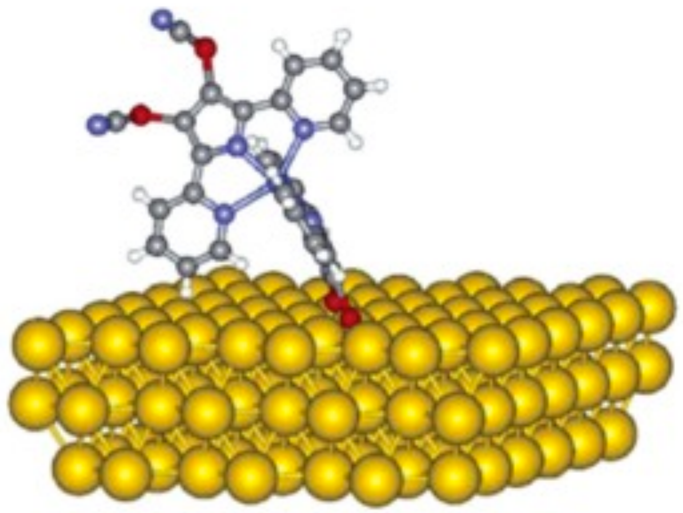
transition metal complex on a surface



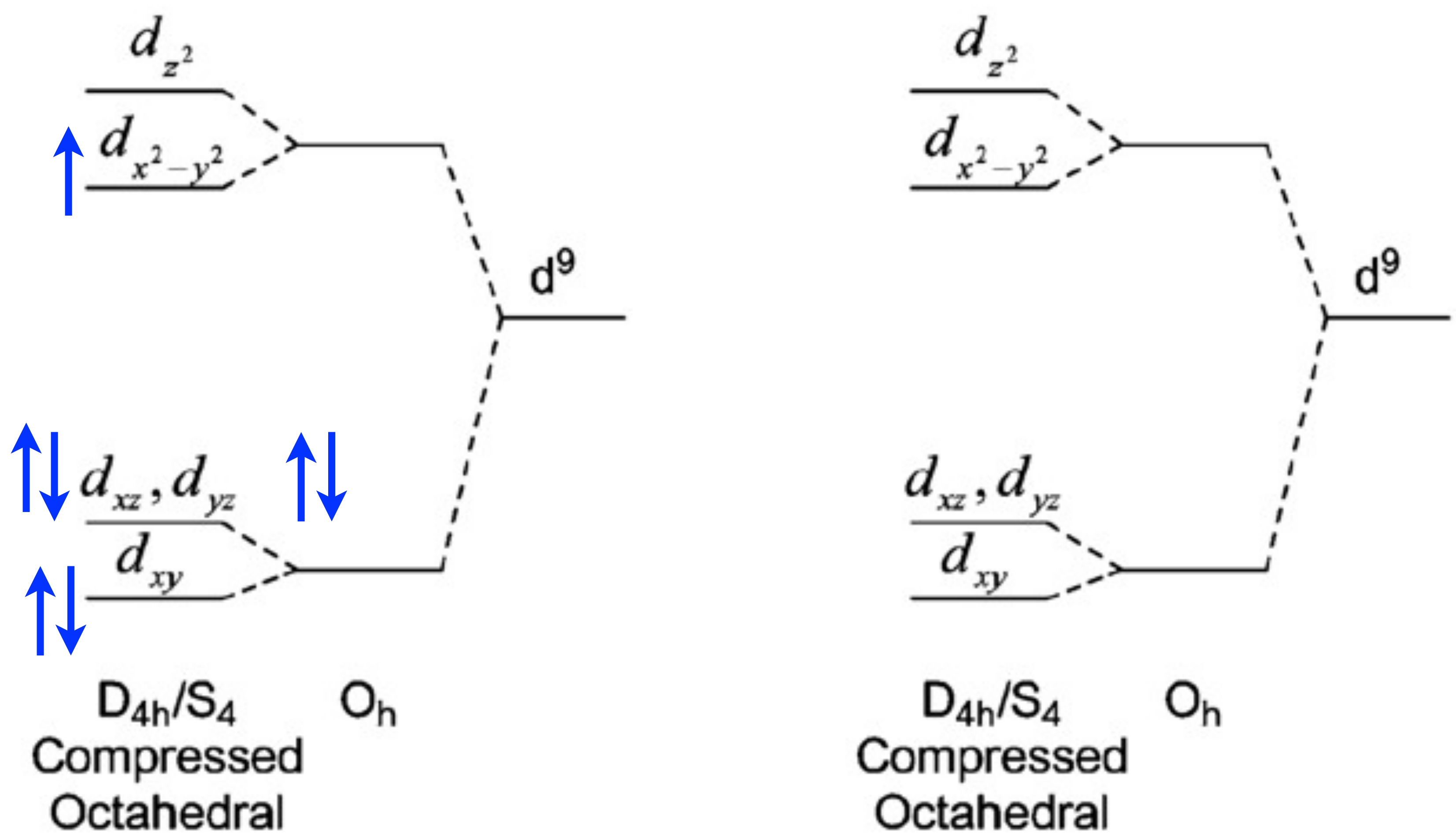
Co^{2+} low spin S=1/2

example impurity

$$H_{imp} = \sum_{i\sigma} \varepsilon_i^d n_{i\sigma}^d + \sum_{\substack{\sigma\sigma' \\ mnpq}} U_{mnpq} d_{n\sigma}^\dagger d_{m\sigma'}^\dagger d_{p\sigma'} d_{n\sigma}$$



transition metal complex on a surface

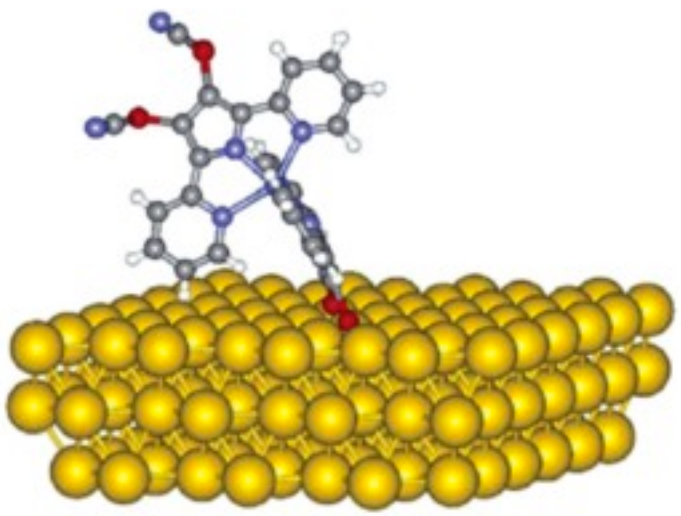


Co^{2+} low spin $S=1/2$

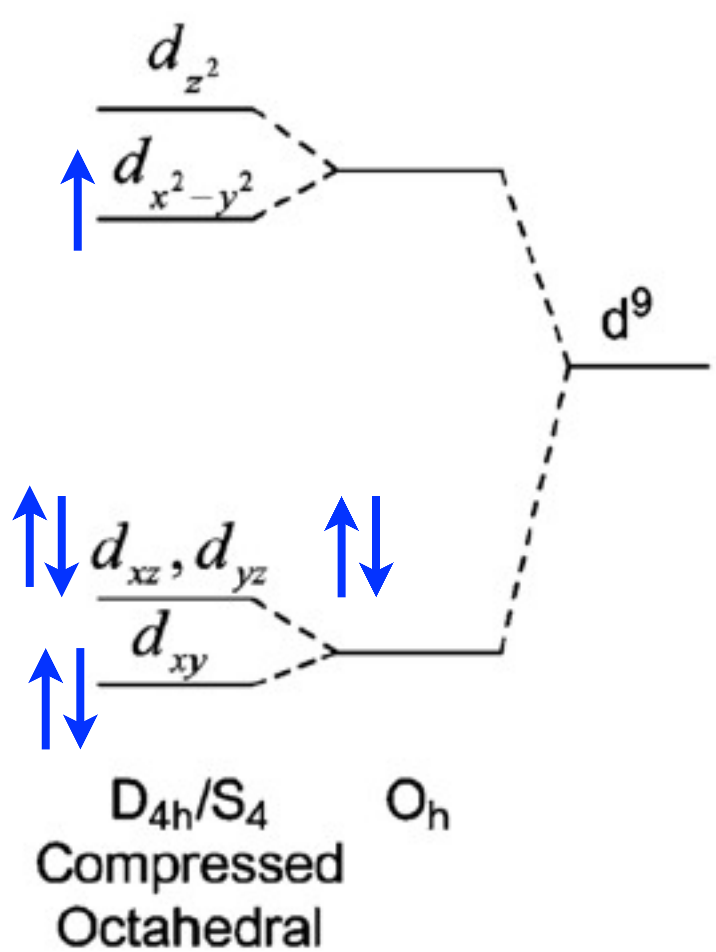
high spin $S=3/2$

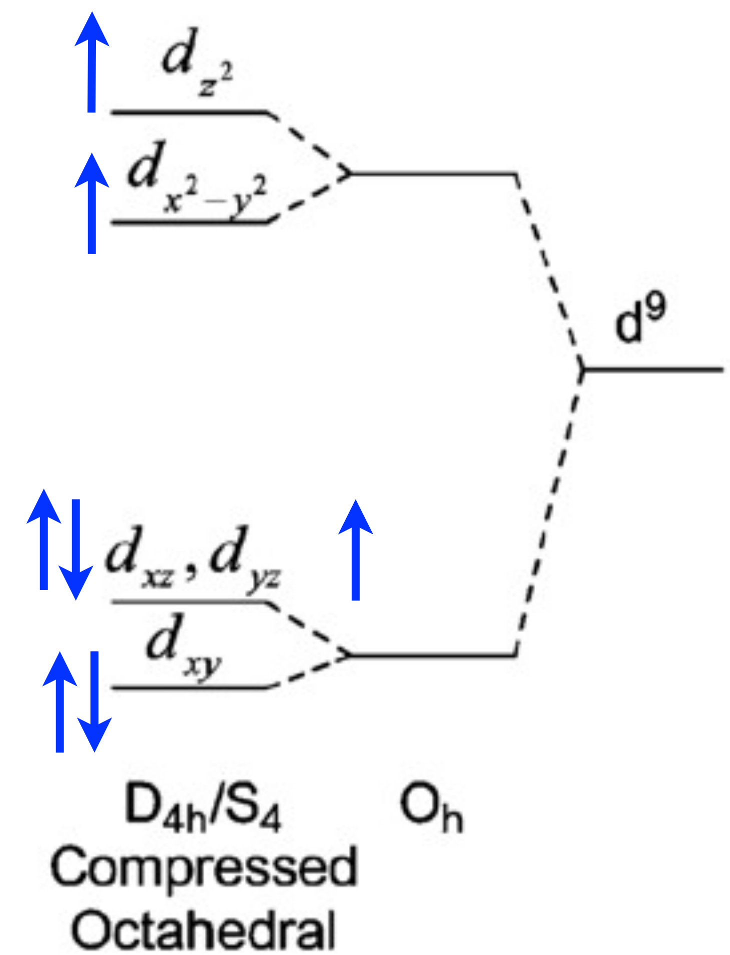
example impurity

$$H_{imp} = \sum_{i\sigma} \varepsilon_i^d n_{i\sigma}^d + \sum_{\substack{\sigma\sigma' \\ mnpq}} U_{mnpq} d_{n\sigma}^\dagger d_{m\sigma'}^\dagger d_{p\sigma'} d_{n\sigma}$$



transition metal complex on a surface





Co^{2+} low spin S=1/2

high spin S=3/2

Renormalization Group

renormalization group

three steps of renormalization:

renormalization group

three steps of renormalization:

1. elimination of high energy modes

three steps of renormalization:

1. elimination of high energy modes
2. rescaling of all parameters

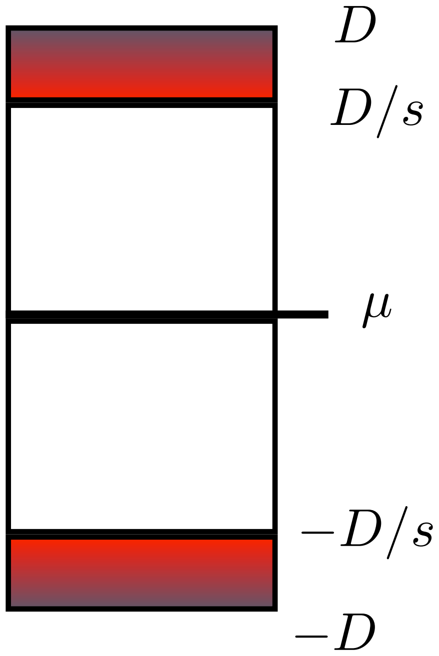
renormalization group

three steps of renormalization:

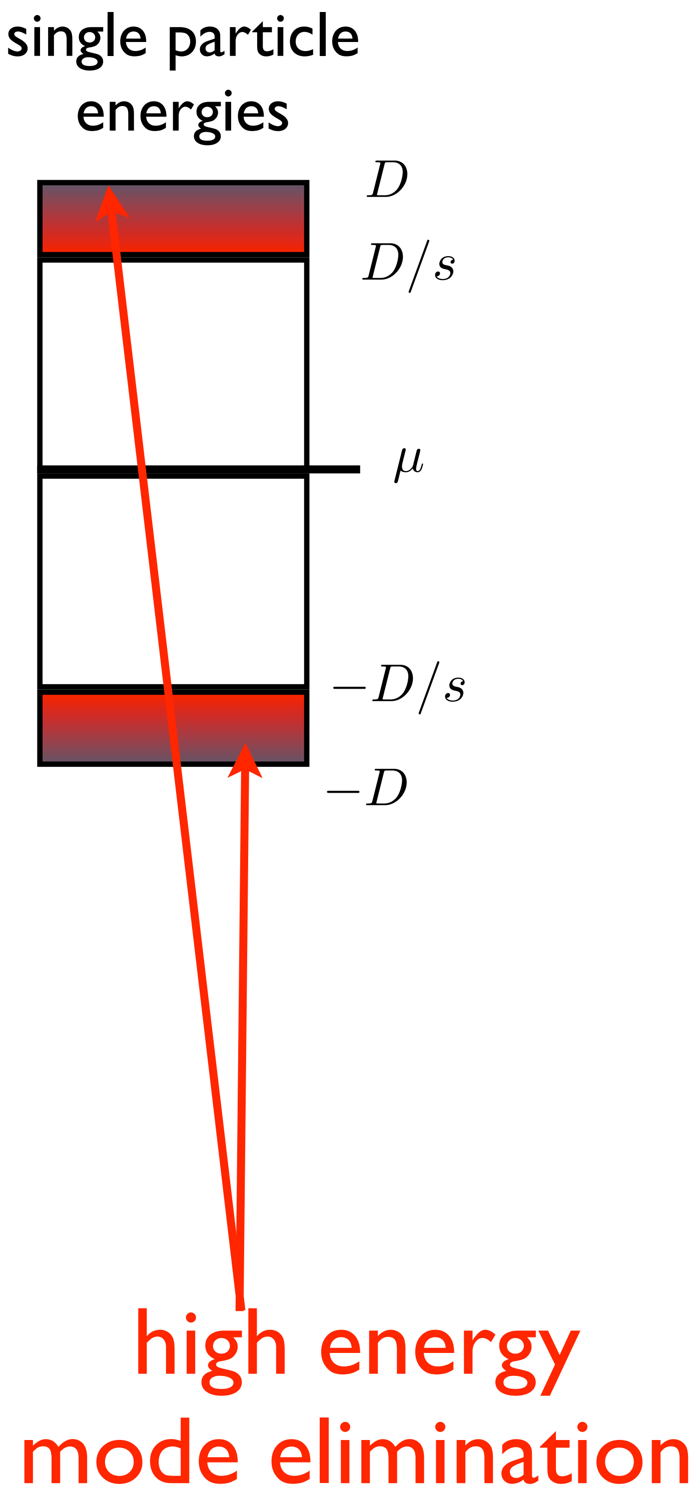
1. elimination of high energy modes
2. rescaling of all parameters
3. rescaling of the quantum fields

example: free electron gas

single particle energies

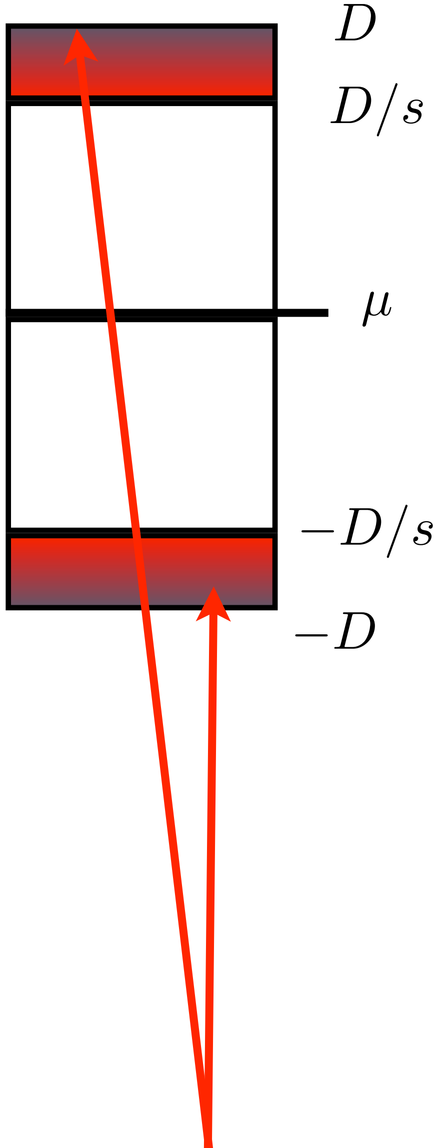


example: free electron gas



example: free electron gas

single particle energies



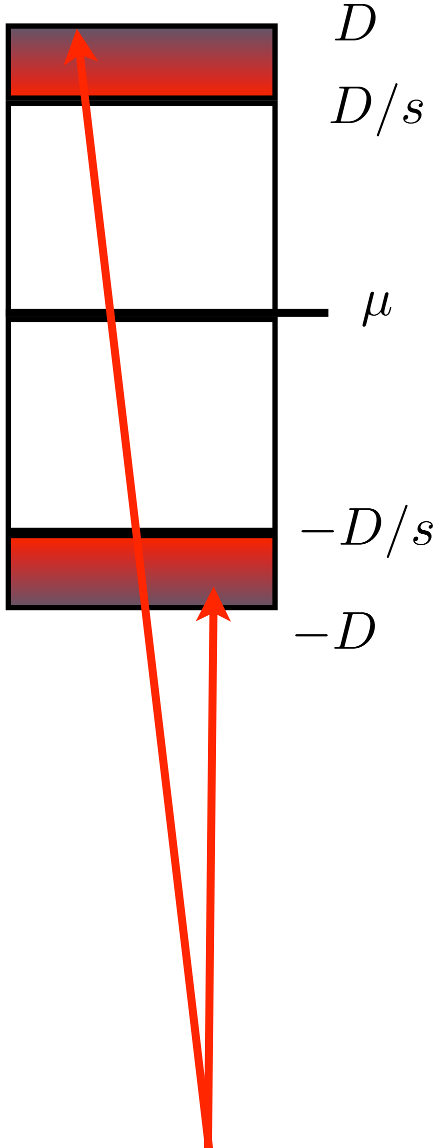
high energy mode elimination

1. mode elimination

$$\frac{H}{D} = \int_{-1}^{1} dx\, x\, c_x^\dagger c_x = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x + \left(\int_{-1}^{-\frac{1}{s}} + \int_{\frac{1}{s}}^{1} \right) dx\, x\, c_x^\dagger c_x$$

example: free electron gas

single particle energies



high energy mode elimination

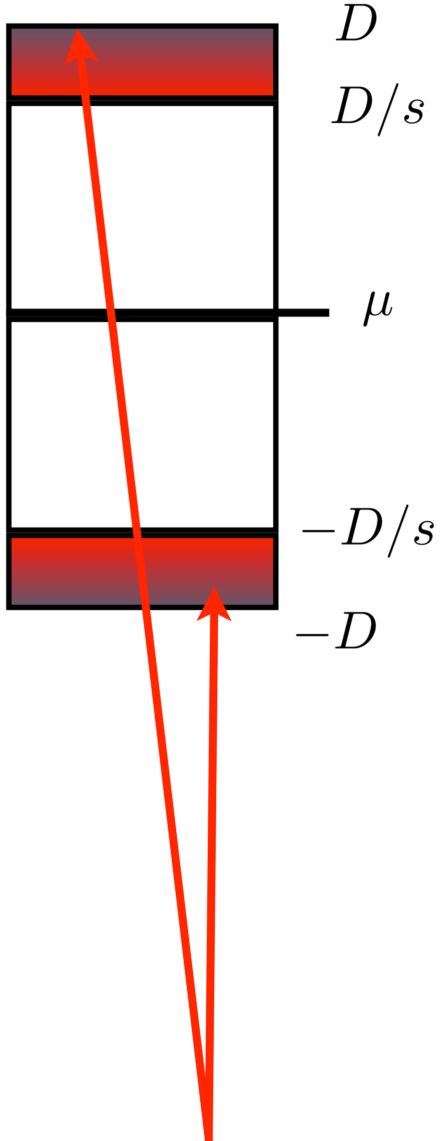
1. mode elimination

$$\frac{H}{D} = \int_{-1}^{1} dx\, x\, c_x^\dagger c_x = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x + \left(\int_{-1}^{-\frac{1}{s}} + \int_{\frac{1}{s}}^{1} \right) dx\, x\, c_x^\dagger c_x$$

kept

example: free electron gas

single particle energies



high energy mode elimination

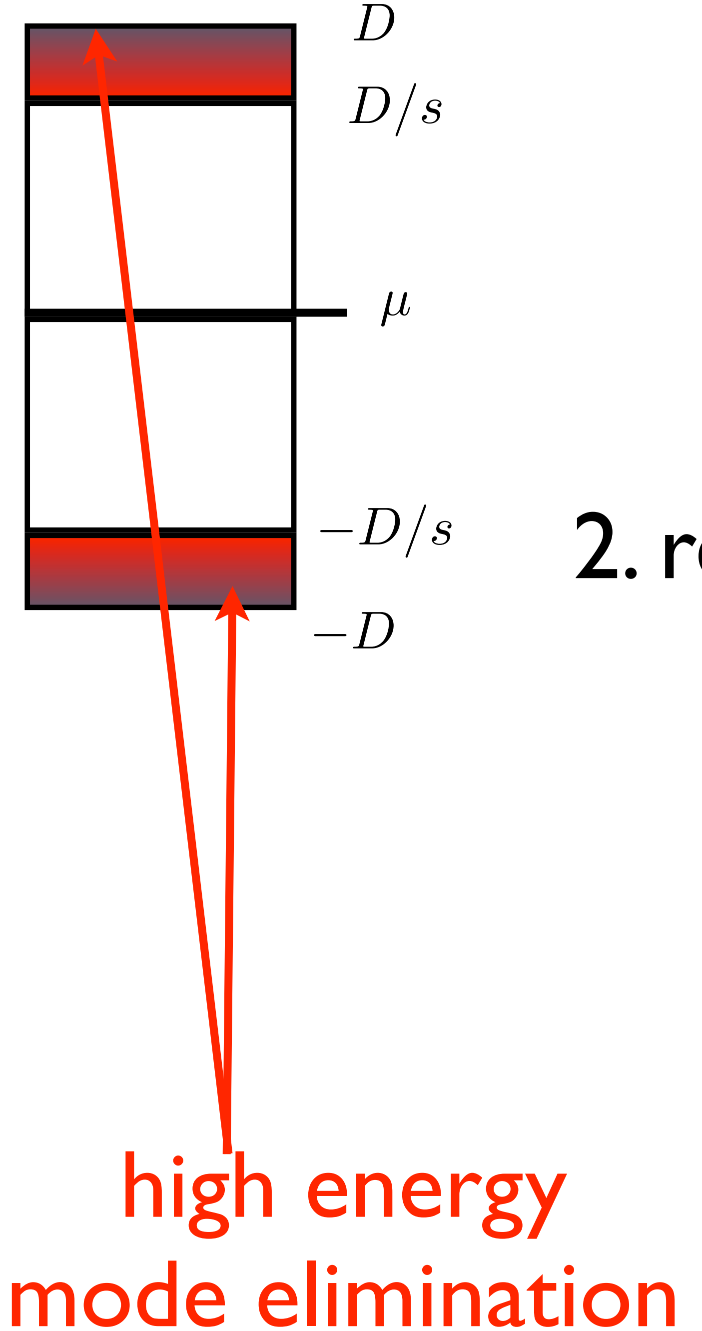
1. mode elimination

$$\frac{H}{D} = \int_{-1}^{1} dx\, x\, c_x^\dagger c_x = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x + \left(\int_{-1}^{-\frac{1}{s}} + \int_{\frac{1}{s}}^{1} \right) dx\, x\, c_x^\dagger c_x$$

kept — discarded

example: free electron gas

single particle energies



1. mode elimination

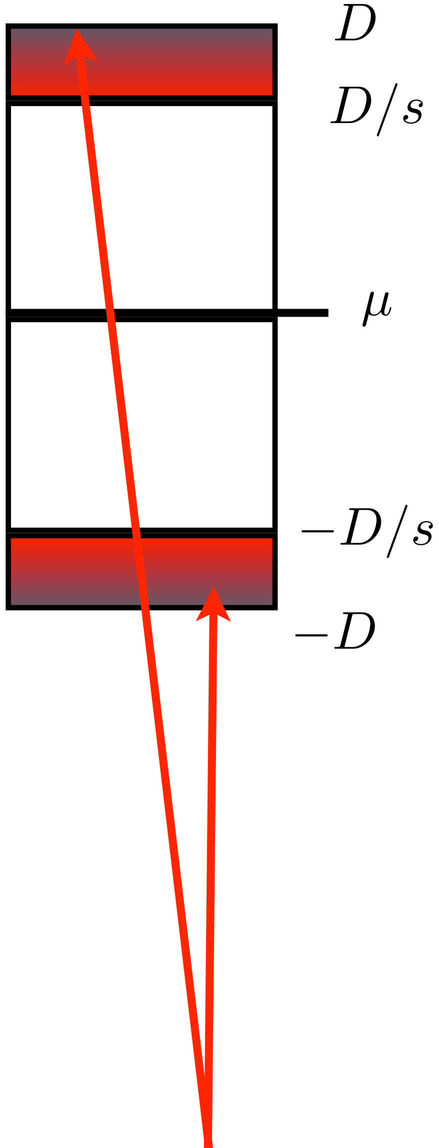
$$\frac{H}{D} = \int_{-1}^{1} dx\, x\, c_x^\dagger c_x = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x + \left(\int_{-1}^{-\frac{1}{s}} + \int_{\frac{1}{s}}^{1} \right) dx\, x\, c_x^\dagger c_x$$

kept | discarded

2. rescaling of the parameters: $x \to x' = sx$

example: free electron gas

single particle energies



high energy mode elimination

1. mode elimination

$$\frac{H}{D} = \int_{-1}^{1} dx\, x\, c_x^\dagger c_x = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x + \left(\int_{-1}^{-\frac{1}{s}} + \int_{\frac{1}{s}}^{1} \right) dx\, x\, c_x^\dagger c_x$$

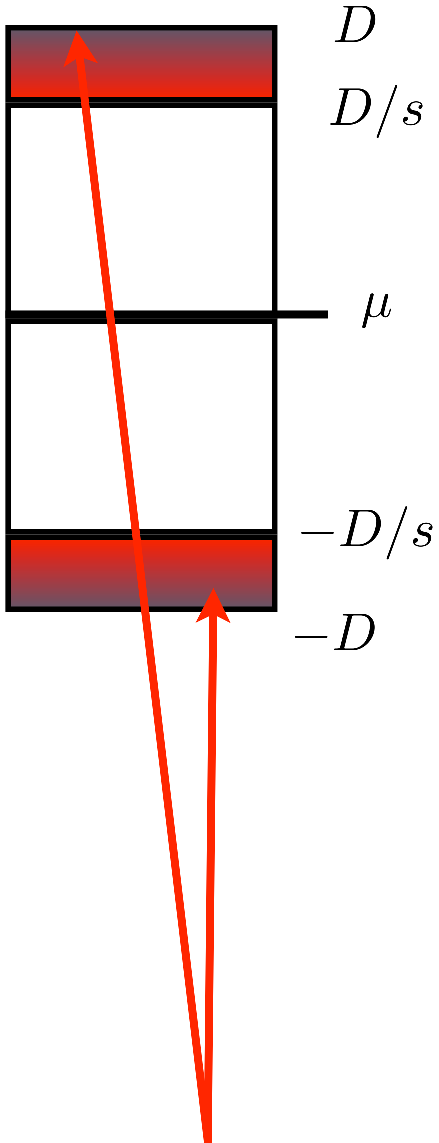
kept — discarded

2. rescaling of the parameters: $x \to x' = sx$

$$\frac{H'}{D} = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x = s^{-2} \int_{-1}^{1} dx'\, x'\, c_{x'(x)}^\dagger c_{x'(x)}$$

example: free electron gas

single particle energies



high energy mode elimination

1. mode elimination

$$\frac{H}{D} = \int_{-1}^{1} dx\, x\, c_x^\dagger c_x = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x + \left(\int_{-1}^{-\frac{1}{s}} + \int_{\frac{1}{s}}^{1} \right) dx\, x\, c_x^\dagger c_x$$

kept — discarded

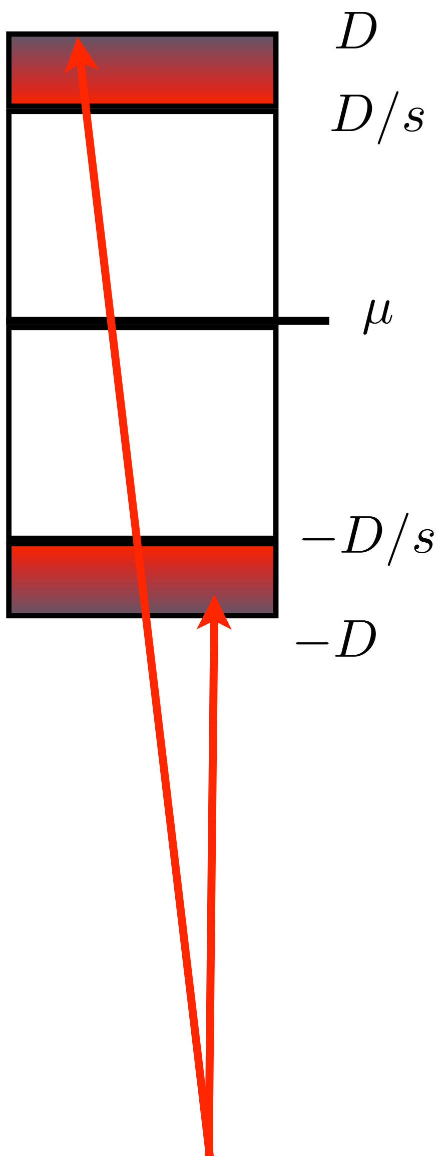
2. rescaling of the parameters: $x \to x' = sx$

$$\frac{H'}{D} = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x = s^{-2} \int_{-1}^{1} dx'\, x'\, c_{x'(x)}^\dagger c_{x'(x)}$$

3. rescaling of field operators: $c_{x'} \to \frac{1}{\sqrt{s}} c_{x'(x)}$

example: free electron gas

single particle energies



1. mode elimination

$$\frac{H}{D} = \int_{-1}^{1} dx\, x\, c_x^\dagger c_x = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x + \left(\int_{-1}^{-\frac{1}{s}} + \int_{\frac{1}{s}}^{1} \right) dx\, x\, c_x^\dagger c_x$$

kept — discarded

2. rescaling of the parameters: $x \to x' = sx$

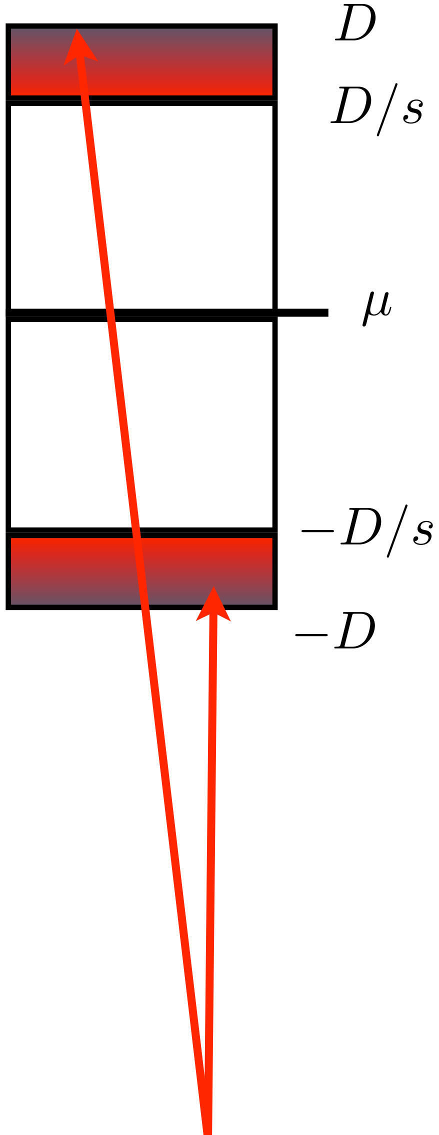
$$\frac{H'}{D} = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x = s^{-2} \int_{-1}^{1} dx'\, x'\, c_{x'(x)}^\dagger c_{x'(x)}$$

3. rescaling of field operators: $c_{x'} \to \frac{1}{\sqrt{s}} c_{x'(x)}$

$$\frac{H'}{D} = s^{-1} \int_{-1}^{1} dx'\, x'\, c_{x'}^\dagger c_{x'} \Rightarrow \frac{H'}{D'} = \int_{-1}^{1} dx'\, x'\, c_{x'}^\dagger c_{x'}$$

example: free electron gas

single particle energies



high energy mode elimination

1. mode elimination

$$\frac{H}{D} = \int_{-1}^{1} dx\, x\, c_x^\dagger c_x = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x + \left(\int_{-1}^{-\frac{1}{s}} + \int_{\frac{1}{s}}^{1} \right) dx\, x\, c_x^\dagger c_x$$

kept — discarded

2. rescaling of the parameters: $x \to x' = sx$

$$\frac{H'}{D} = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x = s^{-2} \int_{-1}^{1} dx'\, x'\, c_{x'(x)}^\dagger c_{x'(x)}$$

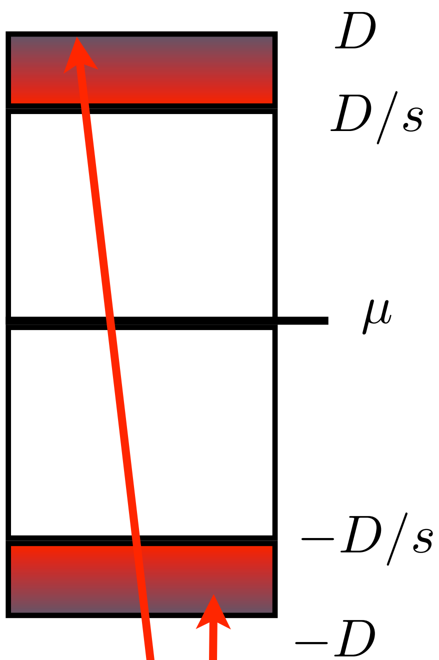
3. rescaling of field operators: $c_{x'} \to \frac{1}{\sqrt{s}} c_{x'(x)}$

$$\frac{H'}{D} = s^{-1} \int_{-1}^{1} dx'\, x'\, c_{x'}^\dagger c_{x'} \Rightarrow \frac{H'}{D'} = \int_{-1}^{1} dx'\, x'\, c_{x'}^\dagger c_{x'}$$

D'=D/s

example: free electron gas

single particle energies



high energy mode elimination

1. mode elimination

$$\frac{H}{D} = \int_{-1}^{1} dx\, x\, c_x^\dagger c_x = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x + \left(\int_{-1}^{-\frac{1}{s}} + \int_{\frac{1}{s}}^{1} \right) dx\, x\, c_x^\dagger c_x$$

kept discarded

2. rescaling of the parameters: $x \to x' = sx$

$$\frac{H'}{D} = \int_{-\frac{1}{s}}^{\frac{1}{s}} dx\, x\, c_x^\dagger c_x = s^{-2} \int_{-1}^{1} dx'\, x'\, c_{x'(x)}^\dagger c_{x'(x)}$$

3. rescaling of field operators: $c_{x'} \to \frac{1}{\sqrt{s}} c_{x'(x)}$

$$\frac{H'}{D} = s^{-1} \int_{-1}^{1} dx'\, x'\, c_{x'}^\dagger c_{x'} \Rightarrow \frac{H'}{D'} = \int_{-1}^{1} dx'\, x'\, c_{x'}^\dagger c_{x'}$$

D'=D/s $\Rightarrow$ RG fixed point

poor man's scaling

Kondo Hamiltonian

$$\frac{H}{D} = \sum_{\sigma} \int_{-1}^{1} dx\; x c^{\dagger}_{x\sigma} c_{x\sigma} + g \int_{-1}^{1} dx \int_{-1}^{1} dx' \sum_{\alpha\beta} c^{\dagger}_{x\alpha} \vec{\sigma} c_{x'\beta} \vec{S}_{imp}$$

Kondo Hamiltonian

$$\frac{H}{D} = \sum_\sigma \int_{-1}^{1} dx\; x c^\dagger_{x\sigma} c_{x\sigma} + g \int_{-1}^{1} dx \int_{-1}^{1} dx' \sum_{\alpha\beta} c^\dagger_{x\alpha} \vec{\sigma} c_{x'\beta} \vec{S}_{imp}$$

Projector onto low energy subspace: $\hat{P}_L$

poor man's scaling

Kondo Hamiltonian

$$\frac{H}{D} = \sum_\sigma \int_{-1}^{1} dx \; x c^\dagger_{x\sigma} c_{x\sigma} + g \int_{-1}^{1} dx \int_{-1}^{1} dx' \sum_{\alpha\beta} c^\dagger_{x\alpha} \vec{\sigma} c_{x'\beta} \vec{S}_{imp}$$

Projector onto low energy subspace: $\hat{P}_L$

Projector onto high energy subspace: $\hat{P}_H = \hat{1} - \hat{P}_L$

poor man's scaling

Kondo Hamiltonian

$$\frac{H}{D} = \sum_\sigma \int_{-1}^{1} dx \; x c^\dagger_{x\sigma} c_{x\sigma} + g \int_{-1}^{1} dx \int_{-1}^{1} dx' \sum_{\alpha\beta} c^\dagger_{x\alpha} \vec{\sigma} c_{x'\beta} \vec{S}_{imp}$$

Projector onto low energy subspace: $\hat{P}_L$

Projector onto high energy subspace: $\hat{P}_H = \hat{1} - \hat{P}_L$

Definitions:

poor man's scaling

Kondo Hamiltonian

$$\frac{H}{D} = \sum_\sigma \int_{-1}^{1} dx\, x c^\dagger_{x\sigma} c_{x\sigma} + g \int_{-1}^{1} dx \int_{-1}^{1} dx' \sum_{\alpha\beta} c^\dagger_{x\alpha} \vec{\sigma} c_{x'\beta} \vec{S}_{imp}$$

Projector onto low energy subspace: $\hat{P}_L$

Projector onto high energy subspace: $\hat{P}_H = \hat{1} - \hat{P}_L$

Definitions:

$$H_d = \hat{P}_L H \hat{P}_L + \hat{P}_H H \hat{P}_H$$

$$\lambda V = \hat{P}_L H \hat{P}_H + \hat{P}_H H \hat{P}_L$$

$$H = \left(\begin{array}{c|c} H_d^L & \lambda V \\ \hline \lambda V & H_d^H \end{array} \right)$$

Kondo Hamiltonian

$$\frac{H_d^L}{D} = \hat{P}_L g \sum_{\alpha\beta} \int_{-1}^{1} dx \int_{-1}^{1} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta} \hat{P}_L$$

$$= g \sum_{\alpha\beta} \int_{-\frac{1}{s}}^{\frac{1}{s}} dx \int_{-\frac{1}{s}}^{\frac{1}{s}} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta}$$

Kondo Hamiltonian

$$\frac{H_d^L}{D} = \hat{P}_L g \sum_{\alpha\beta} \int_{-1}^{1} dx \int_{-1}^{1} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta} \hat{P}_L$$

$$= g \sum_{\alpha\beta} \int_{-\frac{1}{s}}^{\frac{1}{s}} dx \int_{-\frac{1}{s}}^{\frac{1}{s}} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta}$$

low energy part

Kondo Hamiltonian

$$
\begin{aligned}
\frac{H_d^L}{D} &= \hat{P}_L g \sum_{\alpha\beta} \int_{-1}^{1} dx \int_{-1}^{1} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta} \hat{P}_L \\
&= g \sum_{\alpha\beta} \int_{-\frac{1}{s}}^{\frac{1}{s}} dx \int_{-\frac{1}{s}}^{\frac{1}{s}} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta} \\
&= s^{-2} g \sum_{\alpha\beta} \int_{-1}^{1} d\bar{x} \int_{1}^{1} d\bar{x}' \, c^\dagger_{x(\bar{x})\alpha} c_{x'(\bar{x}')\beta} [\vec{\sigma}]_{\alpha\beta}
\end{aligned}
$$

Kondo Hamiltonian

$$\frac{H_d^L}{D} = \hat{P}_L g \sum_{\alpha\beta} \int_{-1}^{1} dx \int_{-1}^{1} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta} \hat{P}_L$$

$$= g \sum_{\alpha\beta} \int_{-\frac{1}{s}}^{\frac{1}{s}} dx \int_{-\frac{1}{s}}^{\frac{1}{s}} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta}$$

$$= s^{-2} g \sum_{\alpha\beta} \int_{-1}^{1} d\bar{x} \int_{1}^{1} d\bar{x}' \, c^\dagger_{x(\bar{x})\alpha} c_{x'(\bar{x}')\beta} [\vec{\sigma}]_{\alpha\beta}$$

rescale energies

Kondo Hamiltonian

$$
\begin{aligned}
\frac{H_d^L}{D} &= \hat{P}_L g \sum_{\alpha\beta} \int_{-1}^{1} dx \int_{-1}^{1} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta} \hat{P}_L \\
&= g \sum_{\alpha\beta} \int_{-\frac{1}{s}}^{\frac{1}{s}} dx \int_{-\frac{1}{s}}^{\frac{1}{s}} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta} \\
&= s^{-2} g \sum_{\alpha\beta} \int_{-1}^{1} d\bar{x} \int_{1}^{1} d\bar{x}' \, c^\dagger_{x(\bar{x})\alpha} c_{x'(\bar{x}')\beta} [\vec{\sigma}]_{\alpha\beta} \\
&= s^{-1} s^{-1+1} g \sum_{\alpha\beta} \int_{-1}^{1} d\bar{x} \int_{1}^{1} d\bar{x}' \, c^\dagger_{\bar{x}\alpha} c_{\bar{x}'\beta} [\vec{\sigma}]_{\alpha\beta}
\end{aligned}
$$

Wednesday, September 5, 2012

Kondo Hamiltonian

$$\frac{H_d^L}{D} = \hat{P}_L g \sum_{\alpha\beta} \int_{-1}^{1} dx \int_{-1}^{1} dx'\, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta} \hat{P}_L$$

$$= g \sum_{\alpha\beta} \int_{-\frac{1}{s}}^{\frac{1}{s}} dx \int_{-\frac{1}{s}}^{\frac{1}{s}} dx'\, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta}$$

$$= s^{-2}\, g \sum_{\alpha\beta} \int_{-1}^{1} d\bar{x} \int_{1}^{1} d\bar{x}'\, c^\dagger_{x(\bar{x})\alpha} c_{x'(\bar{x}')\beta} [\vec{\sigma}]_{\alpha\beta}$$

$$= s^{-1} s^{-1+1}\, g \sum_{\alpha\beta} \int_{-1}^{1} d\bar{x} \int_{1}^{1} d\bar{x}'\, c^\dagger_{\bar{x}\alpha} c_{\bar{x}'\beta} [\vec{\sigma}]_{\alpha\beta}$$

D’=D/s

Wednesday, September 5, 2012

Kondo Hamiltonian

$$\frac{H_d^L}{D} = \hat{P}_L g \sum_{\alpha\beta} \int_{-1}^{1} dx \int_{-1}^{1} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta} \hat{P}_L$$

$$= g \sum_{\alpha\beta} \int_{-\frac{1}{s}}^{\frac{1}{s}} dx \int_{-\frac{1}{s}}^{\frac{1}{s}} dx' \, c^\dagger_{x\alpha} c_{x'\beta} [\vec{\sigma}]_{\alpha\beta}$$

$$= s^{-2} g \sum_{\alpha\beta} \int_{-1}^{1} d\bar{x} \int_{1}^{1} d\bar{x}' \, c^\dagger_{x(\bar{x})\alpha} c_{x'(\bar{x}')\beta} [\vec{\sigma}]_{\alpha\beta}$$

$$= s^{-1} s^{-1+1} g \sum_{\alpha\beta} \int_{-1}^{1} d\bar{x} \int_{1}^{1} d\bar{x}' \, c^\dagger_{\bar{x}\alpha} c_{\bar{x}'\beta} [\vec{\sigma}]_{\alpha\beta}$$

D'=D/s

Kondo interaction: marginal operator

Example: Coulomb interaction

$$\frac{H_C^L}{D} = \hat{P}_L u \sum_{\sigma\sigma'} \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1} dx_3 \int_{-1}^{1} dx_4 c^\dagger_{x_1\sigma} c^\dagger_{x_2\sigma'} c_{x_3\sigma'} c_{x_4\sigma} \hat{P}_L$$

$$= s^{-4} u \sum_{\sigma\sigma'} \int_{-1}^{1} d\bar{x}_1 \int_{-1}^{1} d\bar{x}_2 \int_{-1}^{1} d\bar{x}_3 \int_{-1}^{1} d\bar{x}_4 c^\dagger_{x_1(\bar{x}_1)\sigma} c^\dagger_{x_2(\bar{x}_2)\sigma'} c_{x_3(\bar{x}_3)\sigma'} c_{x_4(\bar{x}_4)\sigma}$$

Example: Coulomb interaction

$$\frac{H_C^L}{D} = \hat{P}_L u \sum_{\sigma\sigma'} \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1} dx_3 \int_{-1}^{1} dx_4 c^\dagger_{x_1\sigma} c^\dagger_{x_2\sigma'} c_{x_3\sigma'} c_{x_4\sigma} \hat{P}_L$$

$$= s^{-4} u \sum_{\sigma\sigma'} \int_{-1}^{1} d\bar{x}_1 \int_{-1}^{1} d\bar{x}_2 \int_{-1}^{1} d\bar{x}_3 \int_{-1}^{1} d\bar{x}_4 c^\dagger_{x_1(\bar{x}_1)\sigma} c^\dagger_{x_2(\bar{x}_2)\sigma'} c_{x_3(\bar{x}_3)\sigma'} c_{x_4(\bar{x}_4)\sigma}$$

rescale energies

Example: Coulomb interaction

$$\frac{H_C^L}{D} = \hat{P}_L u \sum_{\sigma\sigma'} \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1} dx_3 \int_{-1}^{1} dx_4 c^\dagger_{x_1\sigma} c^\dagger_{x_2\sigma'} c_{x_3\sigma'} c_{x_4\sigma} \hat{P}_L$$

$$= s^{-4} u \sum_{\sigma\sigma'} \int_{-1}^{1} d\bar{x}_1 \int_{-1}^{1} d\bar{x}_2 \int_{-1}^{1} d\bar{x}_3 \int_{-1}^{1} d\bar{x}_4 c^\dagger_{x_1(\bar{x}_1)\sigma} c^\dagger_{x_2(\bar{x}_2)\sigma'} c_{x_3(\bar{x}_3)\sigma'} c_{x_4(\bar{x}_4)\sigma}$$

$$= s^{-1} s^{-3+2} u \sum_{\sigma\sigma'} \int_{-1}^{1} d\bar{x}_1 \int_{-1}^{1} d\bar{x}_2 \int_{-1}^{1} d\bar{x}_3 \int_{-1}^{1} d\bar{x}_4 c^\dagger_{\bar{x}_1\sigma} c^\dagger_{\bar{x}_2\sigma'} c_{\bar{x}_3\sigma'} c_{\bar{x}_4\sigma}$$

Example: Coulomb interaction

$$\frac{H_C^L}{D} = \hat{P}_L u \sum_{\sigma\sigma'} \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1} dx_3 \int_{-1}^{1} dx_4 c^\dagger_{x_1\sigma} c^\dagger_{x_2\sigma'} c_{x_3\sigma'} c_{x_4\sigma} \hat{P}_L$$

$$= s^{-4} u \sum_{\sigma\sigma'} \int_{-1}^{1} d\bar{x}_1 \int_{-1}^{1} d\bar{x}_2 \int_{-1}^{1} d\bar{x}_3 \int_{-1}^{1} d\bar{x}_4 c^\dagger_{x_1(\bar{x}_1)\sigma} c^\dagger_{x_2(\bar{x}_2)\sigma'} c_{x_3(\bar{x}_3)\sigma'} c_{x_4(\bar{x}_4)\sigma}$$

$$= s^{-1} s^{-3+2} u \sum_{\sigma\sigma'} \int_{-1}^{1} d\bar{x}_1 \int_{-1}^{1} d\bar{x}_2 \int_{-1}^{1} d\bar{x}_3 \int_{-1}^{1} d\bar{x}_4 c^\dagger_{\bar{x}_1\sigma} c^\dagger_{\bar{x}_2\sigma'} c_{\bar{x}_3\sigma'} c_{\bar{x}_4\sigma}$$

$$u' = s^{-1} u$$

Example: Coulomb interaction

$$\frac{H_C^L}{D} = \hat{P}_L u \sum_{\sigma\sigma'} \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1} dx_3 \int_{-1}^{1} dx_4 c^\dagger_{x_1\sigma} c^\dagger_{x_2\sigma'} c_{x_3\sigma'} c_{x_4\sigma} \hat{P}_L$$

$$= s^{-4} u \sum_{\sigma\sigma'} \int_{-1}^{1} d\bar{x}_1 \int_{-1}^{1} d\bar{x}_2 \int_{-1}^{1} d\bar{x}_3 \int_{-1}^{1} d\bar{x}_4 c^\dagger_{x_1(\bar{x}_1)\sigma} c^\dagger_{x_2(\bar{x}_2)\sigma'} c_{x_3(\bar{x}_3)\sigma'} c_{x_4(\bar{x}_4)\sigma}$$

$$= s^{-1} s^{-3+2} u \sum_{\sigma\sigma'} \int_{-1}^{1} d\bar{x}_1 \int_{-1}^{1} d\bar{x}_2 \int_{-1}^{1} d\bar{x}_3 \int_{-1}^{1} d\bar{x}_4 c^\dagger_{\bar{x}_1\sigma} c^\dagger_{\bar{x}_2\sigma'} c_{\bar{x}_3\sigma'} c_{\bar{x}_4\sigma}$$

$u' = s^{-1} u$ irrelevant interaction

poor man's scaling

Kondo Hamiltonian

$$\frac{H}{D} = \sum_\sigma \int_{-1}^{1} dx\; x c^\dagger_{x\sigma} c_{x\sigma} + g \int_{-1}^{1} dx \int_{-1}^{1} dx' \sum_{\alpha\beta} c^\dagger_{x\alpha} \vec{\sigma} c_{x'\beta} \vec{S}_{imp}$$

Transformation: elimination of modes

$$\hat{H}' = \hat{U}^\dagger H \hat{U} = e^{\lambda \hat{S}} \hat{H} e^{-\lambda \hat{S}} = \hat{H}_d + \lambda \hat{V} + \lambda [\hat{S}, \hat{H}_d] + \lambda^2 [\hat{S}, \hat{V}] + \sum_{n=2} \frac{\lambda^n}{n!} [\hat{S}, \hat{H}]_n$$

poor man's scaling

Kondo Hamiltonian

$$\frac{H}{D} = \sum_{\sigma} \int_{-1}^{1} dx \; x c^{\dagger}_{x\sigma} c_{x\sigma} + g \int_{-1}^{1} dx \int_{-1}^{1} dx' \sum_{\alpha\beta} c^{\dagger}_{x\alpha} \vec{\sigma} c_{x'\beta} \vec{S}_{imp}$$

Transformation: elimination of modes

$$\hat{H}' = \hat{U}^{\dagger} H \hat{U} = e^{\lambda \hat{S}} \hat{H} e^{-\lambda \hat{S}} = \hat{H}_d + \lambda \hat{V} + \lambda [\hat{S}, \hat{H}_d] + \lambda^2 [\hat{S}, \hat{V}] + \sum_{n=2} \frac{\lambda^n}{n!} [\hat{S}, \hat{H}]_n$$

poor man's scaling

Kondo Hamiltonian

$$\frac{H}{D} = \sum_\sigma \int_{-1}^{1} dx\; x c^\dagger_{x\sigma} c_{x\sigma} + g \int_{-1}^{1} dx \int_{-1}^{1} dx' \sum_{\alpha\beta} c^\dagger_{x\alpha} \vec{\sigma} c_{x'\beta} \vec{S}_{imp}$$

Transformation: elimination of modes

$$\hat{H}' = \hat{U}^\dagger H \hat{U} = e^{\lambda \hat{S}} \hat{H} e^{-\lambda \hat{S}} = \hat{H}_d + \lambda \hat{V} + \lambda [\hat{S}, \hat{H}_d] + \lambda^2 [\hat{S}, \hat{V}] + \sum_{n=2} \frac{\lambda^n}{n!} [\hat{S}, \hat{H}]_n$$

$$\hat{V} + [\hat{S}, \hat{H}_d] = 0$$

poor man's scaling

Kondo Hamiltonian

$$\frac{H}{D} = \sum_{\sigma} \int_{-1}^{1} dx\; x c^{\dagger}_{x\sigma} c_{x\sigma} + g \int_{-1}^{1} dx \int_{-1}^{1} dx' \sum_{\alpha\beta} c^{\dagger}_{x\alpha} \vec{\sigma} c_{x'\beta} \vec{S}_{imp}$$

Transformation: elimination of modes

$$\hat{H}' = \hat{U}^{\dagger} H \hat{U} = e^{\lambda \hat{S}} \hat{H} e^{-\lambda \hat{S}} = \hat{H}_d + \lambda \hat{V} + \lambda [\hat{S}, \hat{H}_d] + \lambda^2 [\hat{S}, \hat{V}] + \sum_{n=2} \frac{\lambda^n}{n!} [\hat{S}, \hat{H}]_n$$

$$\hat{V} + [\hat{S}, \hat{H}_d] = 0$$

determines S

poor man's scaling

Kondo Hamiltonian

$$\frac{H}{D} = \sum_{\sigma} \int_{-1}^{1} dx \; x c_{x\sigma}^{\dagger} c_{x\sigma} + g \int_{-1}^{1} dx \int_{-1}^{1} dx' \sum_{\alpha\beta} c_{x\alpha}^{\dagger} \vec{\sigma} c_{x'\beta} \vec{S}_{imp}$$

Transformation: elimination of modes

$$\hat{H}' = \hat{U}^{\dagger} H \hat{U} = e^{\lambda \hat{S}} \hat{H} e^{-\lambda \hat{S}} = \hat{H}_d + \lambda \hat{V} + \lambda [\hat{S}, \hat{H}_d] + \lambda^2 [\hat{S}, \hat{V}] + \sum_{n=2} \frac{\lambda^n}{n!} [\hat{S}, \hat{H}]_n$$

$$\hat{V} + [\hat{S}, \hat{H}_d] = 0$$

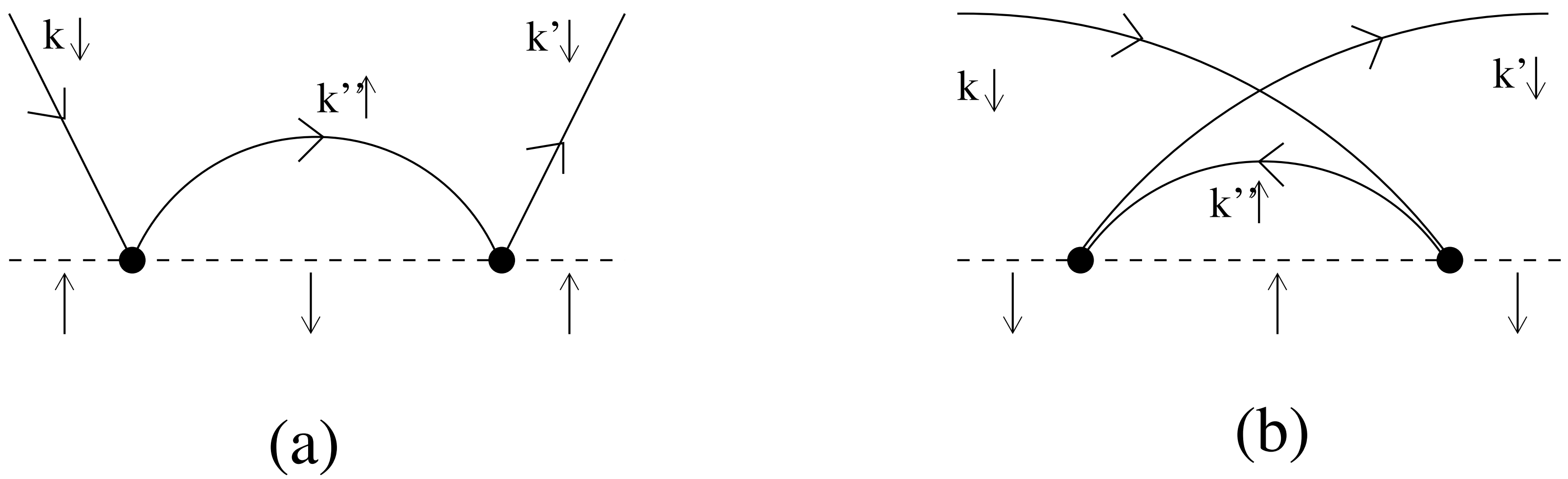
determines S

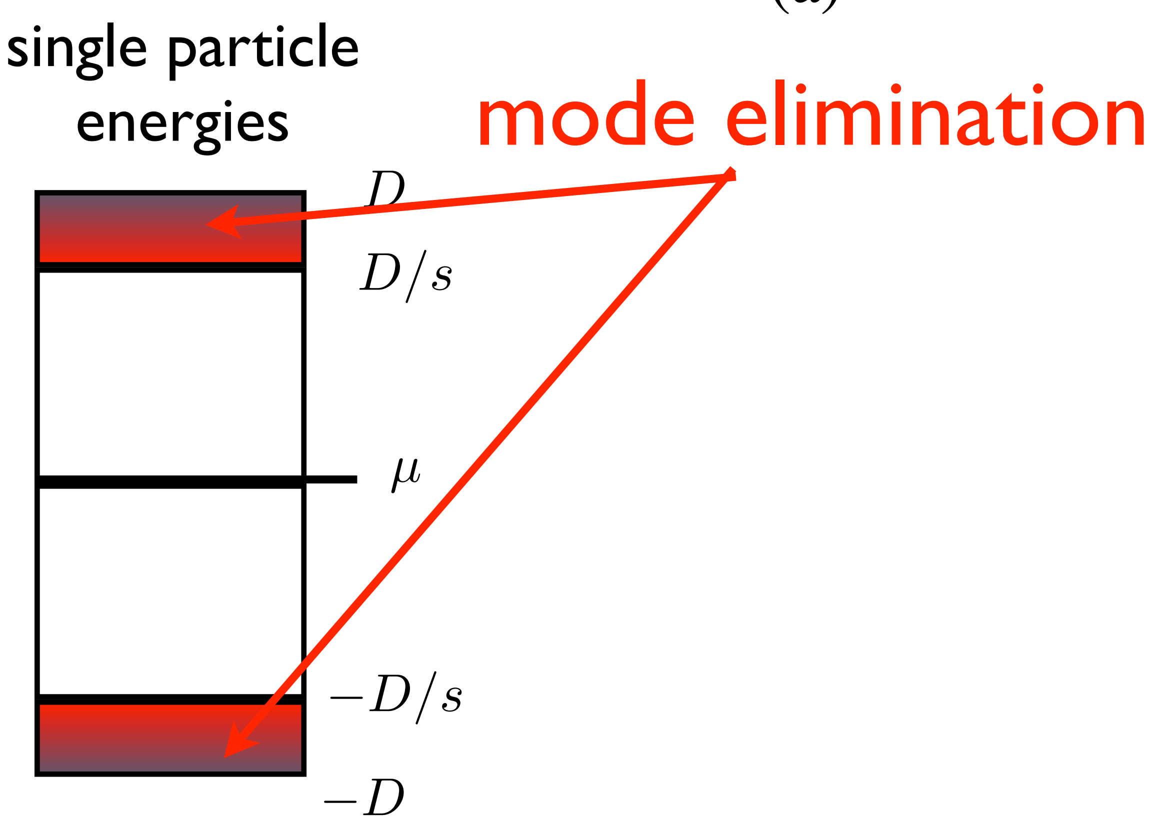
$$H' = H_d + \frac{\lambda^2}{2} [S, V] + O(\lambda^3)$$

Schrieffer-Wolff transformation

poor man's scaling

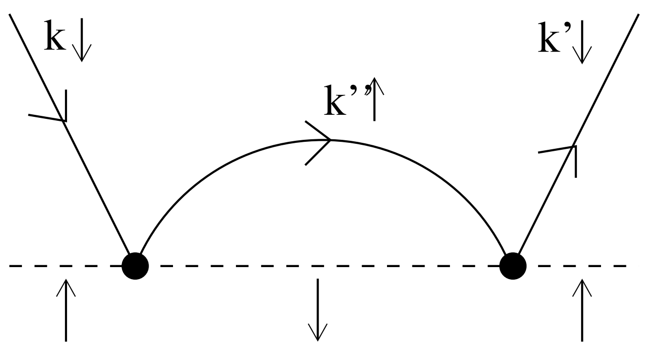
effective Kondo coupling: diagrammatic approach



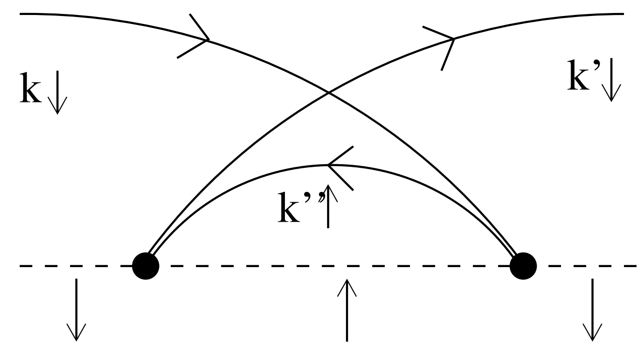


poor man’s scaling

effective Kondo coupling: diagrammatic approach



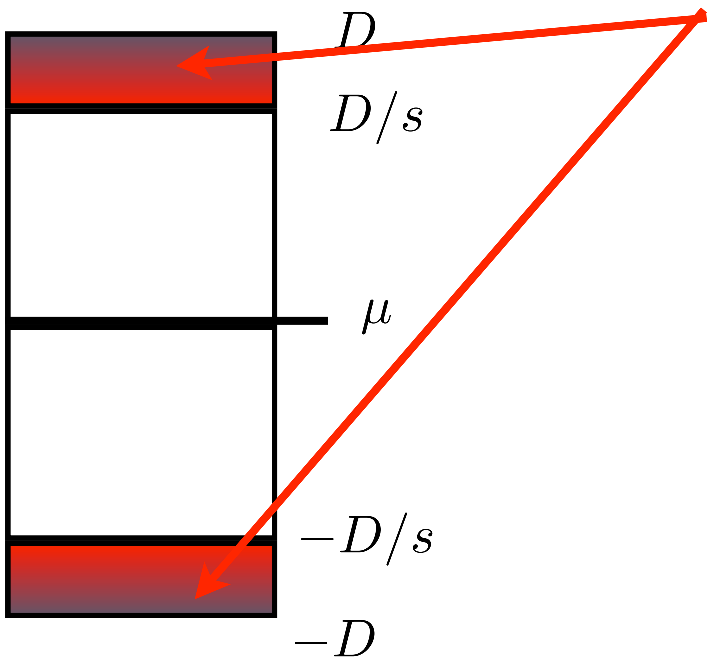
(a)



(b)

single particle energies

mode elimination



scaling equations

$$\frac{dg_\perp}{d\ln\mathcal{D}} = -2g_\perp g^z \quad ; \qquad \frac{dg^z}{d\ln\mathcal{D}} = -2g_\perp^2$$

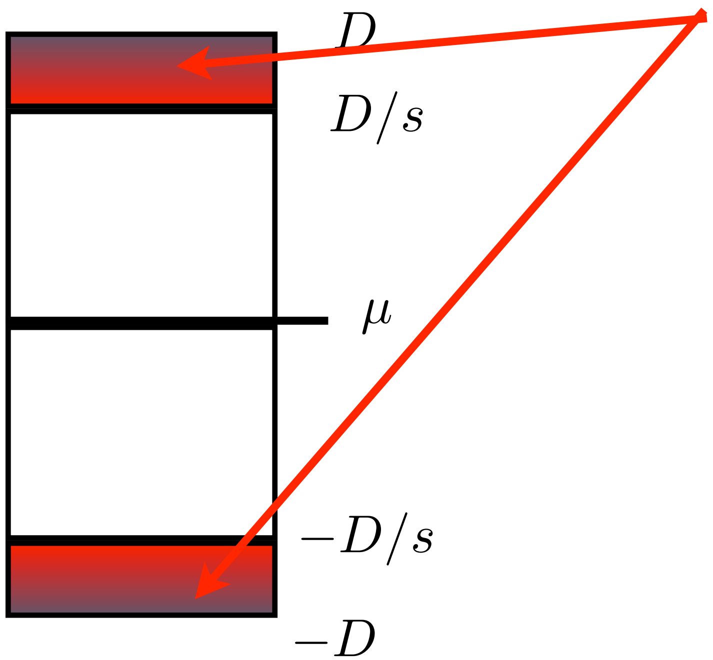
poor man's scaling

effective Kondo coupling: diagrammatic approach

(a) (b)

single particle energies

mode elimination



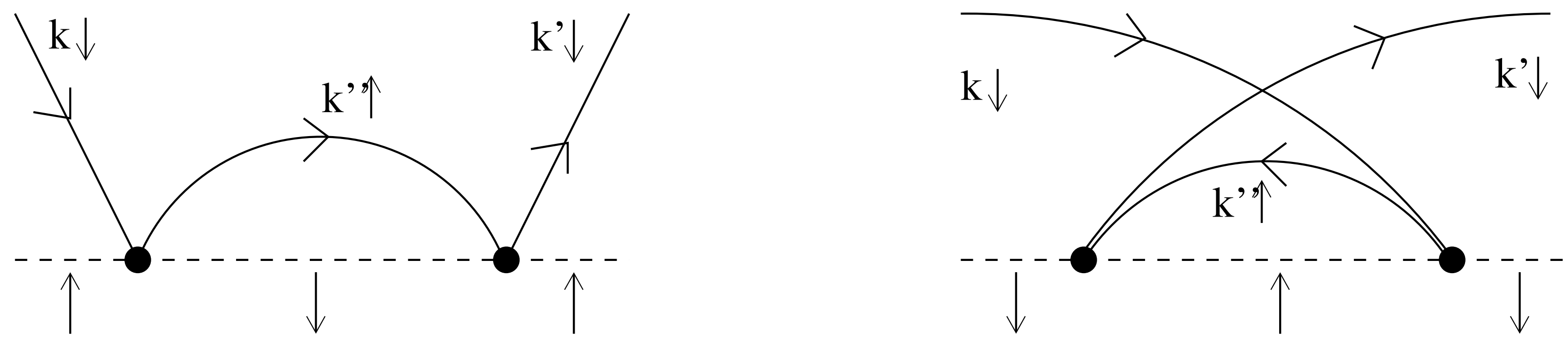
scaling equations

$$\frac{dg_\perp}{d\ln\mathcal{D}} = -2g_\perp g^z \quad ; \quad \frac{dg^z}{d\ln\mathcal{D}} = -2g_\perp^2$$

$$g(D') = \frac{g_0}{1 + 2g_0 \ln(D_0/D')}$$

poor man's scaling

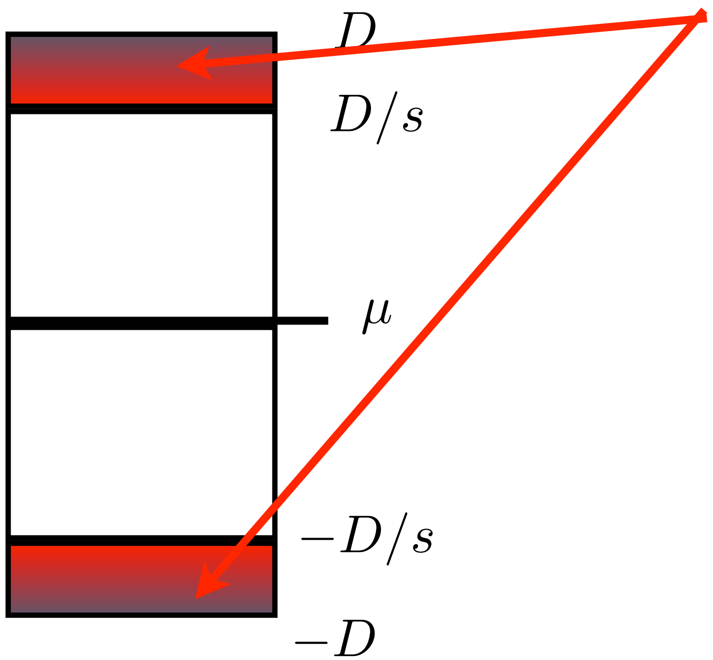
effective Kondo coupling: diagrammatic approach



(a) (b)

single particle energies

mode elimination



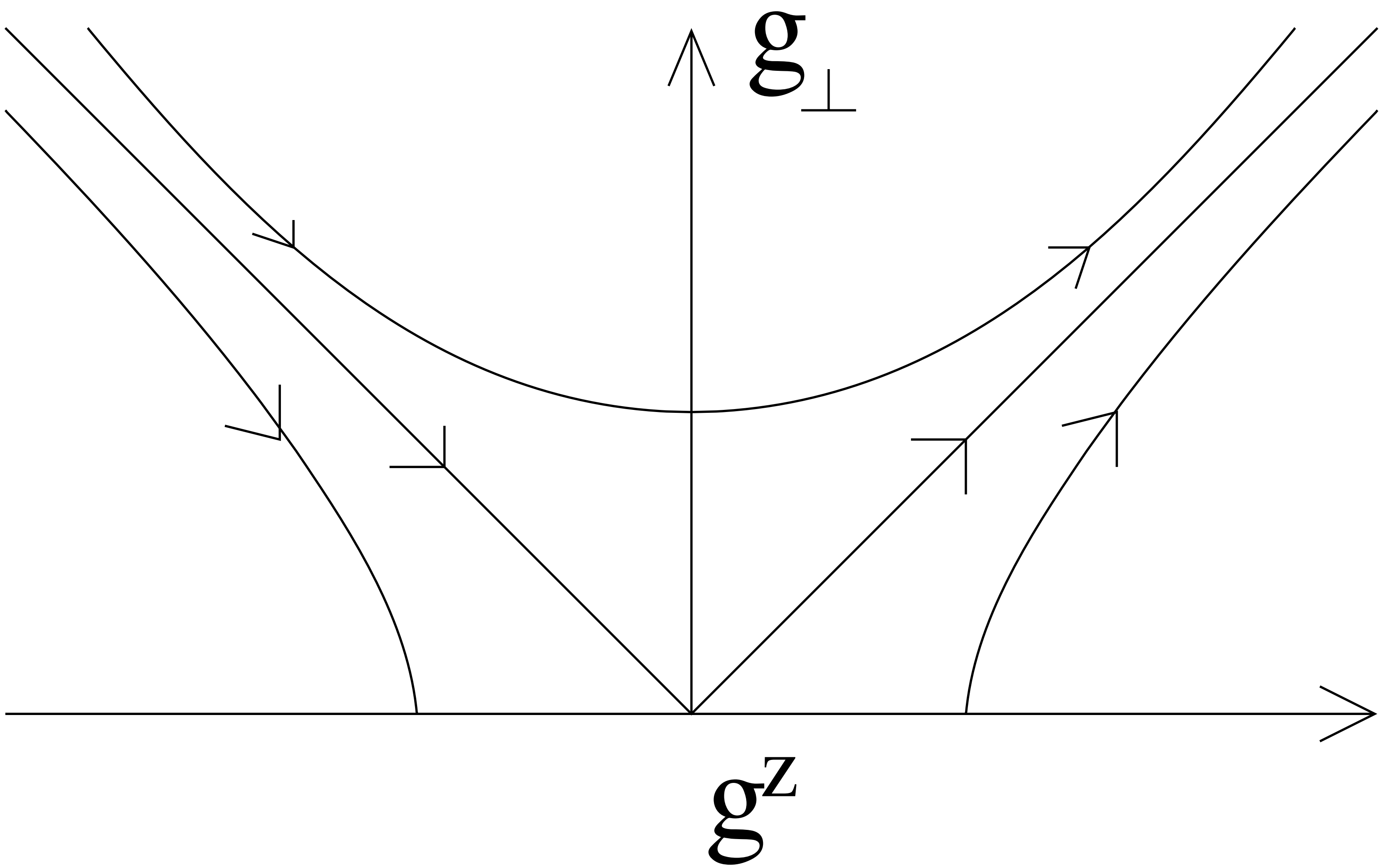
scaling equations

$$\frac{dg_\perp}{d\ln\mathcal{D}} = -2g_\perp g^z \quad ; \quad \frac{dg^z}{d\ln\mathcal{D}} = -2g_\perp^2$$

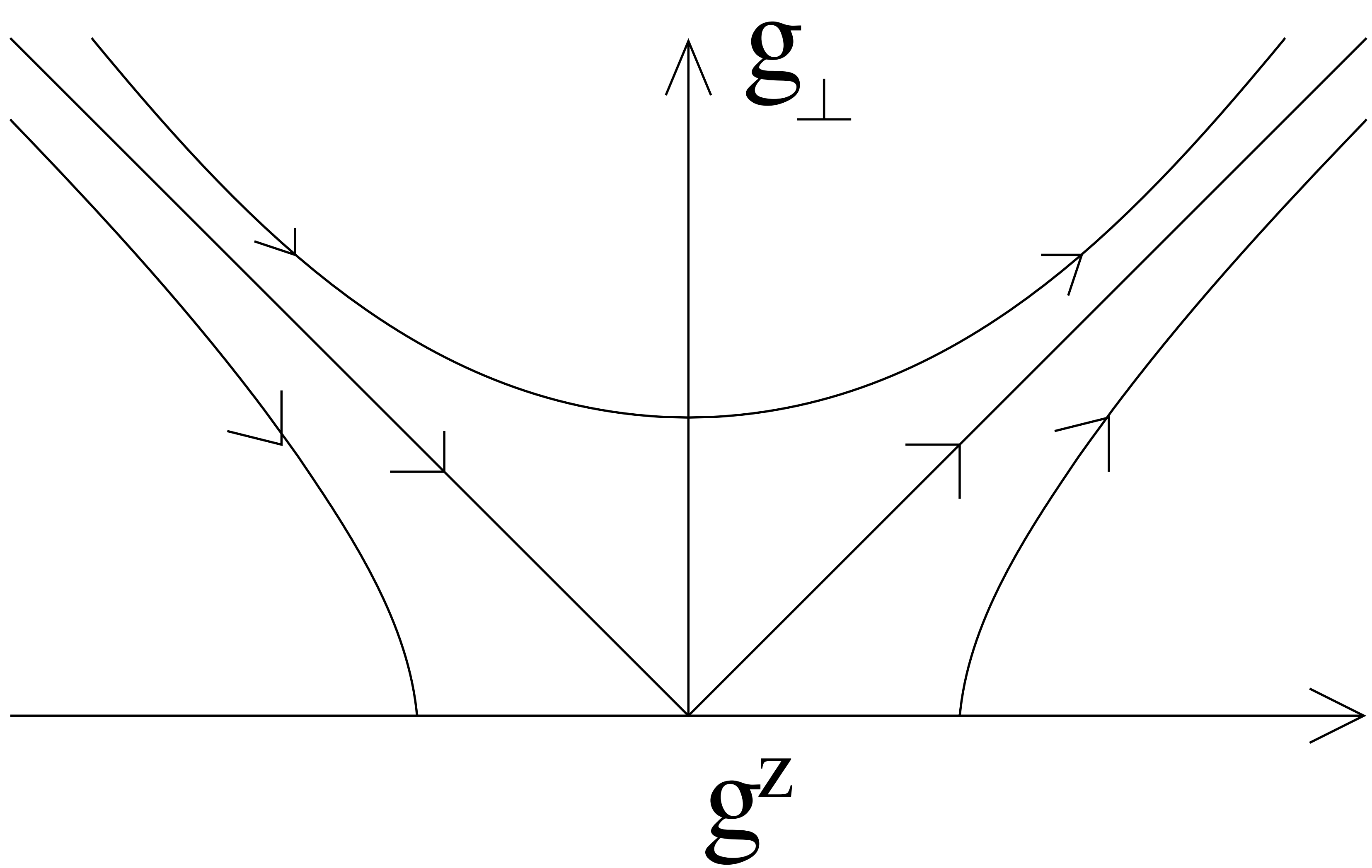
$$g(D') = \frac{g_0}{1 + 2g_0 \ln(D_0/D')}$$

Kondo temperature $T_K = D_0 e^{-1/2g_0} = D_0 e^{-1/\rho_0 J}$

RG Flow

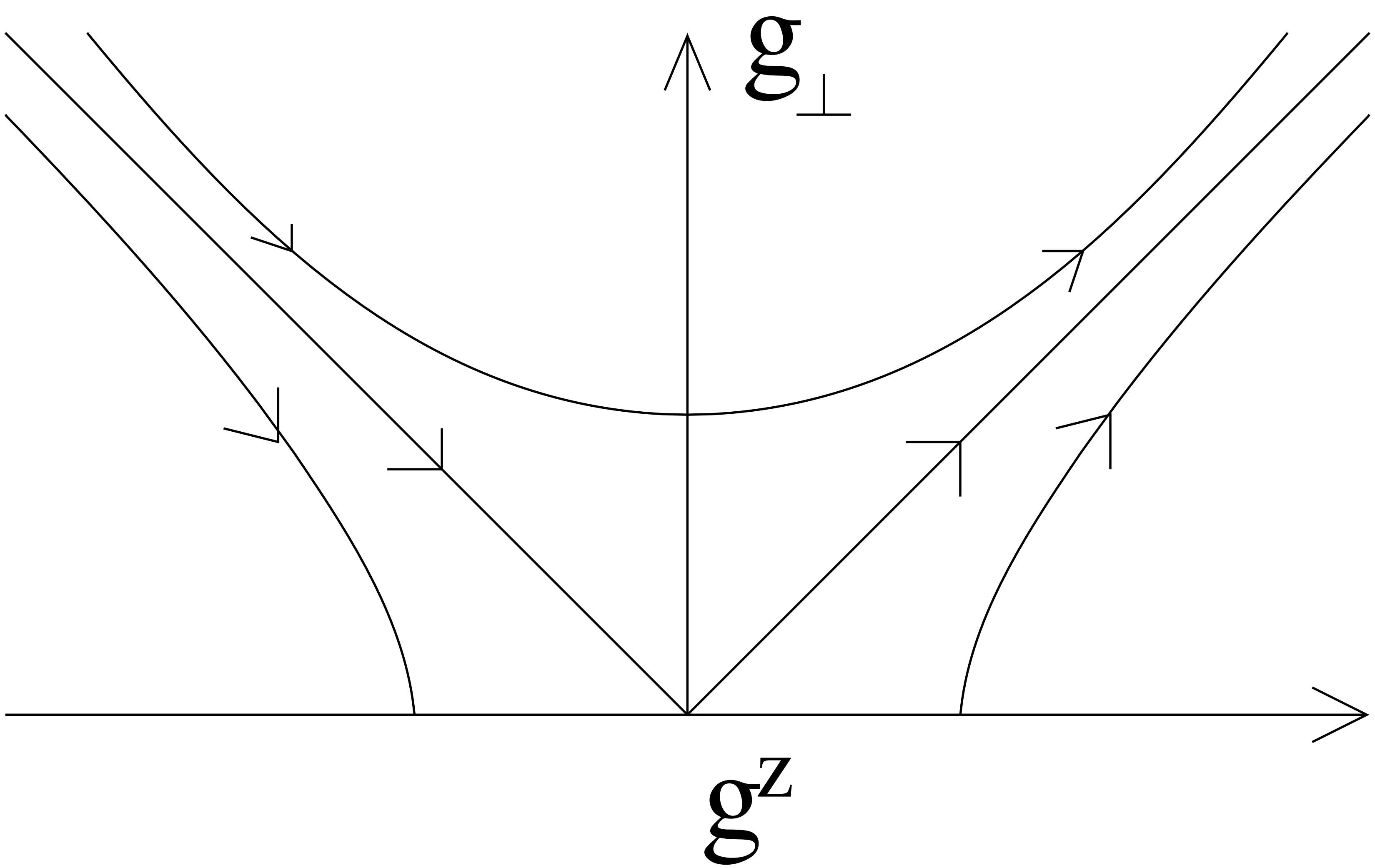


RG Flow



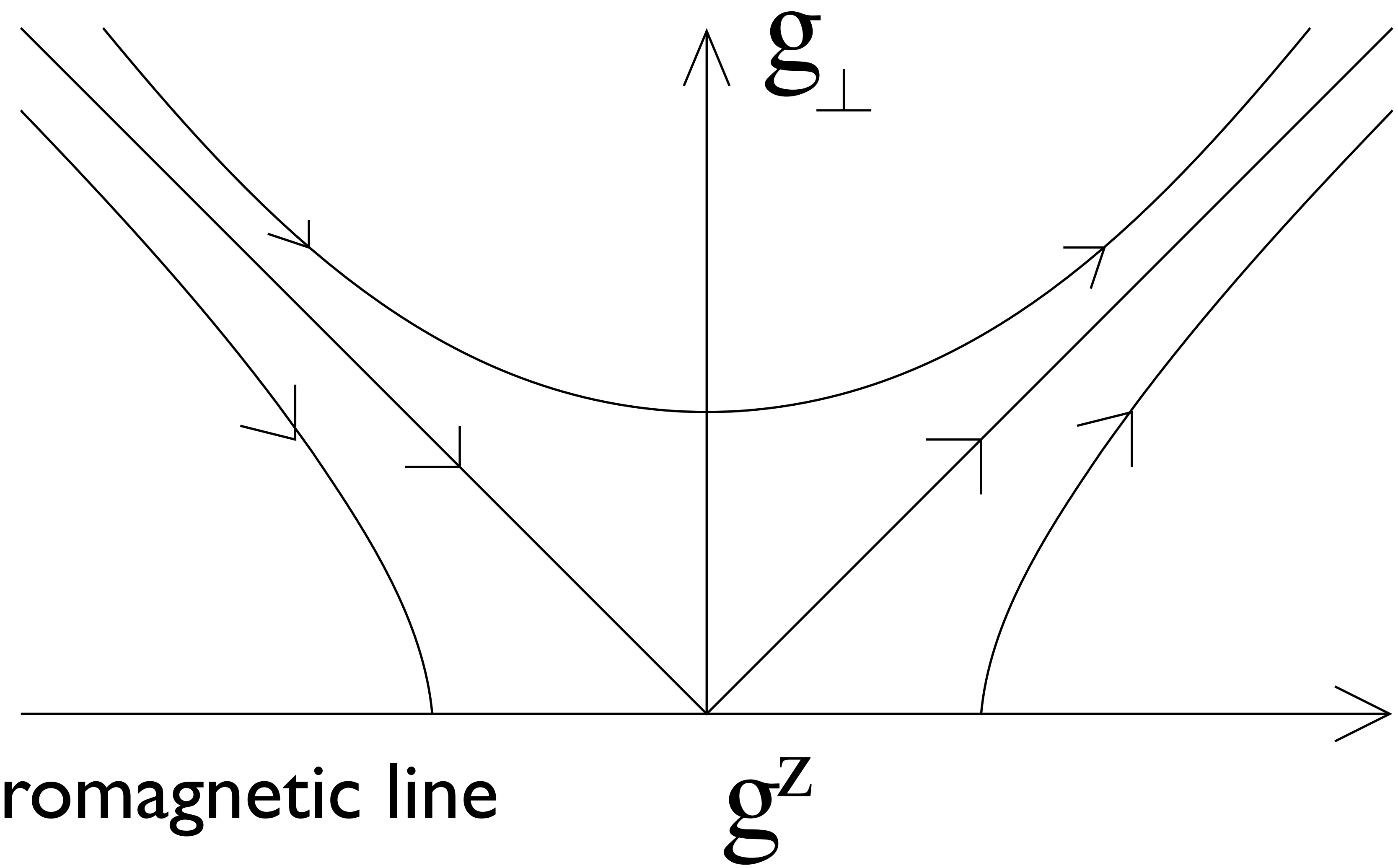
$$\frac{dg_\perp}{d\ln\mathcal{D}} = -2g_\perp g^z \quad ; \qquad \frac{dg^z}{d\ln\mathcal{D}} = -2g_\perp^2$$

RG Flow



$$\frac{dg_\perp}{d\ln\mathcal{D}} = -2g_\perp g^z \quad ; \quad \frac{dg^z}{d\ln\mathcal{D}} = -2g_\perp^2 \longrightarrow [g^z]^2 - g_\perp^2 = \text{const}$$

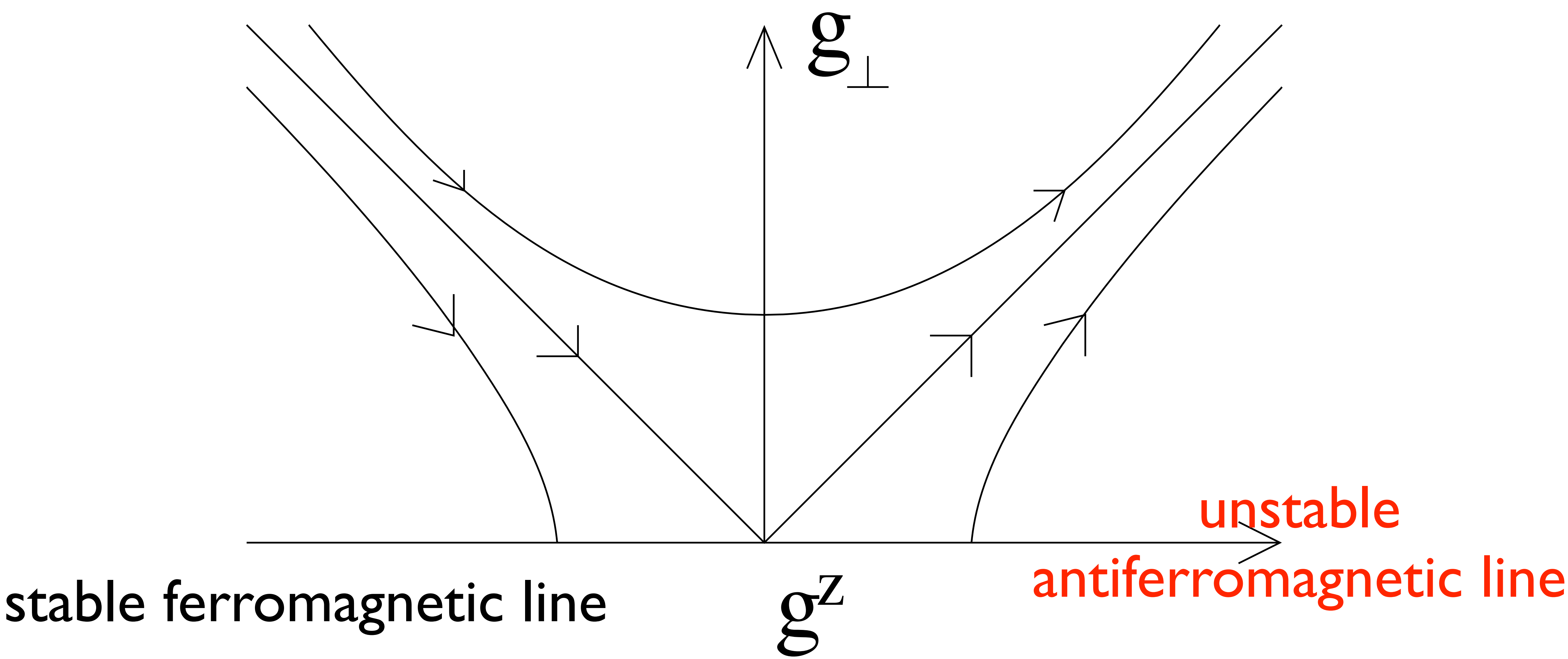
RG Flow



stable ferromagnetic line

$$\frac{dg_\perp}{d\ln\mathcal{D}} = -2g_\perp g^z \quad ; \quad \frac{dg^z}{d\ln\mathcal{D}} = -2g_\perp^2 \longrightarrow [g^z]^2 - g_\perp^2 = \text{const}$$

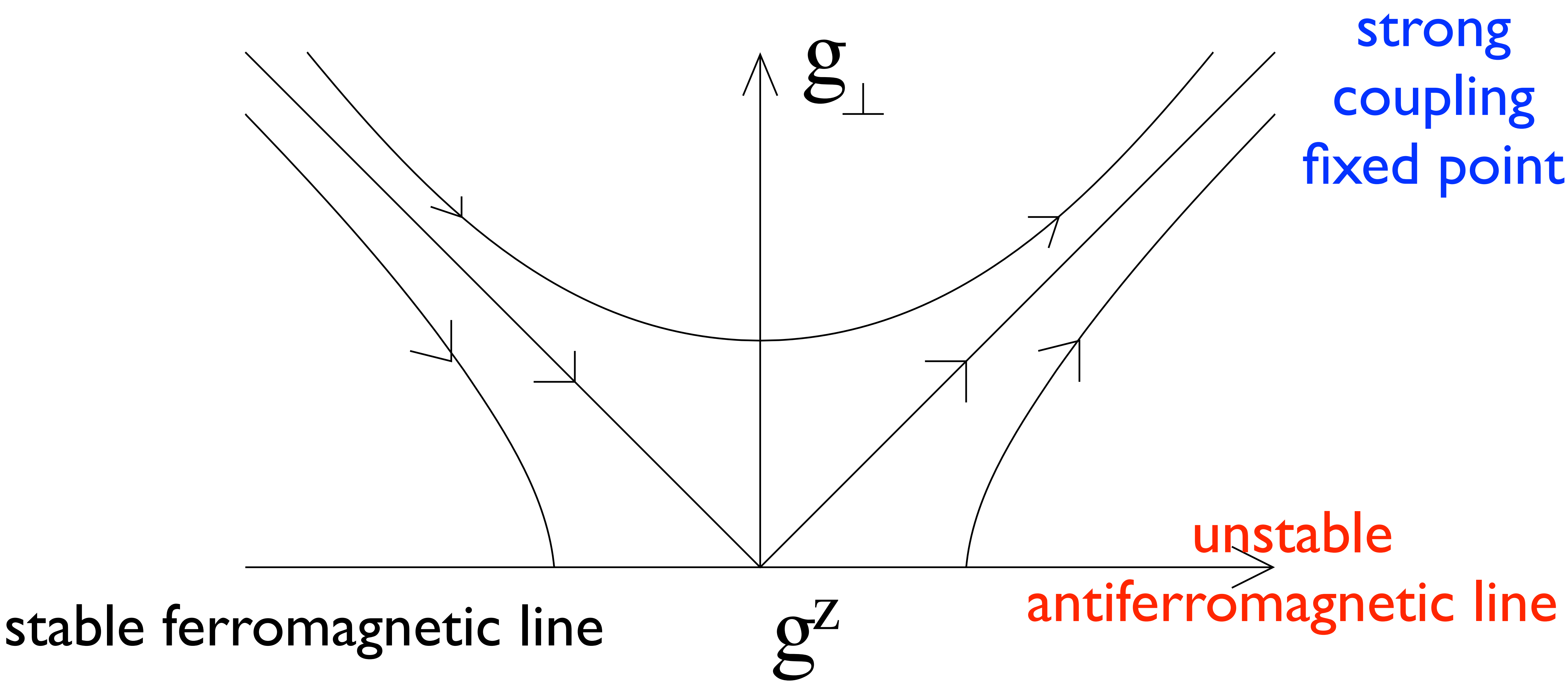
RG Flow



$$\frac{dg_\perp}{d\ln\mathcal{D}} = -2g_\perp g^z \quad ; \quad \frac{dg^z}{d\ln\mathcal{D}} = -2g_\perp^2 \longrightarrow [g^z]^2 - g_\perp^2 = \mathrm{const}$$

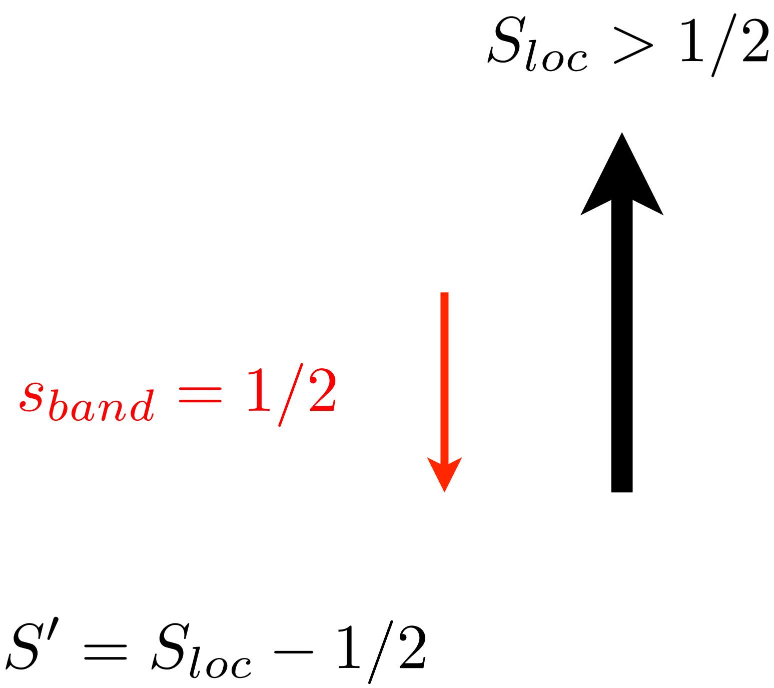
Wednesday, September 5, 2012

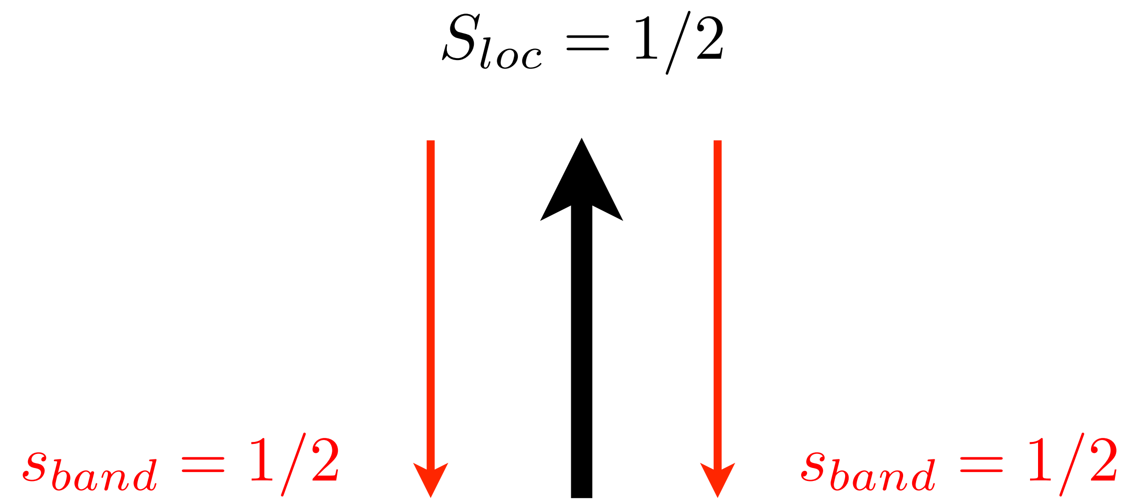
RG Flow



$$\frac{dg_\perp}{d\ln\mathcal{D}} = -2g_\perp g^z \quad ; \qquad \frac{dg^z}{d\ln\mathcal{D}} = -2g_\perp^2 \longrightarrow [g^z]^2 - g_\perp^2 = \text{const}$$

Exotic Kondo effects





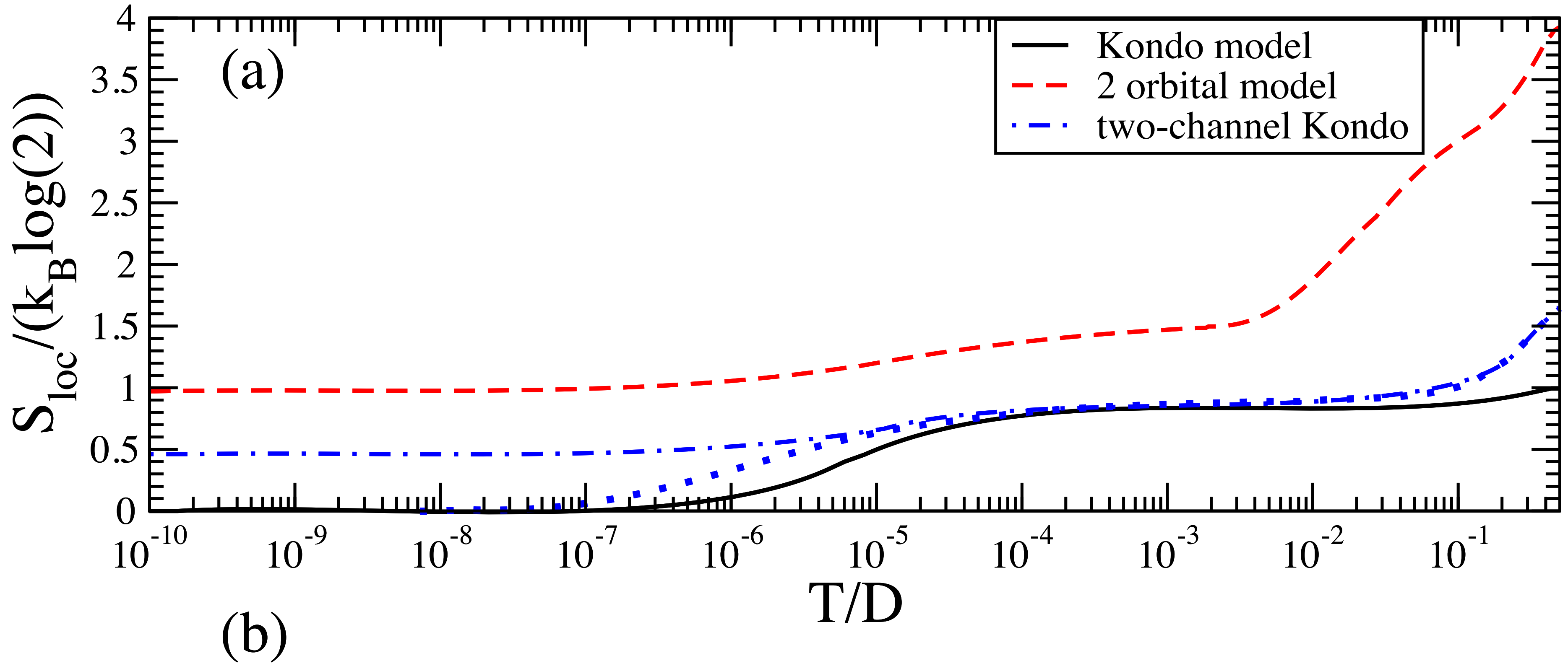
under-screened Kondo:

- residual entropy: log(S')
- singular Fermi liquid: free local spin+strong coupling fixed point

over-screened Kondo

- residual entropy: log(2)/2
- non Fermi liquid

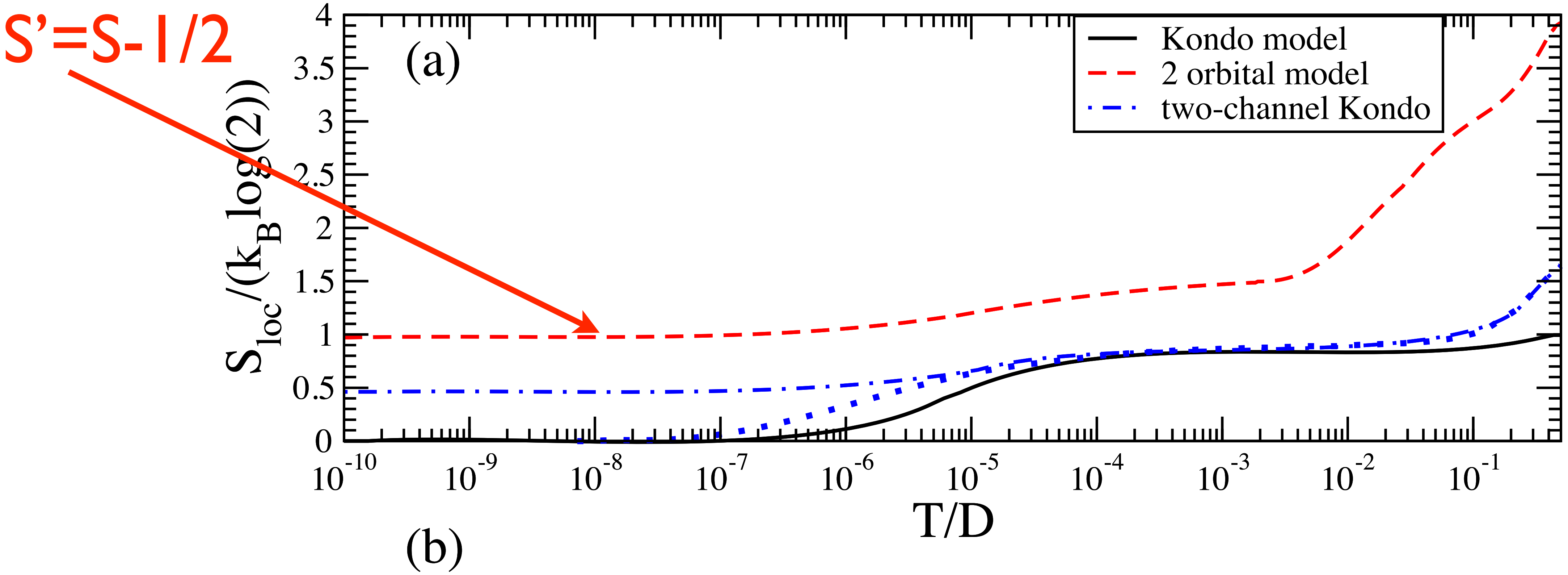
Exotic Kondo effects

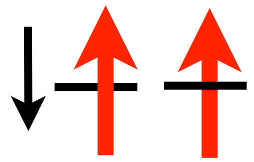


NRG calculations

Wednesday, September 5, 2012

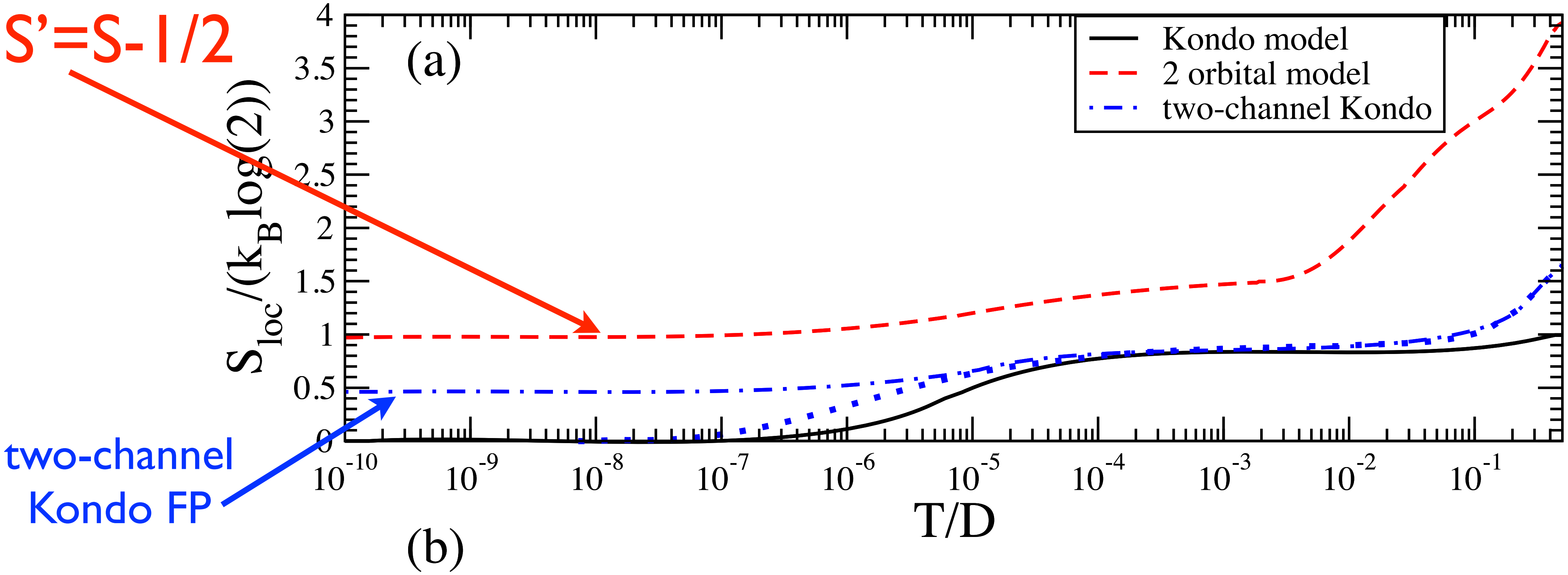
Exotic Kondo effects

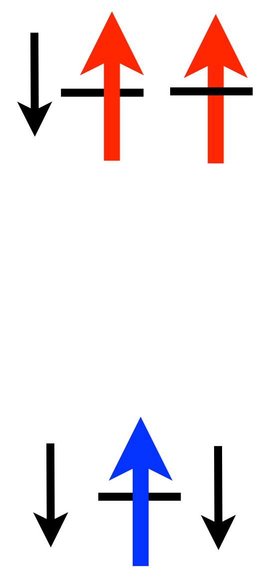




NRG calculations

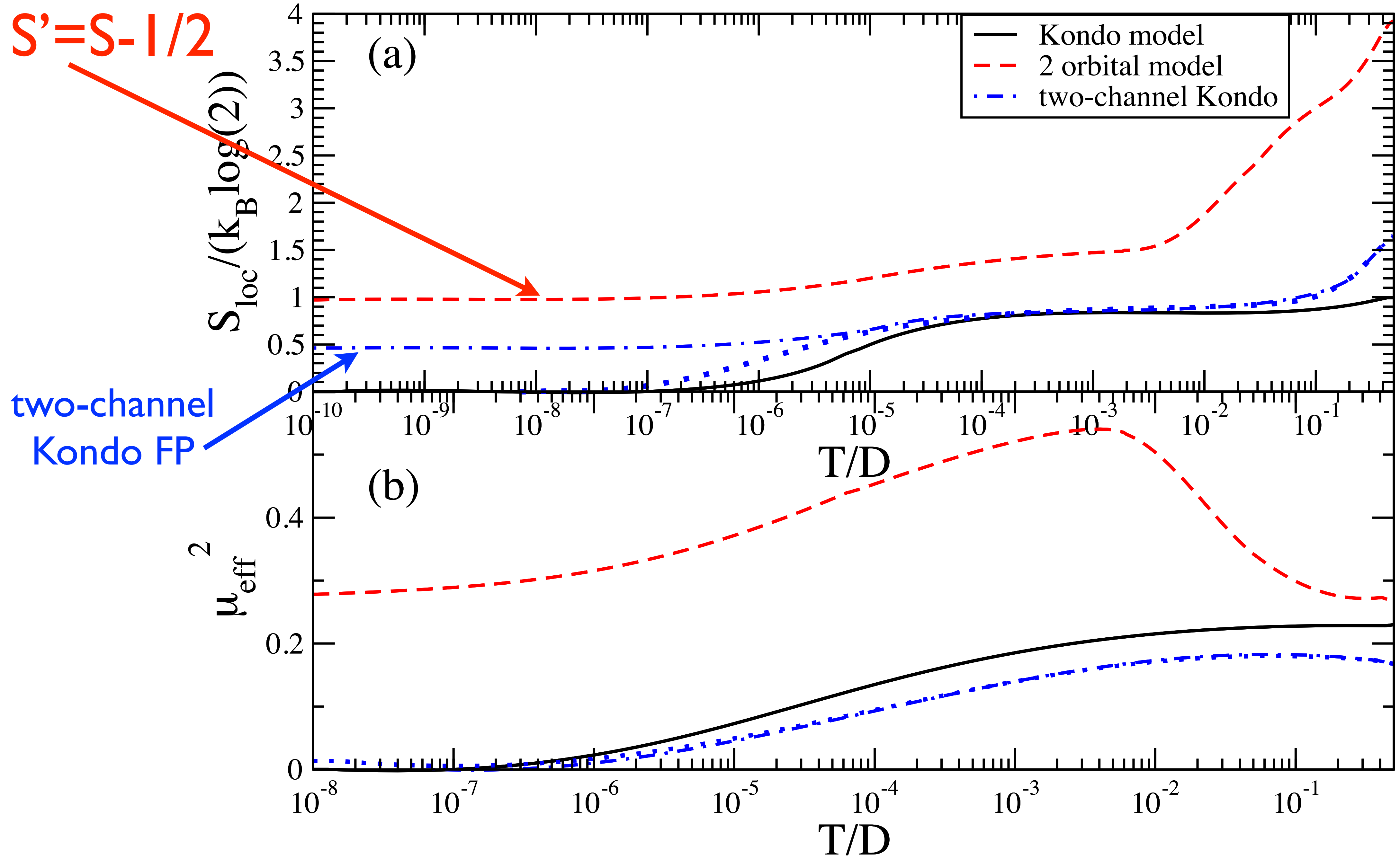
Exotic Kondo effects

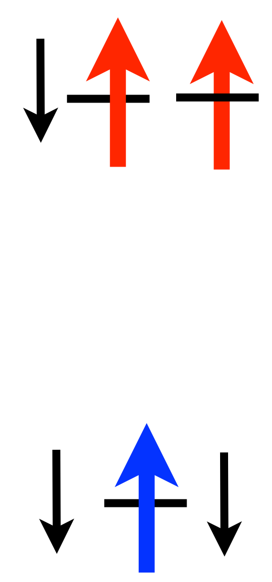




NRG calculations

Exotic Kondo effects

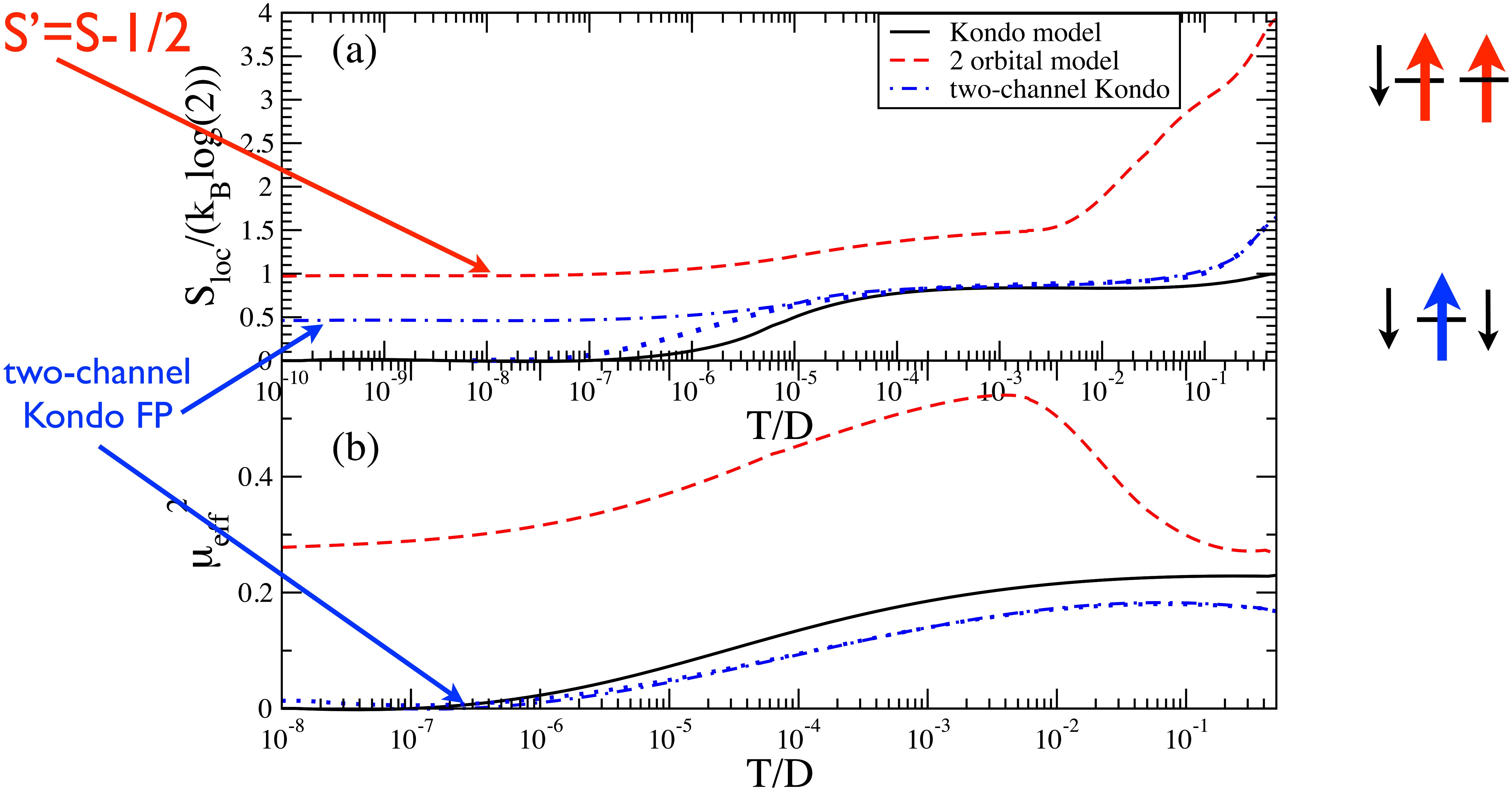




NRG calculations

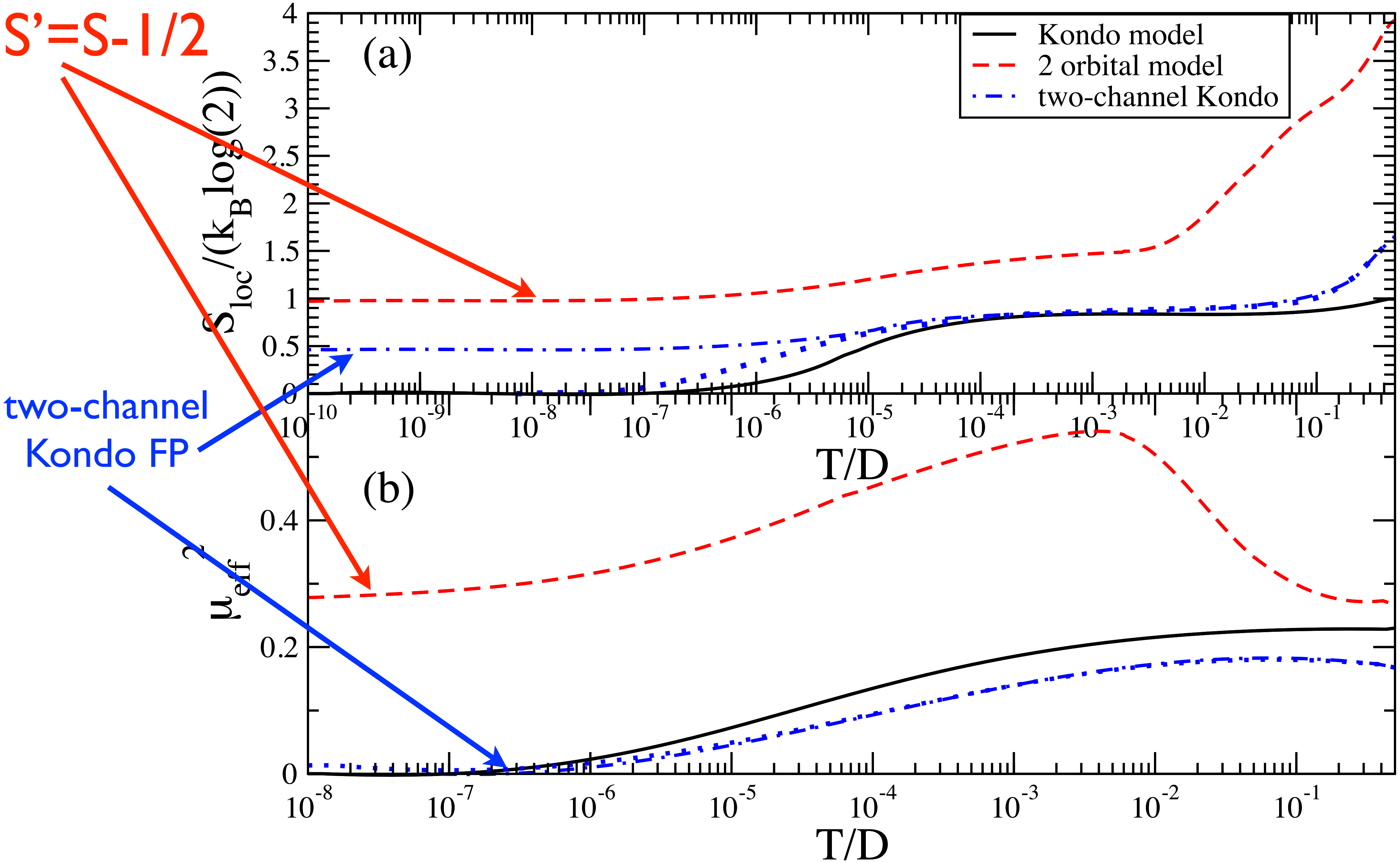
Wednesday, September 5, 2012

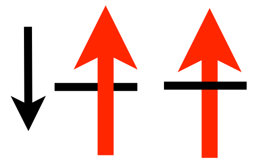
Exotic Kondo effects

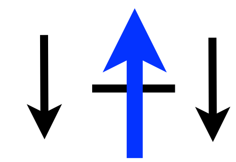


NRG calculations

Exotic Kondo effects



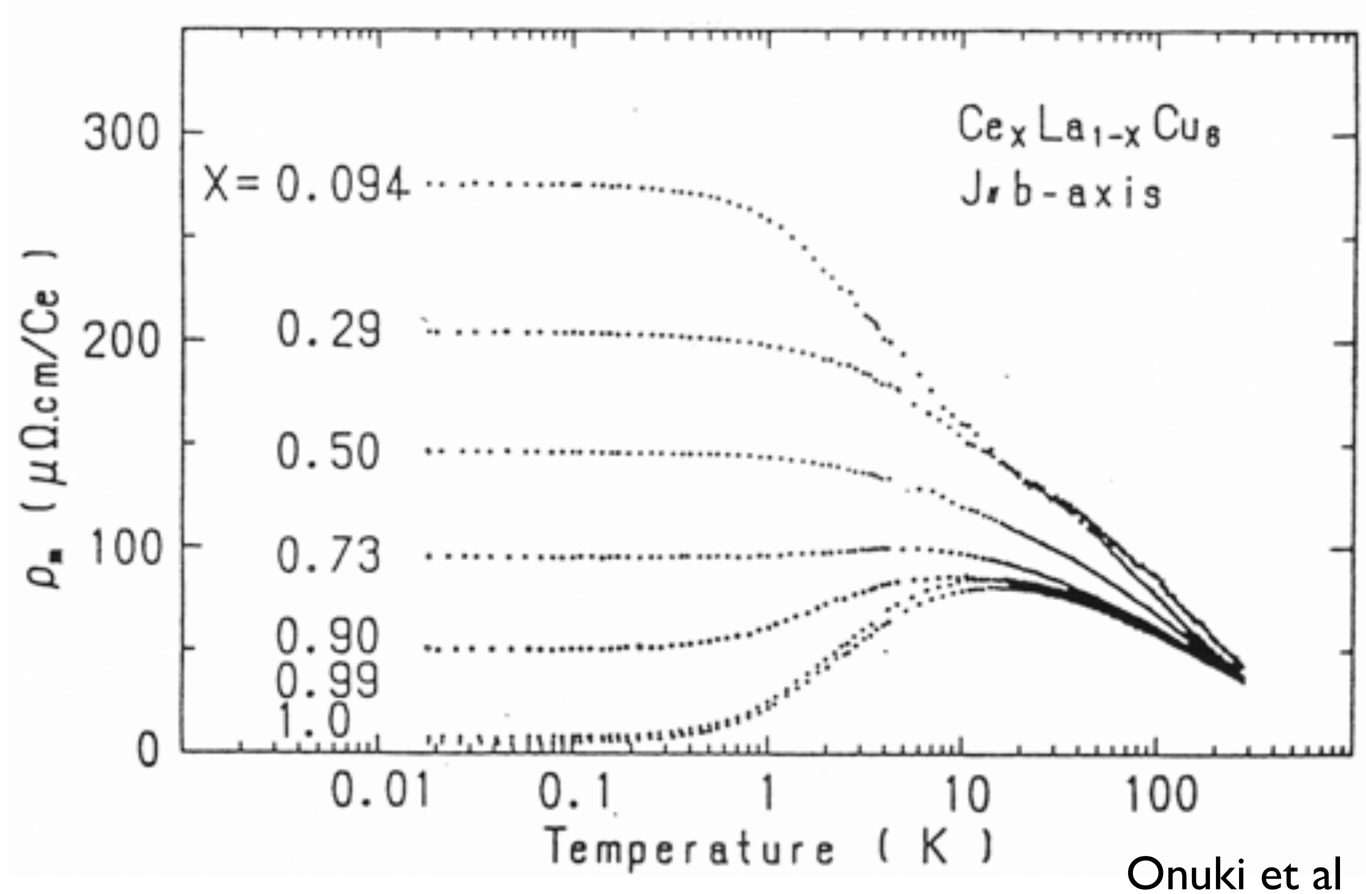




NRG calculations

Kondo effect in Lattice systems

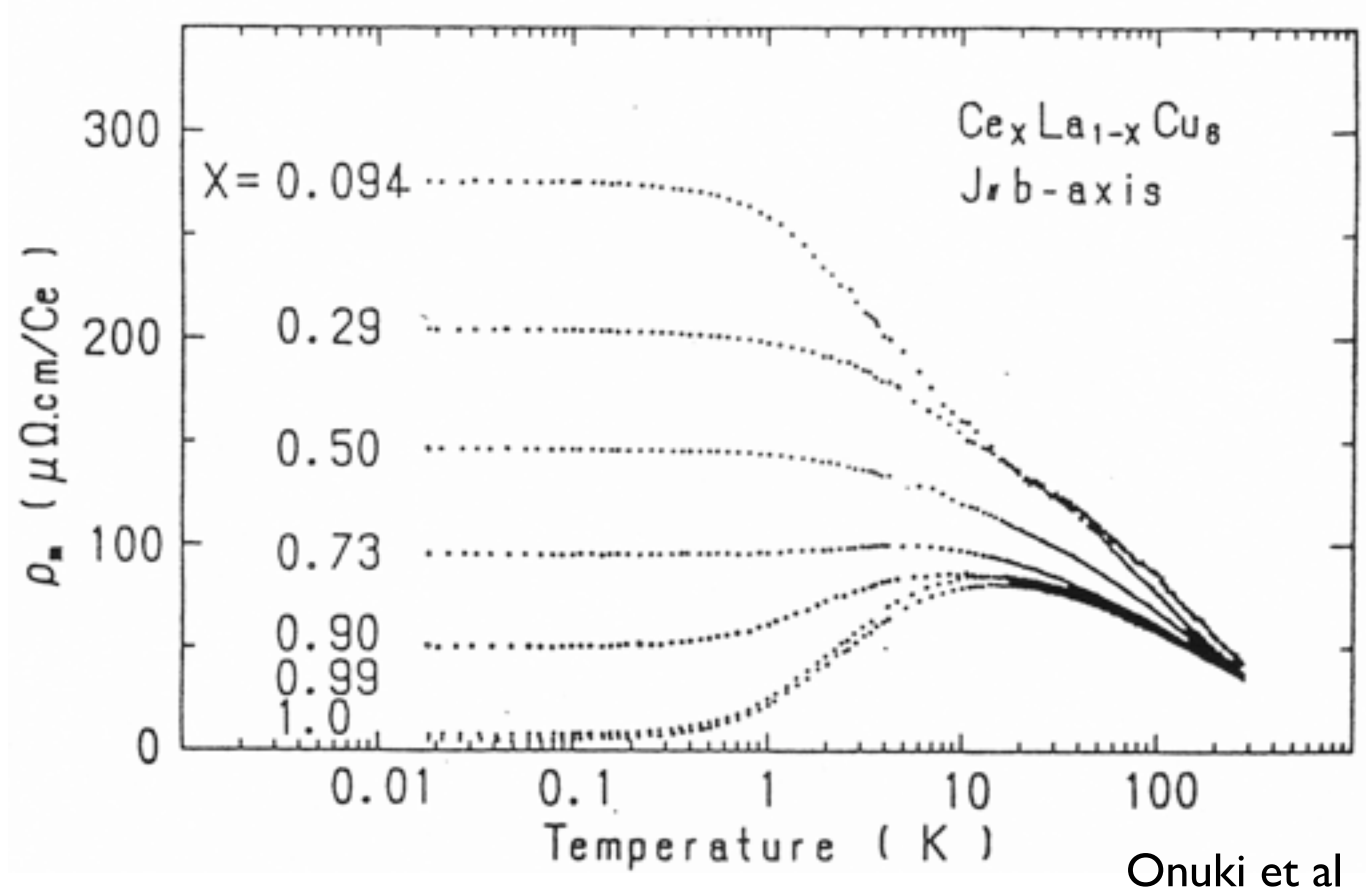
Heavy Fermions



Onuki et al 1987

Heavy Fermions

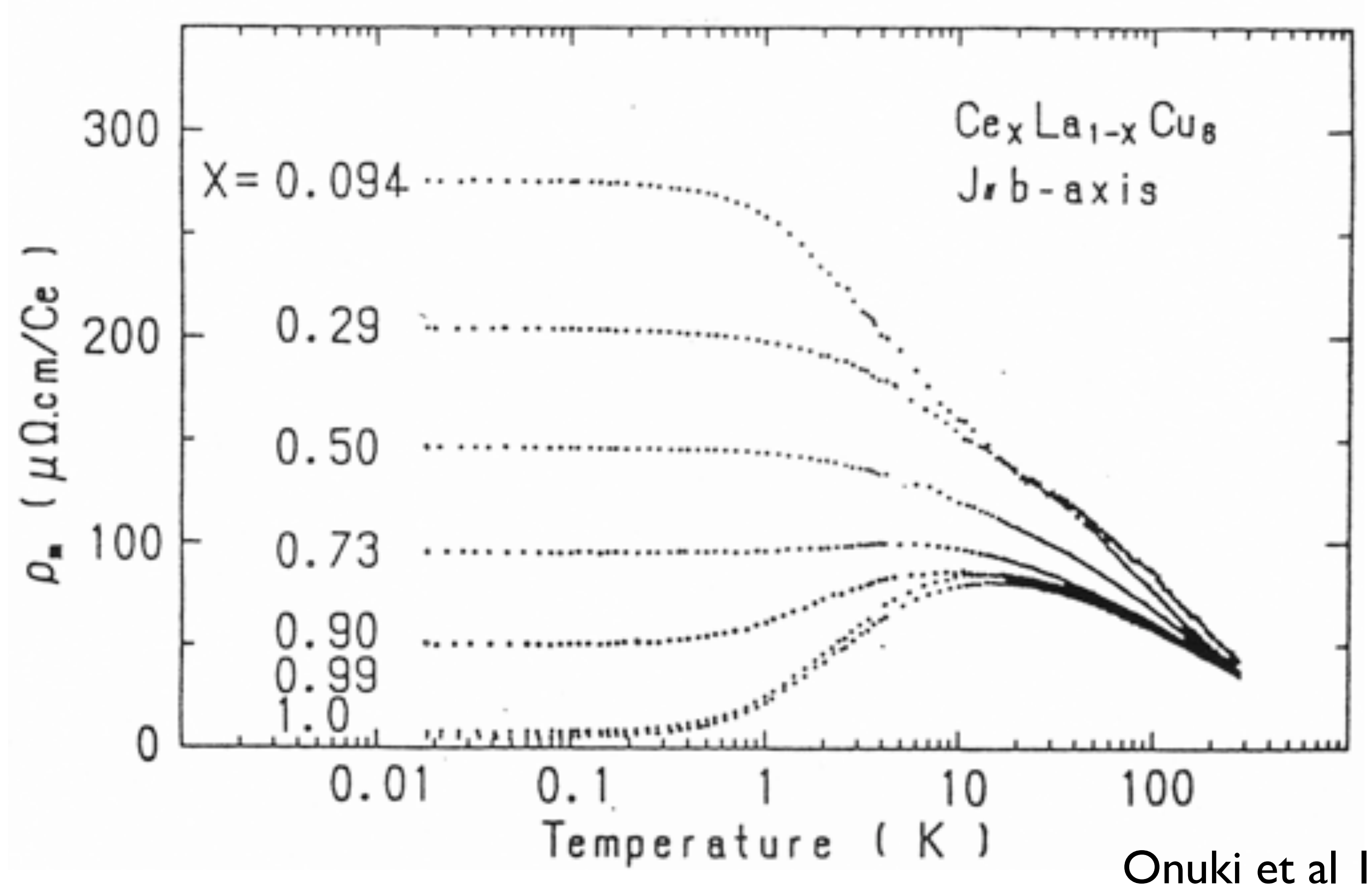
➡ Ce, Yb or Uranium based alloys



Onuki et al 1987

Heavy Fermions

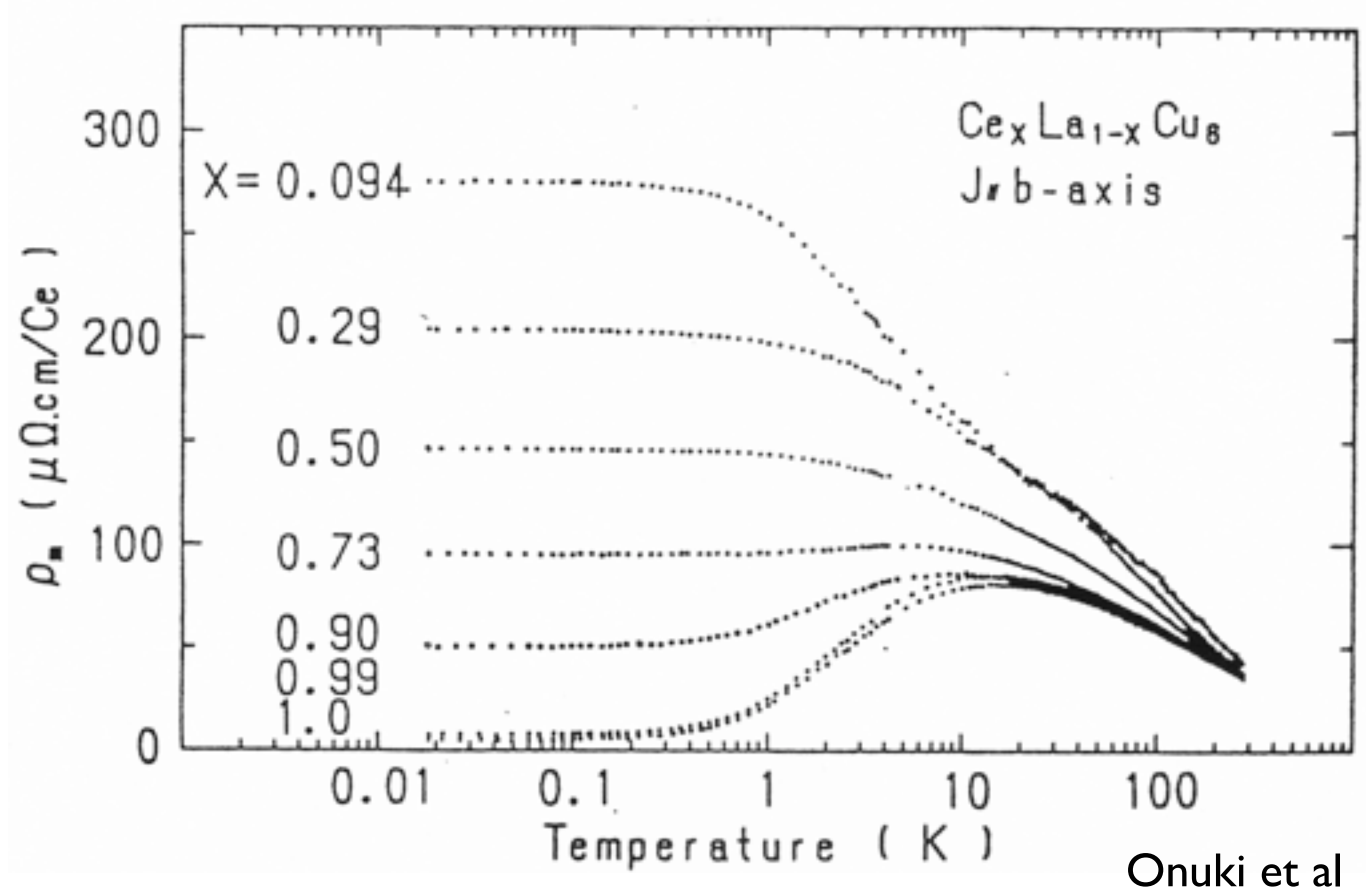
- Ce, Yb or Uranium based alloys
- localized 4f or 5f electrons:



Onuki et al 1987

Heavy Fermions

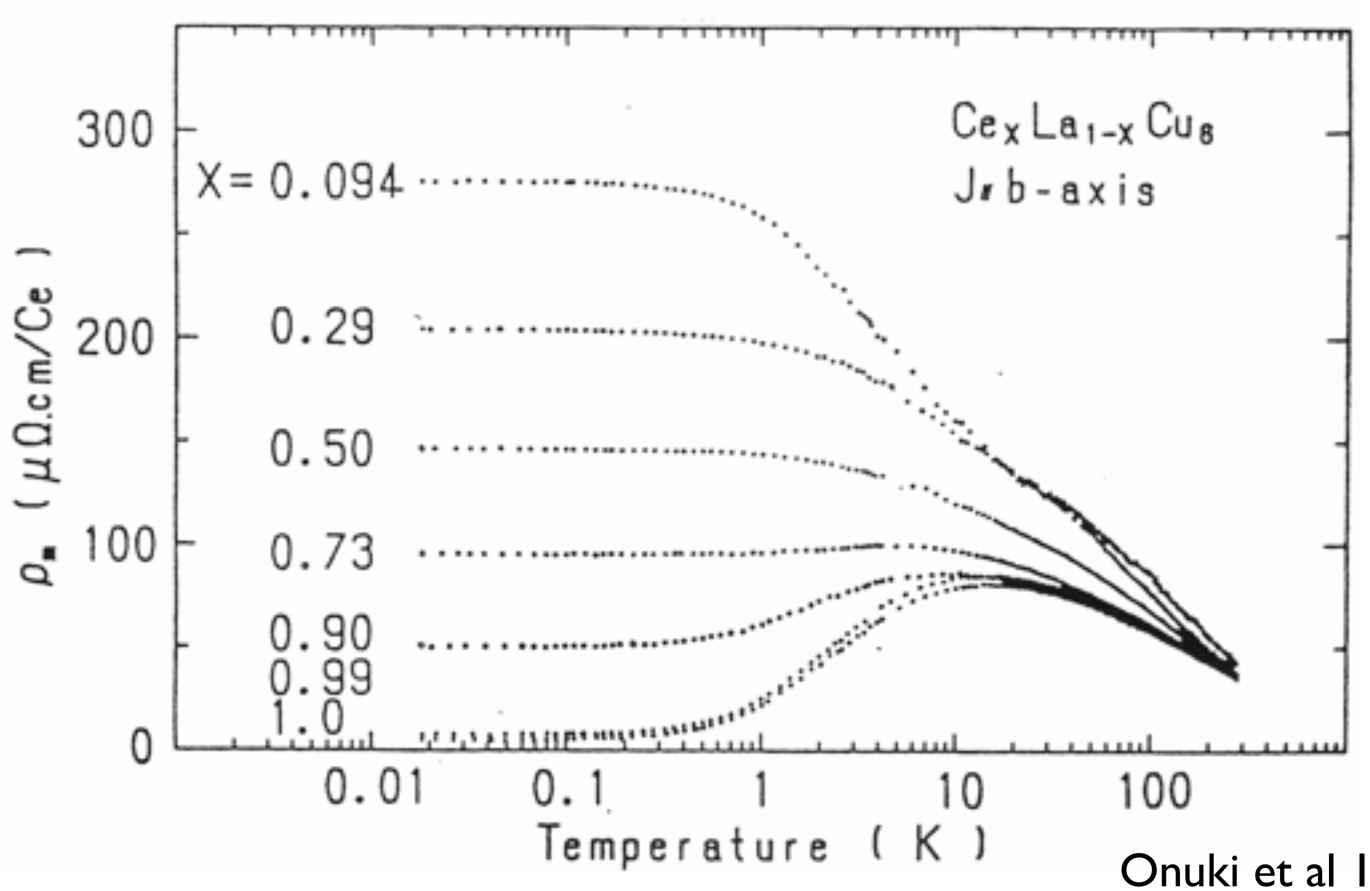
- ➡ Ce, Yb or Uranium based alloys
- ➡ localized 4f or 5f electrons:
- ➡ RKKY interaction mediates magnetic phase



Onuki et al 1987

Heavy Fermions

- ➡ Ce, Yb or Uranium based alloys
- ➡ localized 4f or 5f electrons:
- ➡ RKKY interaction mediates magnetic phase
- ➡ HF superconductivity: heavy quasiparticle form the condensate

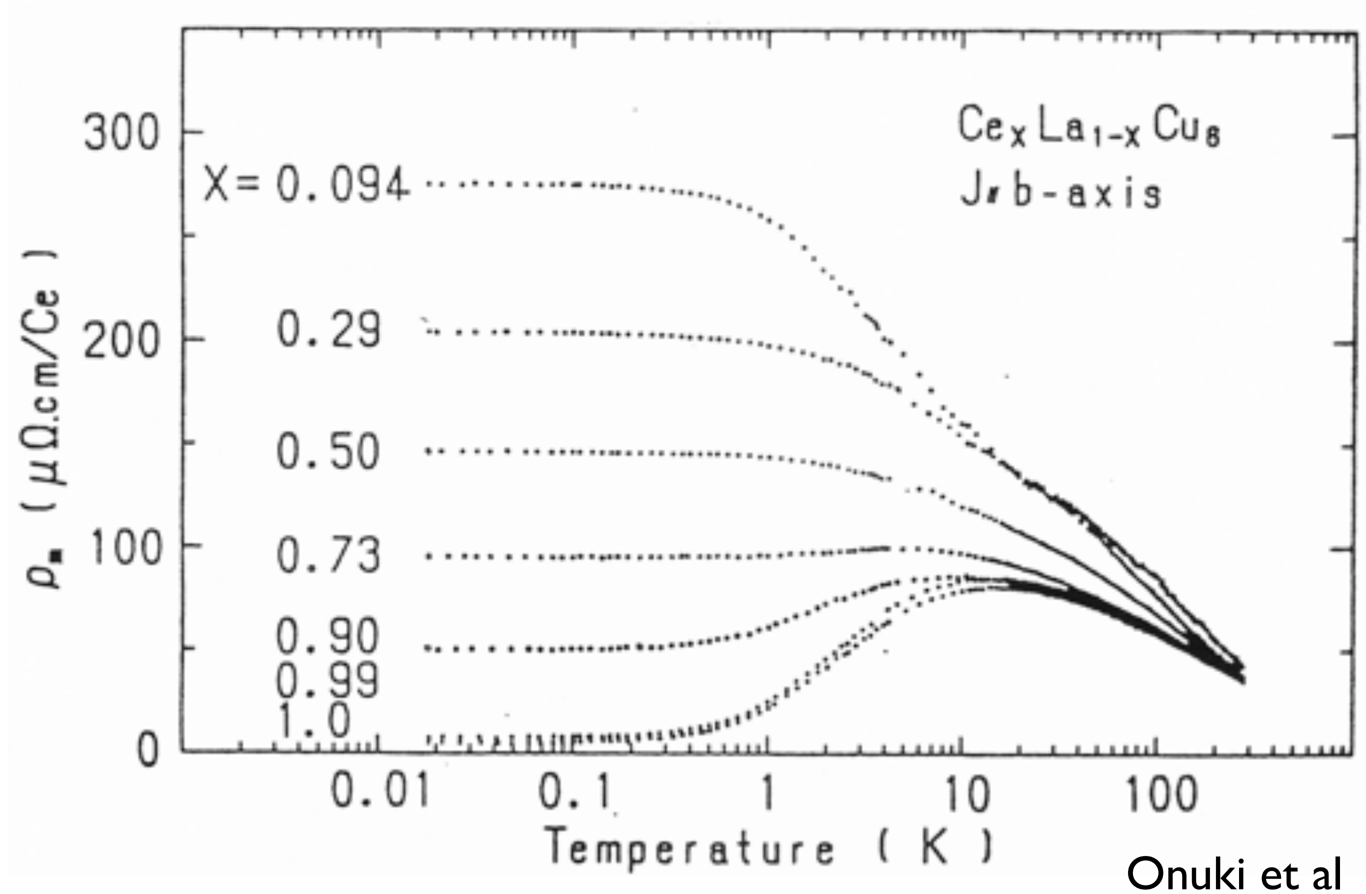


Onuki et al 1987

Wednesday, September 5, 2012

Heavy Fermions

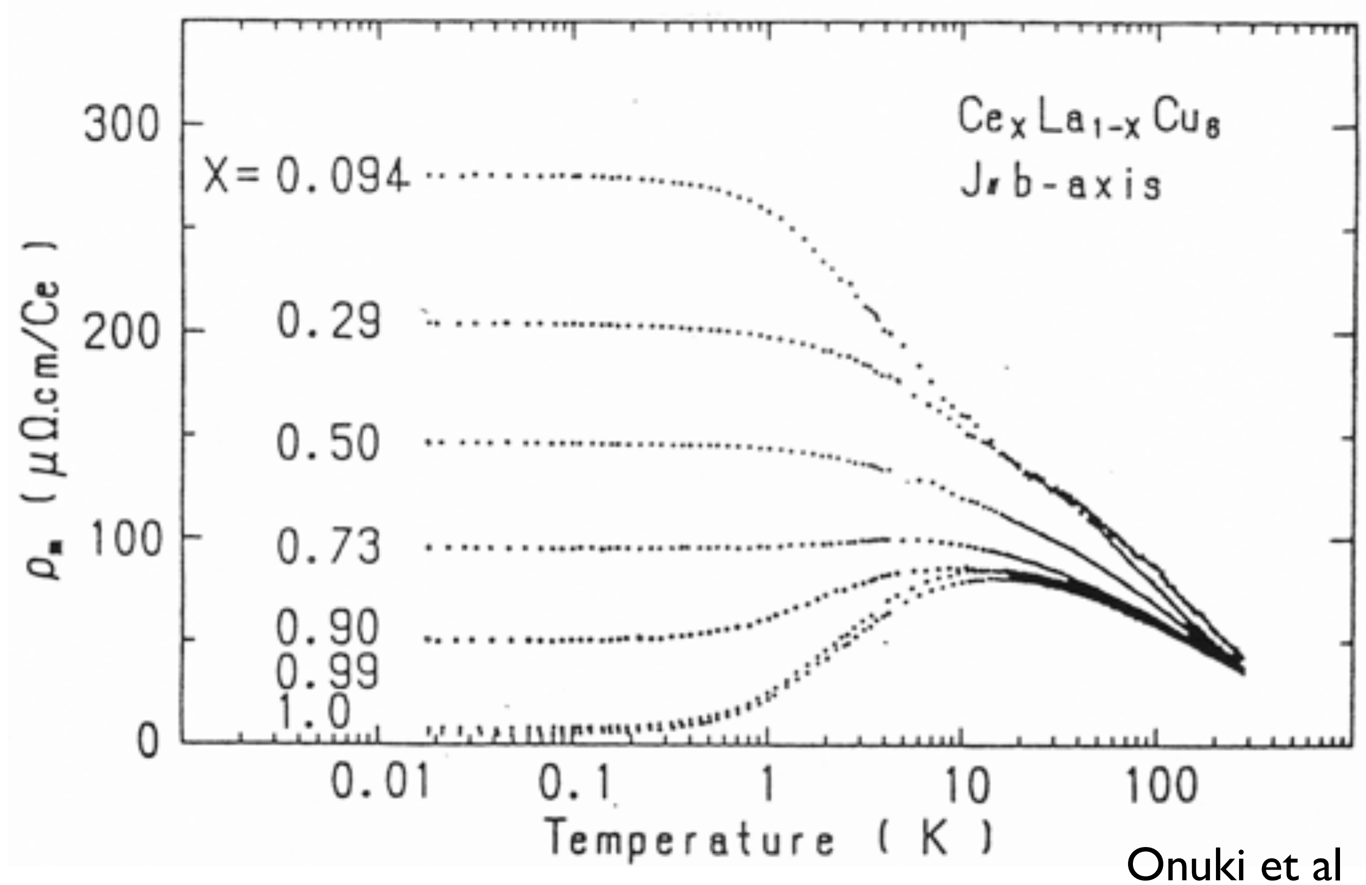
- Ce, Yb or Uranium based alloys
- localized 4f or 5f electrons:
- RKKY interaction mediates magnetic phase
- HF superconductivity: heavy quasiparticle form the condensate
- unconventional order parameter



Onuki et al 1987

Heavy Fermions

- ➡ Ce, Yb or Uranium based alloys
- ➡ localized 4f or 5f electrons:
- ➡ RKKY interaction mediates magnetic phase
- ➡ HF superconductivity: heavy quasiparticle form the condensate
- ➡ unconventional order parameter
- ➡ quantum phase transition

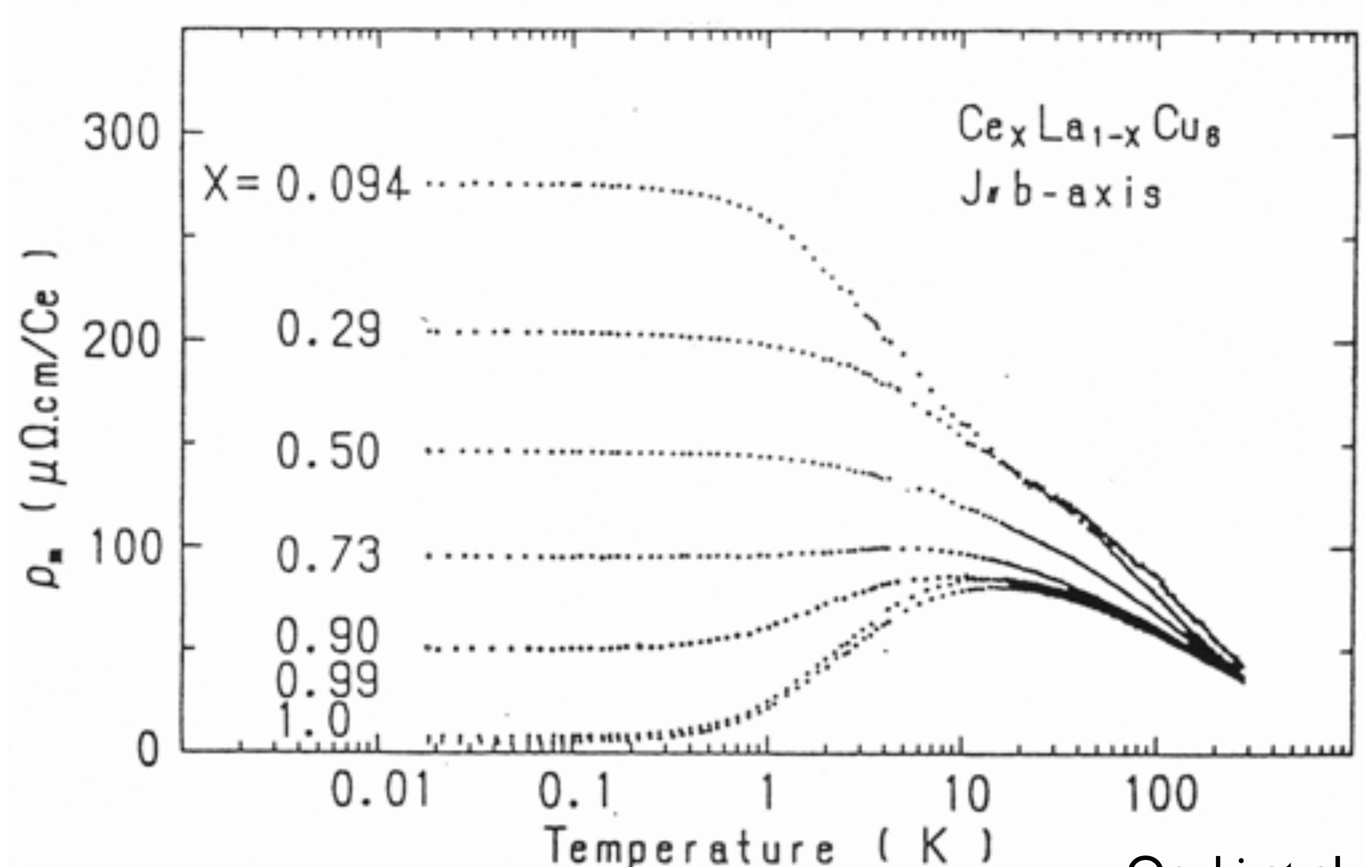


Onuki et al 1987

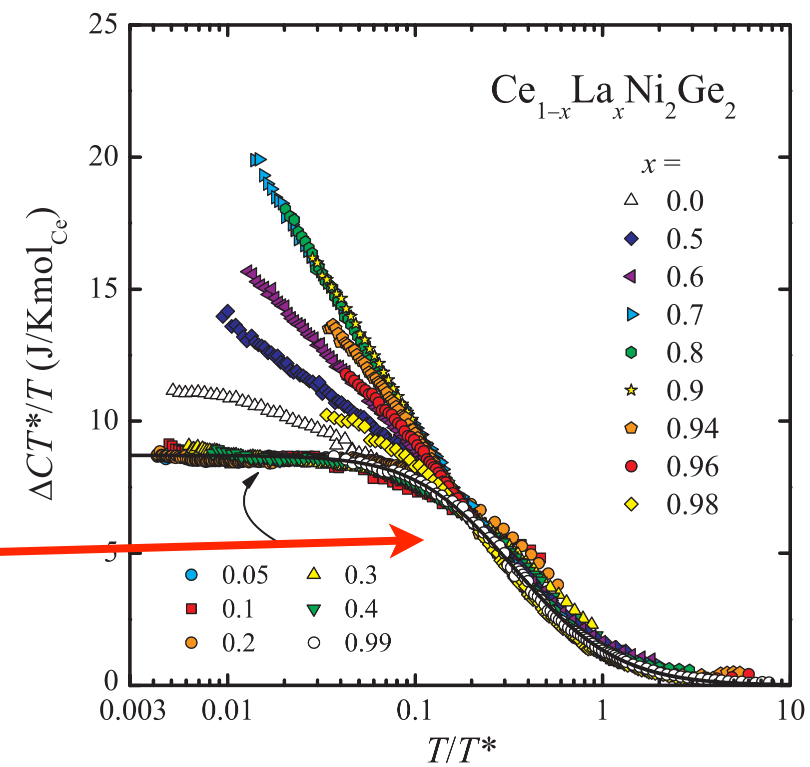
Wednesday, September 5, 2012

Heavy Fermions

- ➡ Ce, Yb or Uranium based alloys
- ➡ localized 4f or 5f electrons:
- ➡ RKKY interaction mediates magnetic phase
- ➡ HF superconductivity: heavy quasiparticle form the condensate
- ➡ unconventional order parameter
- ➡ quantum phase transition
- ➡ but also: single ion physics: scaling of C(T)

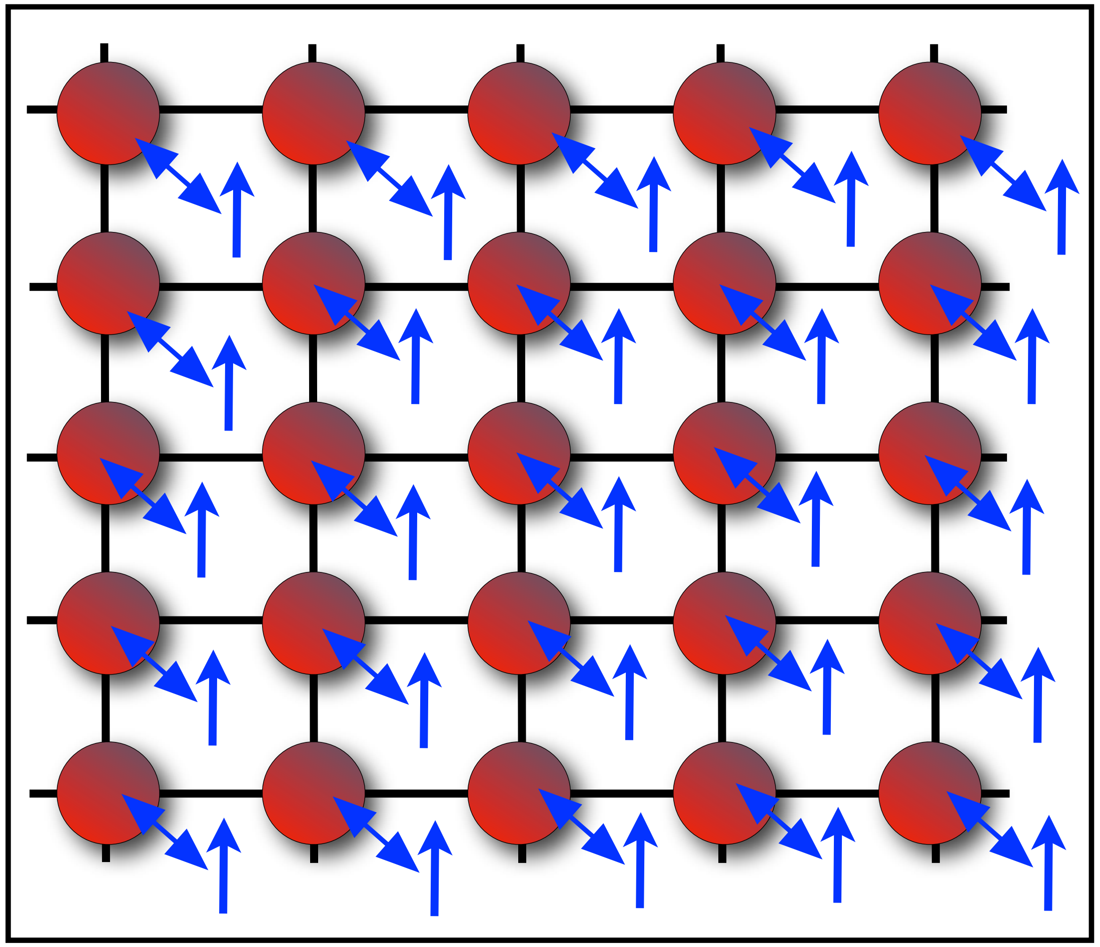


Onuki et al 1987

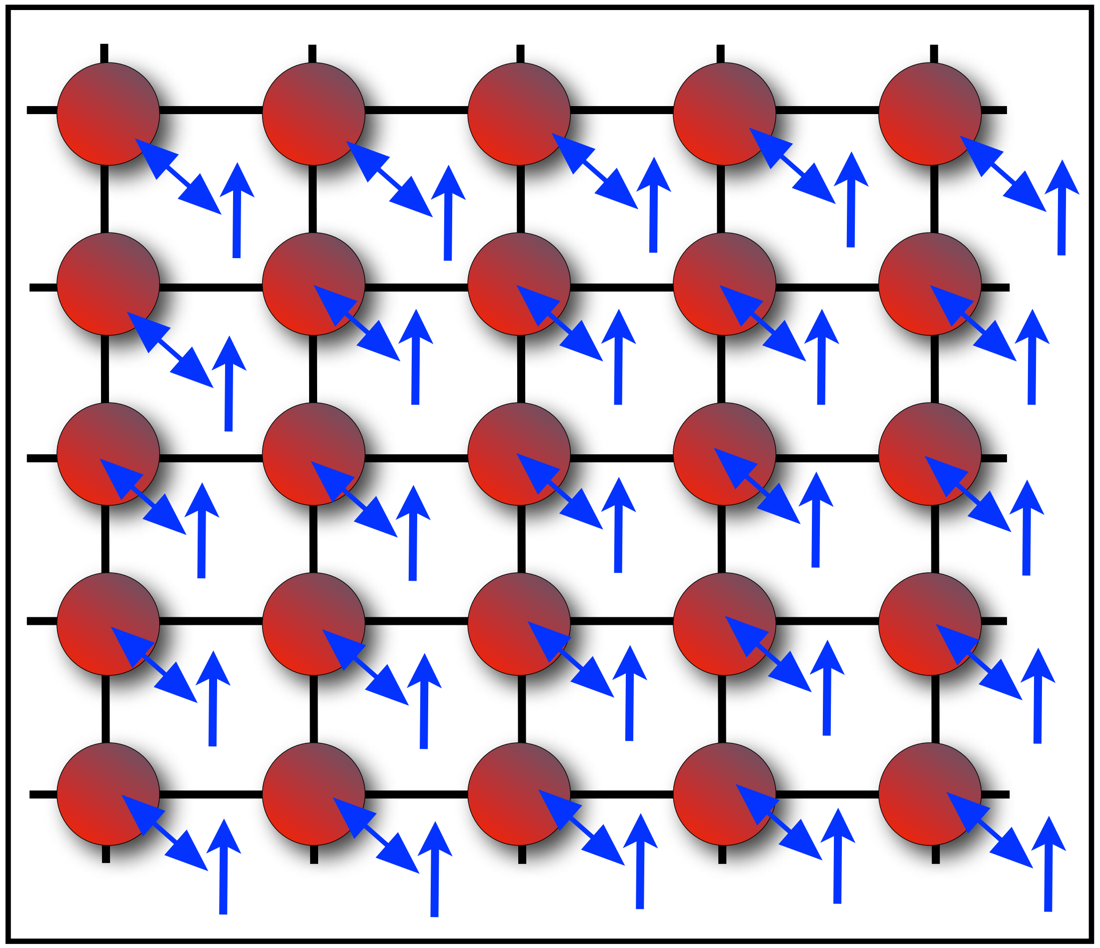


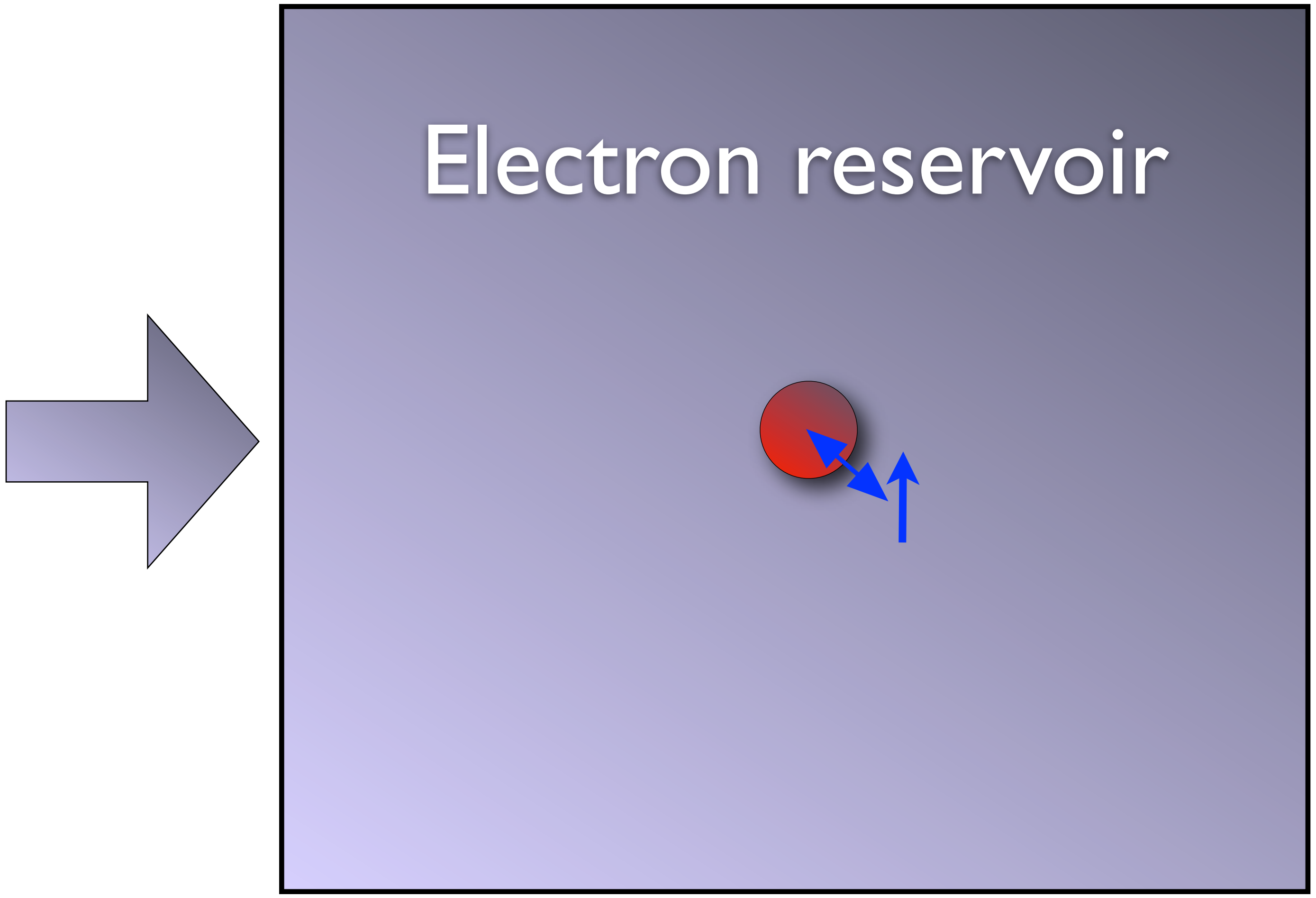
A. P. Pikul et al
PRL 108, 066405 (2012)

Heavy Fermions: modelling



Heavy Fermions: modelling

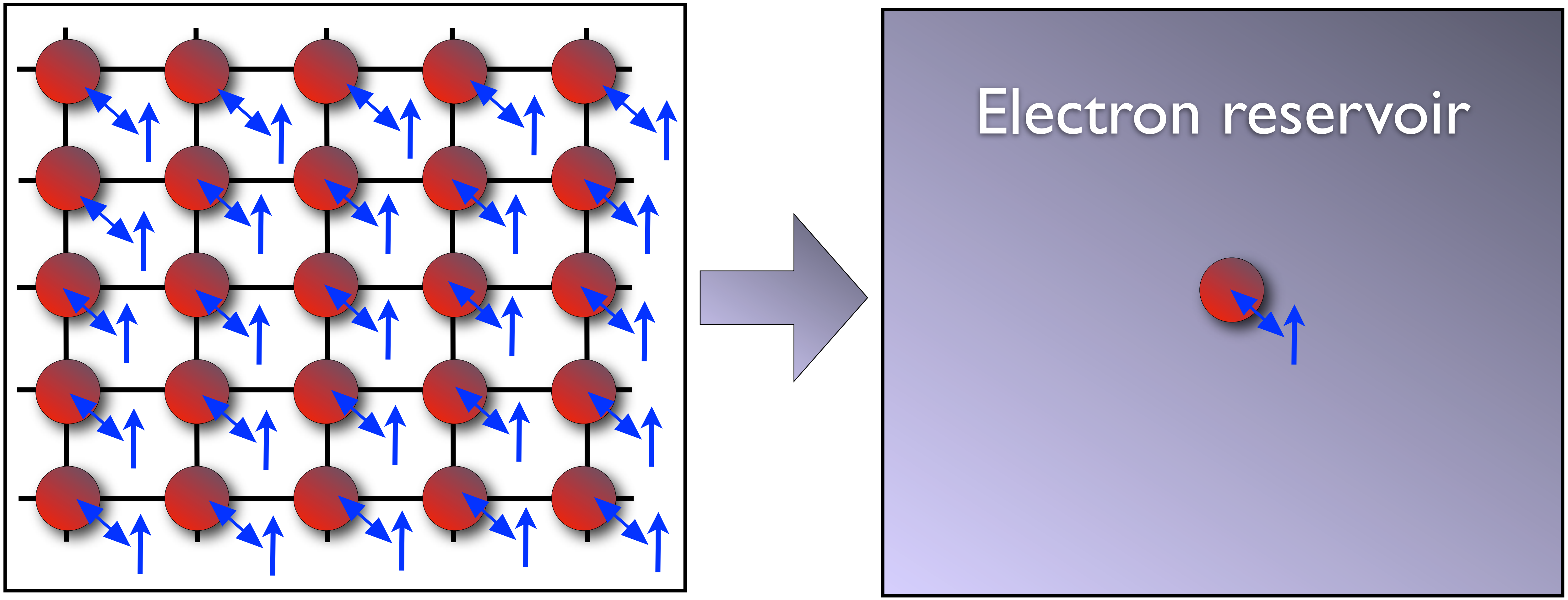




effective site:
impurity problem

f-electrons coupled to a bath

Heavy Fermions: modelling

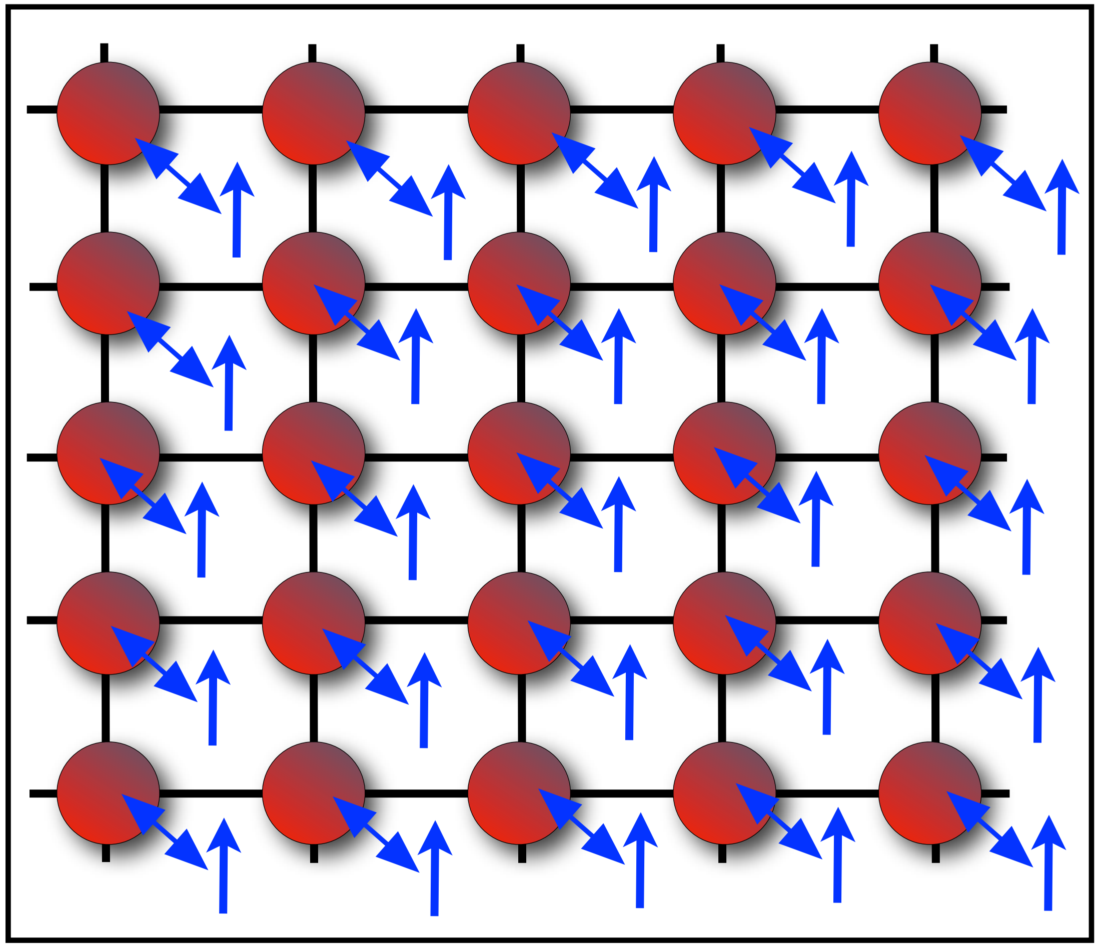


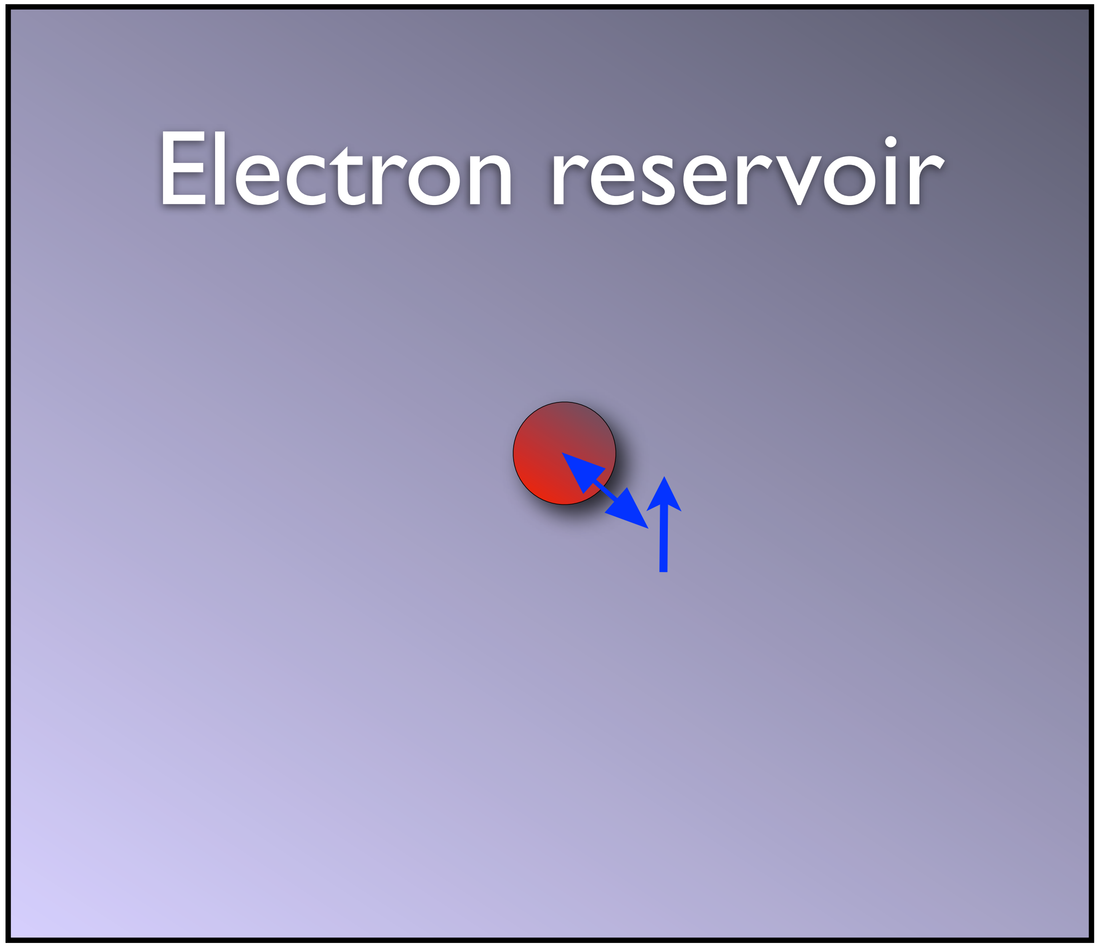
effective site:
impurity problem

f-electrons coupled to a bath

impurity $\Sigma^f(\omega)$= lattice $\Sigma^f(\omega)$

Heavy Fermions: modelling





$$G_\sigma(k,z) = [z - \varepsilon_k - \frac{V^2}{z - \varepsilon_f - \Sigma^f(z)}]^{-1}$$

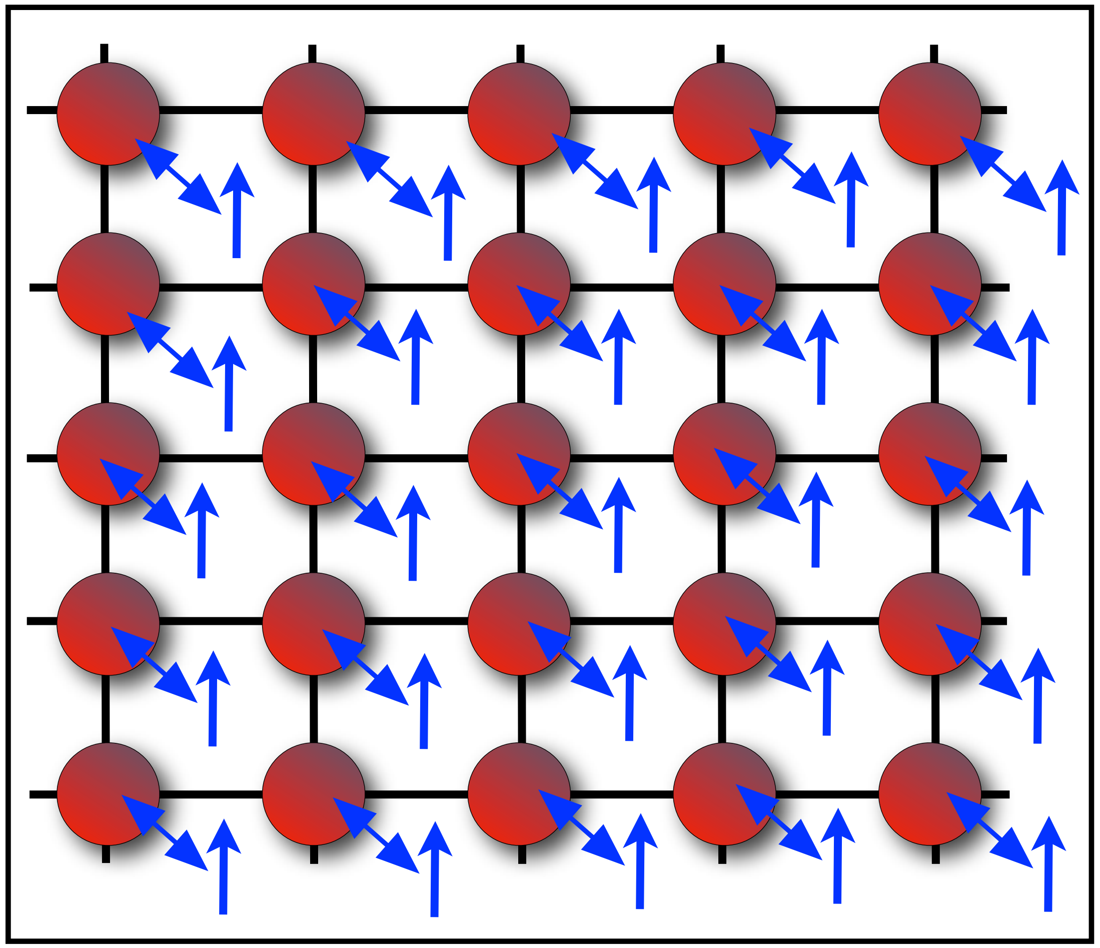
$$F_\sigma(k,z) = [z - \varepsilon_f - \Sigma^f(z) - \frac{V^2}{z - \varepsilon_k}]^{-1}$$

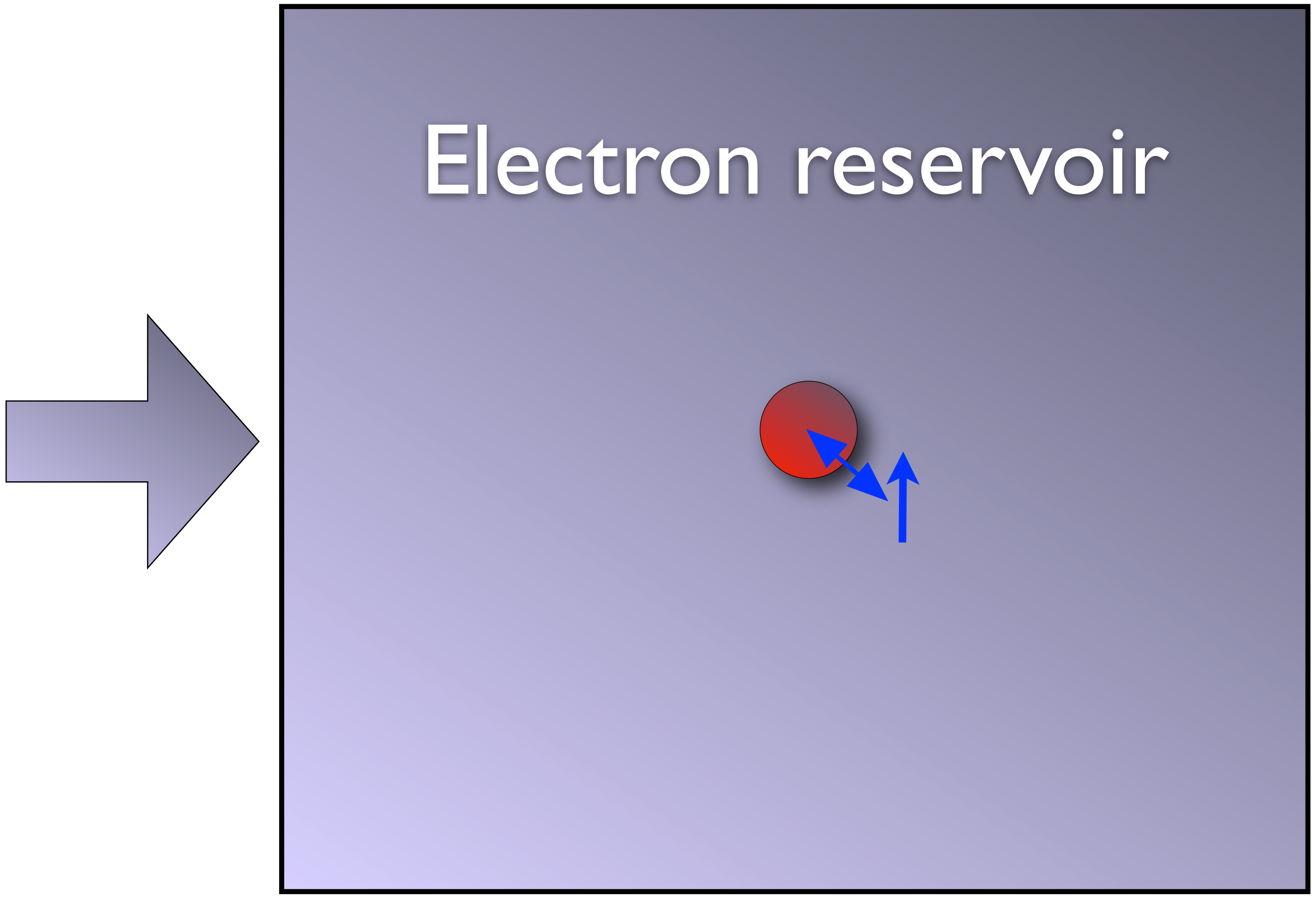
effective site:
impurity problem
f-electrons coupled to a bath

impurity $\Sigma^f(\omega)$= lattice $\Sigma^f(\omega)$

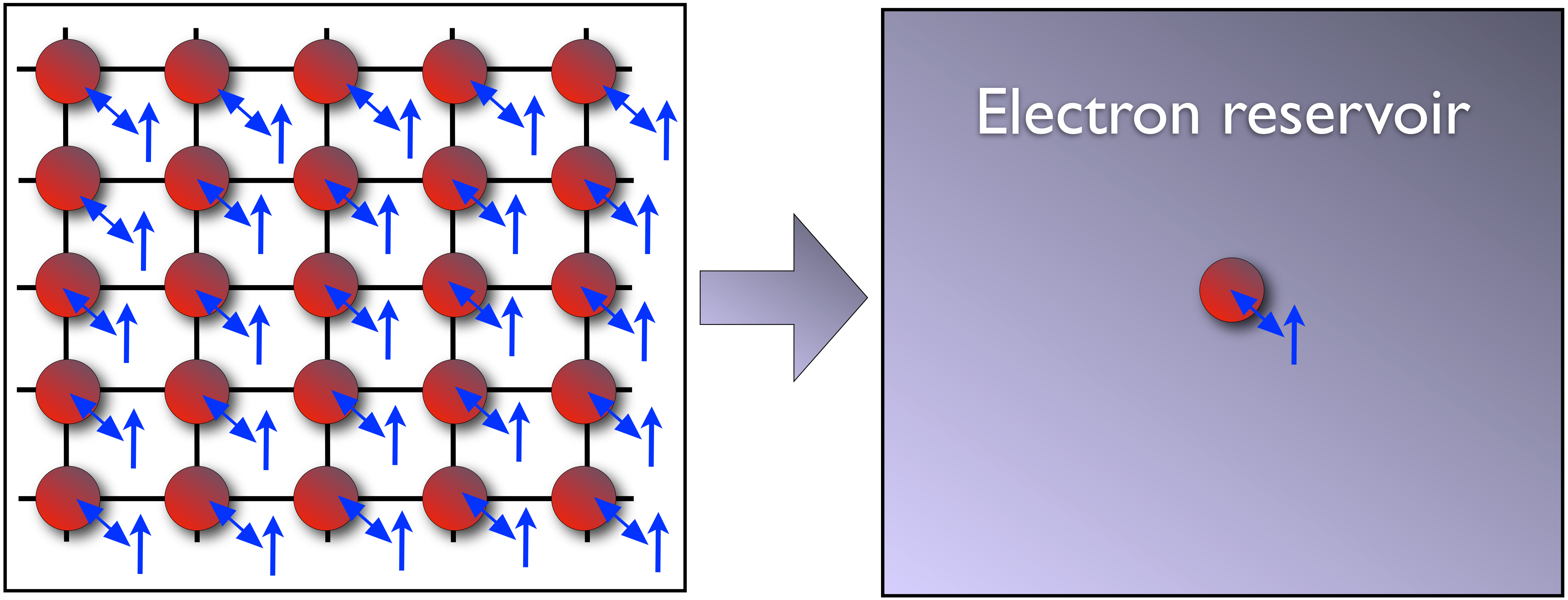
Wednesday, September 5, 2012

Heavy Fermions



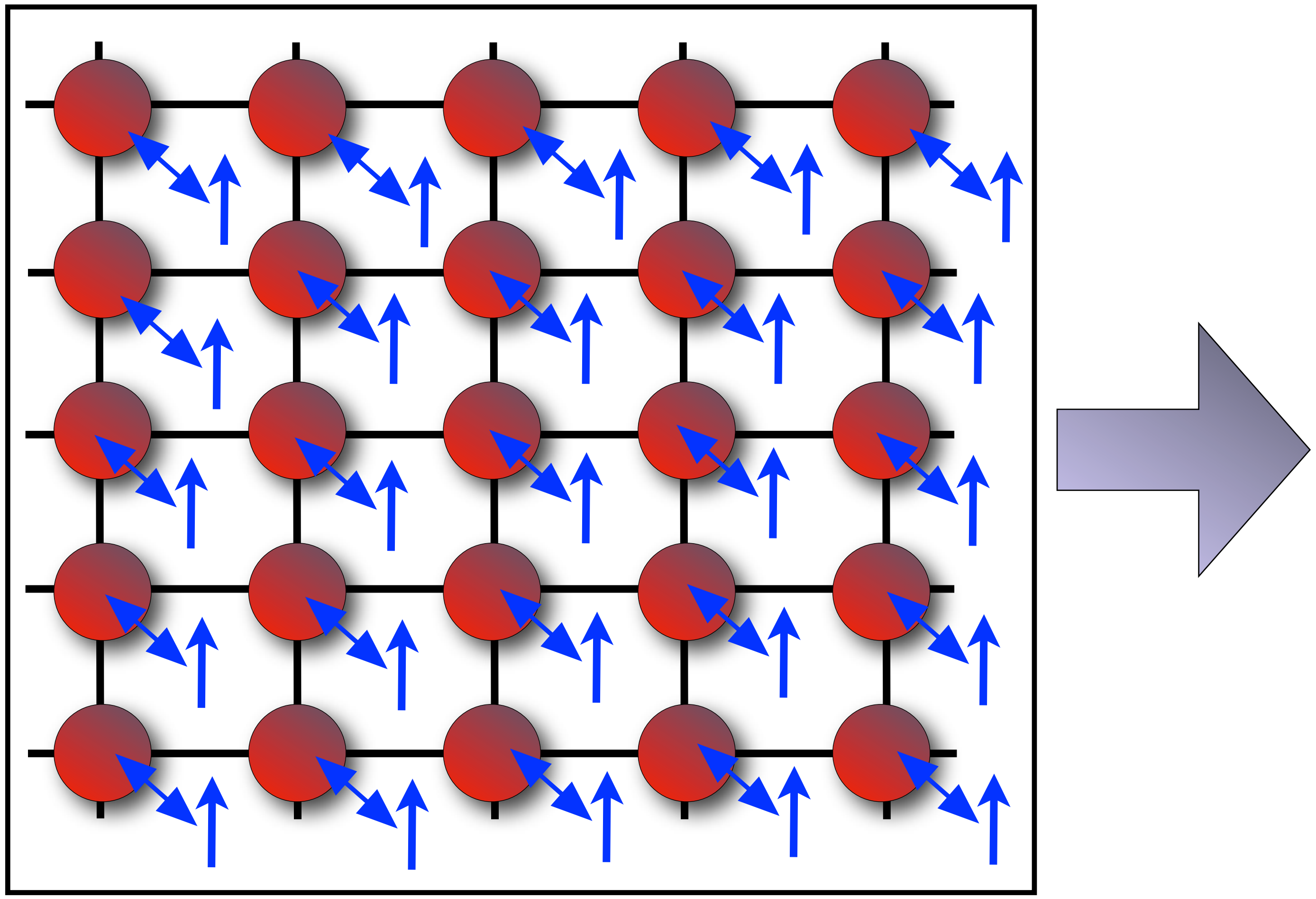


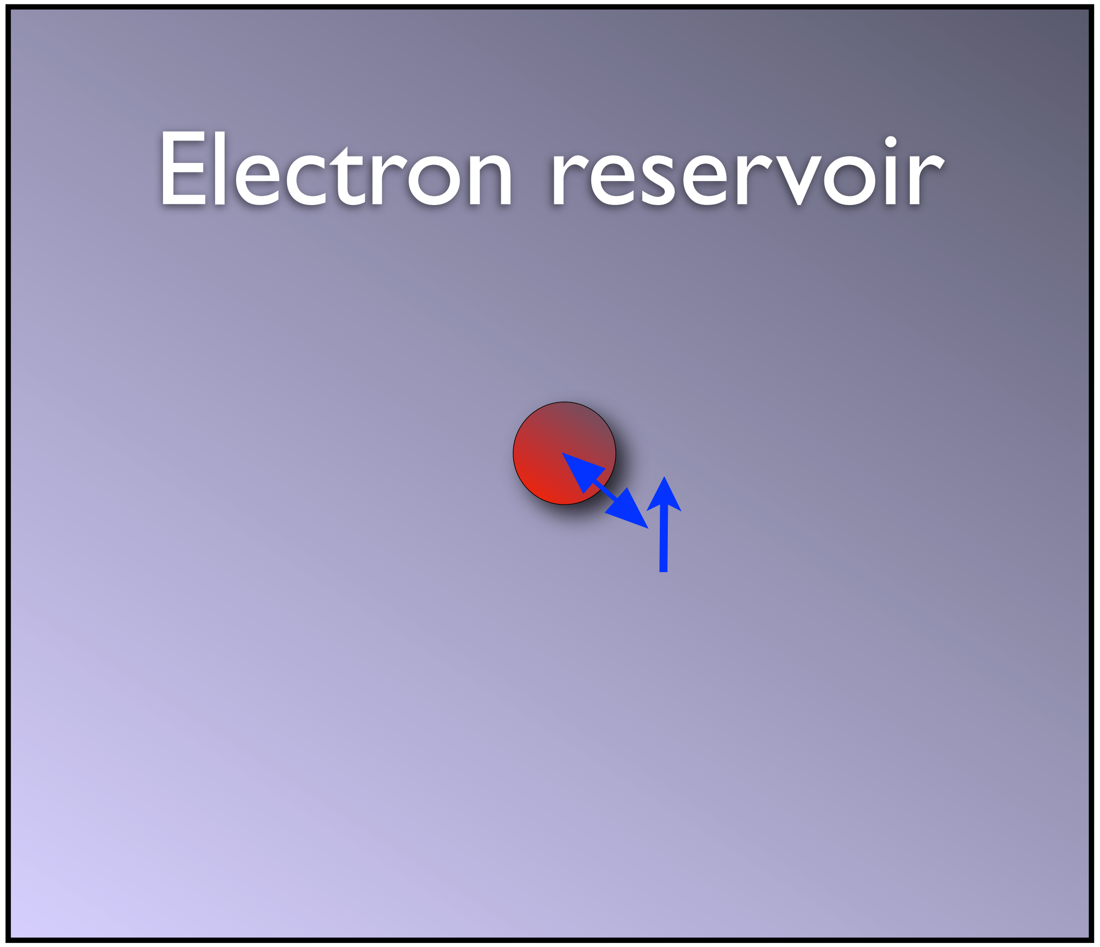
Heavy Fermions



local approximation, two approaches:

Heavy Fermions

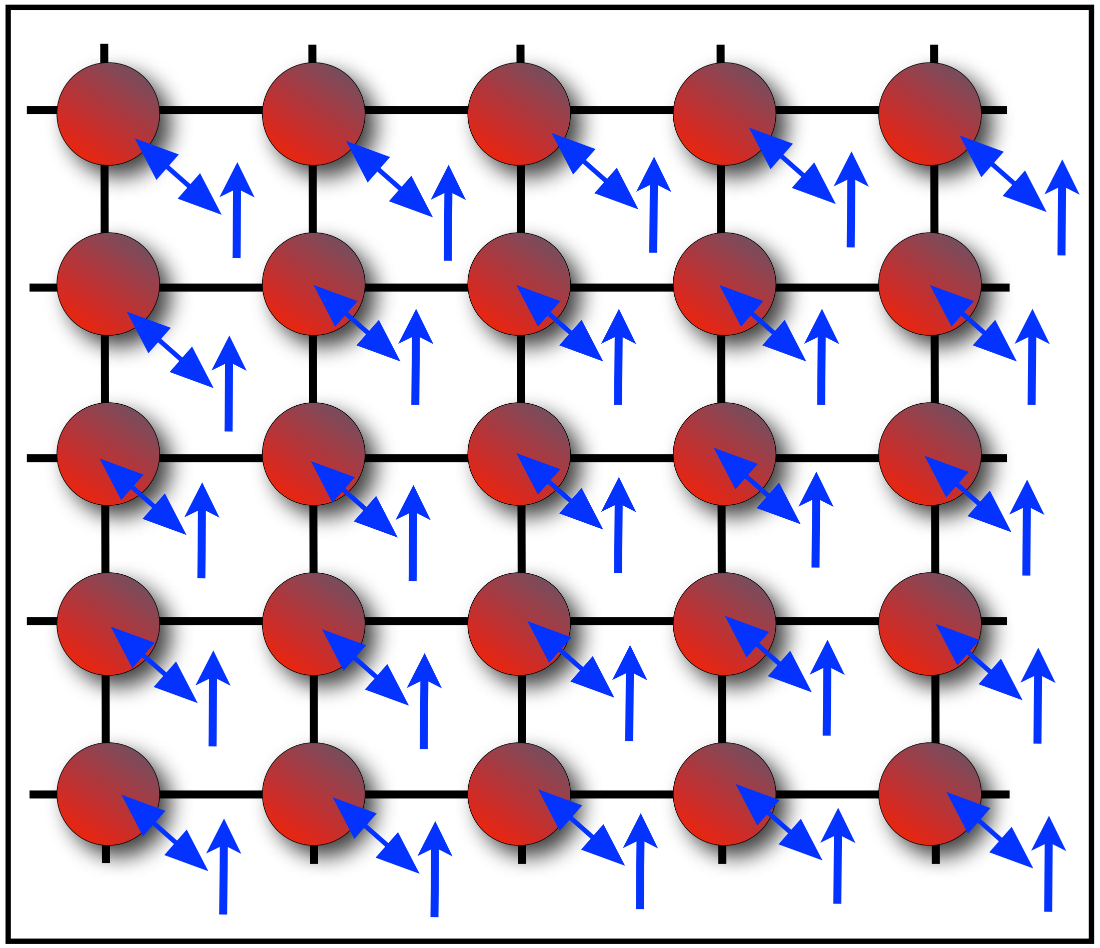


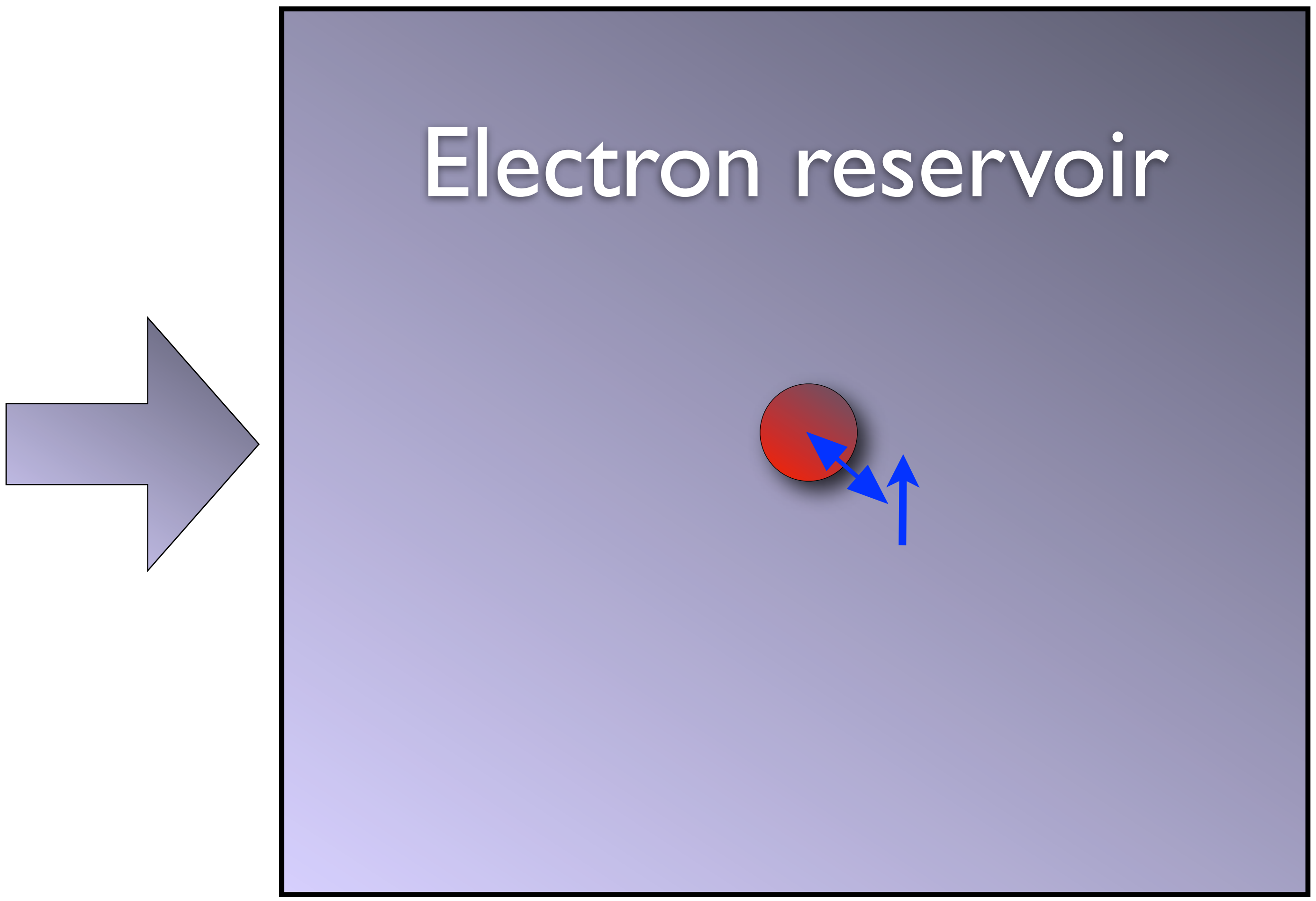


local approximation, two approaches:

- lattice non-crossing approximation (L-NCA) (Grewe 1987)

Heavy Fermions

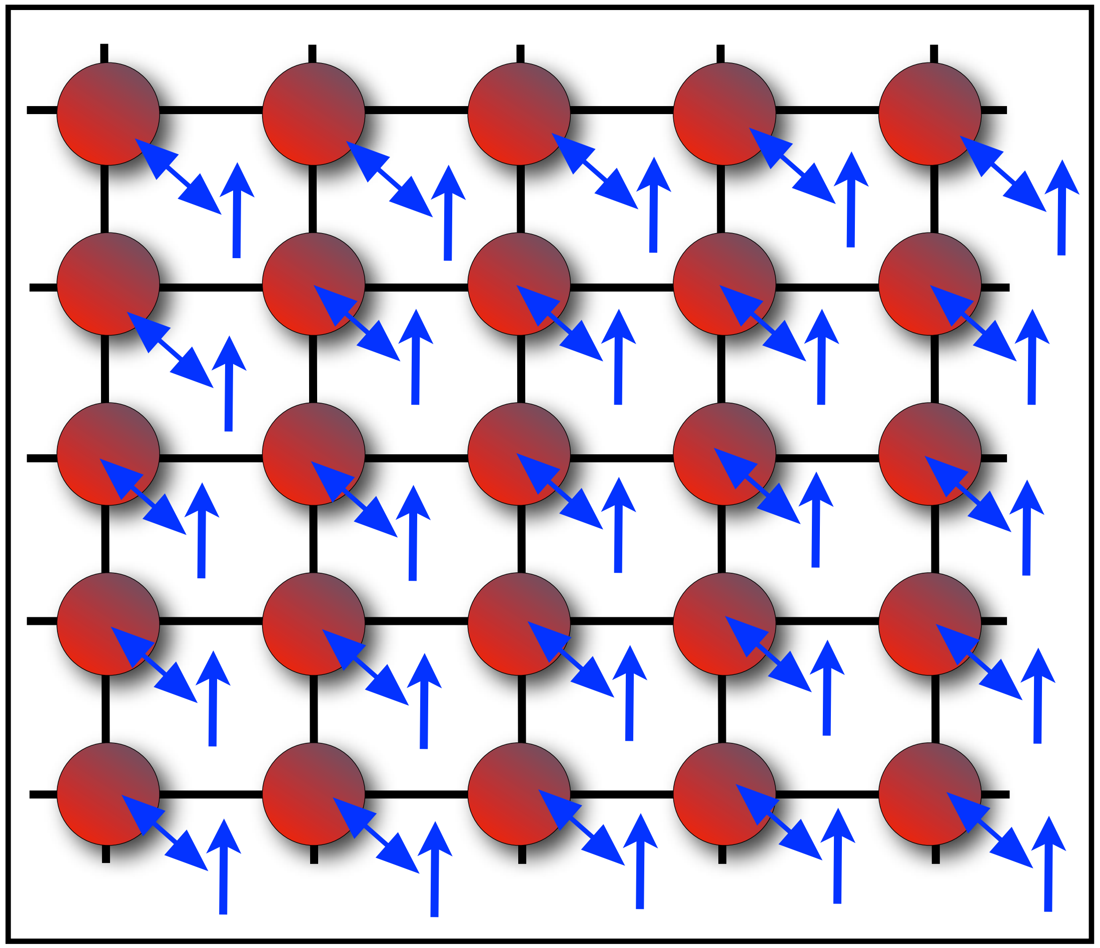


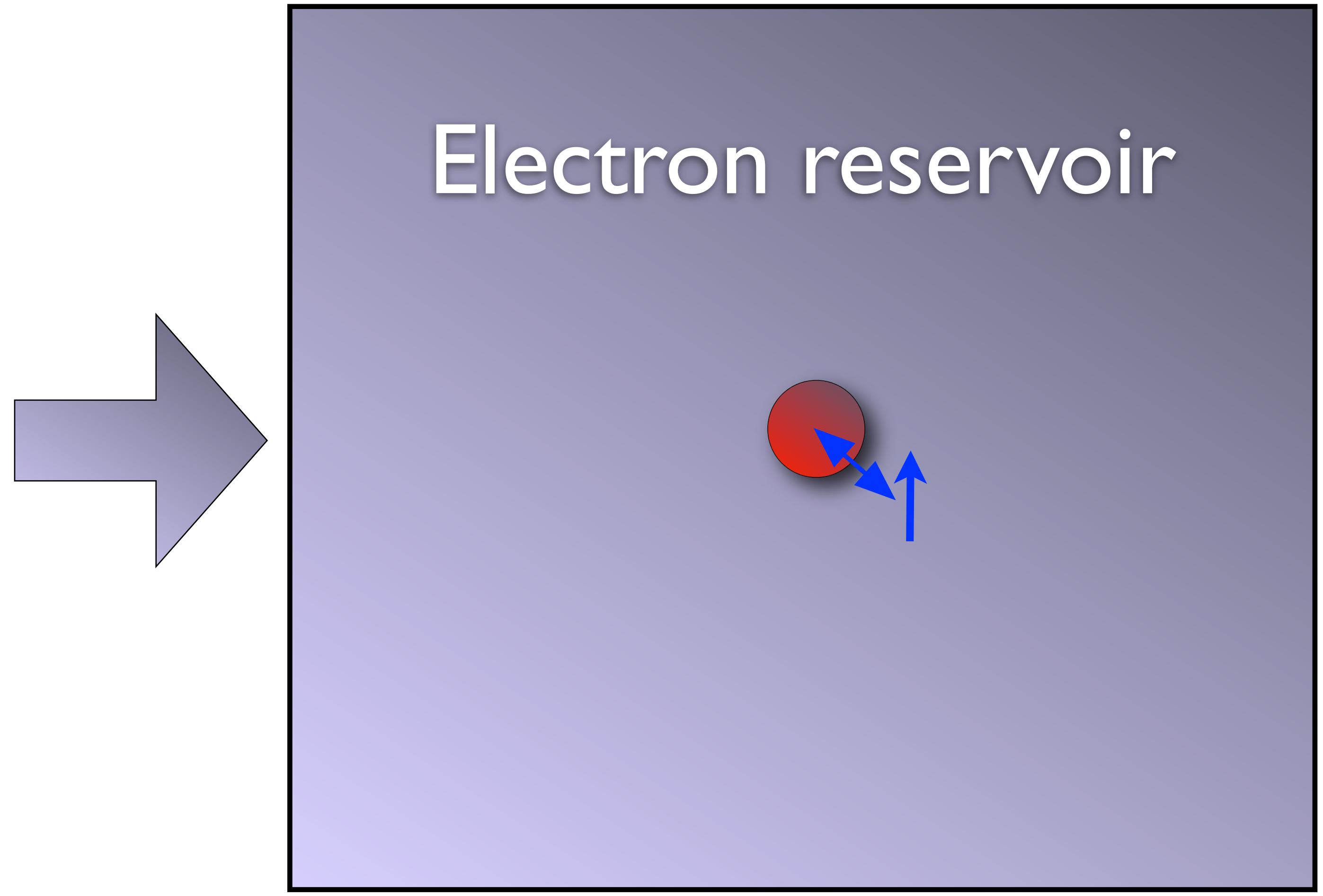


local approximation, two approaches:

- lattice non-crossing approximation (L-NCA) (Grewe 1987)
- eXtended-NCA: Kuramoto 1985-1990

Heavy Fermions

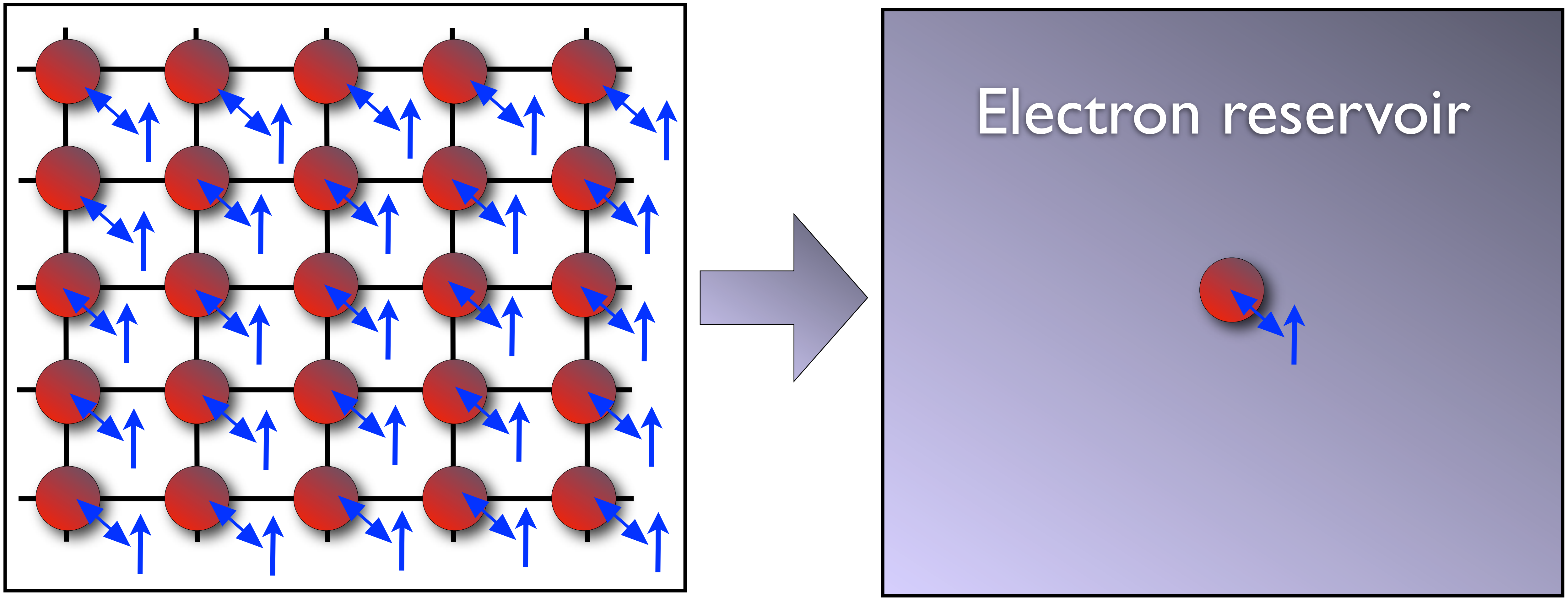




local approximation, two approaches:

- lattice non-crossing approximation (L-NCA) (Grewe 1987)
- eXtended-NCA: Kuramoto 1985-1990
  today: DMFT(NCA)

Heavy Fermions



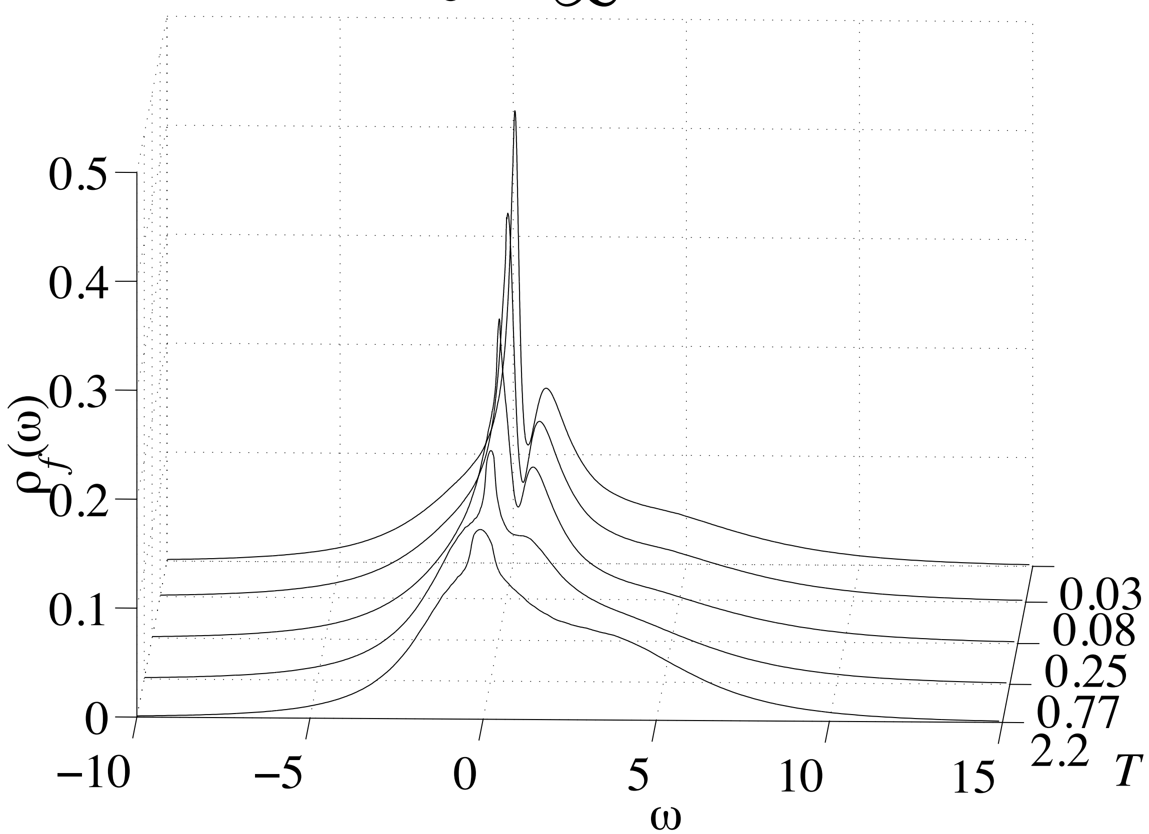
local approximation, two approaches:

- lattice non-crossing approximation (L-NCA) (Grewe 1987)
- eXtended-NCA: Kuramoto 1985-1990
  today: DMFT(NCA)

Metzner/Vollhardt, Müller-Hartmann: (1989) lokal approximation exact in the limit $d \to \infty$

Heavy Fermions

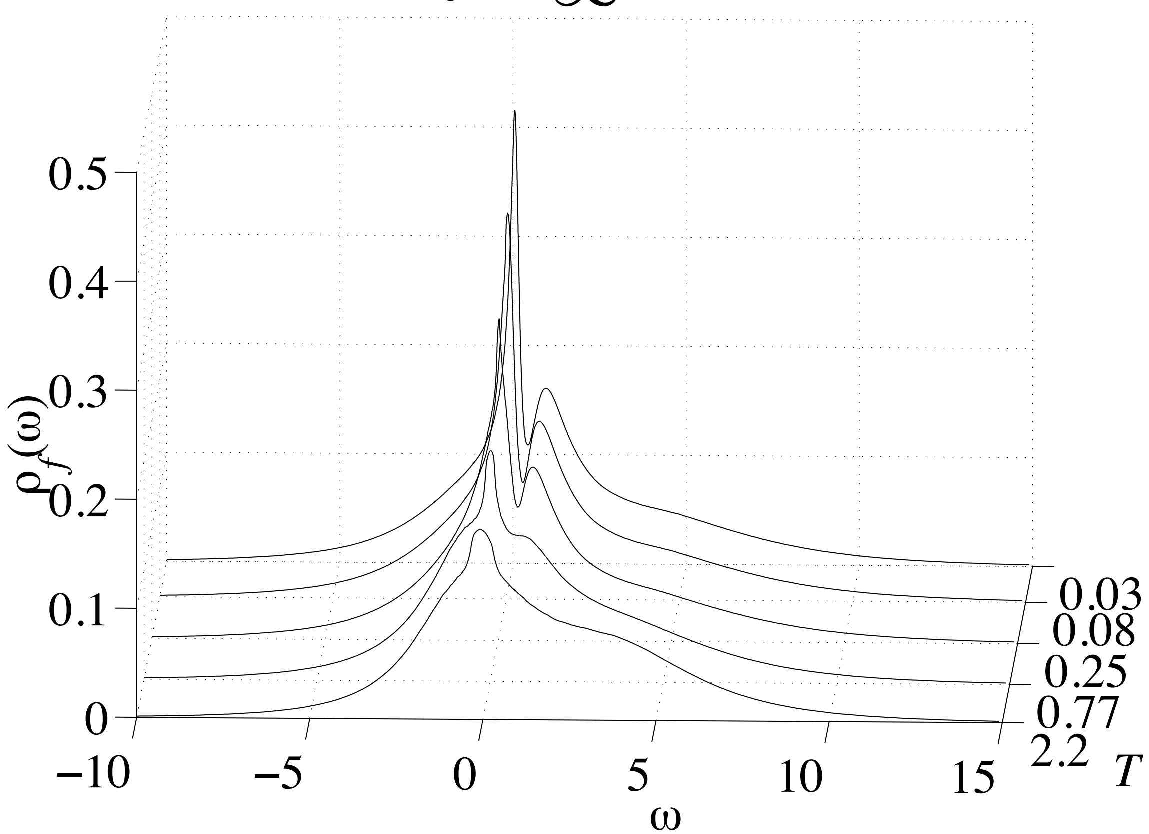
$$\rho_f(\omega) = \int_{-\infty}^{\infty} d\varepsilon\, \rho_\sigma(\omega, \varepsilon_k)$$



local f-density of states

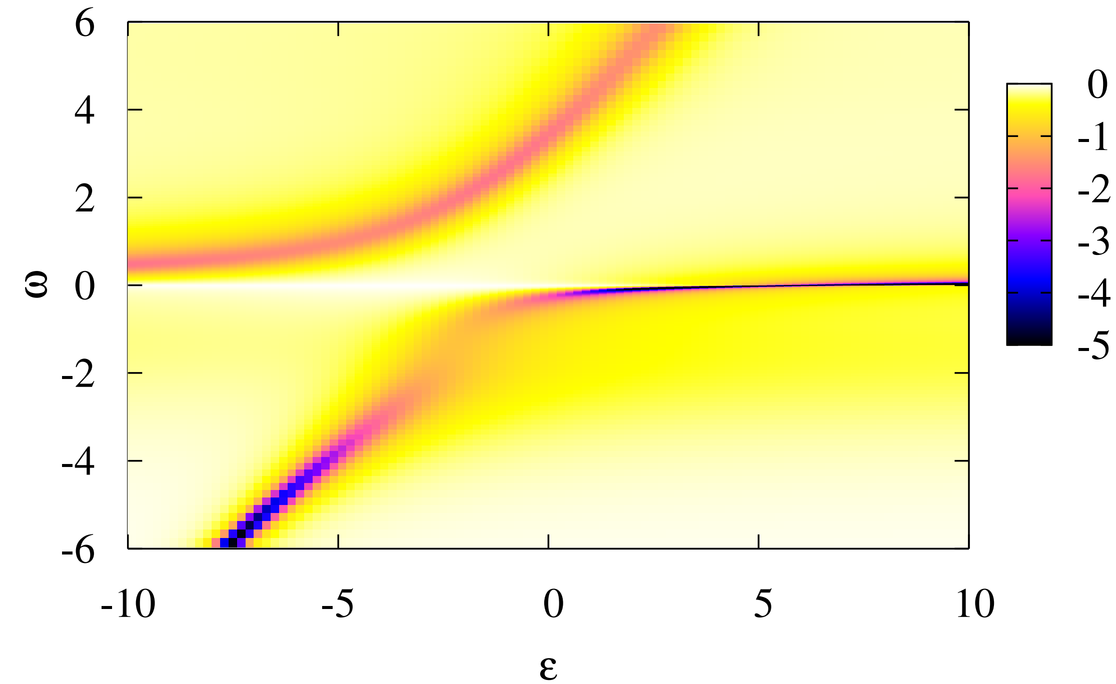
Heavy Fermions

$$\rho_f(\omega) = \int_{-\infty}^{\infty} d\varepsilon\, \rho_\sigma(\omega, \varepsilon_k)$$



local f-density of states

$$\rho_\sigma(\omega, \varepsilon_k)$$

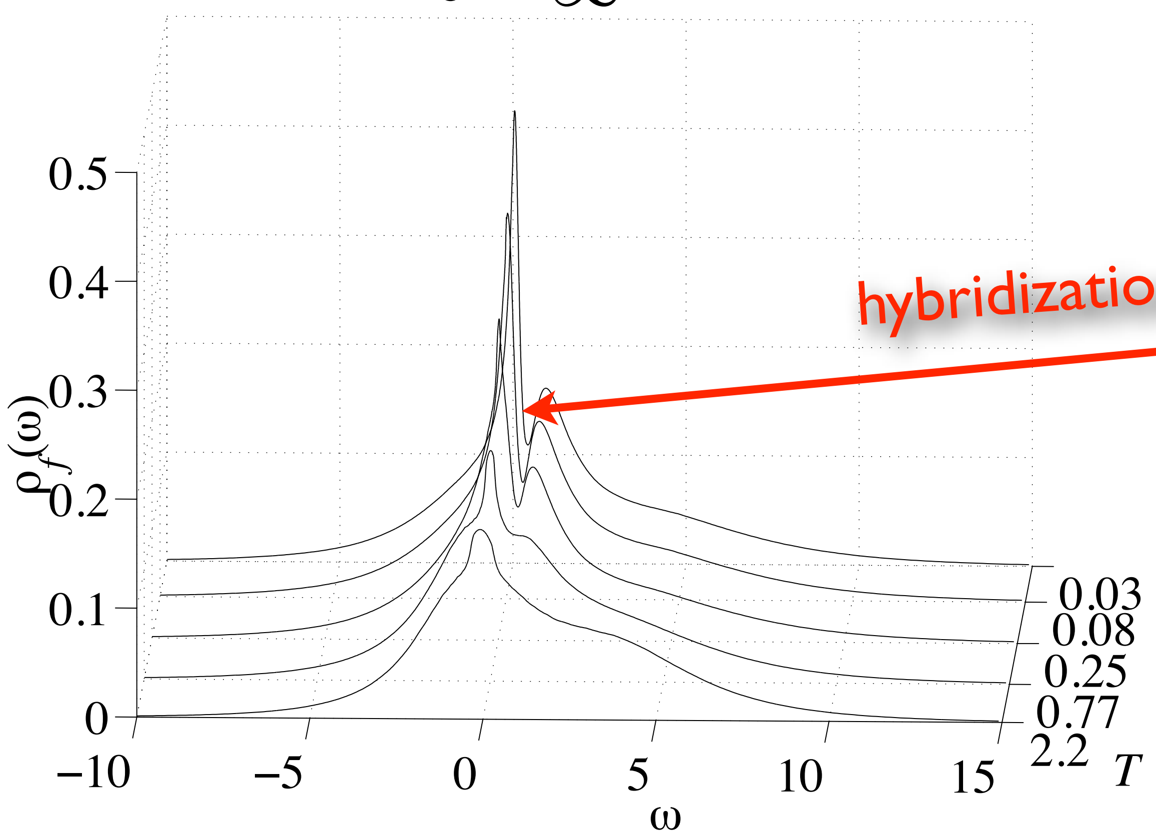


renormalized band structure

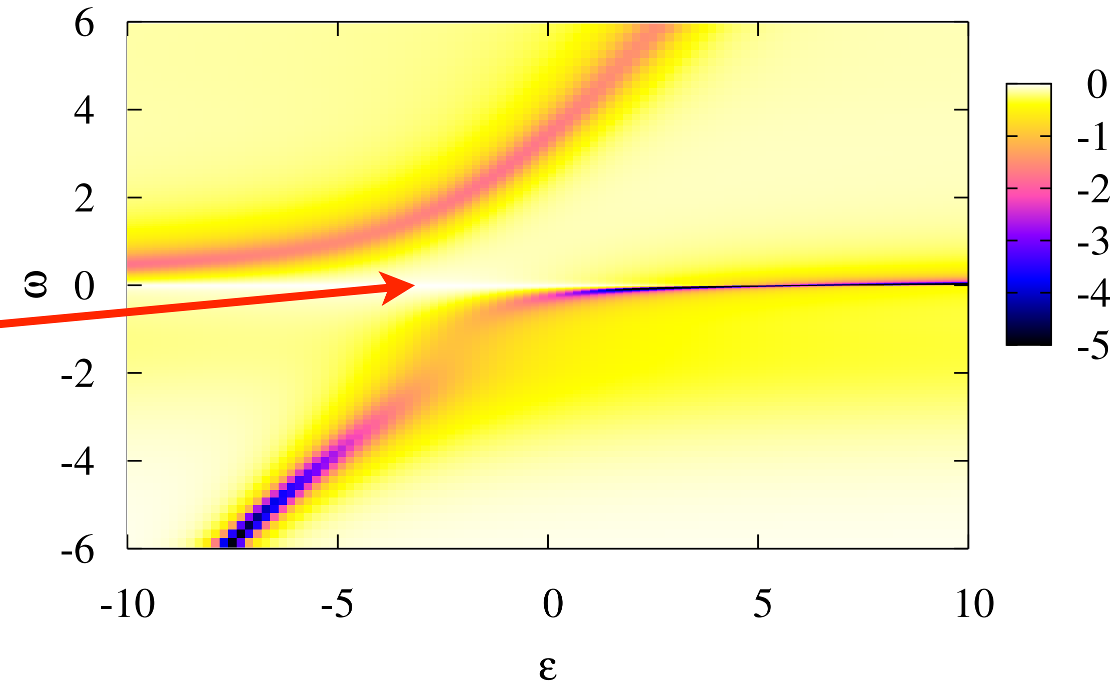
Heavy Fermions

$$\rho_f(\omega) = \int_{-\infty}^{\infty} d\varepsilon\, \rho_\sigma(\omega, \varepsilon_k)$$

$$\rho_\sigma(\omega, \varepsilon_k)$$



hybridization gap

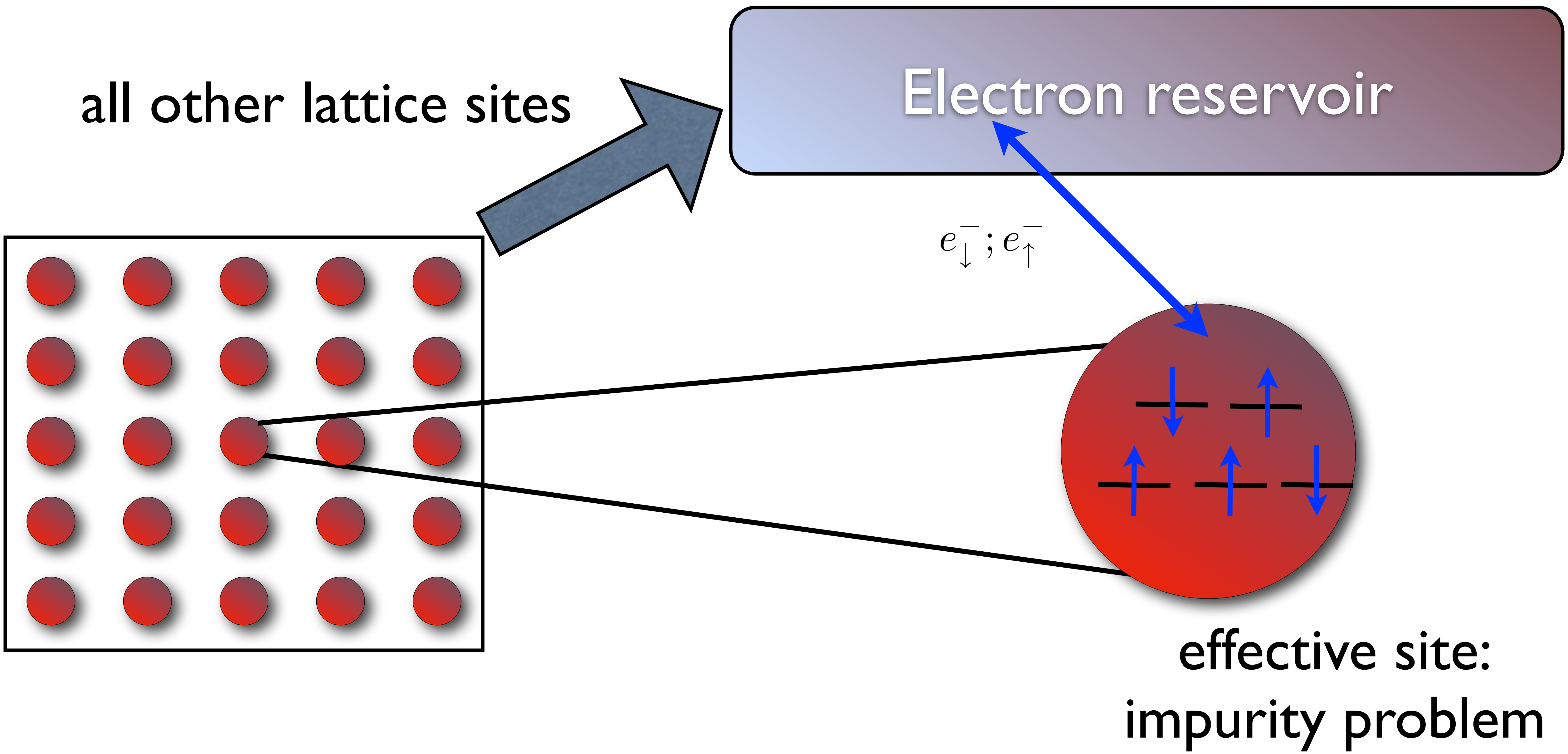


local f-density of states

renormalized band structure

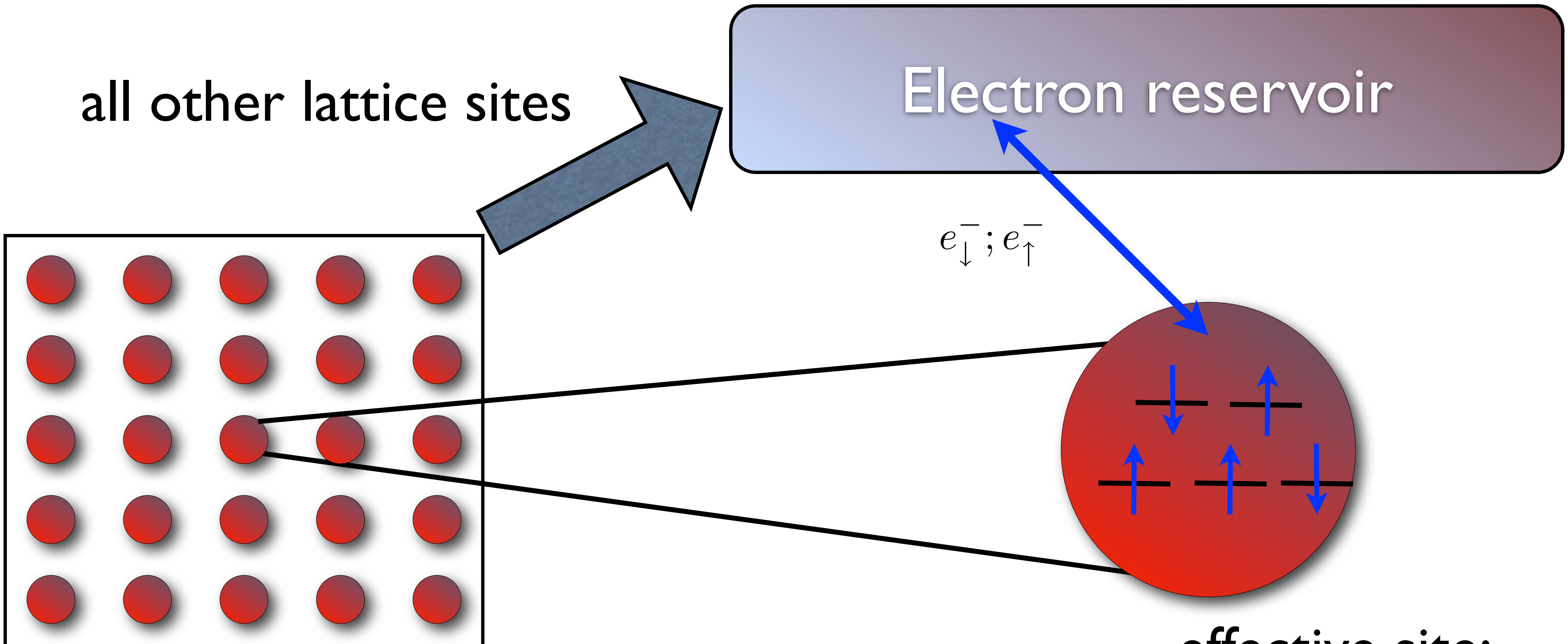
Dynamical mean field theory

Hubbard models: also local $\Sigma(\omega)$



Dynamical mean field theory

Hubbard models: also local $\Sigma(\omega)$

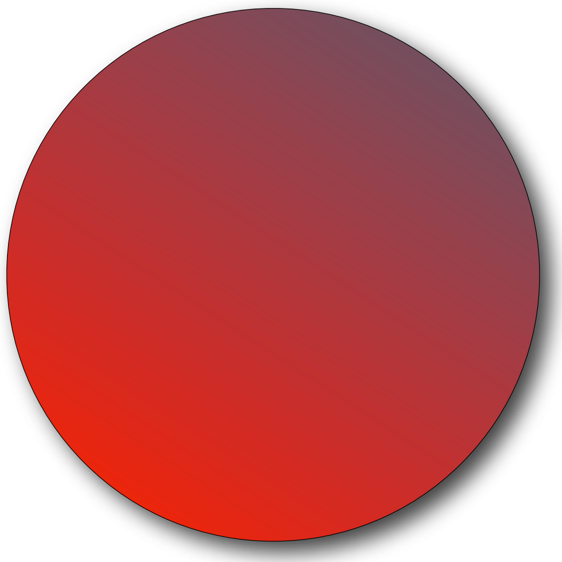


effective site:
impurity problem

$$G_{\rm lat}(z) = \frac{1}{N}\sum_{\vec{k}} G_{\vec{k}}(z) = \frac{1}{N}\sum_{\vec{k}} \frac{1}{z - \varepsilon_{\vec{k}} - \Sigma(z)} = G_{\rm loc}(z)$$

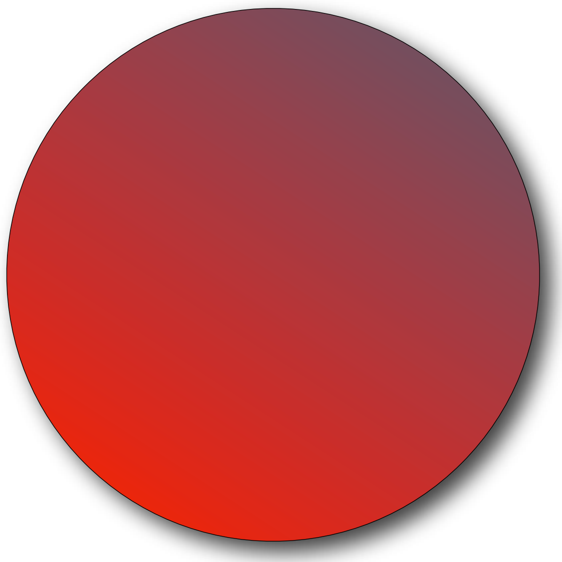
Jarrell, Kotliar, Georges, Pruschke, Vollhardt, and others, 1990s

impurity solver



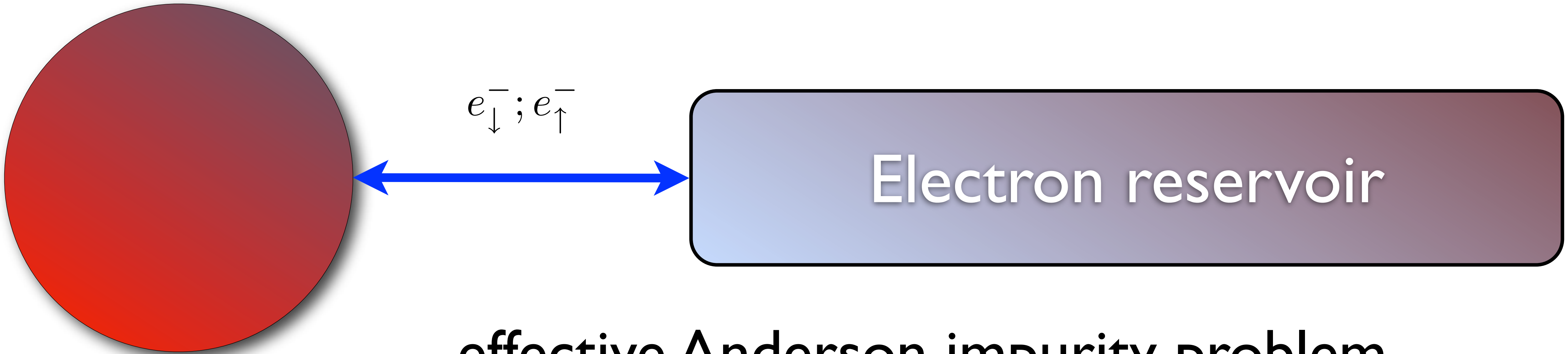
effective Anderson impurity problem

impurity solver



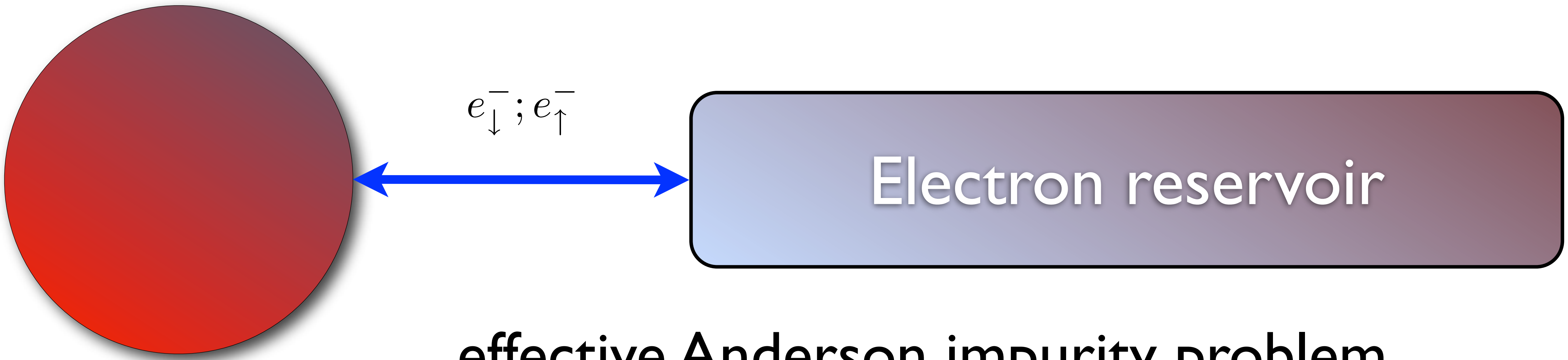
effective Anderson impurity problem

impurity solver



effective Anderson impurity problem

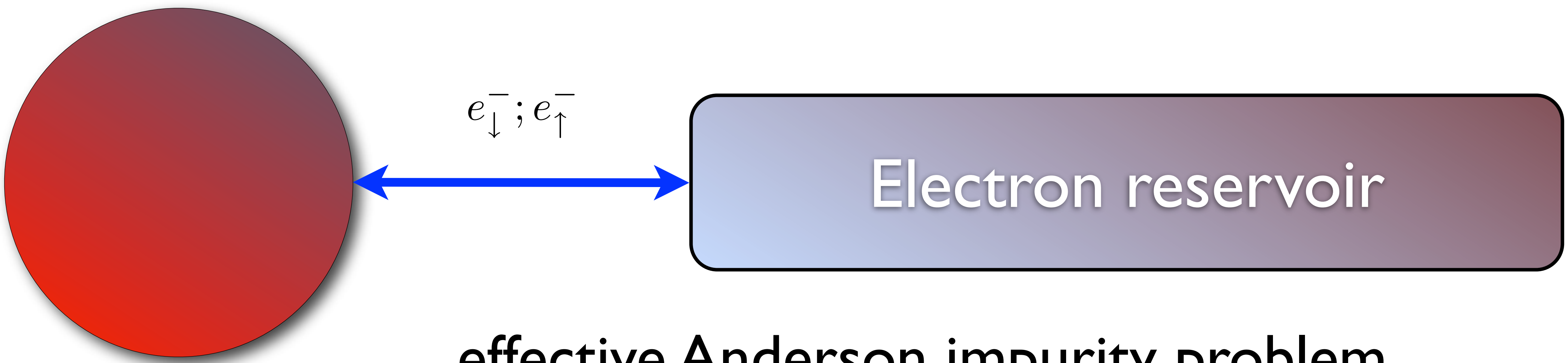
impurity solver



effective Anderson impurity problem

1. perturbation theory (IPT)

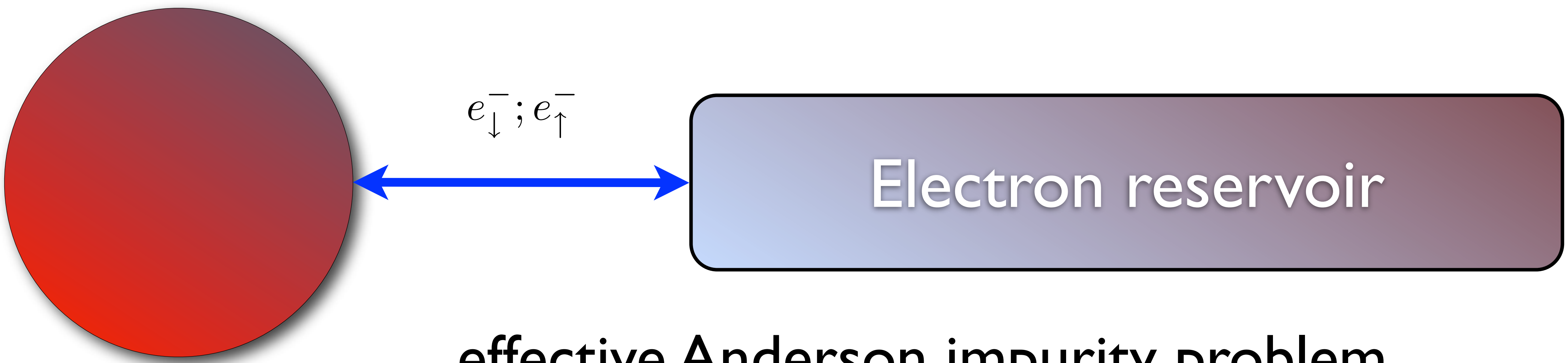
impurity solver



effective Anderson impurity problem

1. perturbation theory (IPT)
2. NCA/Post-NCA

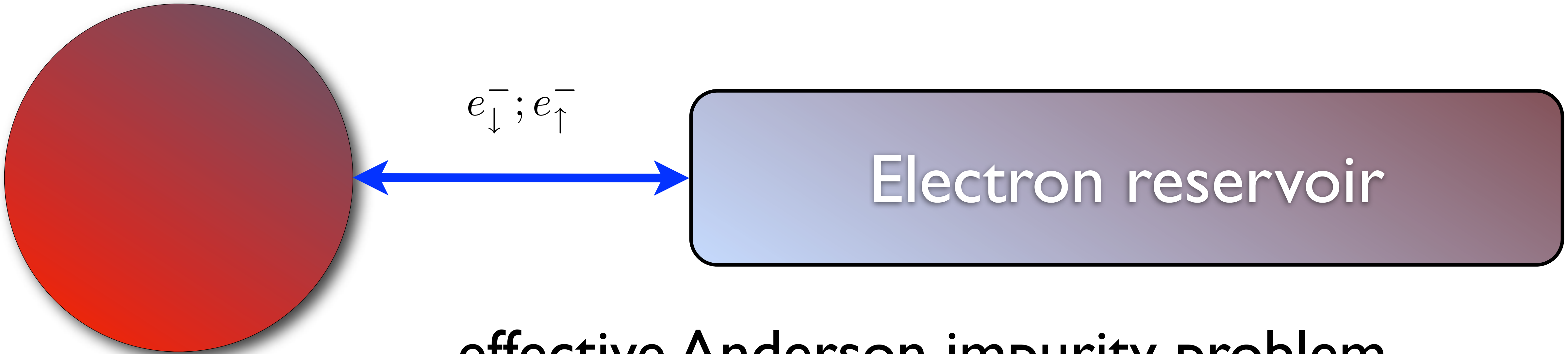
impurity solver



effective Anderson impurity problem

1. perturbation theory (IPT)
2. NCA/Post-NCA
3. Quantum Monte Carlo (Hirsch-Fye, Continuous time)

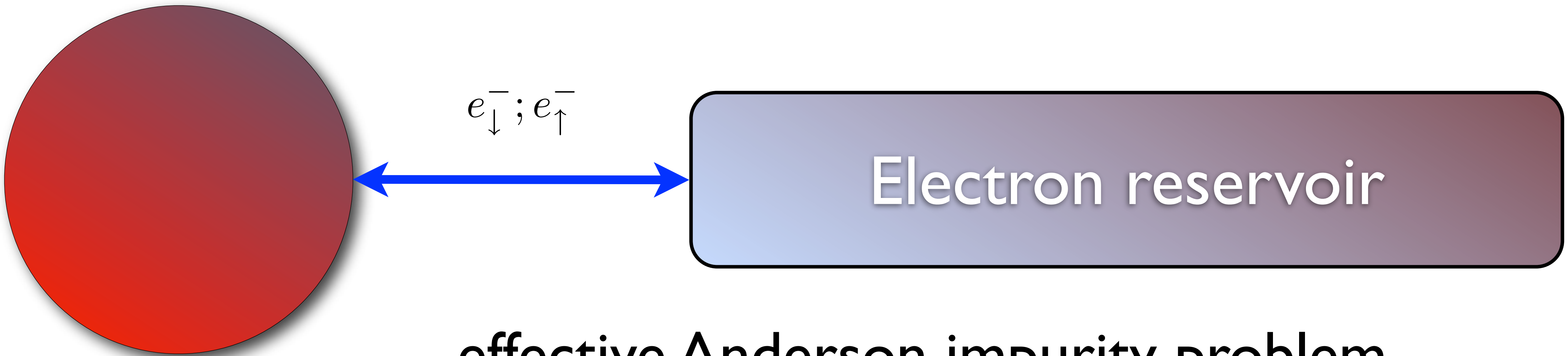
impurity solver



effective Anderson impurity problem

1. perturbation theory (IPT)
2. NCA/Post-NCA
3. Quantum Monte Carlo (Hirsch-Fye, Continuous time)
4. NRG (Ralf Bulla's lecture)

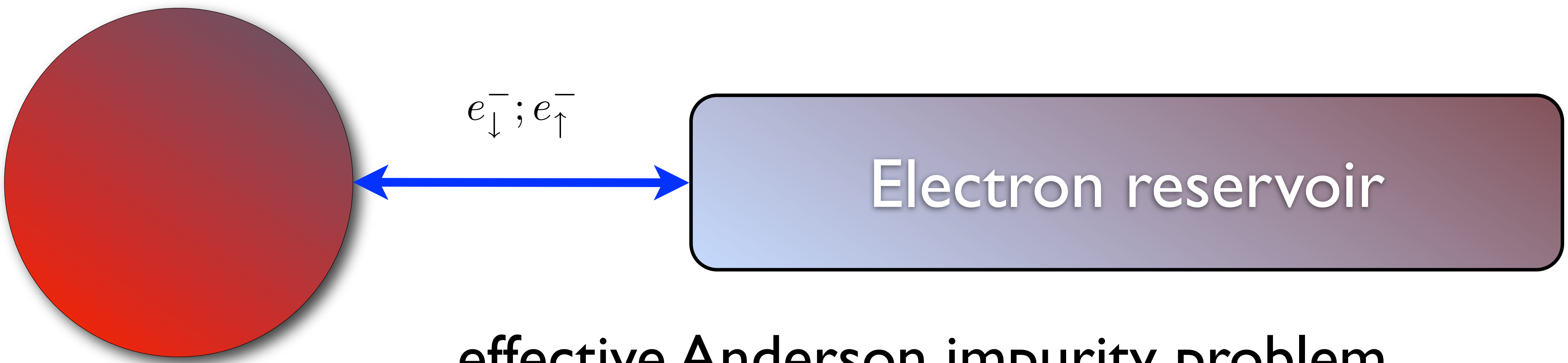
impurity solver



effective Anderson impurity problem

1. perturbation theory (IPT)
2. NCA/Post-NCA
3. Quantum Monte Carlo (Hirsch-Fye, Continuous time)
4. NRG (Ralf Bulla's lecture)
5. DMRG (T=0)

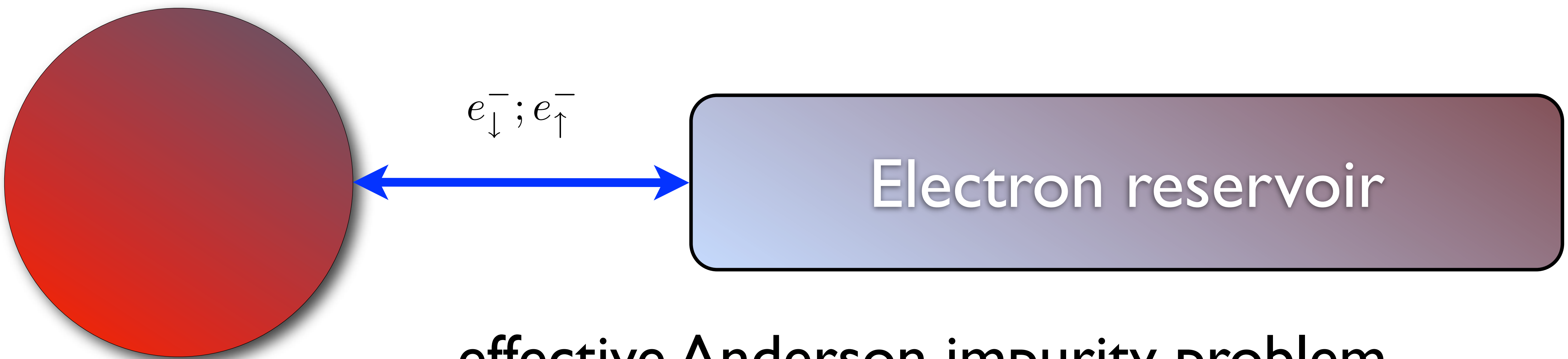
impurity solver



effective Anderson impurity problem

1. perturbation theory (IPT)
2. NCA/Post-NCA
3. Quantum Monte Carlo (Hirsch-Fye, Continuous time)
4. NRG (Ralf Bulla's lecture)
5. DMRG (T=0)
6. Gutzwiller ansatz

impurity solver

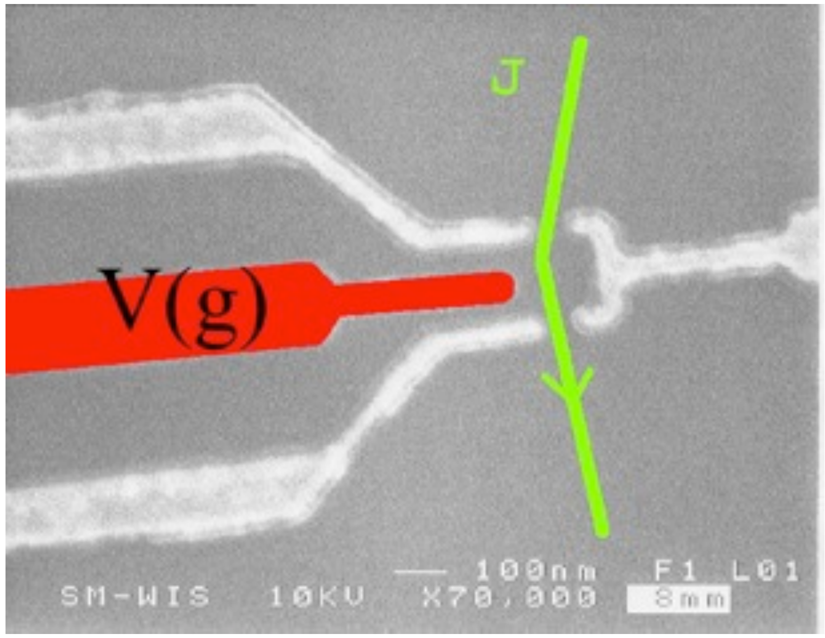


effective Anderson impurity problem

1. perturbation theory (IPT)
2. NCA/Post-NCA
3. Quantum Monte Carlo (Hirsch-Fye, Continuous time)
4. NRG (Ralf Bulla's lecture)
5. DMRG (T=0)
6. Gutzwiller ansatz
7. exact diagonalization (ED)

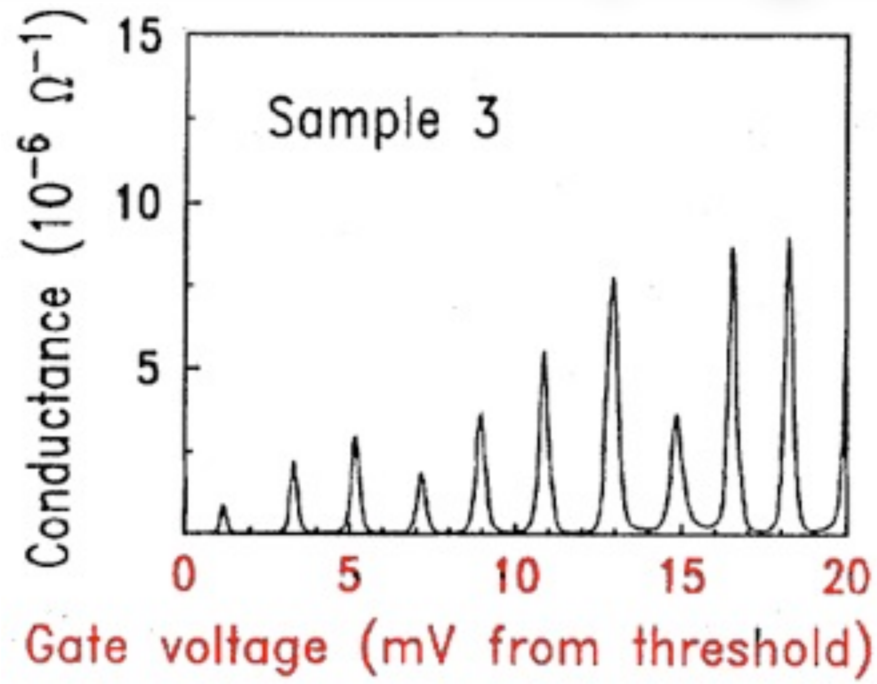
Kondo effect in nano-devices

Single-electron transistor



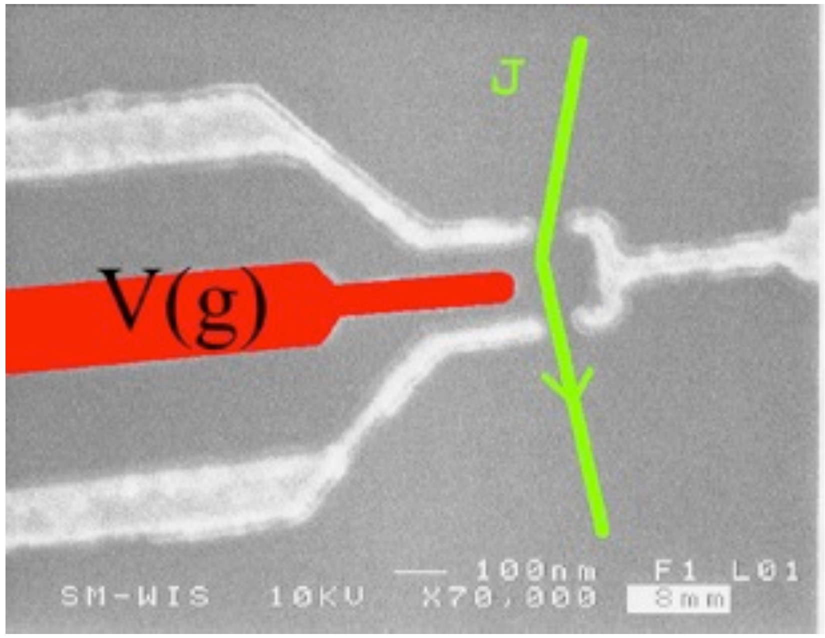
D. Goldhaber-Gordon, Nature 1998

weak coupling



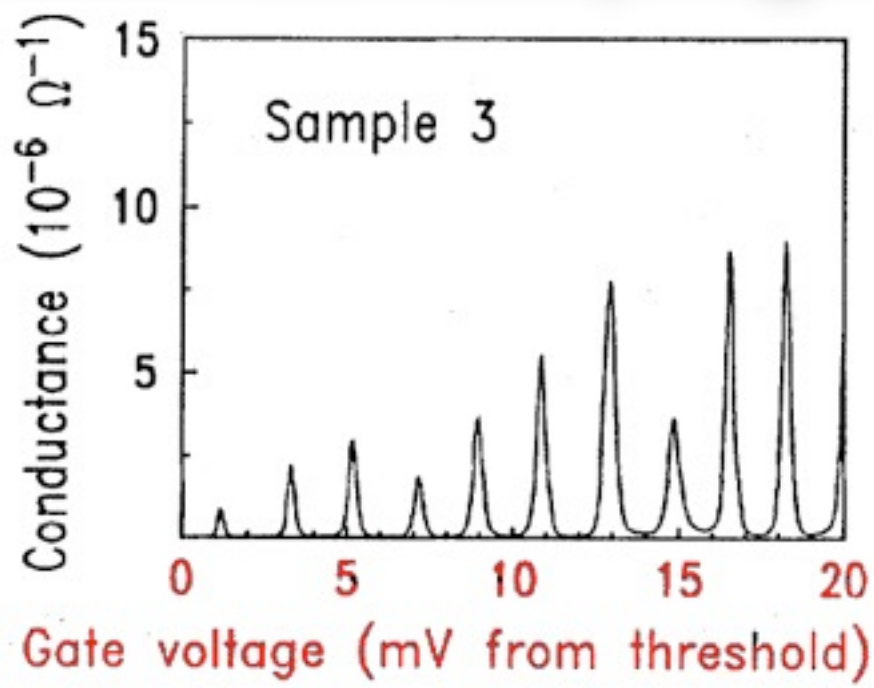
Kastner, RMP 64, 849(1992)

Single-electron transistor



D. Goldhaber-Gordon, Nature 1998

weak coupling

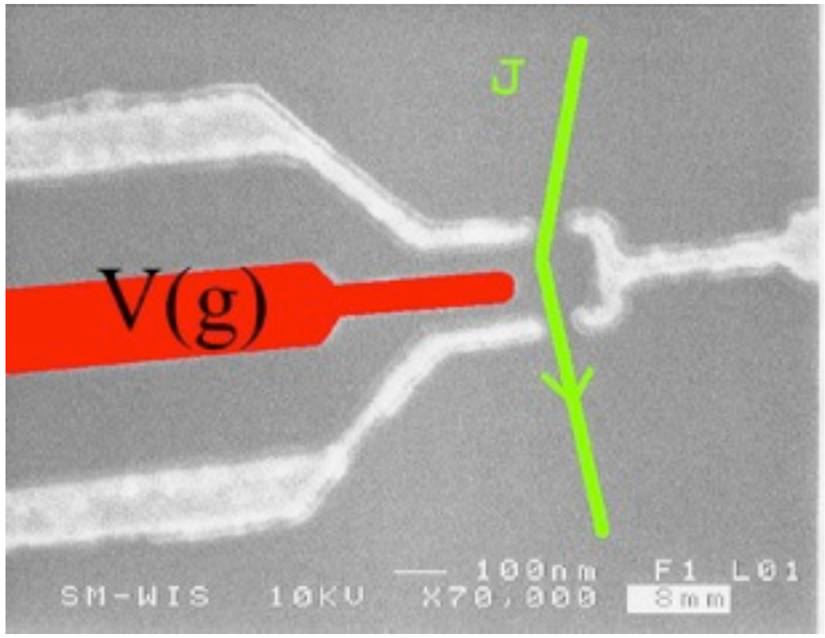


Kastner, RMP 64, 849(1992)

$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

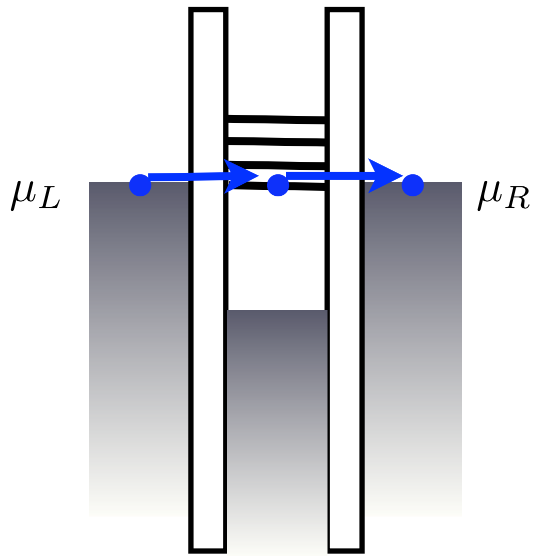
charging energy

Single-electron transistor



D. Goldhaber-Gordon, Nature 1998

weak coupling

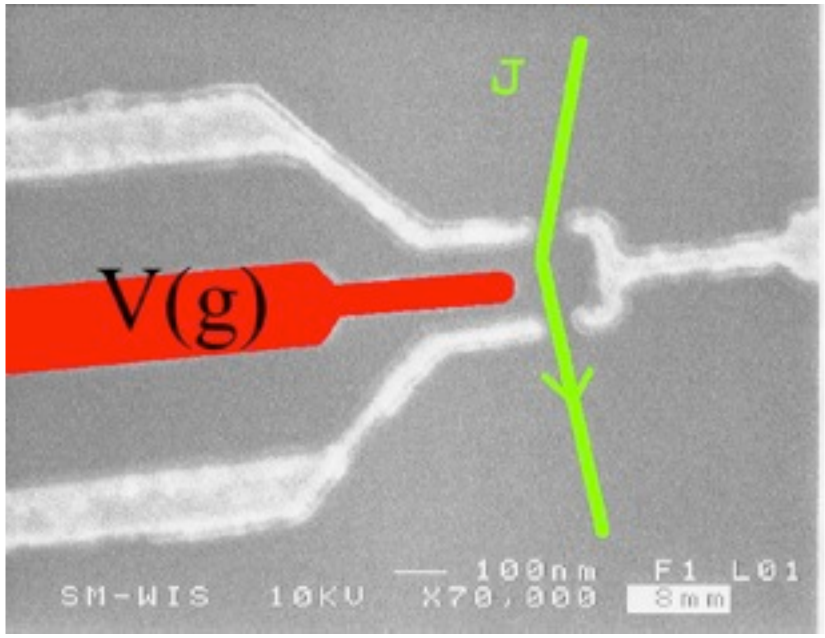


on resonance

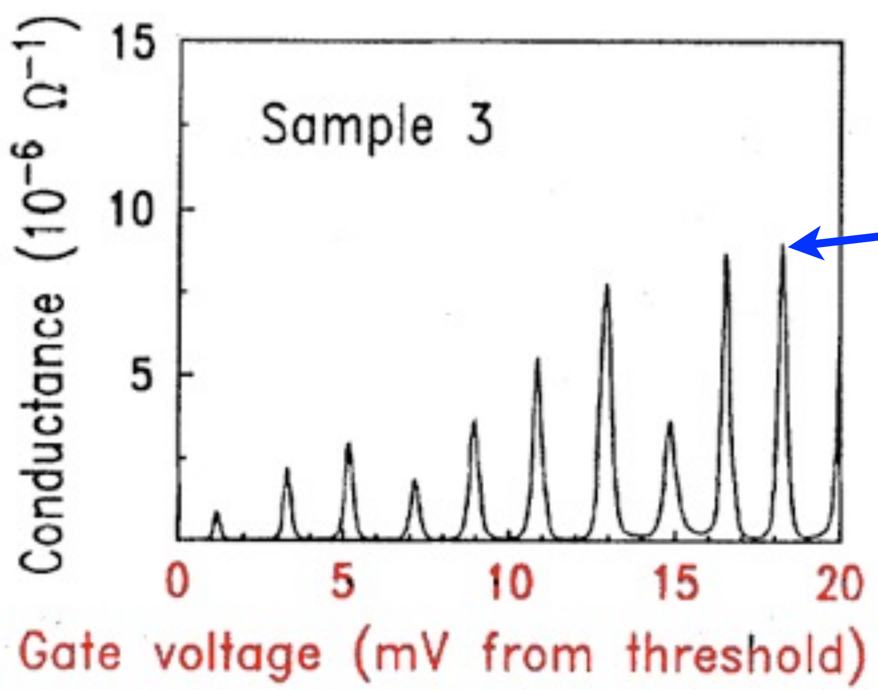
$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

charging energy

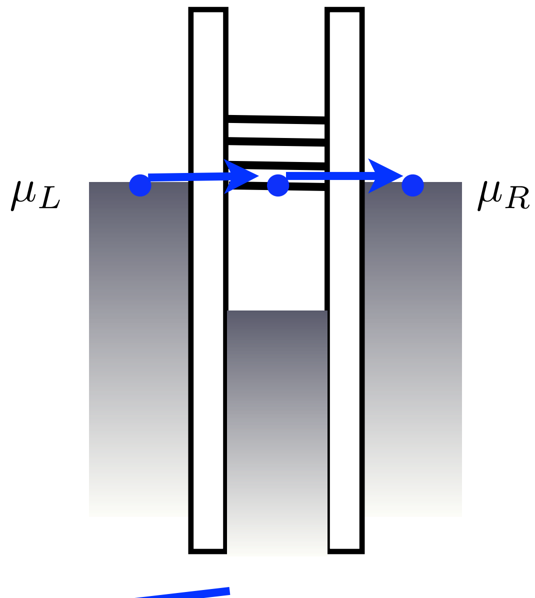
Single-electron transistor



D. Goldhaber-Gordon, Nature 1998



weak coupling

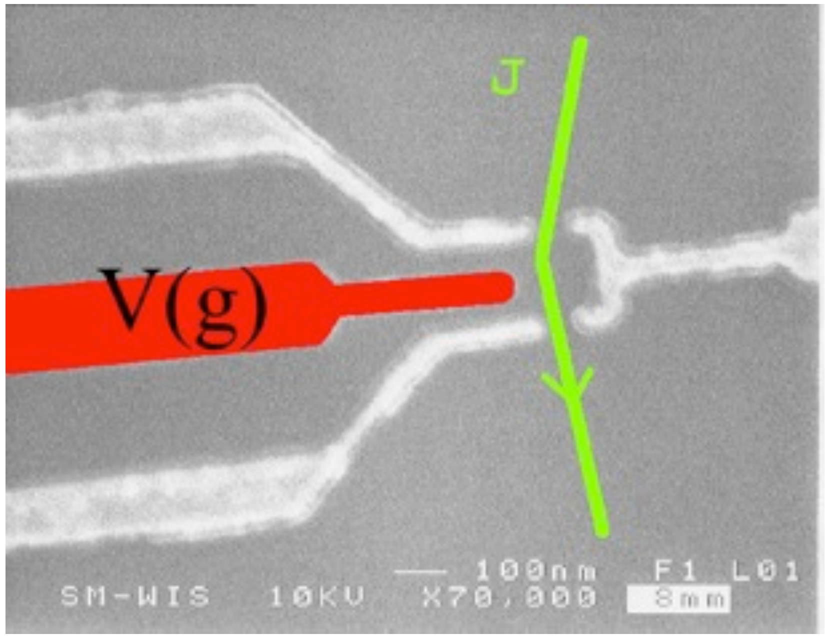


on resonance

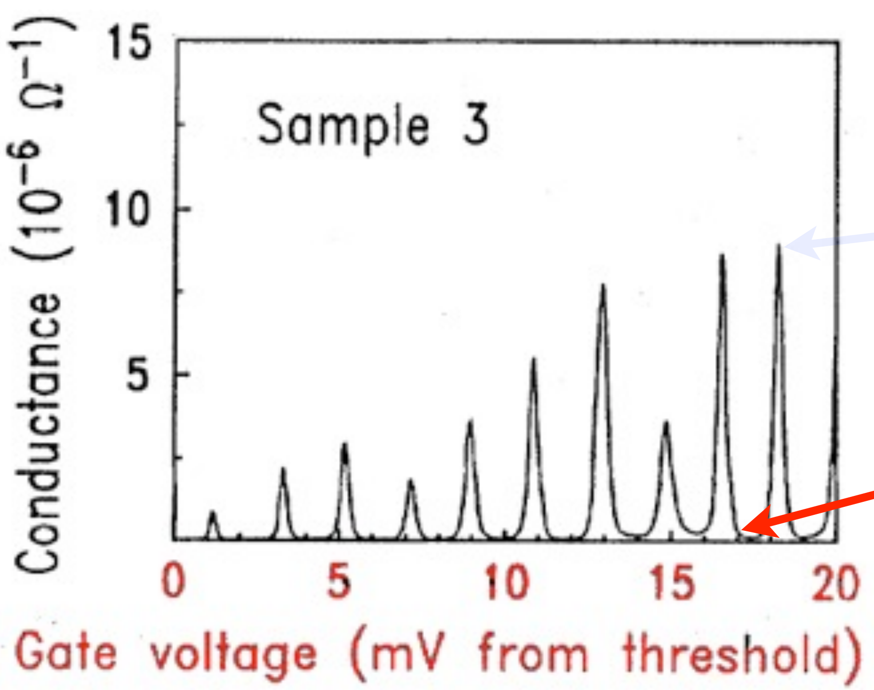
$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

charging energy

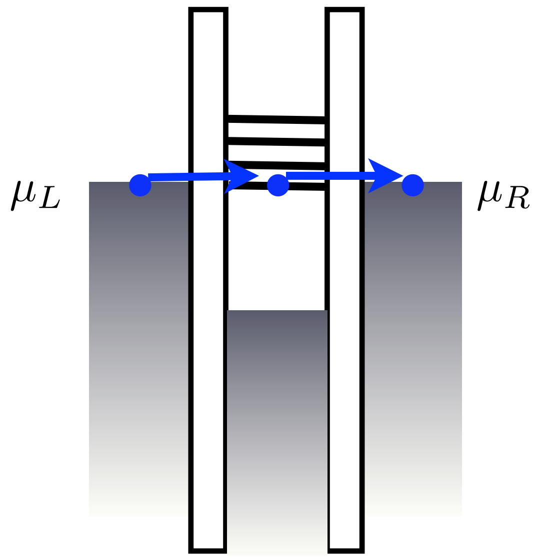
Single-electron transistor

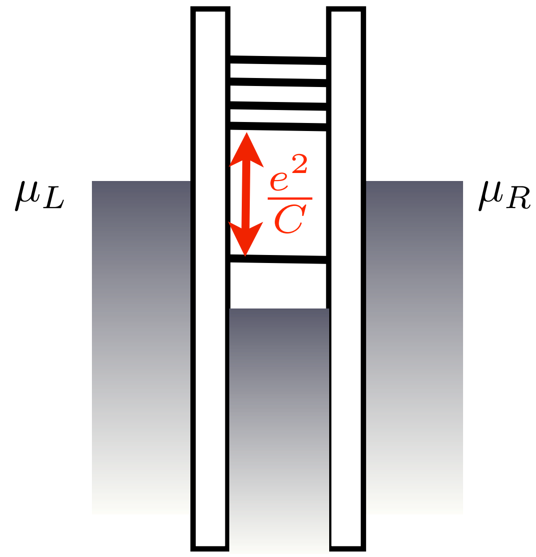


D. Goldhaber-Gordon, Nature 1998



weak coupling



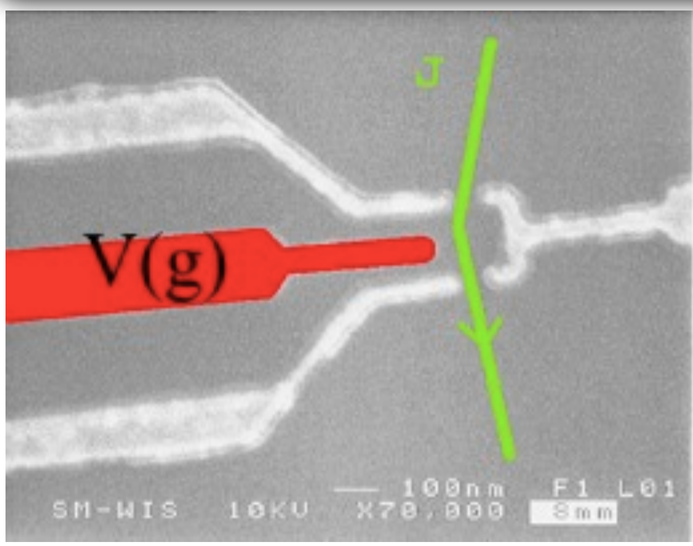


$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

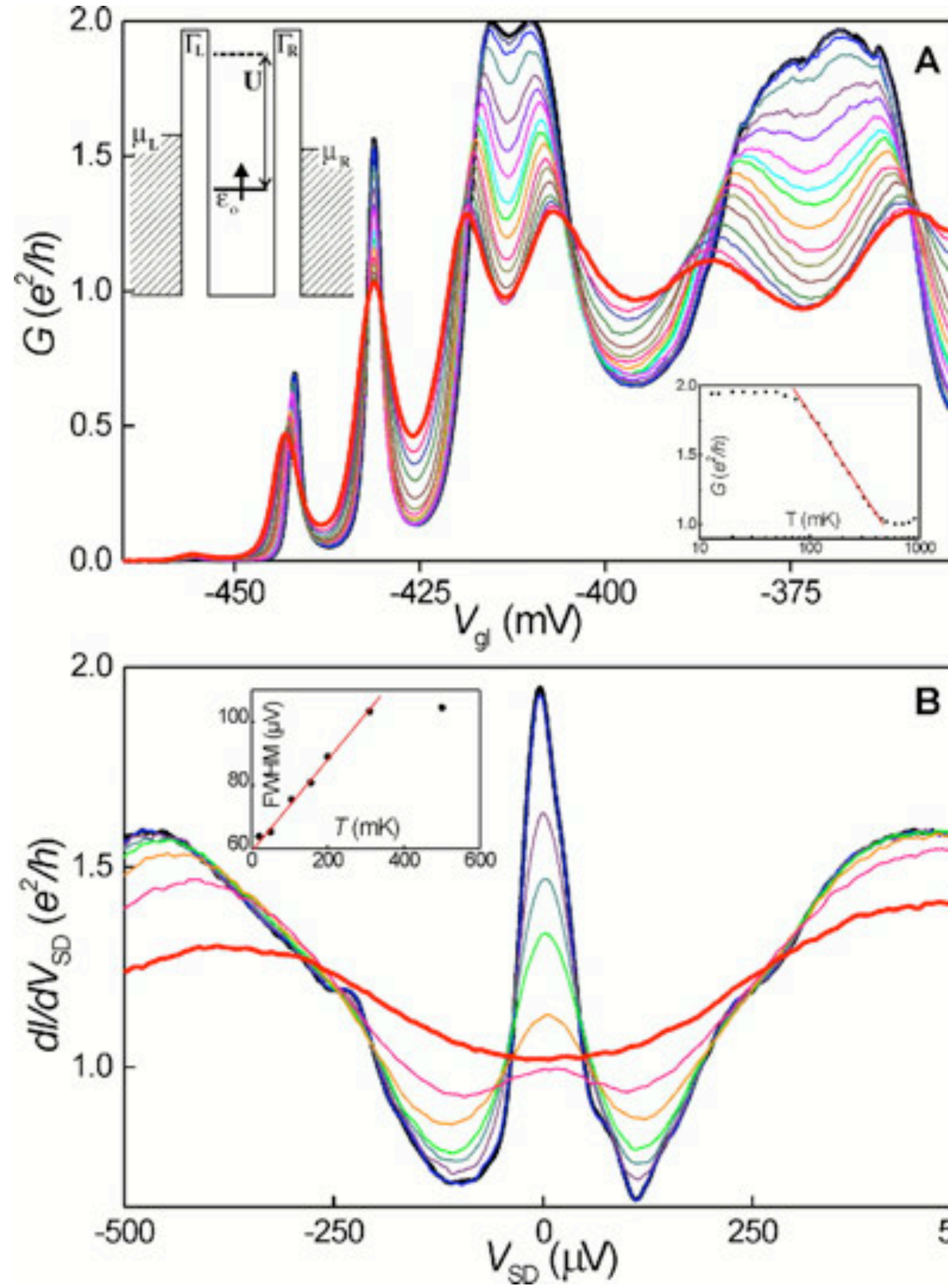
charging energy

Single-electron transistor

strong coupling

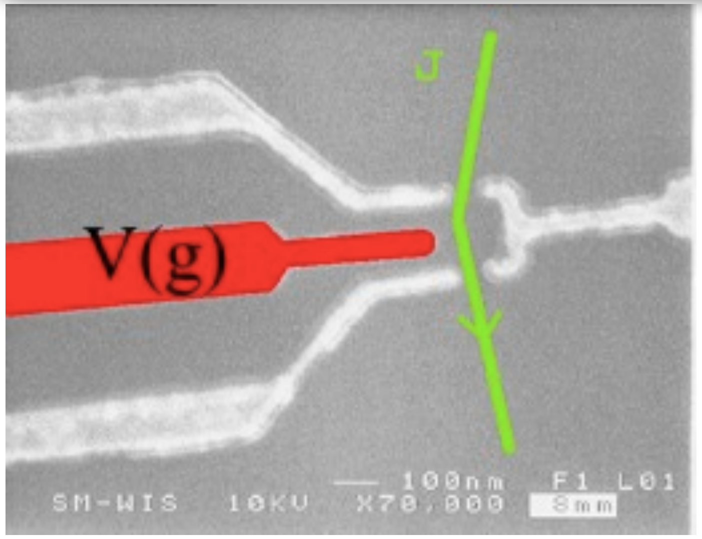


Goldhaber-Gordon, Nature 1998



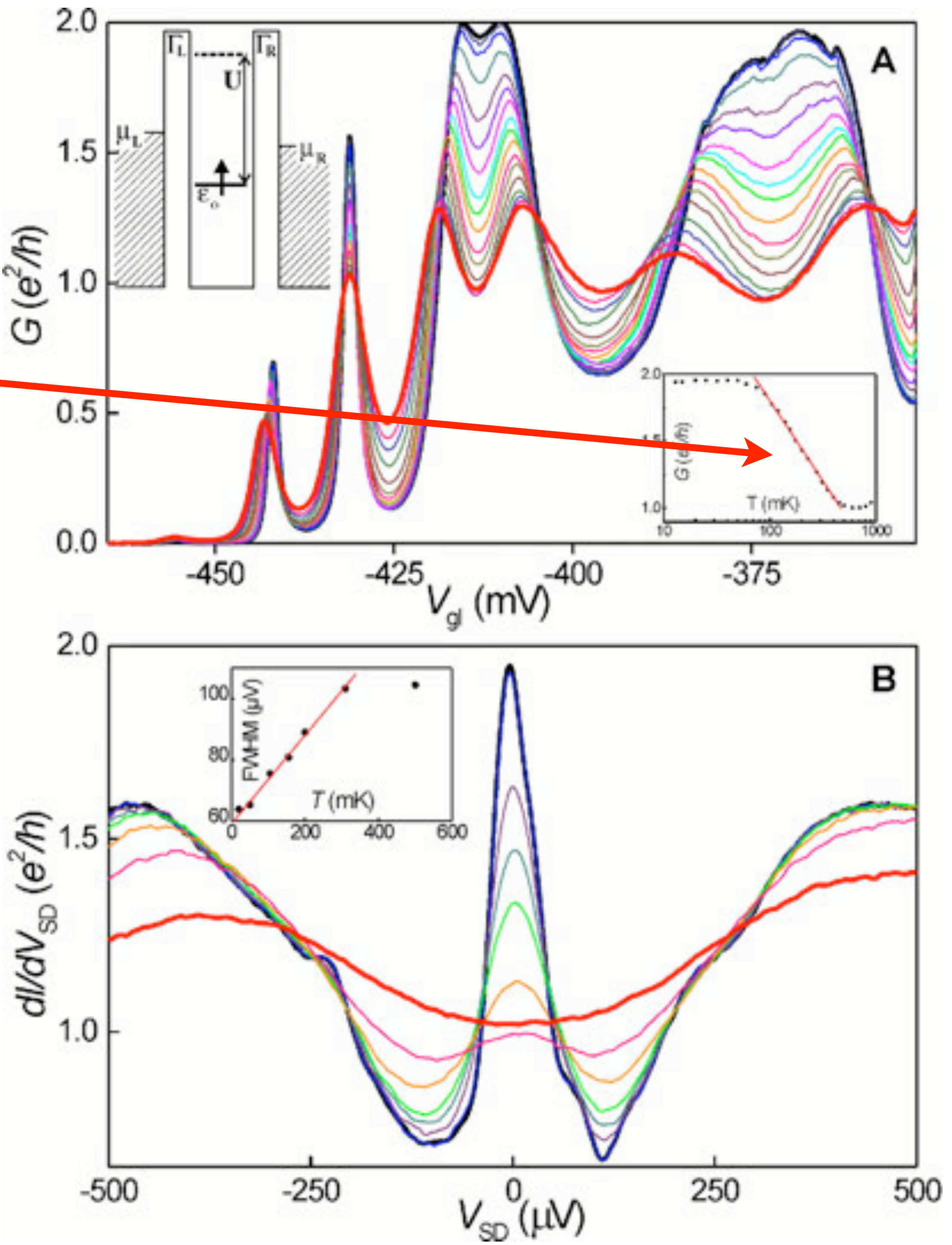
Single-electron transistor

strong coupling



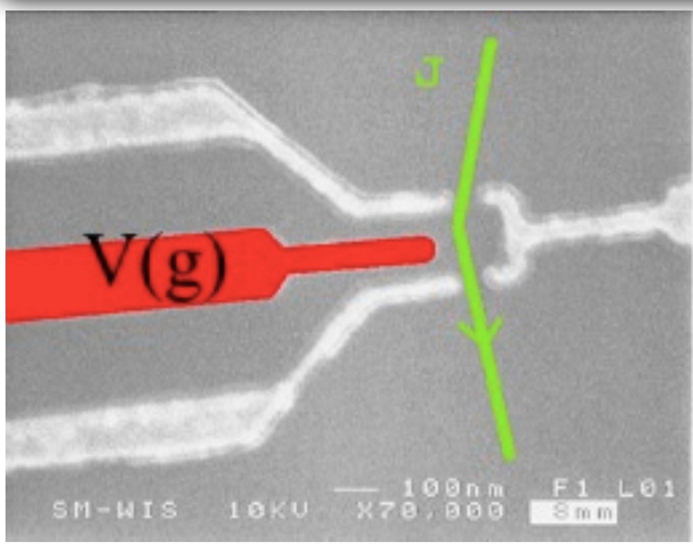
Goldhaber-Gordon, Nature 1998

log. increase in G(T)



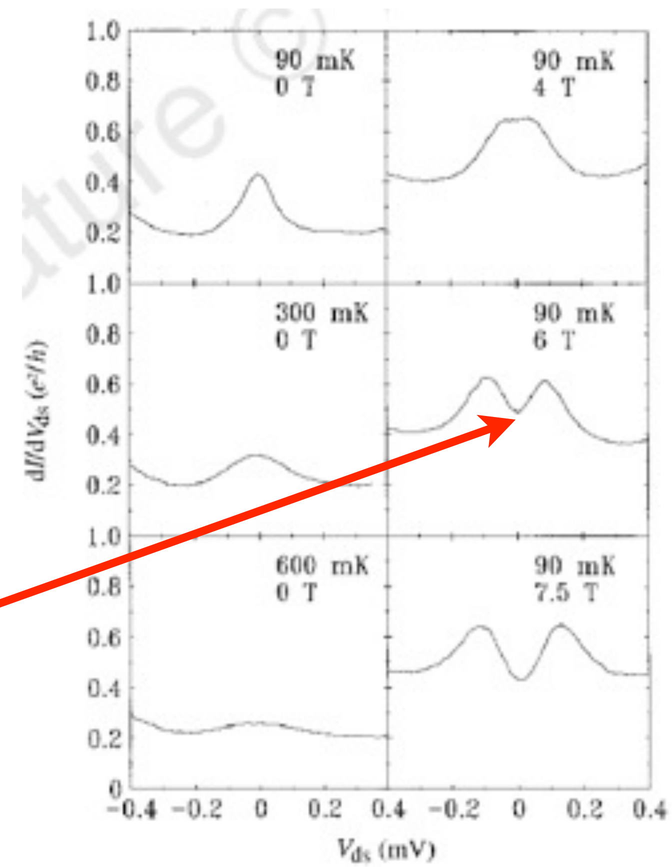
Single-electron transistor

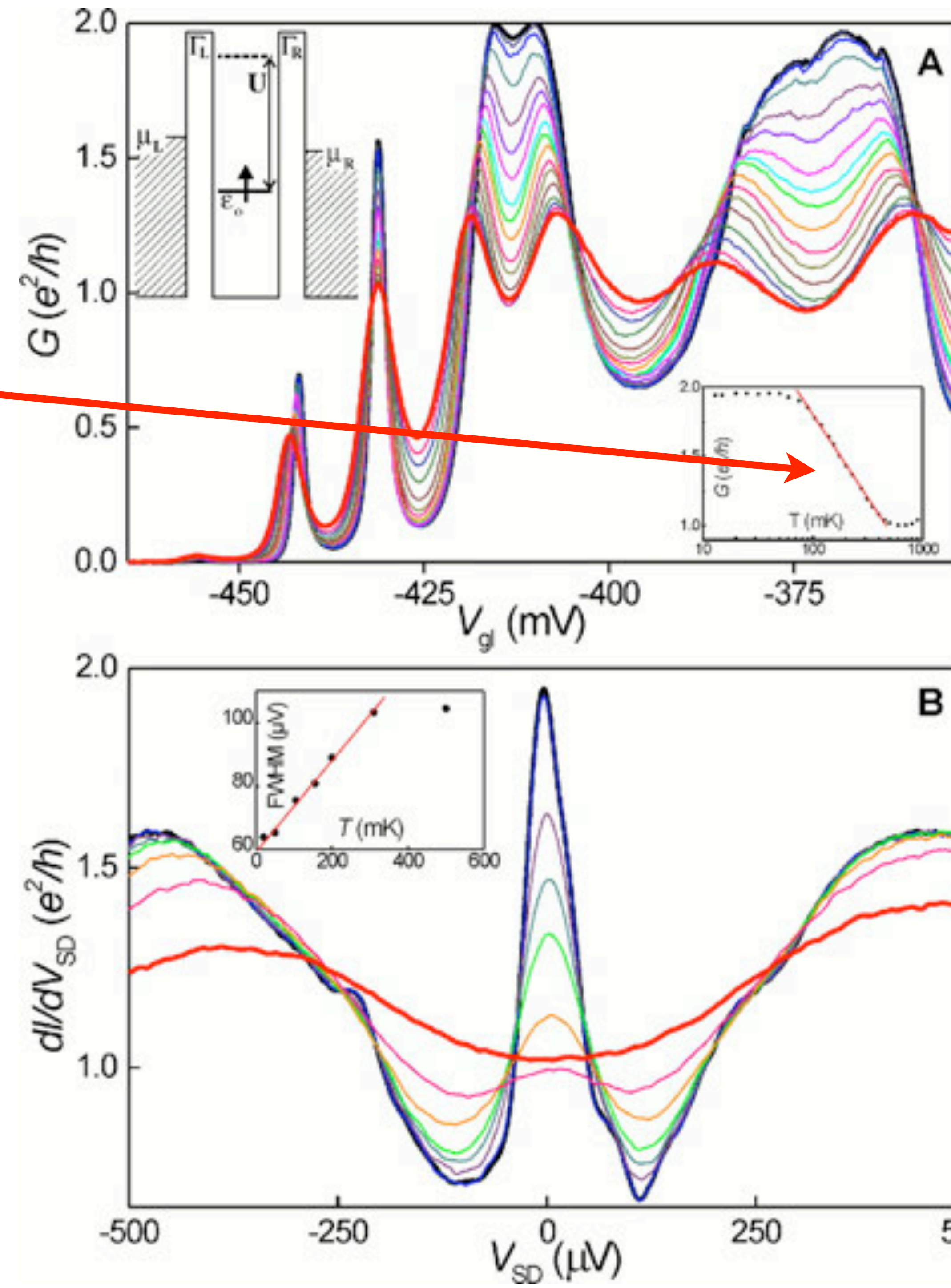
strong coupling



Goldhaber-Gordon, Nature 1998

log. increase in G(T)

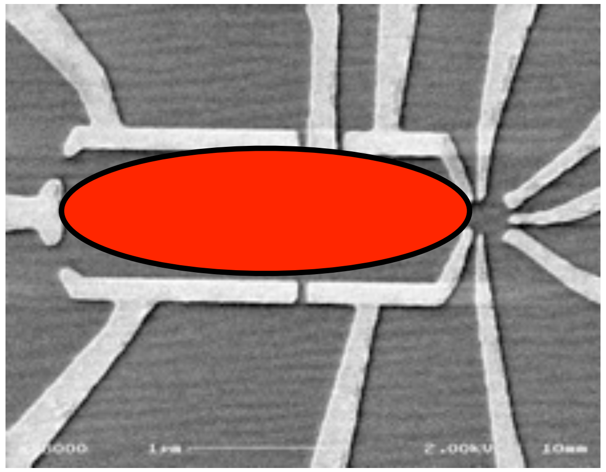




Splitting im Magnetfeld

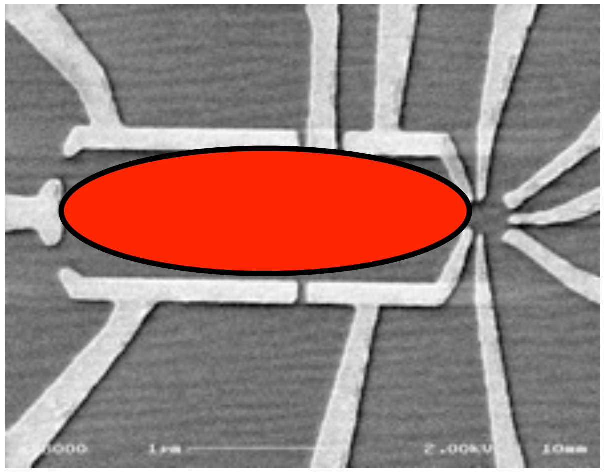
$$H_k = k_B T_K / \mu_B$$

Charging a quantum box



Potok et al. Nature

Charging a quantum box

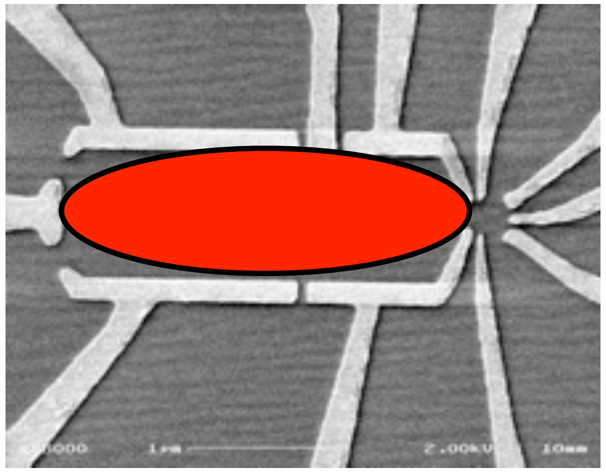


Potok et al. Nature

Charging energy of a capacitor

$$E = \frac{1}{2C}Q^2 - QV_g$$

Charging a quantum box

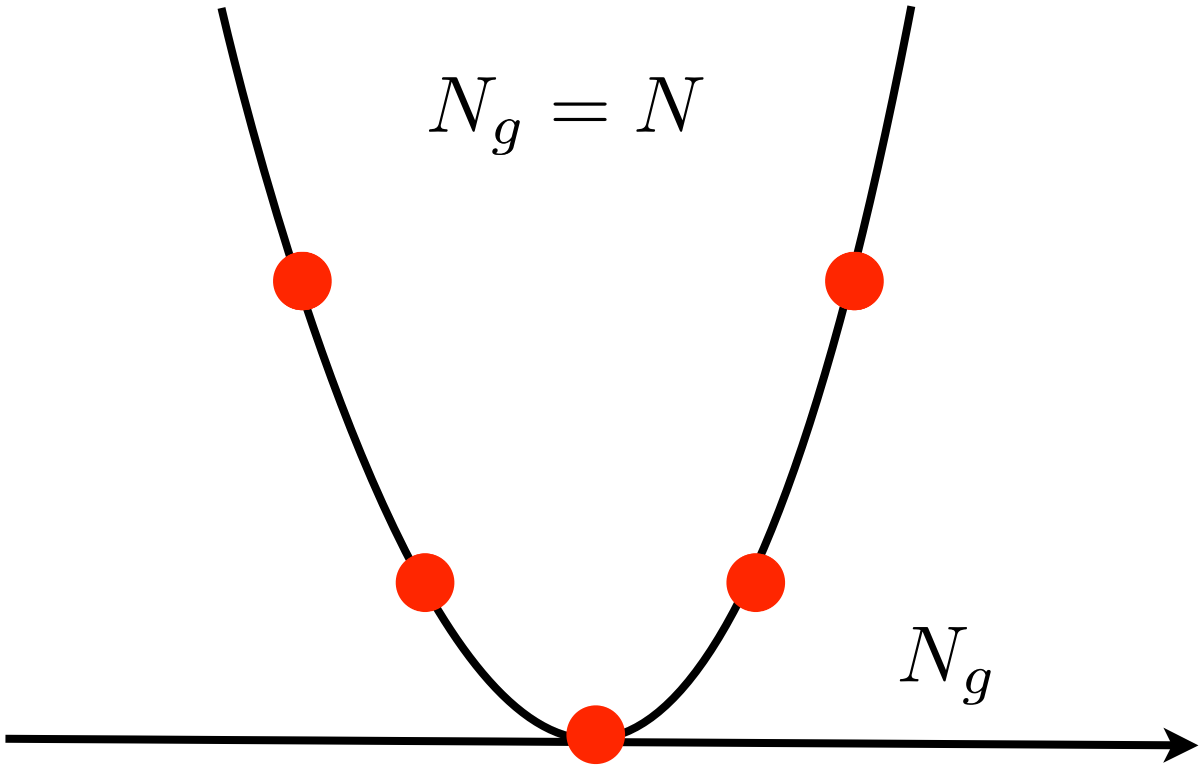


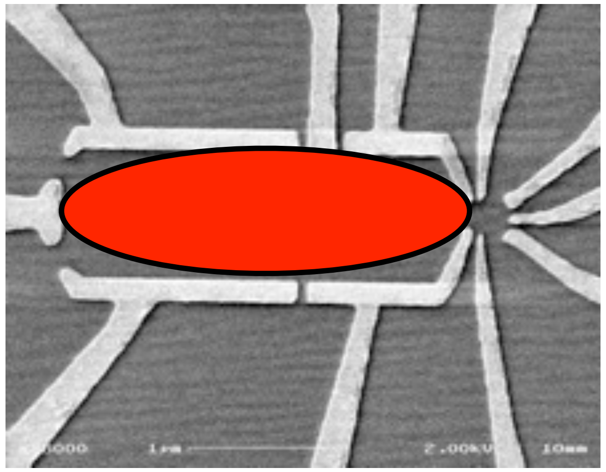
Potok et al. Nature

Charging energy of a capacitor

$$E = \frac{1}{2C}Q^2 - QV_g = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

Charging a quantum box



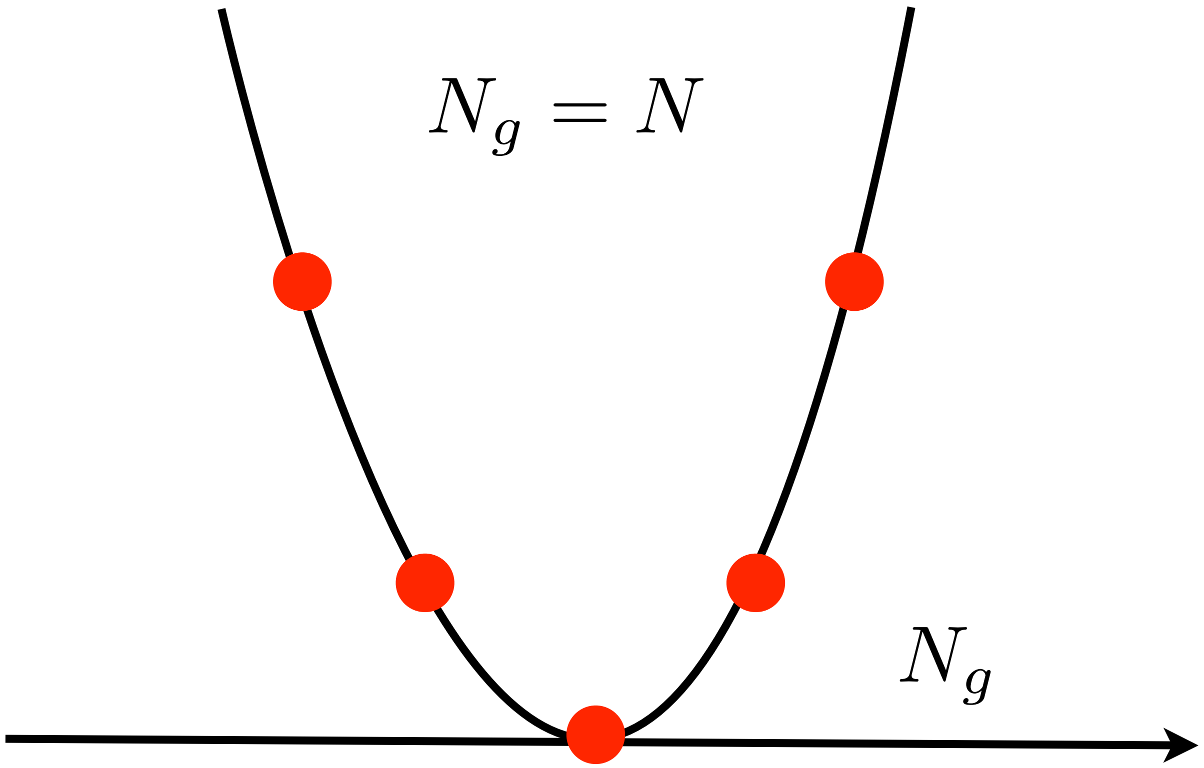


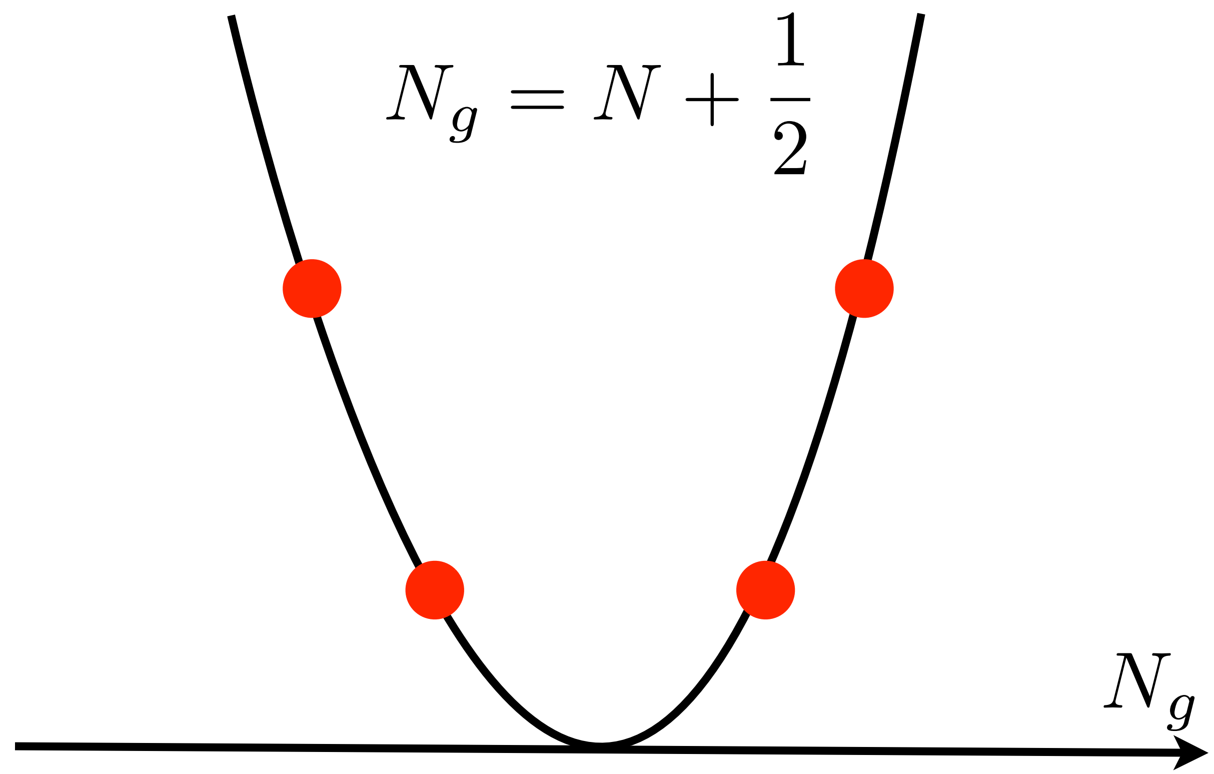
Potok et al. Nature

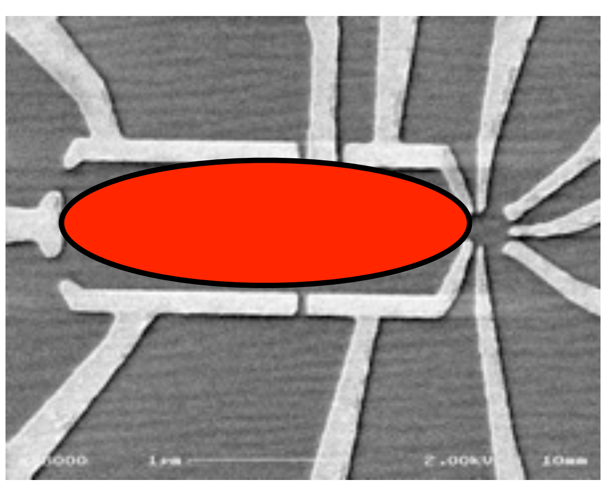
Charging energy of a capacitor

$$E = \frac{1}{2C}Q^2 - QV_g = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

Charging a quantum box





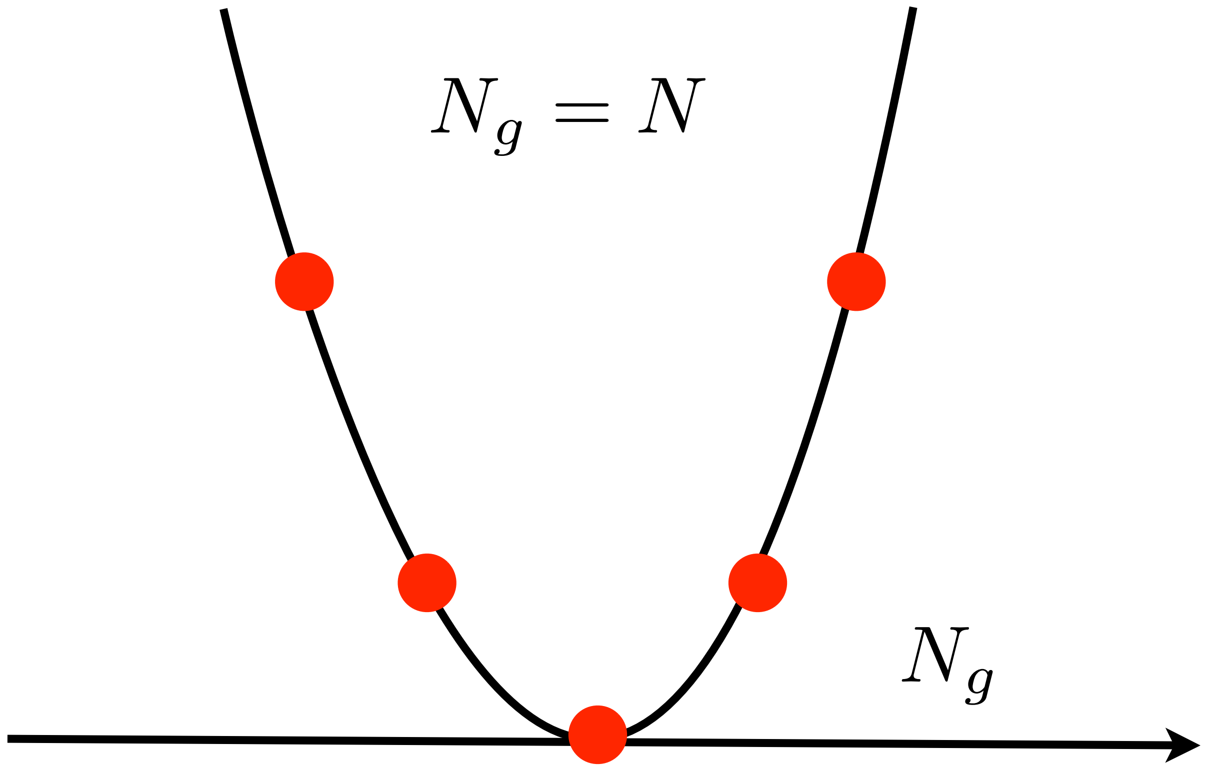


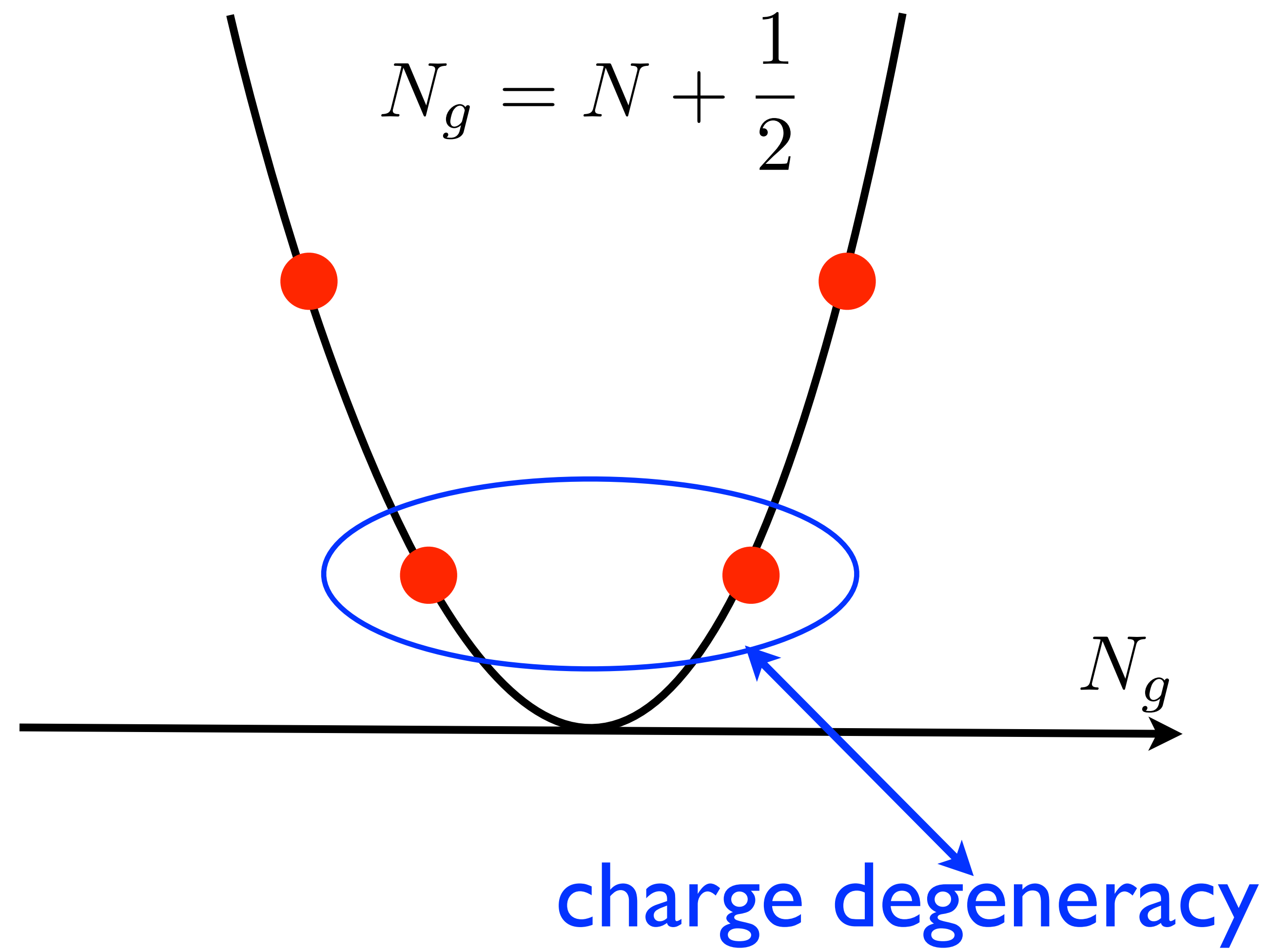
Potok et al. Nature

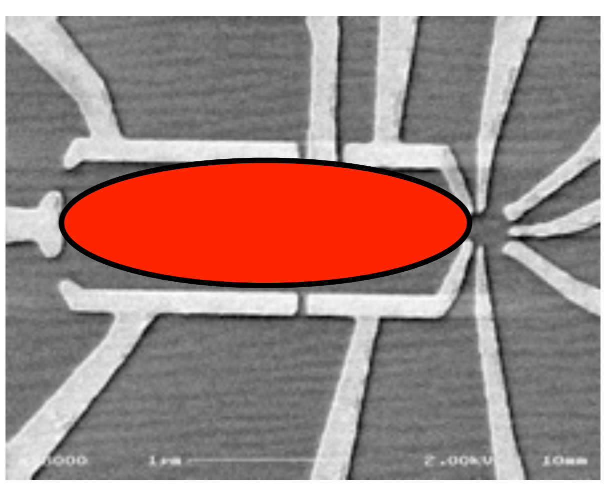
Charging energy of a capacitor

$$E = \frac{1}{2C} Q^2 - QV_g = \frac{e^2}{2C} \left(\hat{N} - N_g \right)^2$$

Charging a quantum box





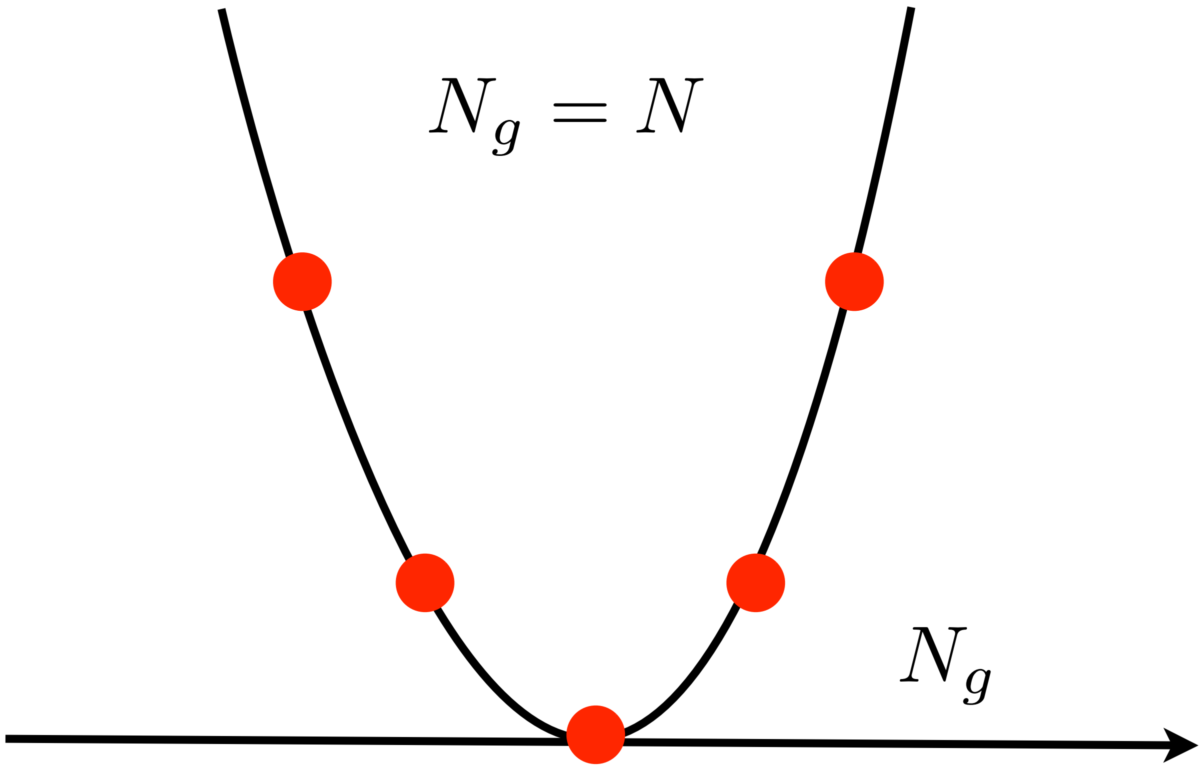


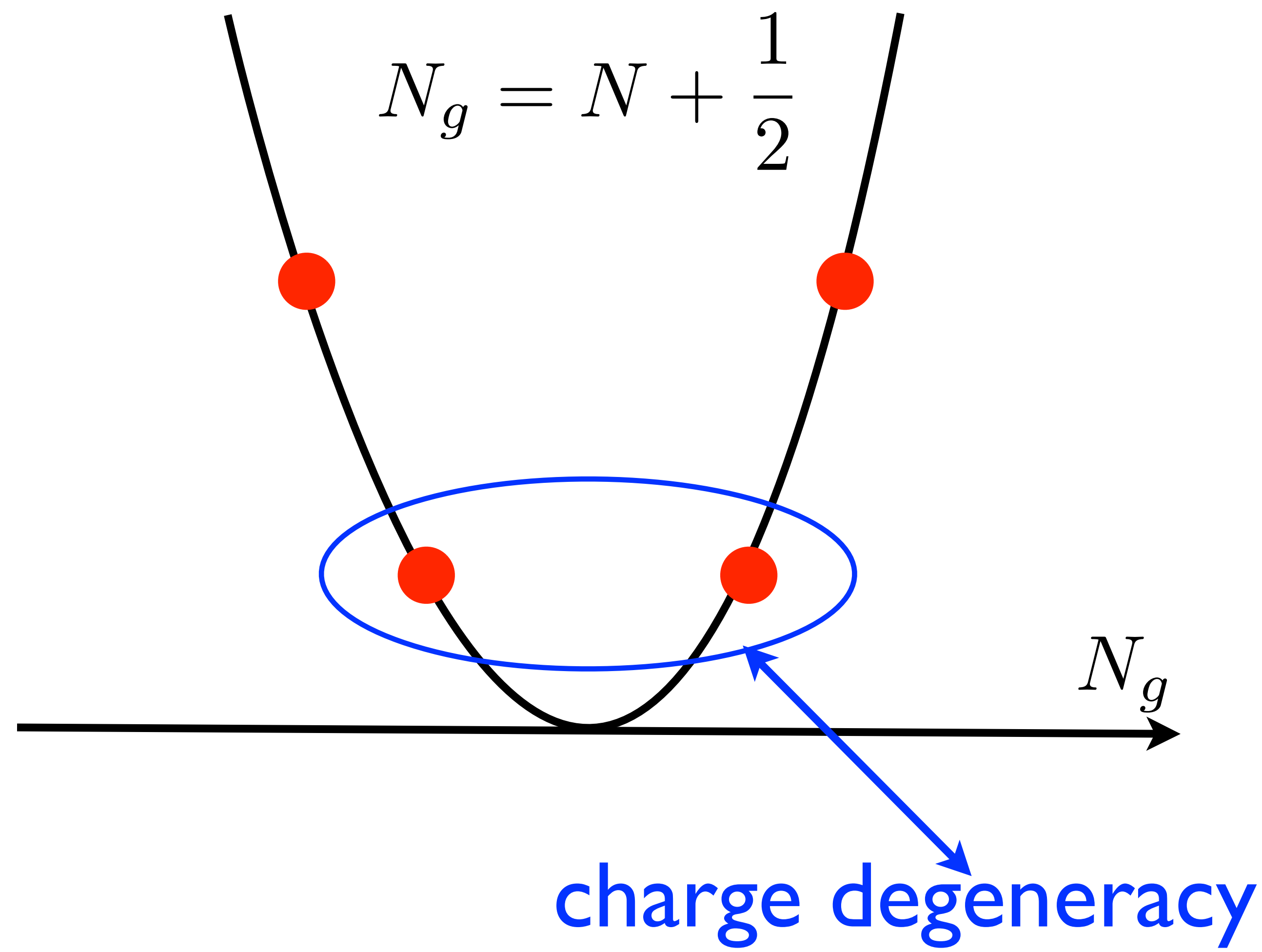
Potok et al. Nature

Charging energy of a capacitor

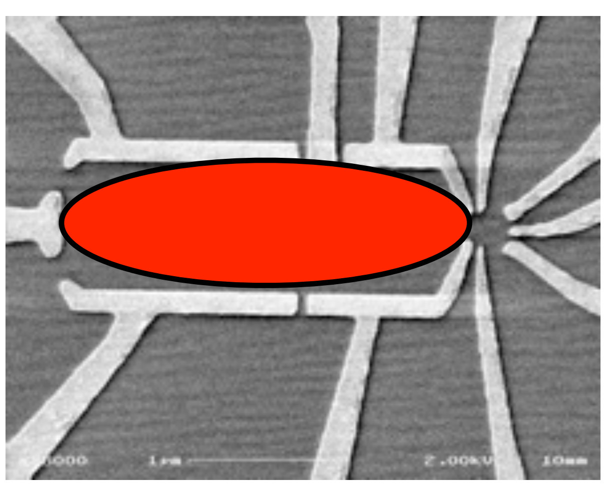
$$E = \frac{1}{2C}Q^2 - QV_g = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

Charge Kondo effect



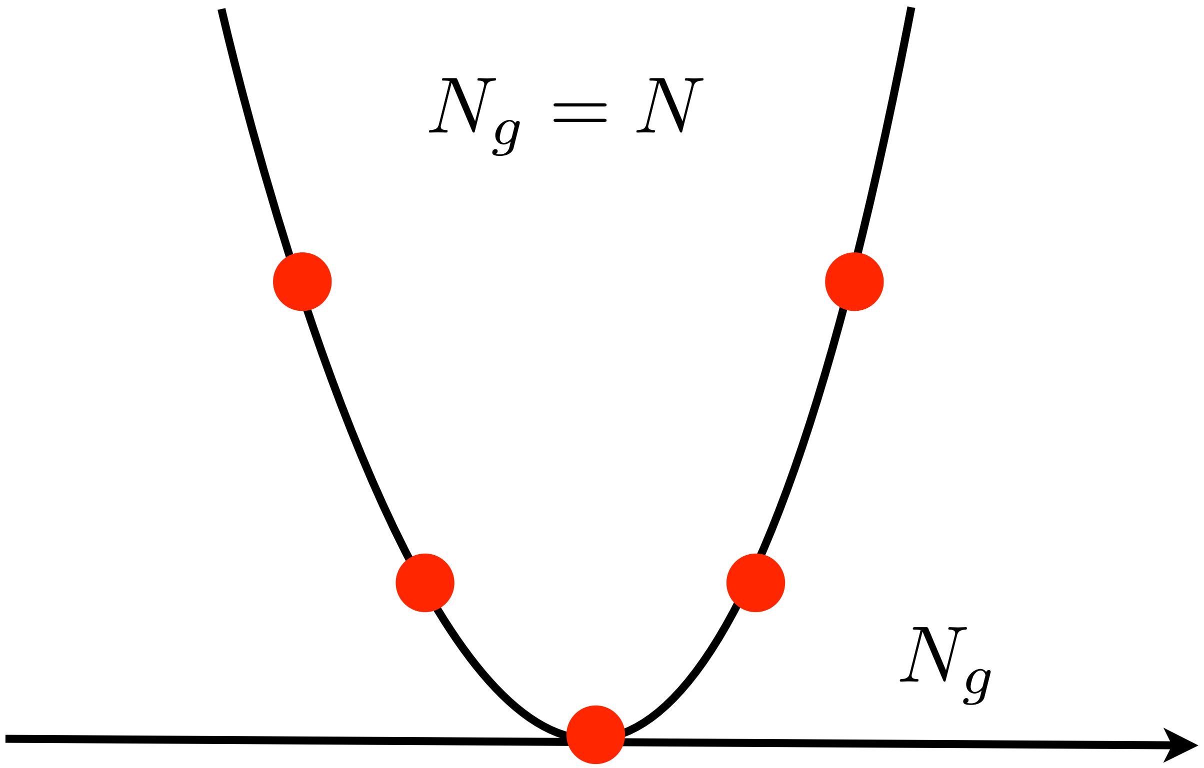


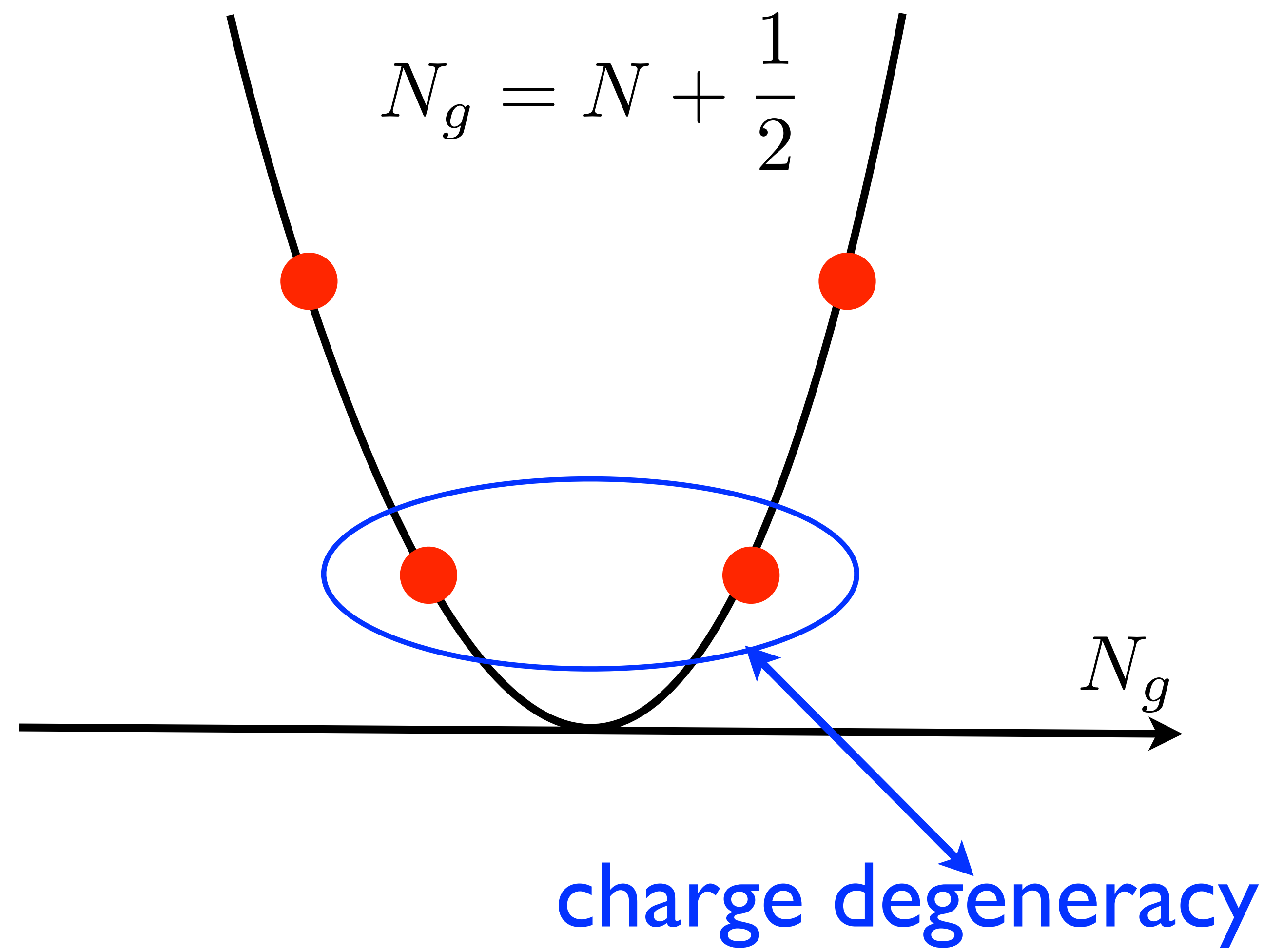
$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$



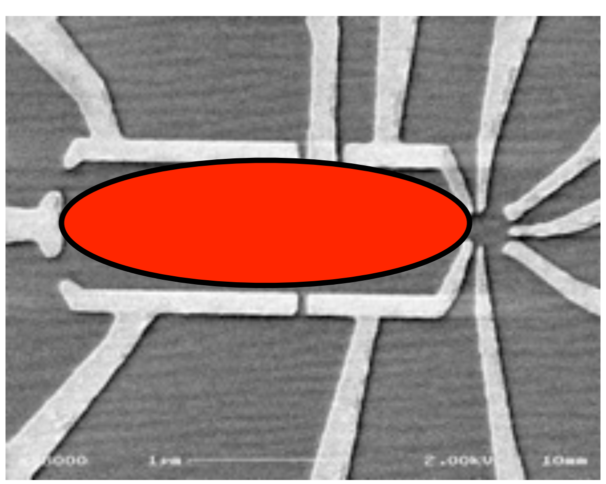
Potok et al. Nature

Charge Kondo effect





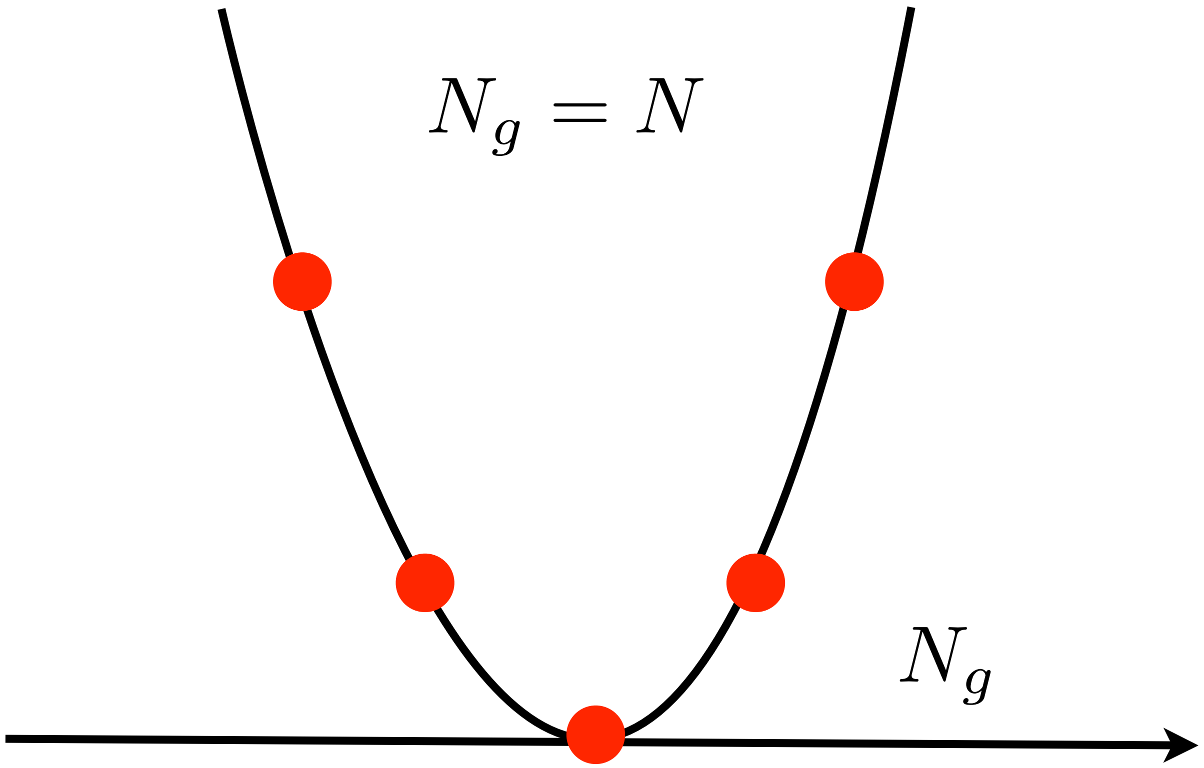
$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

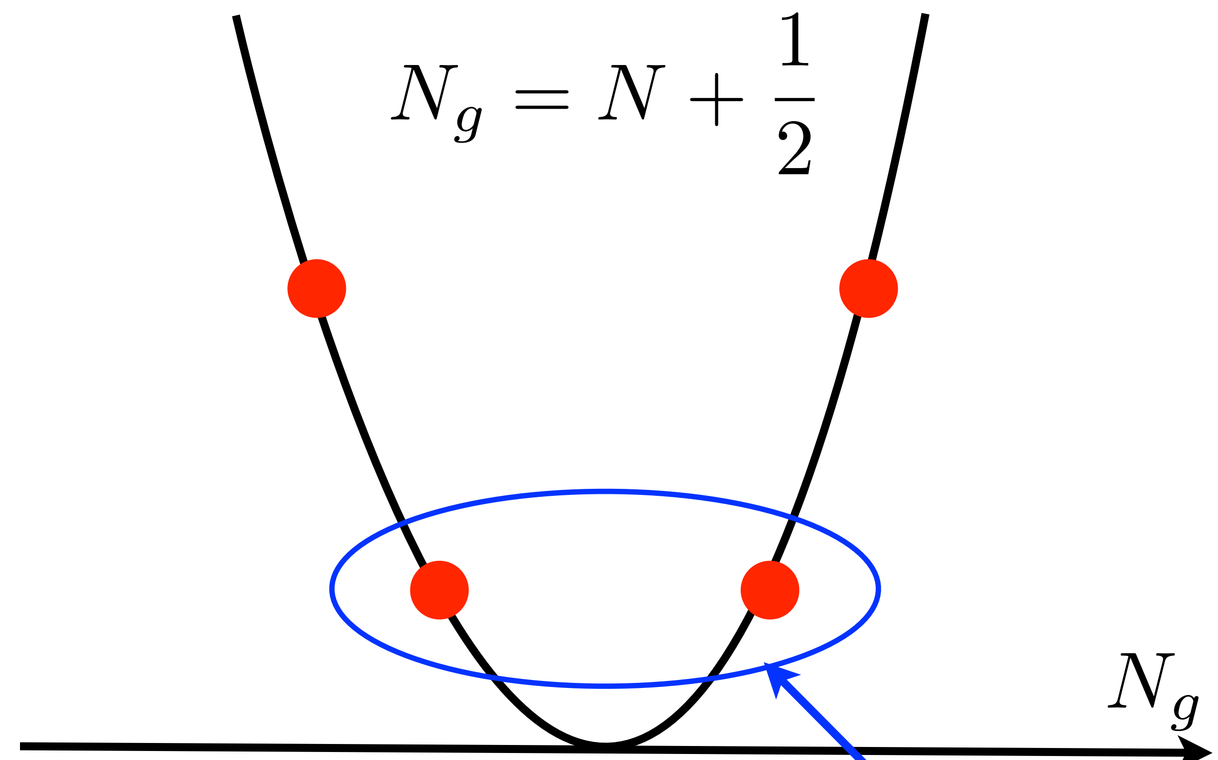


Potok et al. Nature

$$H_{imp} + H_T = \Delta N_g \tau_z + \frac{1}{N}\sum_{\sigma k,q}\left(c^\dagger_{k\sigma B} c_{q\sigma L}\tau^+ + c^\dagger_{q\sigma L} c_{k\sigma B}\tau^-\right)$$

Charge Kondo effect

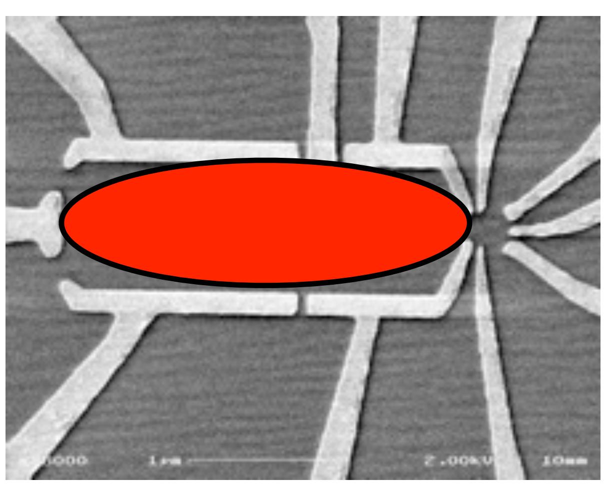




$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

charge degeneracy

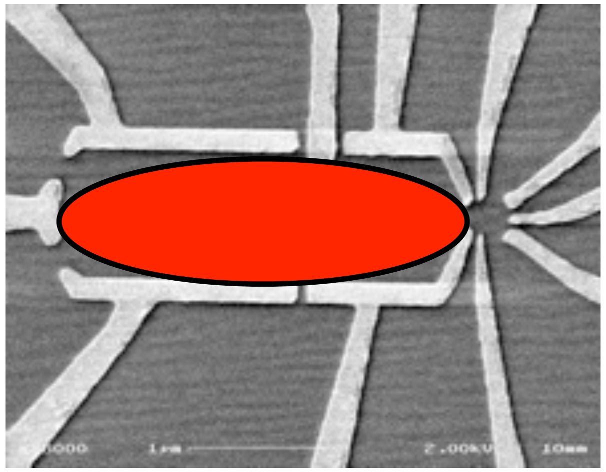
$$H_{imp} + H_T = \Delta N_g \tau_z + \frac{1}{N}\sum_{\sigma k,q}\left(c^\dagger_{k\sigma B} c_{q\sigma L}\tau^+ + c^\dagger_{q\sigma L} c_{k\sigma B}\tau^-\right)$$



Potok et al. Nature

Matveev (1991) two channel Kondo model

Two channel Kondo effect?

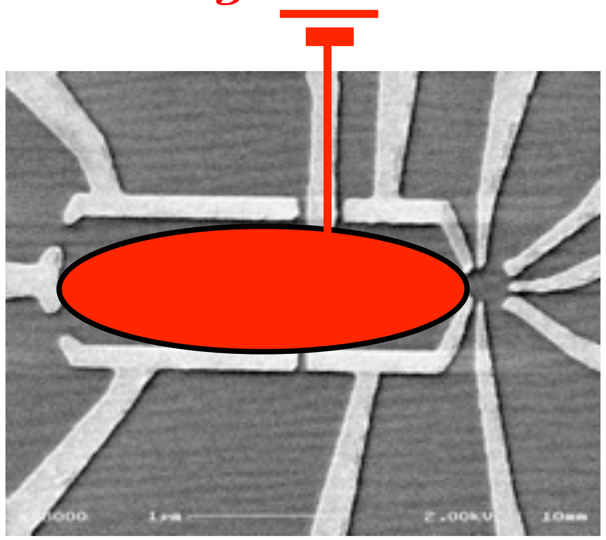


Potok et al, Nature 446, 167 (2007)

Two channel Kondo effect?

$$N_g = bp$$

$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

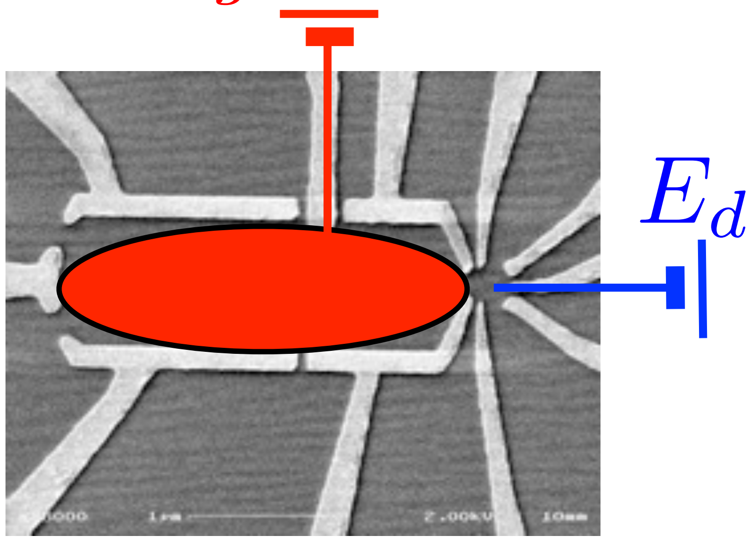


Potok et al, Nature 446, 167 (2007)

Two channel Kondo effect?

$N_g = bp$

$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$



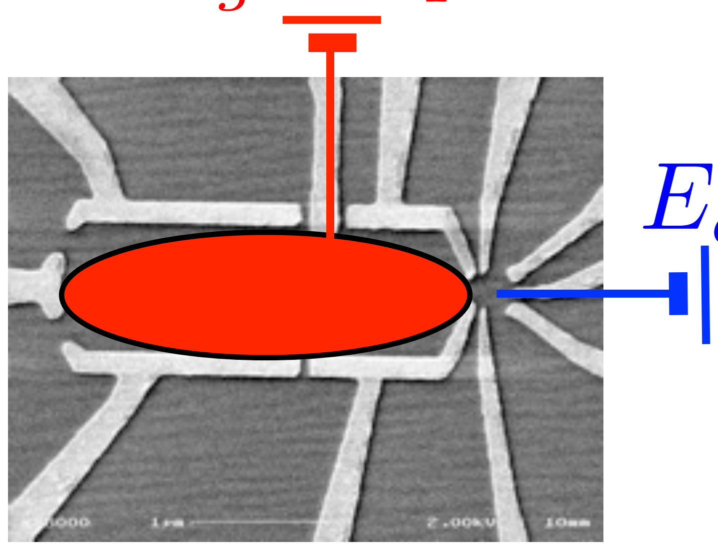
$E_d = V_{ds}$

Potok et al, Nature 446, 167 (2007)

Two channel Kondo effect?

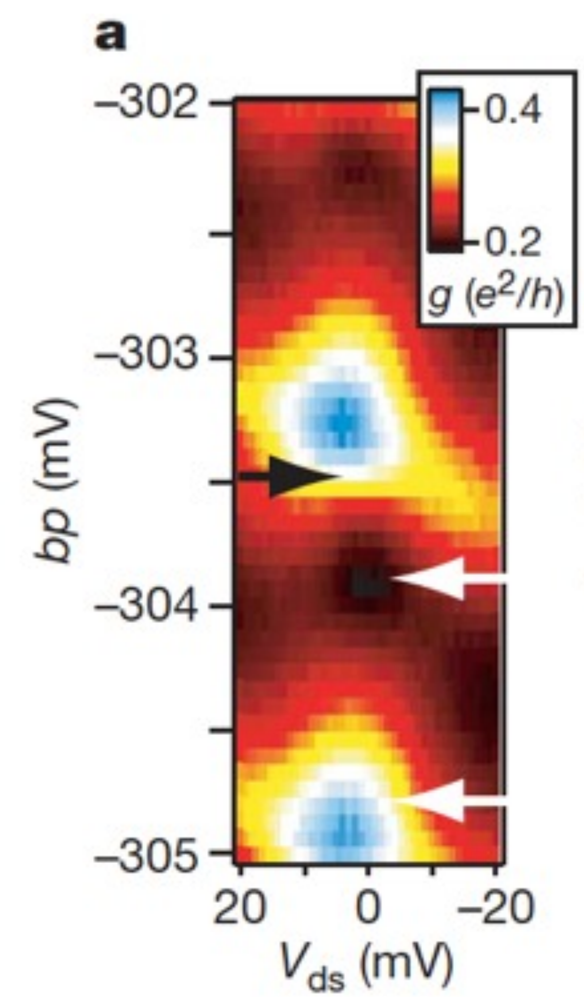
$$N_g = bp$$

$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$



$$E_d = V_{ds}$$

Potok et al, Nature 446, 167 (2007)



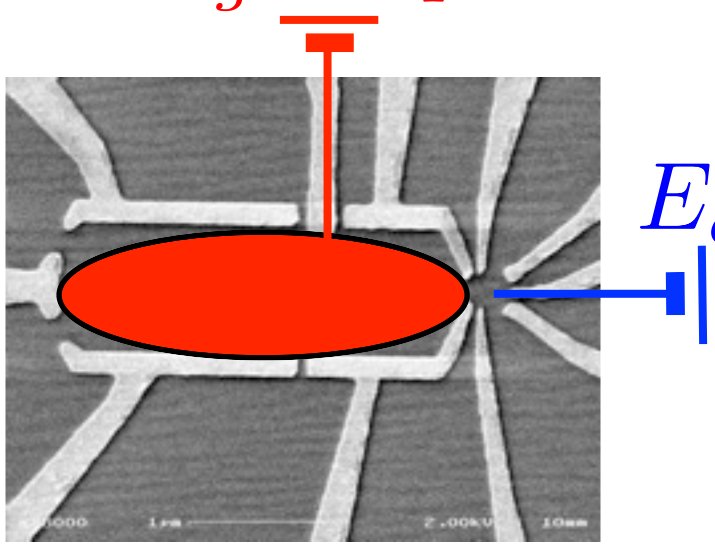
Potok et al, Nature 446, 167 (2007)

technische universität
dortmund

Two channel Kondo effect?

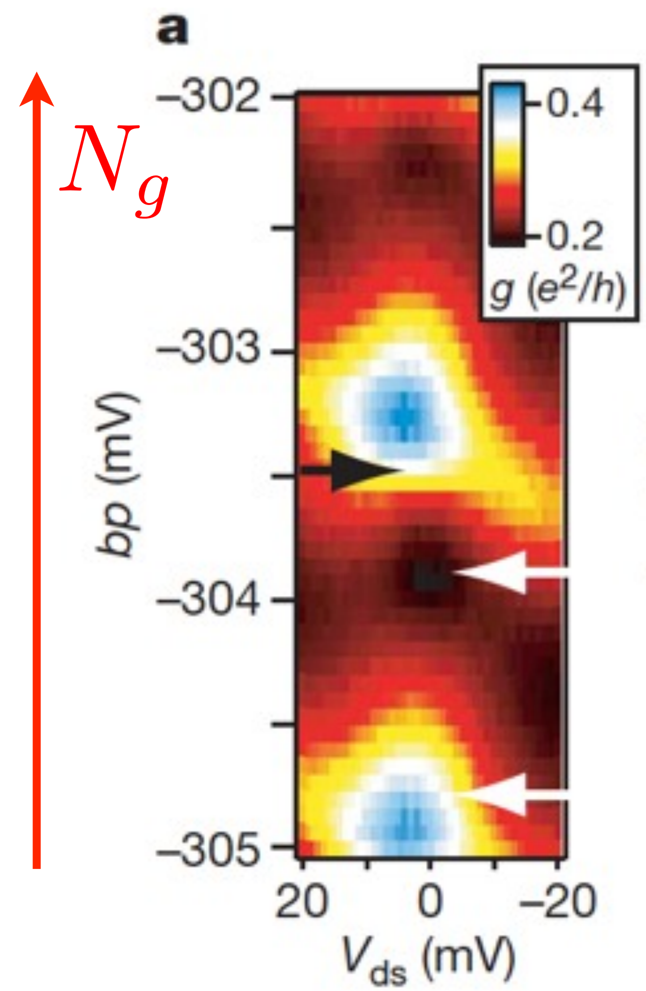
$$N_g = bp$$

$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$



$$E_d = V_{ds}$$

Potok et al, Nature 446, 167 (2007)

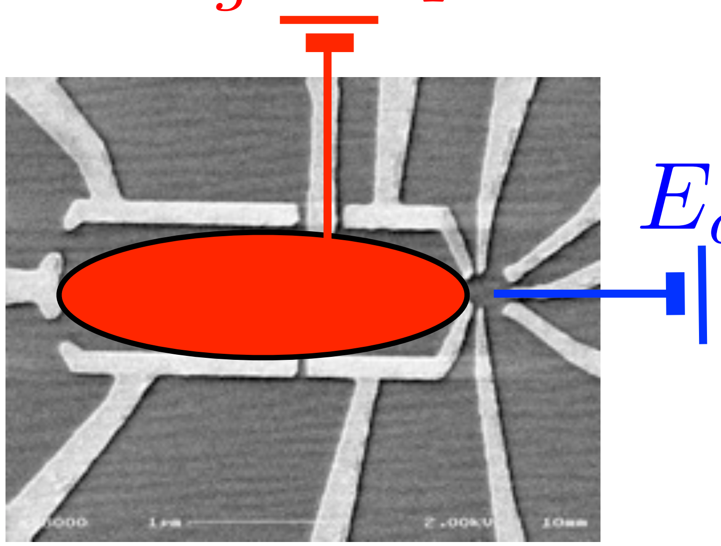


Potok et al, Nature 446, 167 (2007)

Two channel Kondo effect?

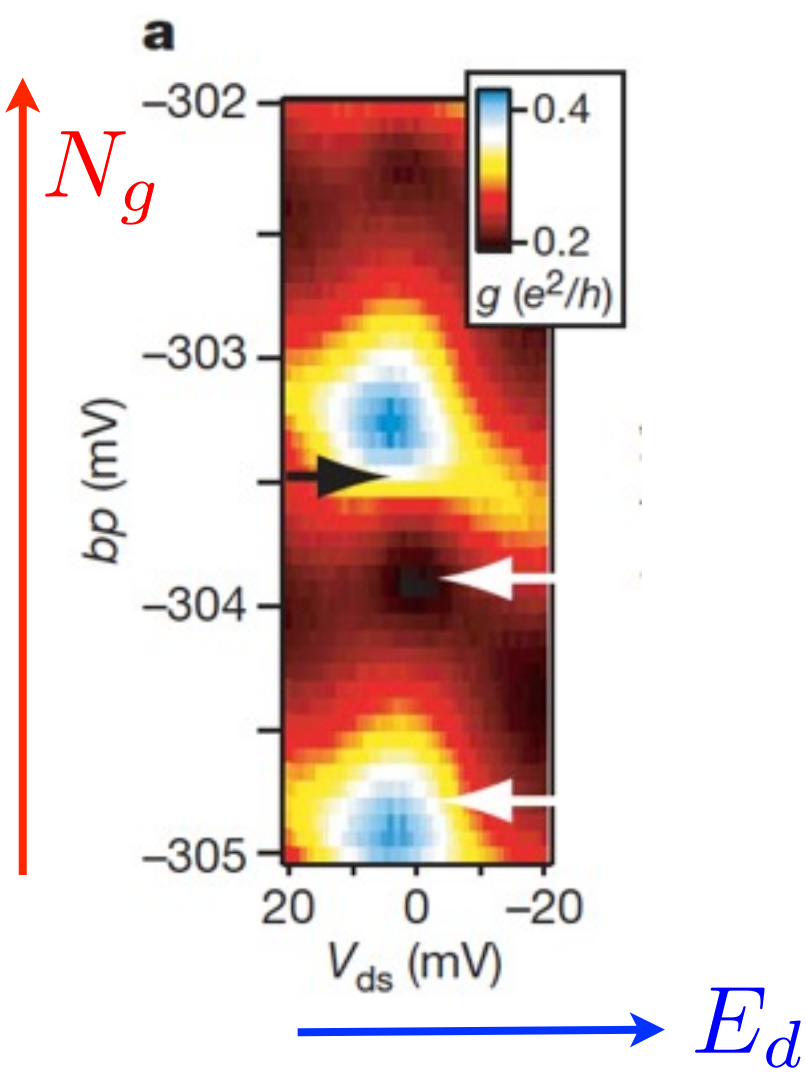
$$N_g = bp$$

$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$



$$E_d = V_{ds}$$

Potok et al, Nature 446, 167 (2007)



Potok et al, Nature 446, 167 (2007)

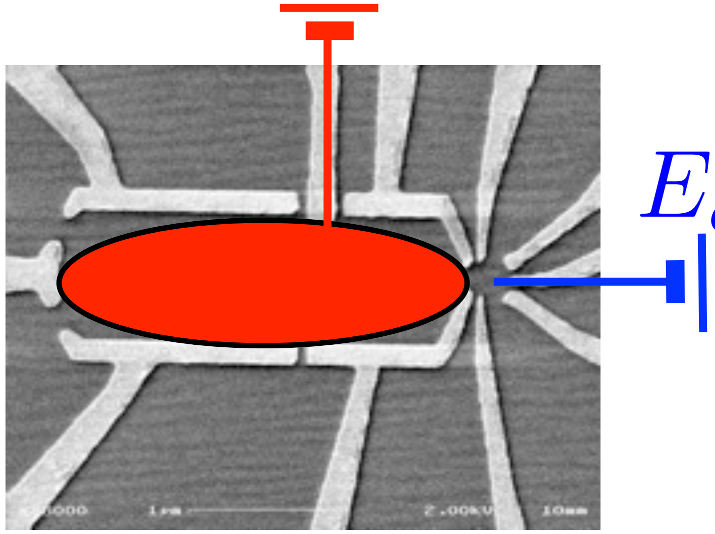
technische universität dortmund

Two channel Kondo effect?

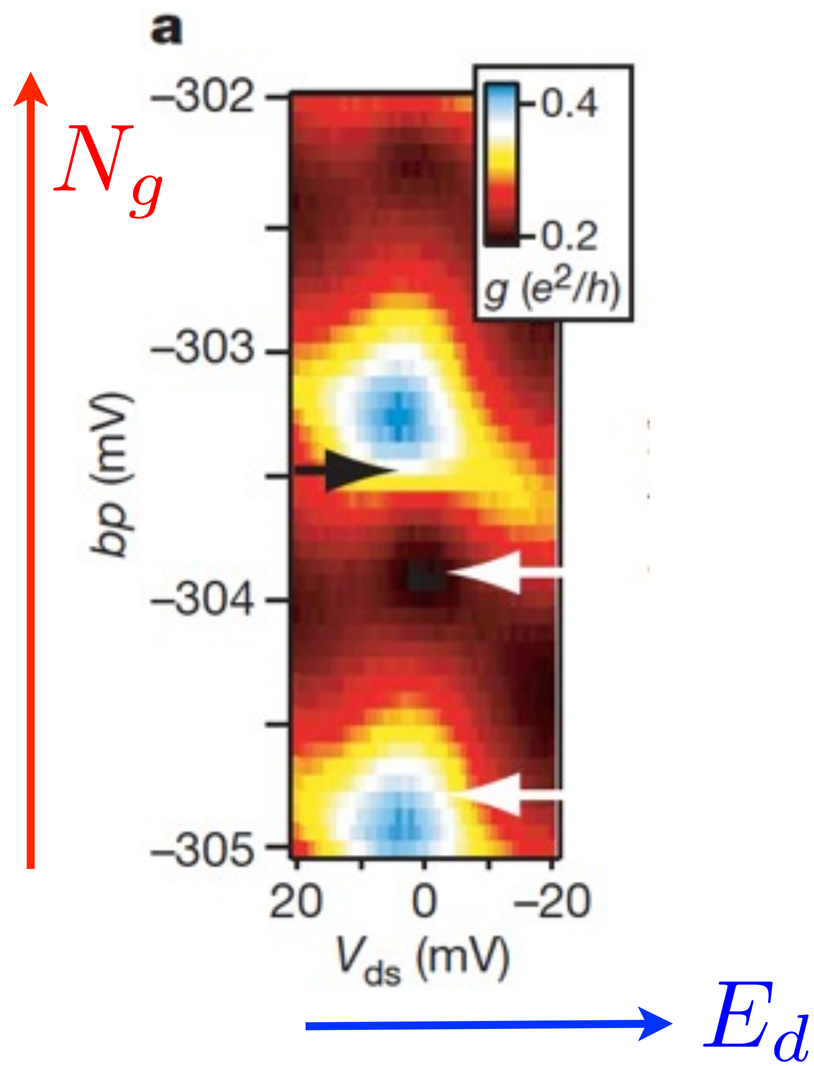
$$N_g = bp$$

$$E = \frac{e^2}{2C}\left(\hat{N} - N_g\right)^2$$

$$E_d = V_{ds}$$

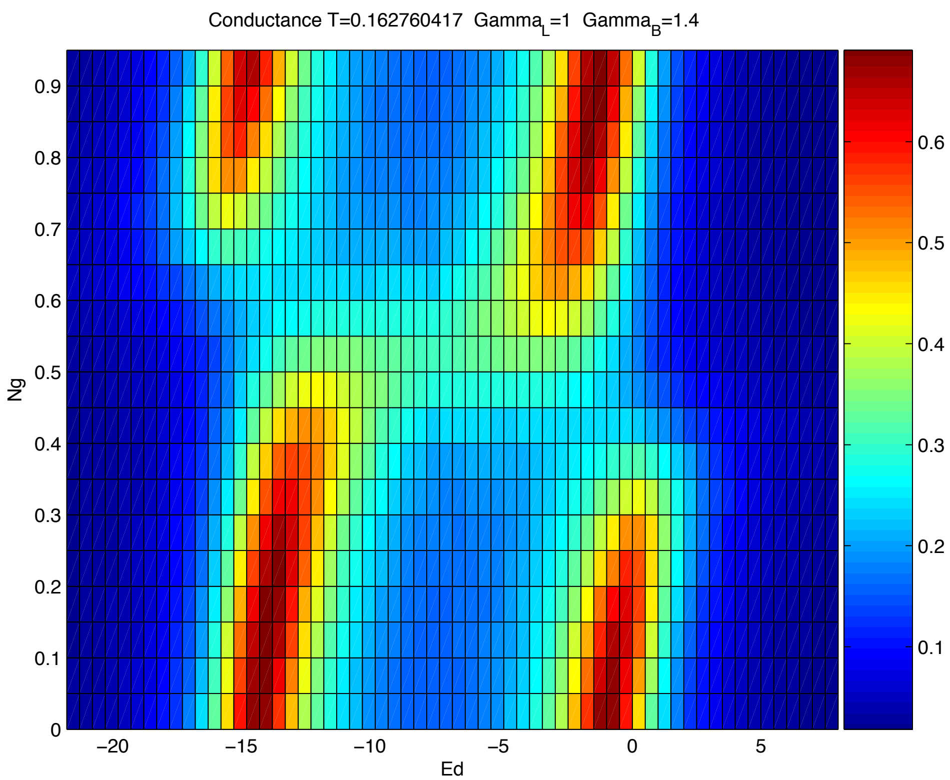


Potok et al, Nature 446, 167 (2007)



Potok et al, Nature 446, 167 (2007)

NRG data T=12mK



Anders 2005 unpublished

Conclusion

Conclusion

Kondo effect

Conclusion

Kondo effect

1. occurs in a variety of different physics contexts

Conclusion

Kondo effect

1. occurs in a variety of different physics contexts
   (i) scattering of electrons on magnetic impurities

Conclusion

Kondo effect

1. occurs in a variety of different physics contexts
   (i) scattering of electrons on magnetic impurities
   (ii) Kondo lattice and HF systems

Conclusion

Kondo effect

1. occurs in a variety of different physics contexts
   - (i) scattering of electrons on magnetic impurities
   - (ii) Kondo lattice and HF systems
   - (iii) effective site of the DMFT

Conclusion

Kondo effect

1. occurs in a variety of different physics contexts
   - (i) scattering of electrons on magnetic impurities
   - (ii) Kondo lattice and HF systems
   - (iii) effective site of the DMFT
   - (iv) single-electron transistors

Conclusion

Kondo effect

1. occurs in a variety of different physics contexts
   (i) scattering of electrons on magnetic impurities
   (ii) Kondo lattice and HF systems
   (iii) effective site of the DMFT
   (iv) single-electron transistors
2. contains an infrared divergent problem

Conclusion

Kondo effect

1. occurs in a variety of different physics contexts
   (i) scattering of electrons on magnetic impurities
   (ii) Kondo lattice and HF systems
   (iii) effective site of the DMFT
   (iv) single-electron transistors
2. contains an infrared divergent problem
   (i) ground state changes; orthogonal to Fermi sea;

Conclusion

Kondo effect

1. occurs in a variety of different physics contexts
   - (i) scattering of electrons on magnetic impurities
   - (ii) Kondo lattice and HF systems
   - (iii) effective site of the DMFT
   - (iv) single-electron transistors
2. contains an infrared divergent problem
   - (i) ground state changes; orthogonal to Fermi sea;
   - (ii) effective moment is screened (partially or perfectly)

Conclusion

Kondo effect

1. occurs in a variety of different physics contexts
   (i) scattering of electrons on magnetic impurities
   (ii) Kondo lattice and HF systems
   (iii) effective site of the DMFT
   (iv) single-electron transistors
2. contains an infrared divergent problem
   (i) ground state changes; orthogonal to Fermi sea;
   (ii) effective moment is screened (partially or perfectly)
   (iii) new energy scale: T_K

Conclusion

Kondo effect

1. occurs in a variety of different physics contexts
   (i) scattering of electrons on magnetic impurities
   (ii) Kondo lattice and HF systems
   (iii) effective site of the DMFT
   (iv) single-electron transistors
2. contains an infrared divergent problem
   (i) ground state changes; orthogonal to Fermi sea;
   (ii) effective moment is screened (partially or perfectly)
   (iii) new energy scale: T_K
   (iv) new fixed points: RG type methods required