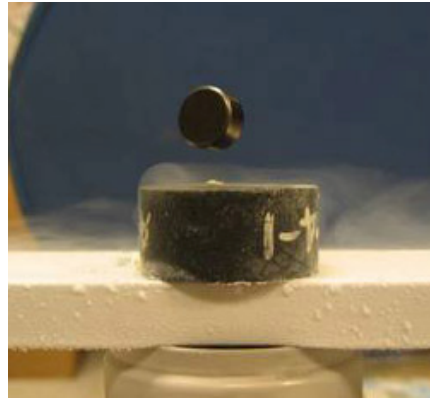
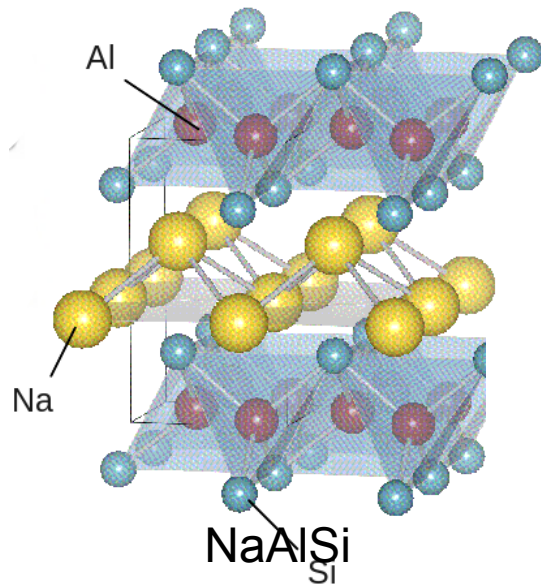


# Superconductivity and two dimensionality: experimental facts, theoretical issues, *novel materials*

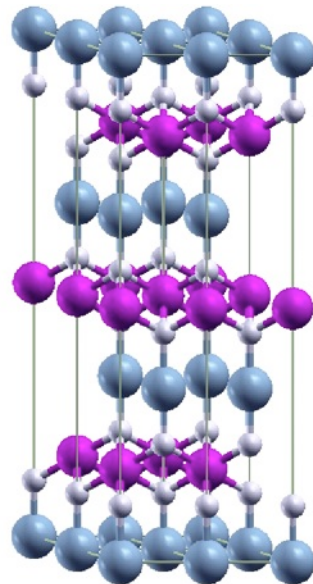


Warren E. Pickett  
University of California Davis

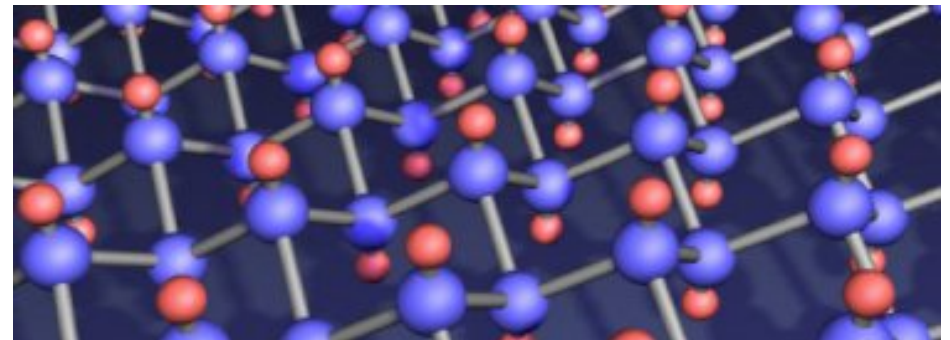
Emphasis: el-ph coupling and  
nonmagnetic el-el coupling  
→ higher  $T_c$  in this talk.



$\text{CuAlO}_2$



Graphane CH



Ch. 11: Superconductors, W. E. Pickett

Coming soon: Lectures on Superconductivity *Theory*

Ch. 10: Correlated Superconductivity, A.-M. Tremblay

Ch. 12: Electron-Phonon Coupling, R. Heid

Ch. 13: Eliashberg Theory, G. A. C. Ummarino

# Educational Sites for Superconductivity

University of Cambridge *Lectures on Superconductivity*

<http://www.msm.cam.ac.uk/ascg/lectures/introduction/welcome.php>

MIT Open Courseware: *Applied Superconductivity*

Very long URL, search for it.

Several others, mostly less extensive.

Also: work through the first 12-15 pages of BCS paper (Phys Rev, 1957)

And the book: *Superconductivity*, by J. R. Schrieffer (1964)

Word to the wise: regarding new, [high temperature superconductors](#) – don't believe all (or even much) of what you find online.



# Workshop on The Possibility of Room Temperature Superconductivity & Related Topics

June 10th, 11th and 12th

## FEATURED SPEAKERS

Alexei A. Abrikosov  
Meigan Aronson  
Neil Ashcroft  
Paul Canfield  
Paul Chu  
Laura Greene  
Arthur Freeman  
John Ketterson  
Gennady Logvenov  
Igor Mazin  
Warren Pickett  
Mohit Randeria  
John Sarrao  
Douglas Scalapino  
Masashi Tachiki  
A. J. Leggett  
(to be confirmed)

## TOPICS

- Routes to higher transition temperature  
- Materials  
- Mechanisms  
- Theoretical limits  
- Exotic condensates

## ORGANIZING COMMITTEE

George Crabtree (ANL)  
Wai-Kwong Kwok (ANL)  
Baldissar Janke (ND)  
Michael R. Norman (ANL)

## CONFERENCE REVIEW

To be published by *Physica C*

## CONFERENCE LOCATION

McKenna Hall  
University of Notre Dame

For further information contact:

Drew Sandler, Administrator | The Institute for Theoretical Sciences | 339C Nieuwland Science Hall | University of Notre Dame | Notre Dame, IN 46556  
email: [dsandler1@nd.edu](mailto:dsandler1@nd.edu) | phone (574) 631-2668 | Fax (574) 631-5259 | [www.theoryinstitute.org/its](http://www.theoryinstitute.org/its)

## The Path to Room Temperature Superconductivity

**Hotel Alexandra**

**Loen, Norway**

**17 - 23 June 2007**

Organizers/Sponsors

Dr. [Harold Weinstock](#), European Office of Aerospace R&D (AFOSR)

Prof. [Paul C. W. Chu](#), Houston International Materials Forum, U. Houston

Prof. [Horst Rogallo](#), U. Twente

**All Presentations** (link added 23 April 2010)



Upcoming Superconductivity Workshop  
(almost certain)

“(Toward) Room Temperature Superconductivity”

Organizers: E K U Gross, A Sanna, L Boeri

June-July 2014, Leiden, The Netherlands

How to attract funding.....

## ASTONISH

Atomic-scale **STudies Of the Nature of and conditions for Inducing Superconductivity at High-temperatures**

A 2013 ERC funded research project

R. Wiesendanger, Univ. of Hamburg

In the framework of the ERC Advanced Grant "ASTONISH" (Atomic-scale **STudies Of the Nature of and conditions for Inducing Superconductivity at High-temperatures**), the University of Hamburg is **seeking outstanding and enthusiastic young researchers with a strong record of creativity and achievements in the area of superconductivity** in MBE grown sample systems. The project aims at a novel surface-science based approach to unconventional superconductivity, combining atomically controlled vertical growth and lateral atomic-scale manipulation tools with a unique set of atomic-resolution characterization techniques, including elastic and inelastic scanning probe spectroscopy with spin resolution at relevant energy scales.....

This announcement is currently active.

# Why 2D? why low-Z? why doped insulators?

## Why 2D?

- examples (following slides): many classes of good sc'ors
- phase space  $[N(E)]$  is more favorable than in 3D
- electronic and phononic structure may be distinctive

## Why low-Z?

- mass is smaller  $\rightarrow$  good for el-ph high  $T_c$
- bonding is stronger  $\rightarrow$  stronger el-ph coupling
- other factors?? many classes of good 2D sc'ors

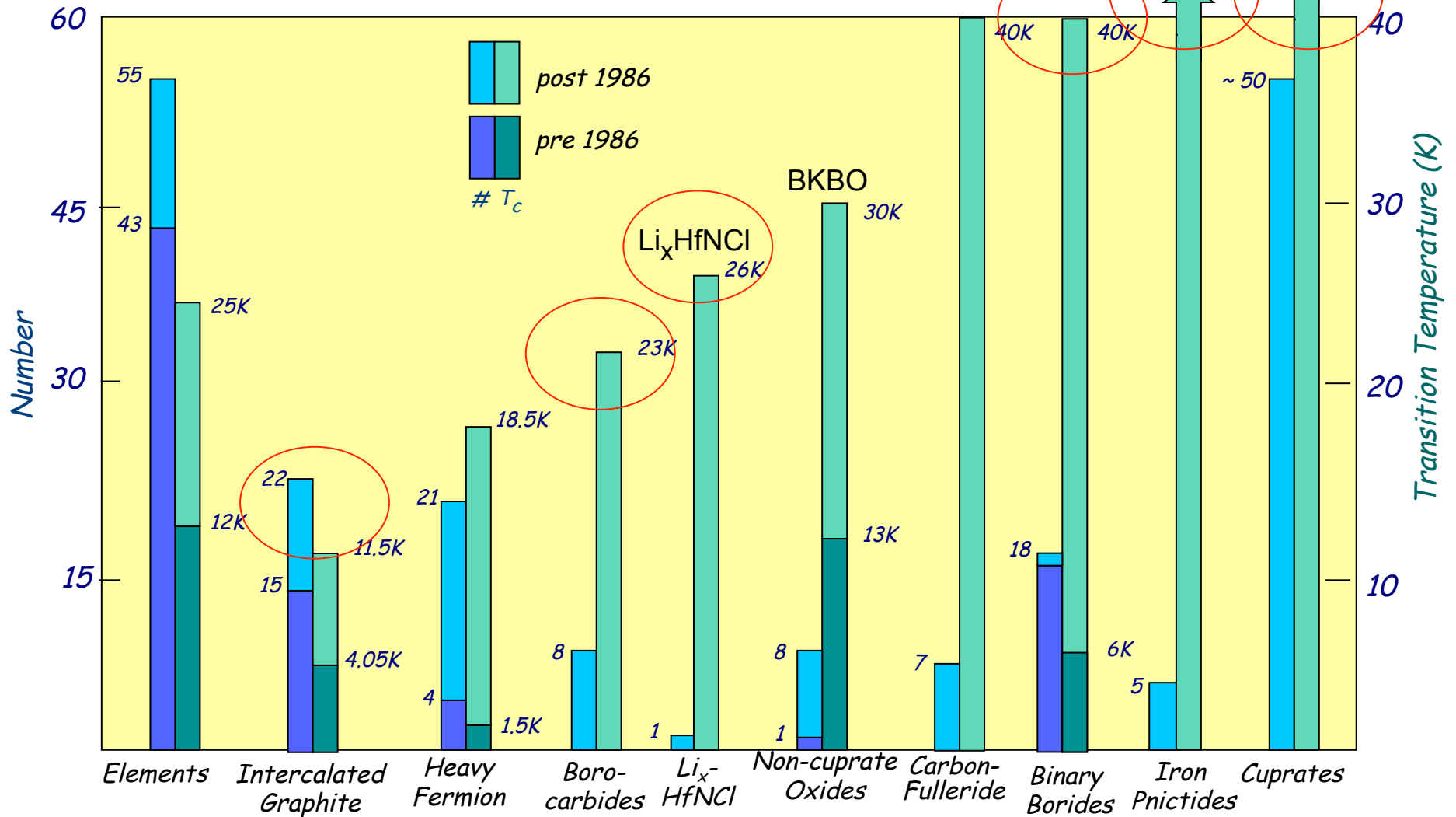
Why doped insulator? Much of the point of this talk

# New Materials: Recent Superconductor Discoveries

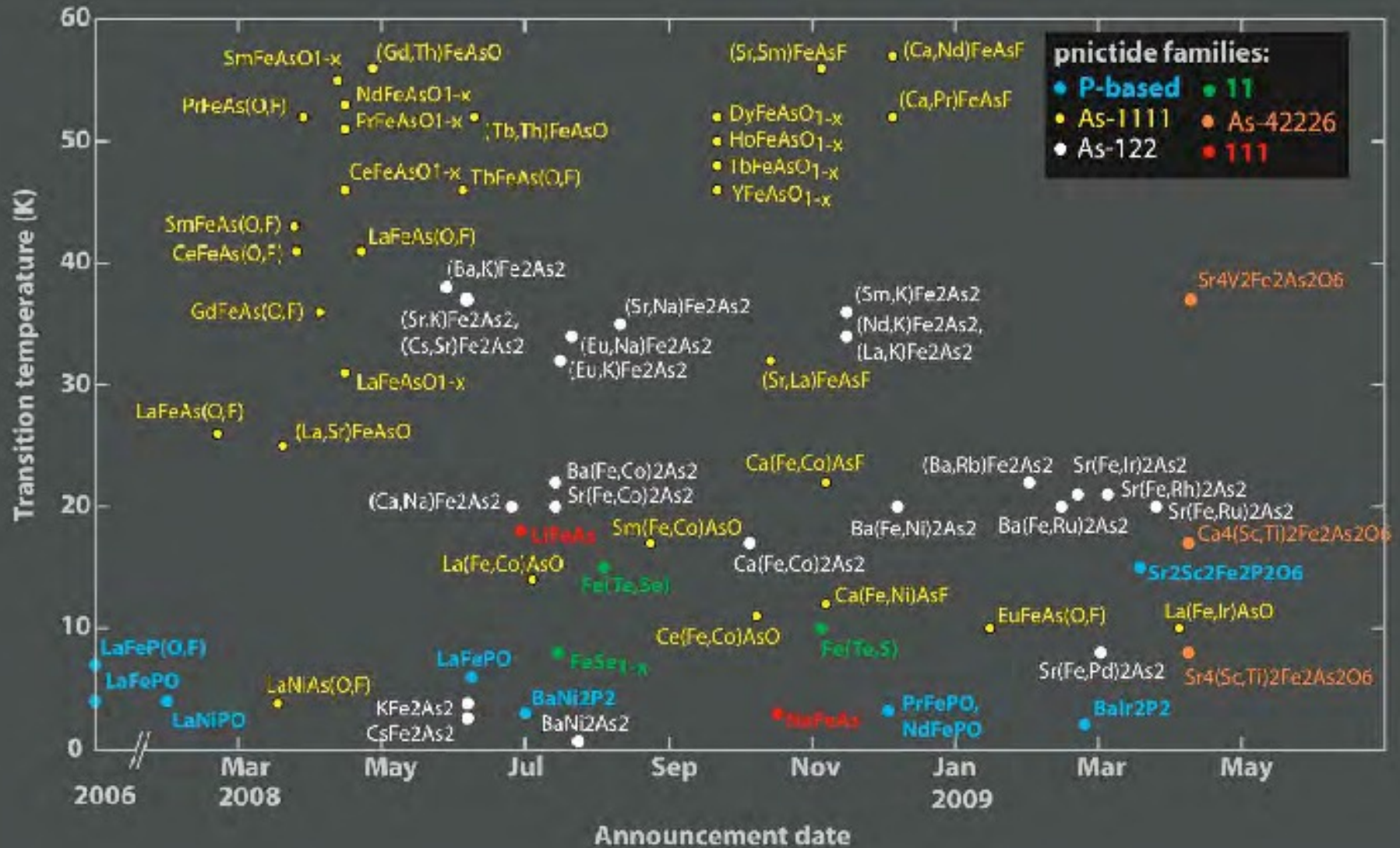
Courtesy:  
G. W. Crabtree

2D materials

Based on BES Report on Basic Research Needs for Superconductivity 2006  
<http://www.sc.doe.gov/bes/reports/abstracts.html#5C>



# A partial history of the pnictide superconductors



(chemical formulas:  $\text{PrFeAs}(\text{O},\text{F}) = \text{PrFeAsO}_{1-x}\text{F}_x$ , e.g.)

Think in terms of new compounds.....

Recall:

- Cuprates began with La-Ba-Cu-O with  $T_C=30$  K
- Fe pnictides began with LaONiP with  $T_C= 5$ K

So: pay attention to new discoveries.

# Bi<sub>4</sub>O<sub>4</sub>S<sub>3</sub> system

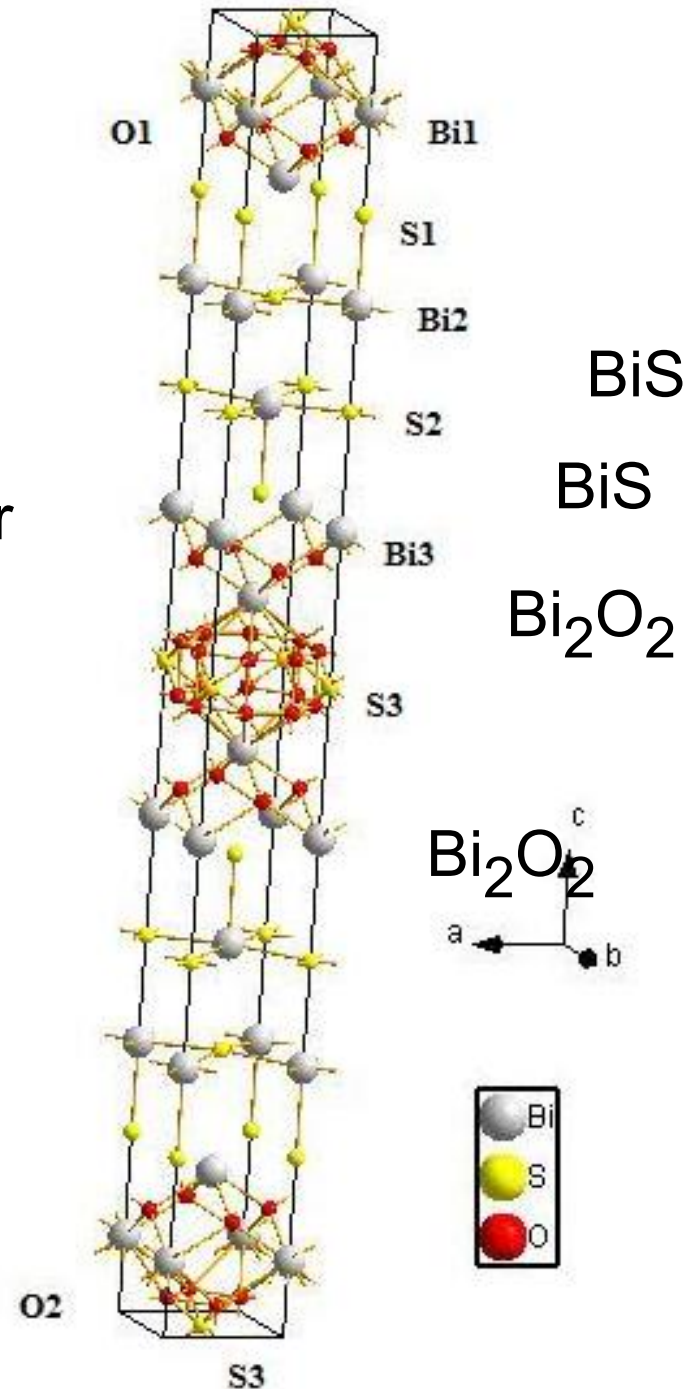
Mizuguchi et al., arXiv:1207.3145  
was an early paper. A dozen or  
two papers available now.

Superconducting in the Bi-S bilayer  
at  $T_c = 4.5\text{K}$

Apparently, electron carriers  
in the Bi  $p_x, p_y$  bands

Nonstoichiometric, i.e.doped.

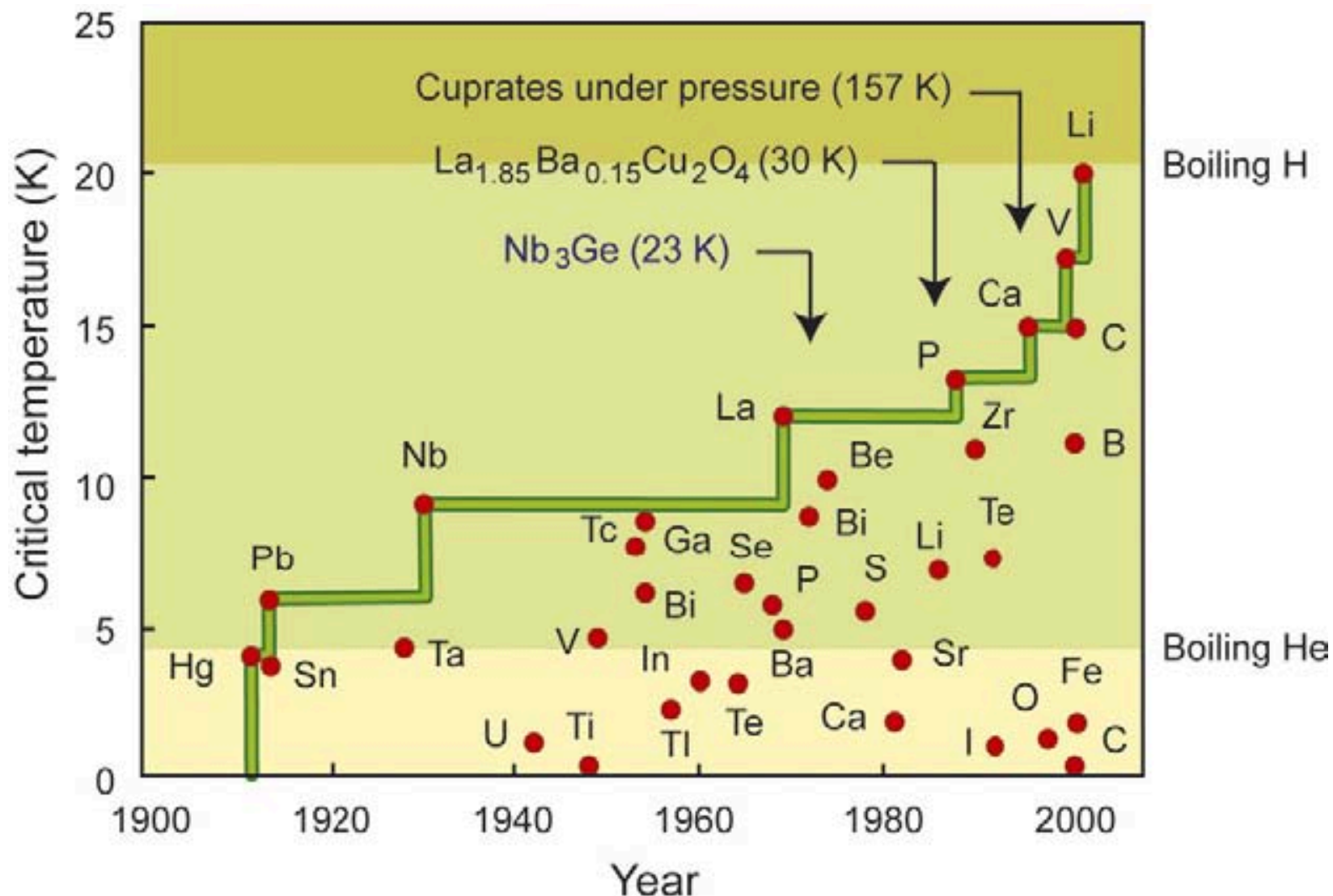
A new platform for higher  $T_c$ ?  
Not yet?





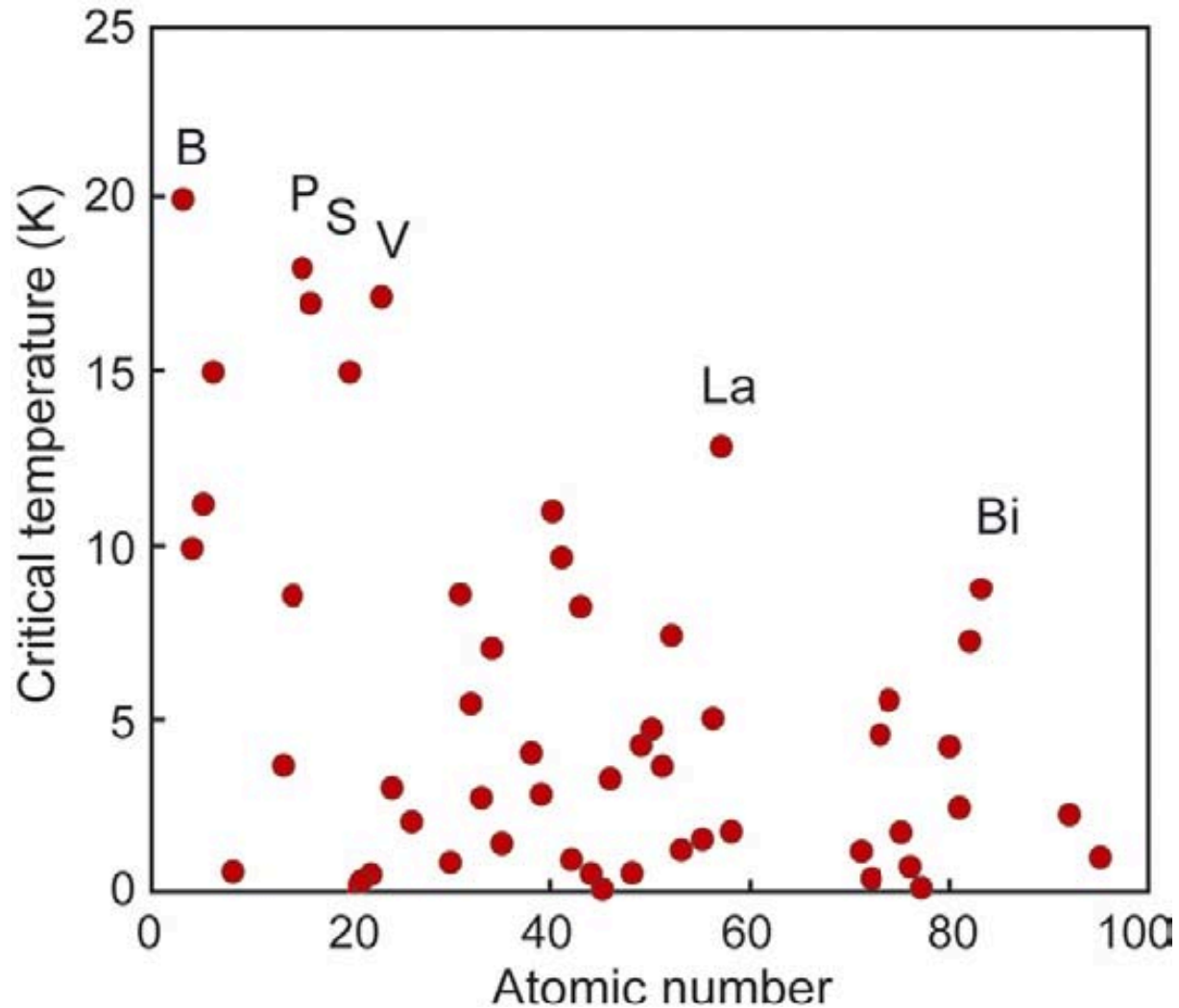


# Elemental Superconductors versus Time



- Historical development of the critical temperature of simple elements (C. Buzea *et al.*, Supercond. Sci. Technol. **18** (2005) R1–R8)

## Elemental Superconductors versus Z



Highest critical temperature of simple elements plotted vs. atomic number.

[C. Buzea *et al.*, *Supercond. Sci. Technol.* **18** (2005) R1–R8]

(missing Li)

# 1972 Nobel Prize in Physics

“For their jointly developed theory of superconductivity,  
usually called the BCS theory.”  
[This is the weak-coupling theory.]



John Bardeen, Leon Cooper, J. Robert Schrieffer

Phys. Rev. 108, 1175-1204 (1957)

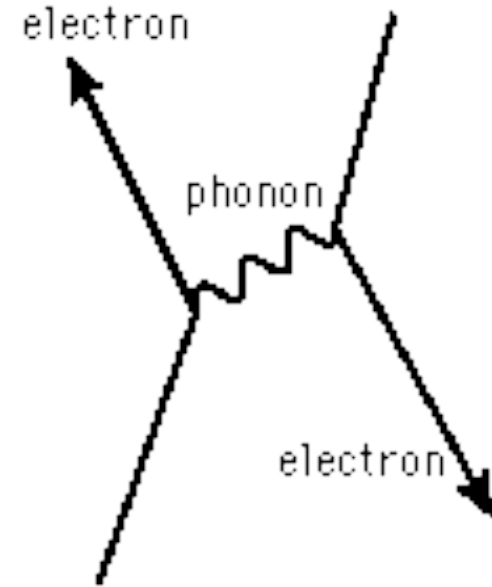
The genuine article: Eliashberg theory for real materials,  
including strong coupling.

Scalapino, Schrieffer, Wilkins, PR ~ 1965

# Electron-Phonon Coupling

Hamiltonian for band electrons coupled to harmonic phonons:

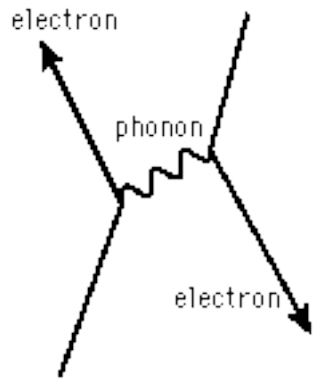
$$\begin{aligned}
 H_{e-ph} = & \sum_{ks} \epsilon_{ks} c_{ks}^\dagger c_{ks} \\
 & + \sum_{q\nu} \hbar\omega_{q\nu} (b_{q\nu}^\dagger b_{q\nu} + \frac{1}{2}) \\
 & + \sum_{ks} \sum_{q\nu} [M_{k,q}^\nu c_{k+q,s}^\dagger c_{k,s} b_{q\nu} + h.c.]
 \end{aligned}$$



Matrix element: amplitude for scattering from the one-electron state  $|\mathbf{k}j\rangle$  to the state  $|\mathbf{k}+\mathbf{Q}j'\rangle$  via the phonon  $\mathbf{Q}\nu$

$$M_{\vec{k}+\vec{Q}j',\vec{k}j}^{\vec{Q}\nu} = \langle \vec{k} + \vec{Q}j' | \delta^{\vec{Q}\nu} V_{eff} | \vec{k}j \rangle$$

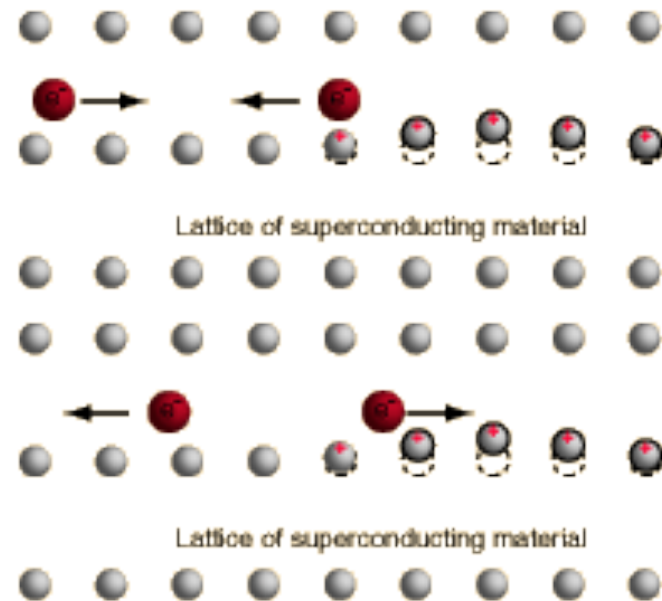
## The scattering diagram



## The physical picture (a crude version)

An electron moving through the lattice disturbs the positions of the ions (electron-phonon interaction), a later electron experiences the deformation, with a net attractive interaction.

$v_{\text{ph}}/v_{\text{el}} \ll 1$  is the expansion parameter in perturbation theory



1<sup>st</sup> three: Bernd Matthias's rules for finding high  $T_c$  superconductors (~1970).

Last four: extended to ~1980 understanding.

1. Must have d electrons (not just s-p, nor f)
2. High symmetry is good, cubic is best
3. Need a peak in density of states at Fermi level
4. Must spread the coupling over all phonons
5. Coulomb interaction must be docile
6. Stay away from oxides
7. Stay away from magnetism



1<sup>st</sup> three: Bernd Matthias's rules for finding high  $T_c$  superconductors (~1970).

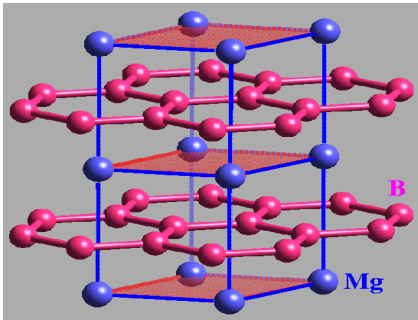
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3. Need a peak in density of states at Fermi level
4. Must spread the coupling over all phonons
5. Coulomb interaction must be docile
6. Stay away from oxides
7. Stay away from magnetism
8. **Stay away from theorists!**

Bernd Matthias's rules for finding high  $T_c$  superconductors (~1970).  
Extended to ~1980 understanding.

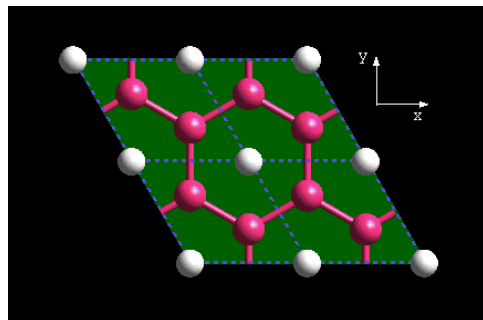
1. Must have d electrons (not just s-p, nor f) X  $\text{MgB}_2$
2. High symmetry is good, cubic is best X Best are 2D!
3. Need a peak in density of states at Fermi level X  $\text{MgB}_2$ , ...
4. Must spread the coupling over all phonons X  $\text{MgB}_2$
5. Coulomb interaction must be docile X Cuprates, FeSCs
6. Stay away from oxides X Cuprates, BKBO
7. Stay away from magnetism X Cuprates, FeSCs, HFs
8. Stay away from rules for finding higher  $T_c$ !

# Akimitsu's Discovery (2001): $T_c=40\text{K}$ in $\text{MgB}_2$



1.  $\text{MgB}_2$ : covalent bonds driven metallic by chemistry
2. Deformation potential  $D=13 \text{ eV/\AA}$   
(amazingly large for a metal)
3. 2D (cylinder) Fermi surfaces focus coupling strength
4. Yet structure remains stable: intrinsic covalency and very strong bonds

$\text{MgB}_2$  is naturally "hole-doped."

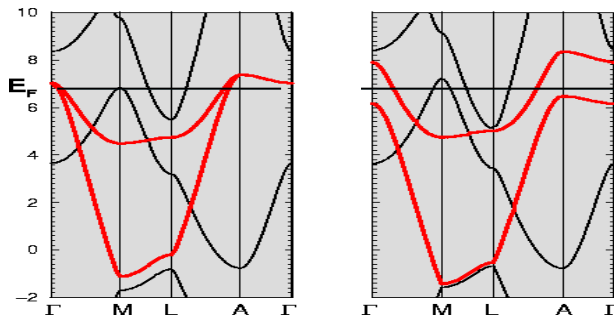


J. M. An and WEP, Phys. Rev. Lett. (2001)  
 J. Kortus et al., Phys. Rev. Lett. (2001)  
 Y. Kong et al., Phys. Rev. B (2001)  
 K.-P. Bohnen et al., Phys. Rev. Lett. (2001)  
 .....more.....

Highly focused EPI

Y. KONG, O. V. DOLGOV, O. JEPSEN, AND O. K. ANDERSEN

PHYSICAL REVIEW B 64 020501(R)



T. Yildirim (NIST)

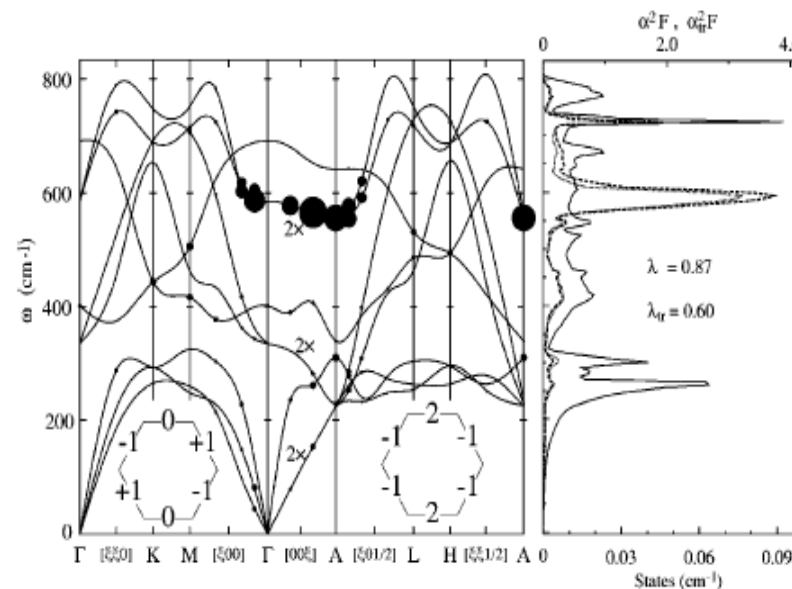


FIG. 1. Left: Calculated phonon dispersion curves in  $\text{MgB}_2$ . The area of each circle is proportional to the mode  $\lambda$ . The insets at the bottom show the two  $\Gamma$ -A  $E$  eigenvectors (not normalized), which apply to the holes at the top of the  $\sigma$  bands (bond-orbital coefficients) as well as to the optical bond-stretching phonons (relative change of bond lengths). Right:  $F(\omega)$  (full curve and bottom scale),  $\alpha^2(\omega)F(\omega)$  (broken), and  $\alpha_T^2(\omega)F(\omega)$  (dotted). See text.

Electron-Phonon Coupling:  
 General Results for  $\lambda_{Q\nu}$  and  $\gamma_{Q\nu}$   
 P. B. Allen, PR B6, 2577 (1972)

$$\lambda = \frac{1}{N_\nu} \sum_{Q,\nu=1}^{N_\nu} \lambda_{Q,\nu} = 2 \int \frac{\alpha^2 F(\omega)}{\omega} d\omega$$

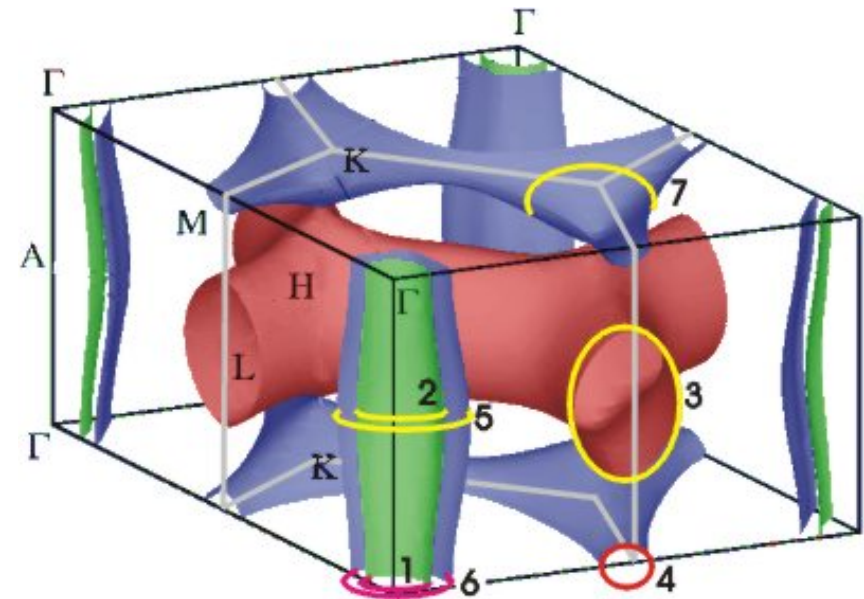
$$\alpha^2 F(\omega) = \frac{2}{\pi N(0)\omega} \sum_{Q\nu} \gamma_{Q\nu} \delta(\omega_{Q\nu} - \omega)$$

$$\omega_{Q\nu}^2 = \Omega_{Q\nu}^2 + 2\Omega_{Q\nu}^2 \text{Re} \Pi(Q, \omega_{Q\nu})$$

$$\gamma_{Q\nu} = \frac{\Omega_{Q\nu}}{\omega_{Q\nu}} \text{Im} \Pi(Q, \omega_{Q\nu})$$

$$= \pi \sum_k |M_{k,k+Q}|^2 \delta(\epsilon_k) \delta(\epsilon_{k+Q})$$

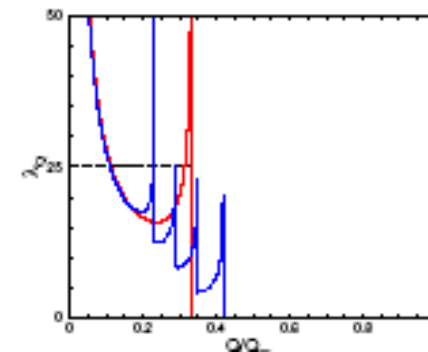
$$\lambda = \frac{1}{N_\nu} \sum_{Q\nu} \lambda_{Q\nu} = \frac{4}{\pi N(0)} \sum_{Q\nu} \frac{\gamma_{Q\nu}}{\omega_{Q\nu}^2}$$



$$\lambda_{Q,\nu} = 4 \frac{2}{\omega_{Q,\nu}} \sum_k |M_{k,k+Q}|^2 \delta(\epsilon_k) \delta(\epsilon_{k+Q})$$

$$= 4 \frac{4\pi V_c |M|^2}{\omega_{Q,\nu} c k_F^2} \frac{1}{x \sqrt{1-x^2}}, x \equiv \frac{Q}{2k_F}$$

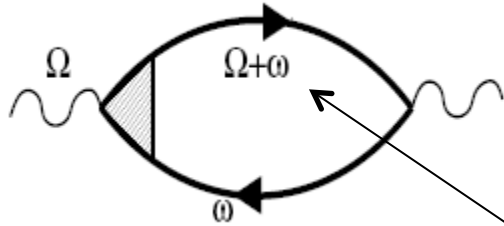
$\lambda_q$  for Li<sub>3</sub>BC Model



3% of phonons have  $\lambda \sim 25!$   
 Rest of phonons have  $\lambda \sim 0.3$

# Phonon Renormalization (Self Energy)

simple and distinctive phase space in 2D



**Extreme Electron-Phonon Coupling:  
Kohn Anomalies**

Cylinder Fermi surface leads to sharp Kohn anomaly

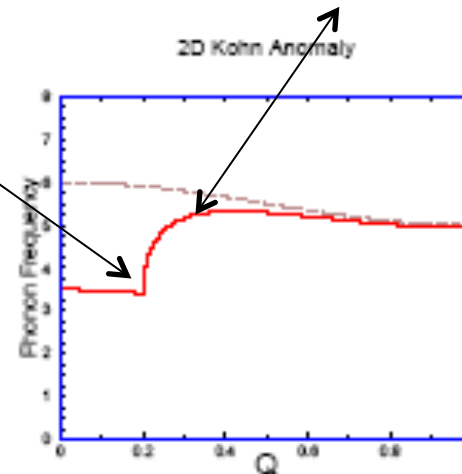
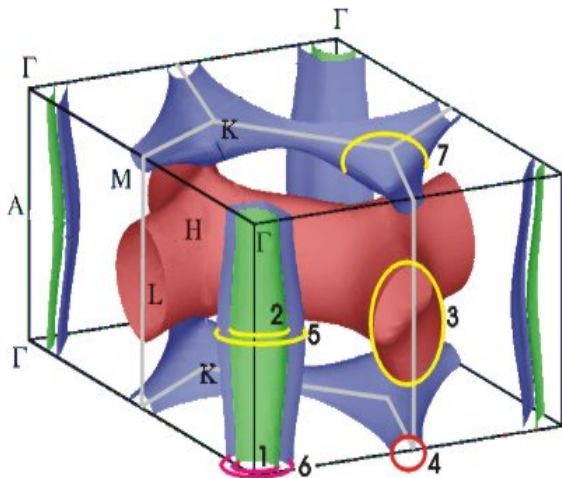
Large matrix elements lead to strong renormalization for  $Q < 2k_F$

$$\omega_{Q\nu}^2 = \Omega_{Q\nu}^2 + 2\Omega_{Q\nu}^2 \Pi(Q, \omega_{Q\nu})$$

$$\Pi(Q, \omega) = -2 \sum_k |M_{k, k+Q}|^2 \frac{f_k - f_{k+Q}}{\epsilon_{k+Q} - \epsilon_k - \omega - i\delta}$$

2D dispersion, slowly varying matrix elements give

$$\Pi_\nu(Q, \omega) = -2|M_\nu|^2 \chi_L^{2D}(Q, \omega)$$



# Phonon softening in MgB<sub>2</sub>

Extreme Electron-Phonon Coupling:  
Kohn Anomalies

$$\omega_{Q\nu}^2 = \Omega_{Q\nu}^2 + 2\Omega_{Q\nu}^2 \Pi(Q, \omega_{Q\nu})$$

$$\Pi(Q, \omega) = -2 \sum_k |M_{k, k+Q}|^2 \frac{f_k - f_{k+Q}}{\epsilon_{k+Q} - \epsilon_k - \omega - i\delta}$$

2D dispersion, slowly varying matrix elements give

$$\Pi_\nu(Q, \omega) = -2|M_\nu|^2 \chi_L^{2D}(Q, \omega)$$

Y. KONG, O. V. DOLGOV, O. JEPSEN, AND O. K. ANDERSEN

PHYSICAL REVIEW B 64 020501(R)

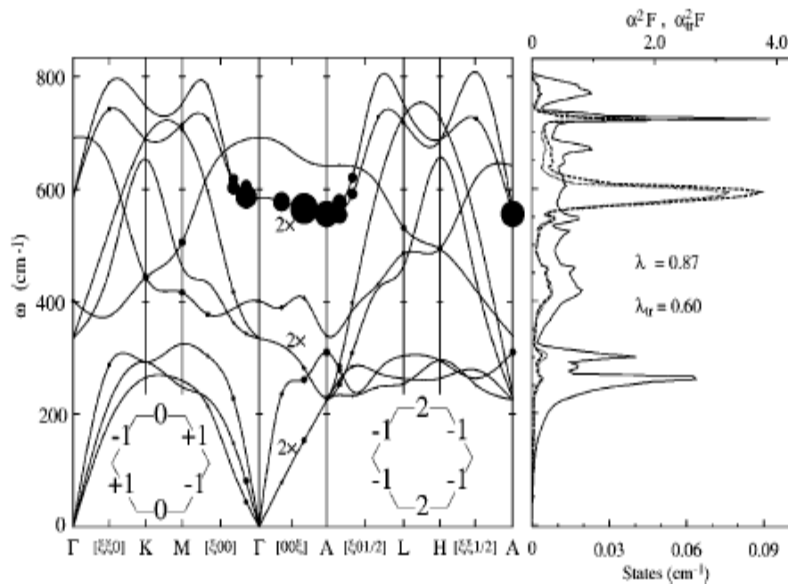
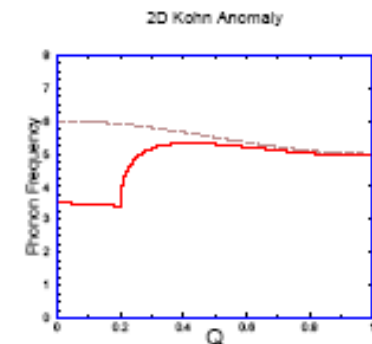


FIG. 1. Left: Calculated phonon dispersion curves in MgB<sub>2</sub>. The area of each circle is proportional to the mode  $\lambda$ . The insets at the bottom show the two  $\Gamma$ -A E eigenvectors (not normalized), which apply to the holes at the top of the  $\sigma$  bands (bond-orbital coefficients) as well as to the optical bond-stretching phonons (relative change of bond lengths). Right:  $F(\omega)$  (full curve and bottom scale),  $\alpha^2(\omega)F(\omega)$  (broken), and  $\alpha^2_\nu(\omega)F(\omega)$  (dotted). See text.



(No) theoretical limit on  $T_c$ . BCS:  $T_c \sim e^{-1/\lambda}$

$$T_c = \frac{\langle \omega \rangle}{1.20} \exp\left(-\frac{1.04(1+\lambda)}{\lambda - \mu^*(1+0.62\lambda)}\right)$$

McMillan equation for  $T_c$   
saturates for  $\lambda > 1$

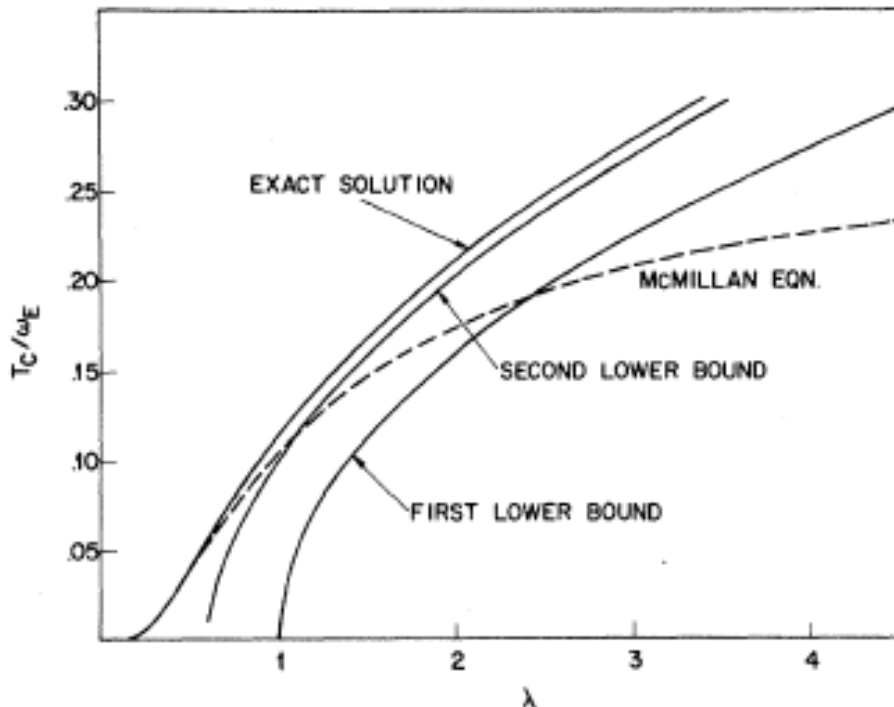
$$T_c = \frac{f_1 f_2 \omega_{1\text{og}}}{1.20} \exp\left(-\frac{1.04(1+\lambda)}{\lambda - \mu^* - 0.62\lambda\mu^*}\right) \quad \text{Allen-Dynes equation for } T_c \text{ (1975)}$$

$$f_1 = [1 + (\lambda/\Lambda_1)^{3/2}]^{1/3},$$

$$\Lambda_1 = 2.46(1 + 3.8\mu^*),$$

$$f_2 = 1 + \frac{(\bar{\omega}_2/\omega_{1\text{og}} - 1)\lambda^2}{\lambda^2 + \Lambda_2^2}$$

$$\Lambda_2 = 1.82(1 + 6.3\mu^*) (\bar{\omega}_2/\omega_{1\text{og}})$$



From full “Eliashberg theory”  
in the strong coupling regime

$$\lambda \gg 1, T_c \rightarrow \lambda^{1/2}$$

Extremely encouraging news  
for those interested in high  $T_c$ !



# Strong Coupling: Good News, Bad News

Good news:  $T_c$  is unbounded in Eliashberg theory

Bad news: real materials are complicated

*P. B. Allen*

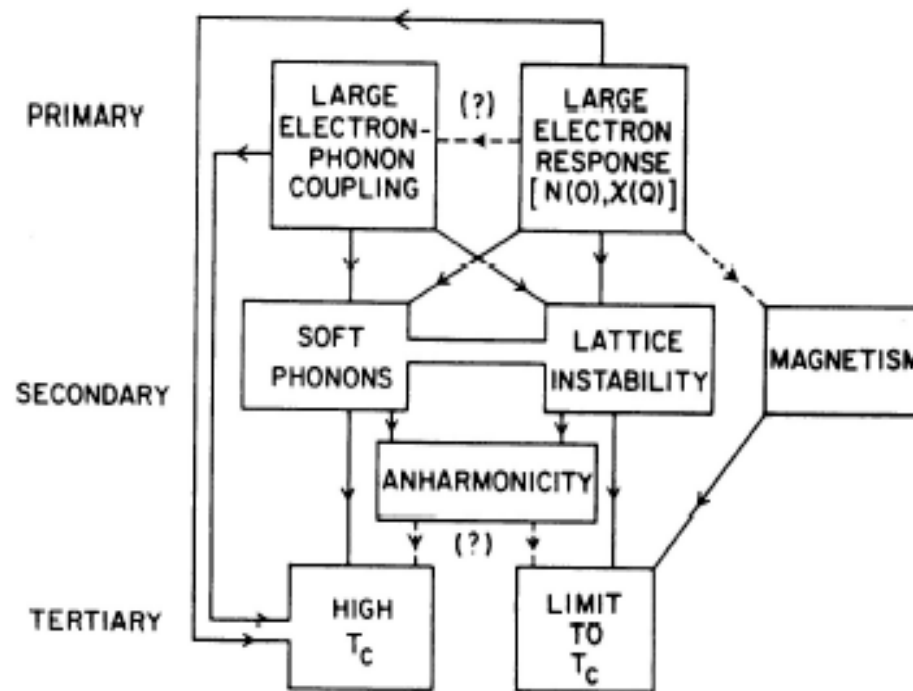


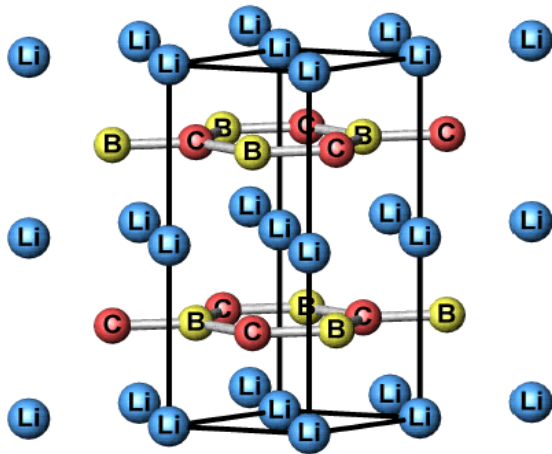
Fig. 3. Causative relations are indicated by arrows. High  $T_c$  is partly a direct consequence of the primary factors, and partly indirect through the phonon dispersion. Dashed lines with arrows represent less certain relationships.

# Extreme

## Electron-Phonon Coupling Strength Calculated for $\text{Li}_{1-x}\text{BC}$

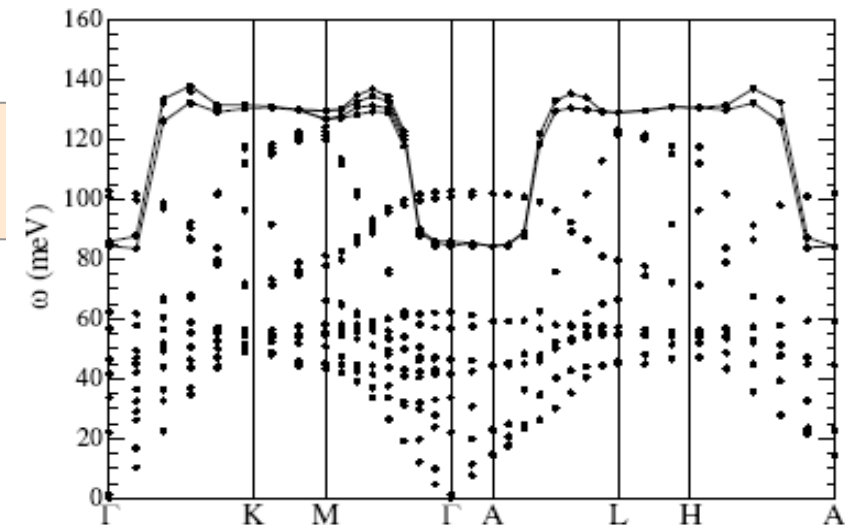
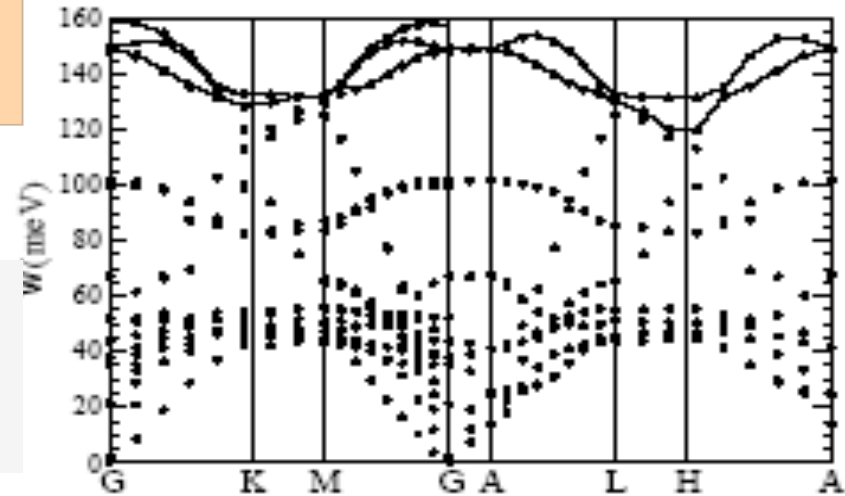
Rosner, Kitiagorodsky, WEP, Phys. Rev. Lett. (2002)

Semiconductor  $x=0$   
Simple vibrational spectrum



$T_c \sim 75$  K  
using same theory  
as for  $\text{MgB}_2$

Metal for  $x=0.25$   
**Extreme** Kohn anomalies



Not so simple experimentally!  
Li is very active chemically in LiBC.  
Hole doping  $\rightarrow$  phase separation.

The “complete understanding” of el-ph mechanism allows a rational search and/or design of new/better examples.

It is the materials that are complex and devious.

Why 2D? why low-Z? why doped insulators?

Why 2D?

- examples (following slides): many classes of good sc'ors
- phase space  $[N(E)]$  is more favorable than in 3D
- electronic and phononic structure may be distinctive

Why low-Z?

- mass is smaller  $\rightarrow$  good for high  $T_c$  (isotope effect)
- bonding is stronger  $\rightarrow$  stronger el-ph coupling
- other factors?? many classes of good 2D sc'ors

Why doped insulator? Much of the point of this talk

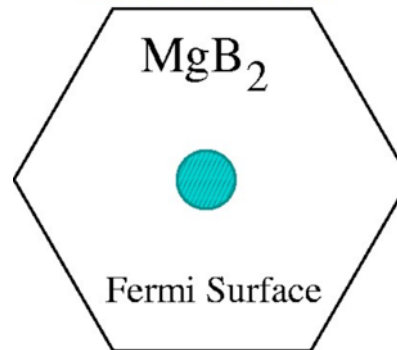
Design of higher  $T_c$  superconductors: is it viable?

## Rational Design/Search for new hTS

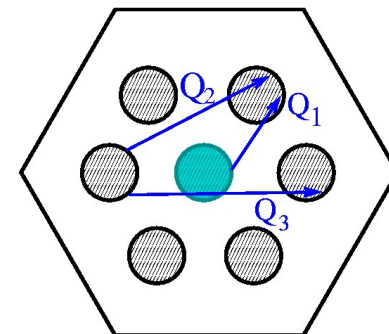
Example of  
one design  
criterion:  
doped 2D  
insulators

Select band structure  
to enable the phonons  
to use more of the  
Brillouin zone

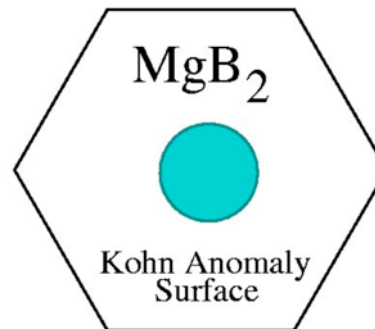
**Electron BZ**



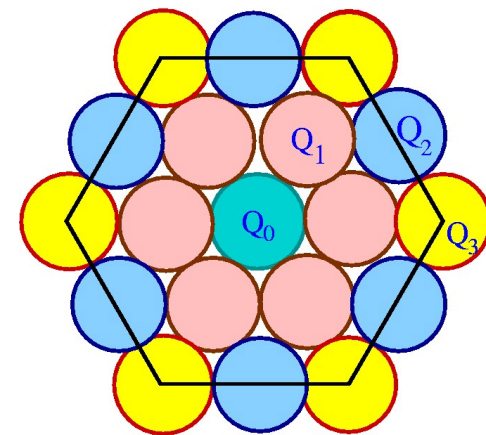
**Electron BZ**



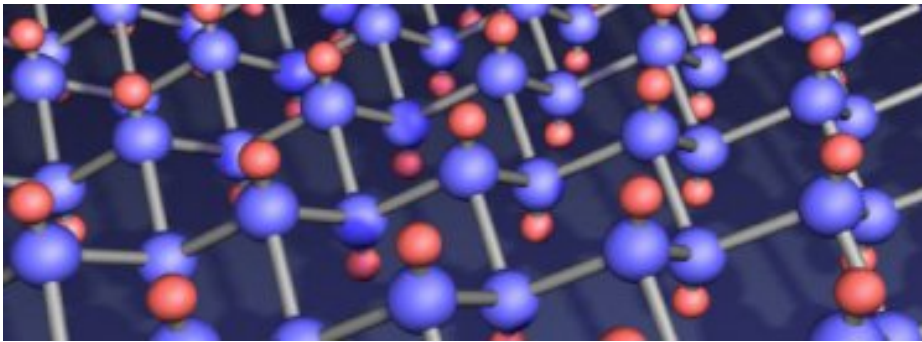
**Phonon BZ**



**Phonon BZ**



MgB<sub>2</sub>-like materials  
(a brief mention)



# Graphane

“up to 90K”

PRL 105, 037002 (2010)

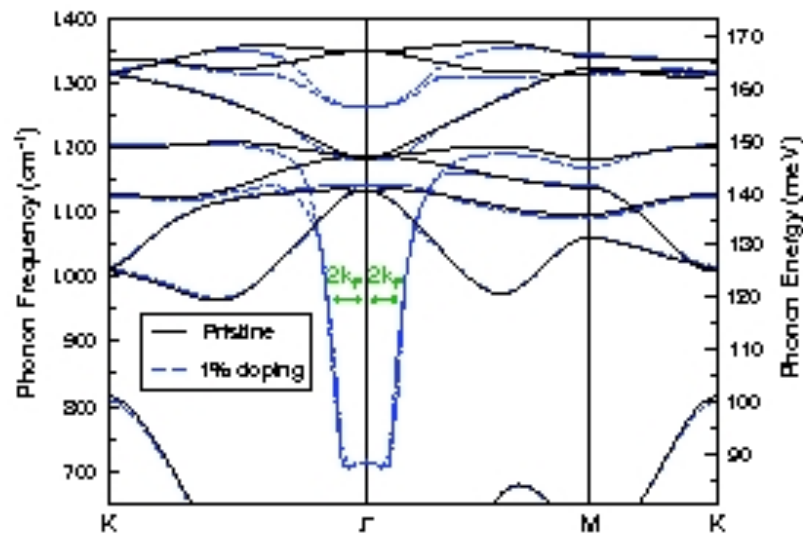
PHYSICAL REVIEW LETTERS

week ending  
16 JULY 2010

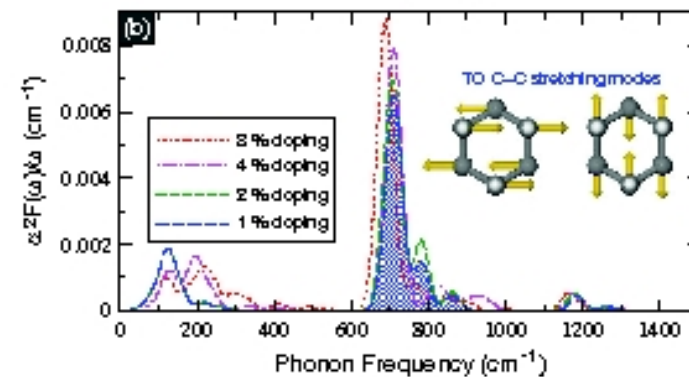
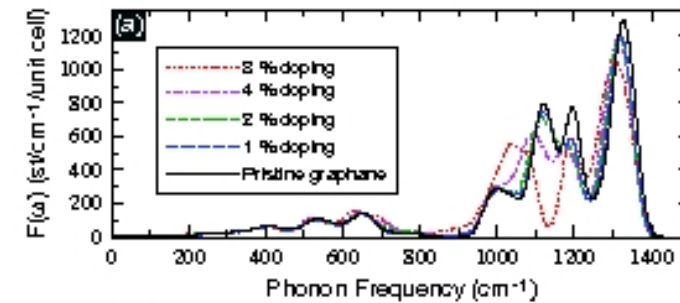


## First-Principles Prediction of Doped Graphane as a High-Temperature Electron-Phonon Superconductor

G. Savini,<sup>1,2</sup> A. C. Ferrari,<sup>1,\*</sup> and Feliciano Giustino<sup>3</sup>



Doped graphane CH



Coupling character

A newer  $\text{MgB}_2$ -like example,  
hole-doped  $\text{BeB}_2\text{C}_2$

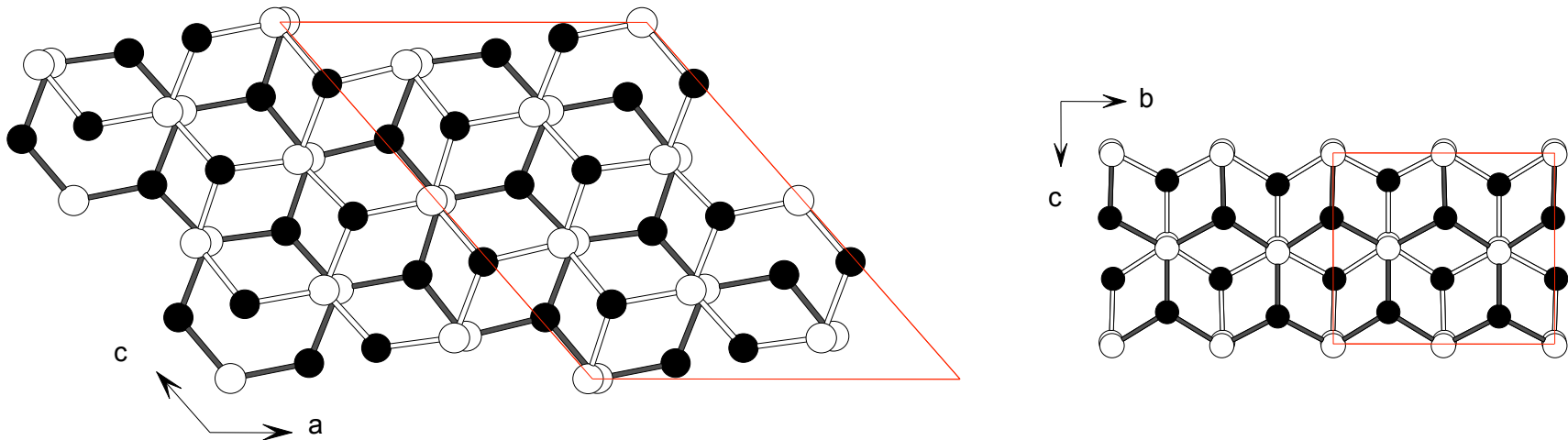


$\text{BeB}_2\text{C}_2$ : isoelectronic with LiBC, isoelectronic with  $\text{MgB}_2$

$\text{MgB}_2\text{C}_2$  has received some study: hex B-C layers, not flat due to Mg positions. Could be quite interesting if hole-doped; no superconductivity yet produced..

$\text{Be}_{1-x}\text{B}_2\text{C}_2$ : Moudden, Eur. Phys. J. B 64, 173 (2008). Gussed at structure(s). Found indications (predictions) of relatively high  $T_c$ .

2010: B. Albert's group determined the structure:  $P2_1/c$ ? No.  $Pmmm$ , yes.



Calculations (Ylvisaker and WEP): when doped, very  $\text{MgB}_2$ -like. Could have  $T_c \sim 50$  K if structure can be retained.

A puzzle in 2D superconductivity  
electron-doped ZrNCl, HfNCl, TiNCl

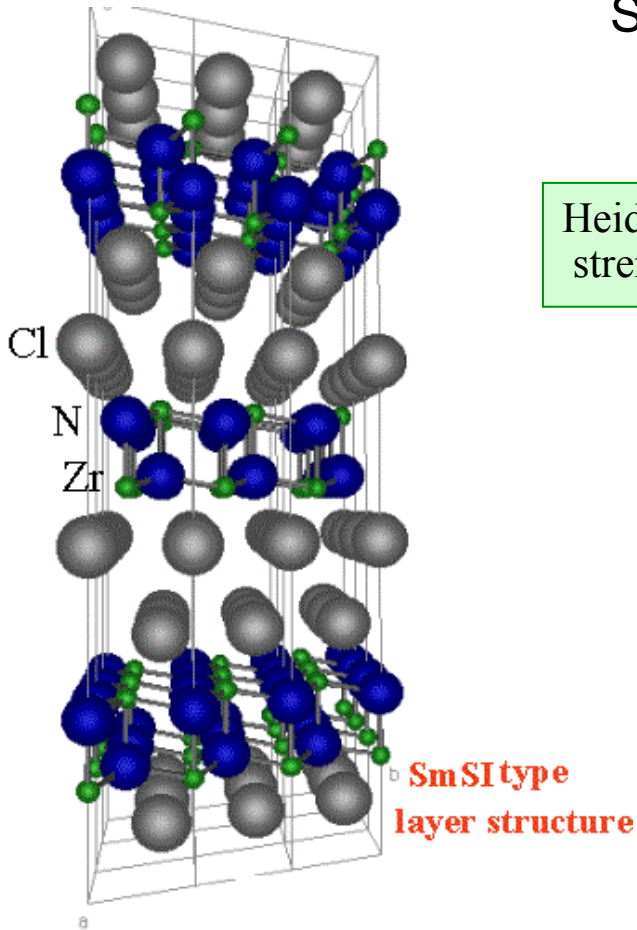
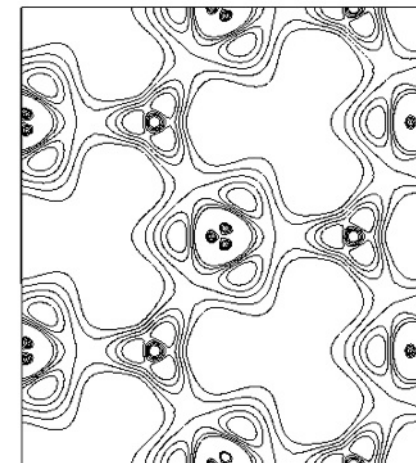
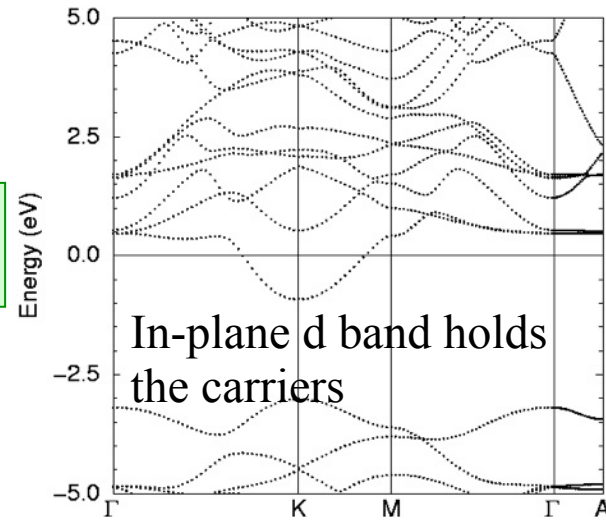
# Alkali-doped ionic insulators: $A_x\text{ZrNCl}$ (15 K), $A_x\text{HfNCl}$ (25 K), $A_x\text{TiNCl}$ (17 K) Yamanaka et al. 1998--; Shamoto et al. 1998

Structure is somewhat  $\text{MgB}_2$ -like; so is it electron-phonon?

Heid & Bohnen (2006) el-ph coupling strength is not large enough

Bill et al. (2003) Coupling to/ screening by low energy plasmons is important

- Double hex Zr-N layer
- Strongly 2D bands
- Electron-doped
- Inverse 'isotope shift'
- Weak el-ph coupling



Superconductor-insulator transition at  $x=0.06$

# Increase in $T_c$ upon Reduction of Doping in $\text{Li}_x\text{ZrNCl}$ Superconductors

PRL, 2009

Y. Taguchi,<sup>1,2</sup> A. Kitora,<sup>1</sup> and Y. Iwasa<sup>1,2</sup>

<sup>1</sup>Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan

<sup>2</sup>CREST, Japan Science and Technology Corporation, Kawaguchi 332-0012, Japan

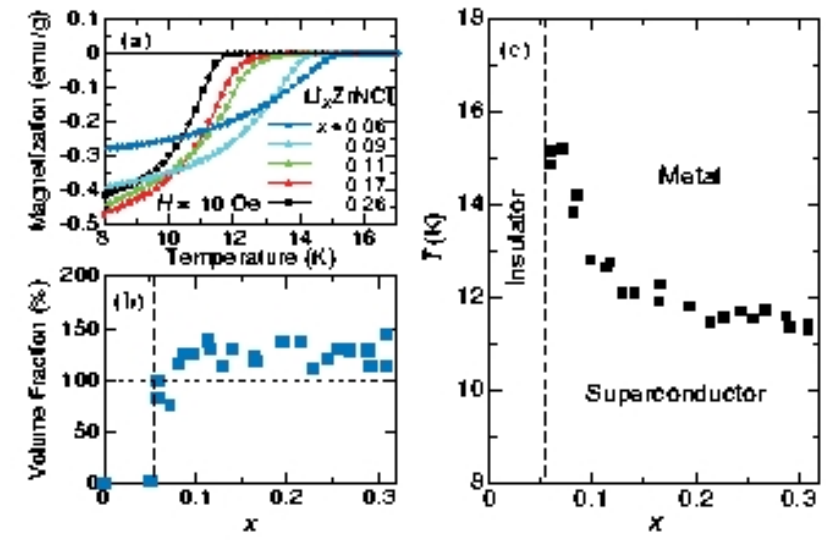
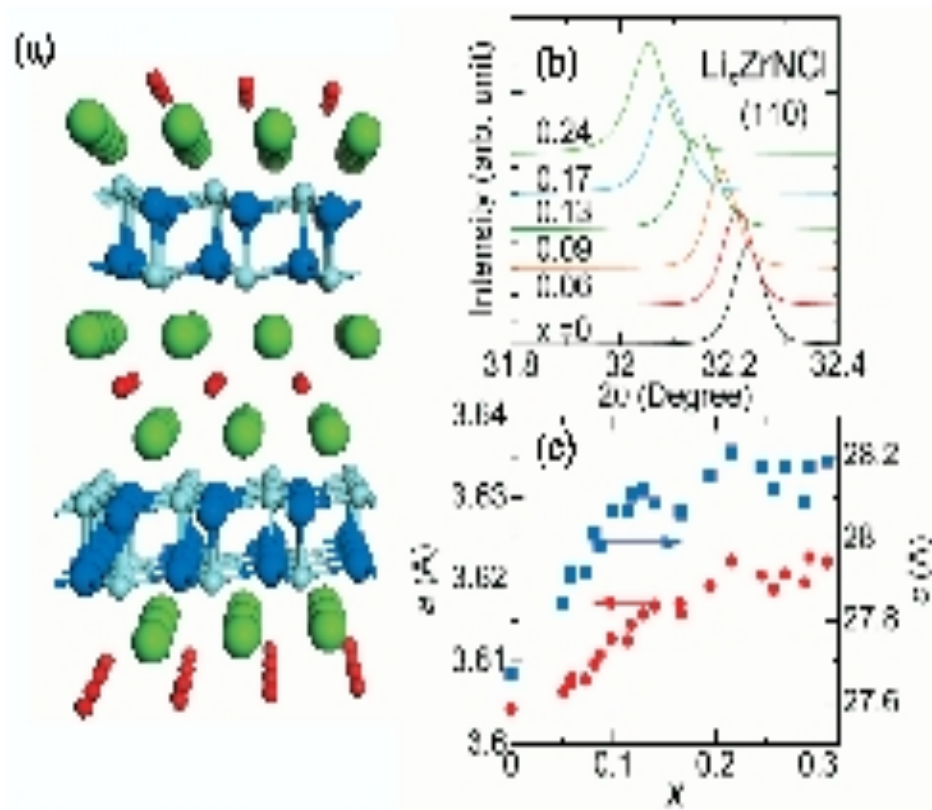


FIG. 4 (color). (a) Magnetization at a field of 10 Oe (without correction for demagnetizing field) is plotted against temperature for selected samples. (b)  $x$  dependence of the superconducting volume fraction determined by  $MH$  measurements at 2 K (or 5 K). Imperfect correction of the demagnetizing field is the reason the estimated volume fraction exceeds 100%. (c)  $x$  dependence of  $T_c$ , exemplifying rapid increase in  $T_c$  below  $x = 0.12$ .

Insulator-superconductor transition to optimal superconducting  $T_c = 15.5$  K at  $x=0.06$ . Weak phonon coupling, no magnetism. Doped  $\text{HfNCl}$ :  $T_c = 26$  K.

# $\text{Li}_x\text{ZrNCl}$ , $x=0$ , $x=1/6$

Phonons and el-ph coupling:  
Heid and Bohnen, PR B (2005)

Nearly circular  
Fermi surfaces at  
the zone corner  
K points: almost  
perfect nesting at K!  
Focusing of scattering

Calculated el-ph  
coupling strength  
is **0.5**, insufficient to  
account for  $T_c=15$  K

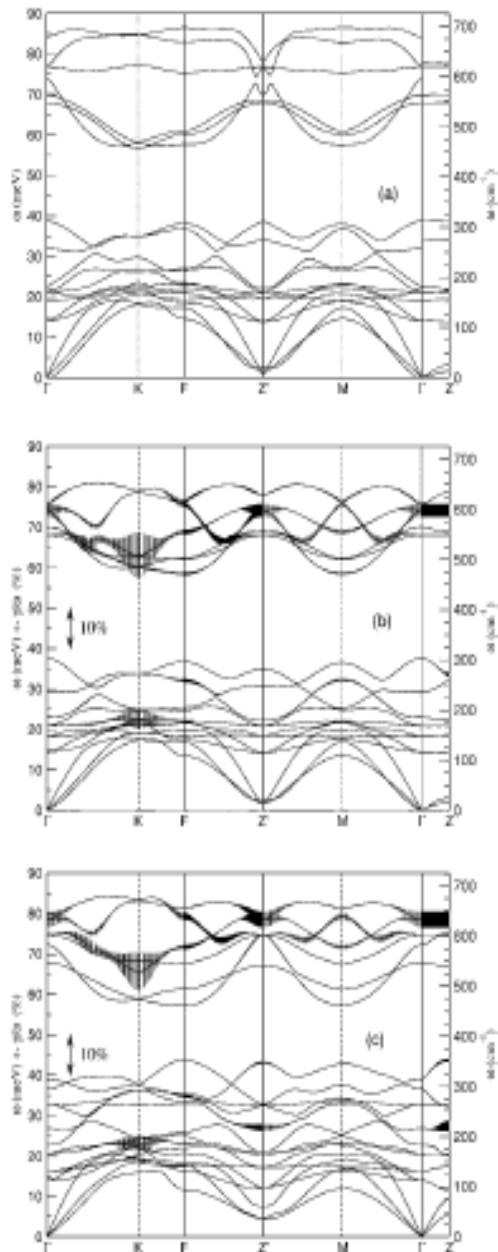


FIG. 5. Phonon dispersions of  $\beta$ -ZrNCl (a) and Li-doped ZrNCl model A (b) and model B (c), respectively. The vertical bars indicate the relative phonon linewidths  $\gamma/\omega$  in % as calculated from the electron-phonon coupling [see Eq. (1)].

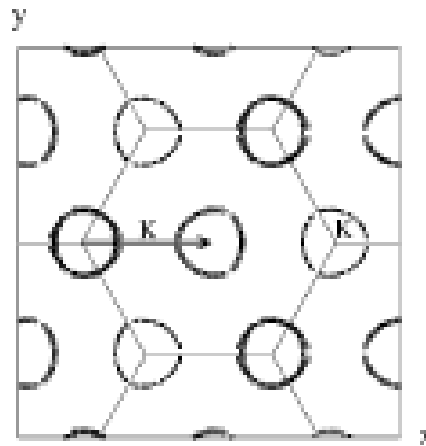
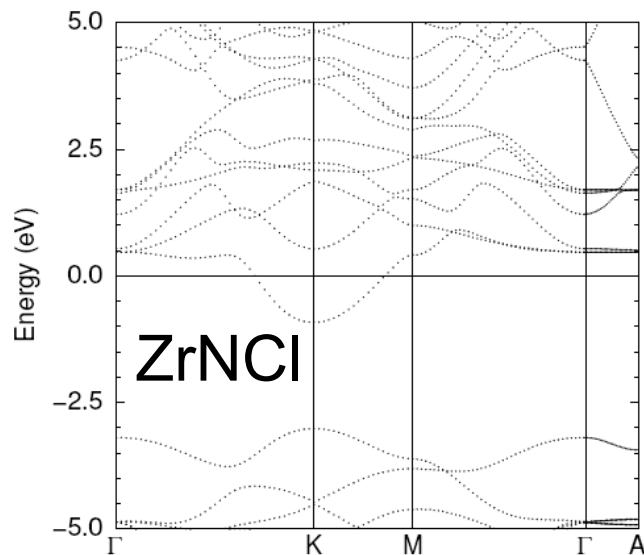
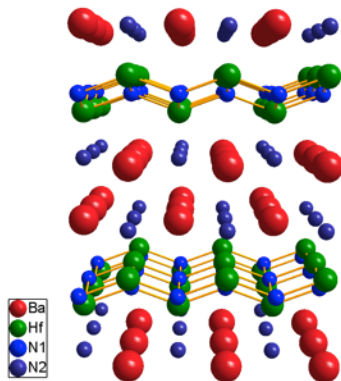


FIG. 7. Cut through the Fermi surface orthogonal to the rhombohedral axis. Full circles indicate the original Fermi surface, open circles the Fermi surface shifted by the momentum  $K$ .

# Electron doping of 2D ionic insulator $\text{BaHfN}_2$ : $\text{Ba}^{2+}$ , $\text{Hf}^{4+}$ , $\text{N}^{3-}$

HfN bilayer is related to, but different from, ZrNCl.

$\text{BaHfN}_2$  has only one reactive cation, allowing vapor phase growth.

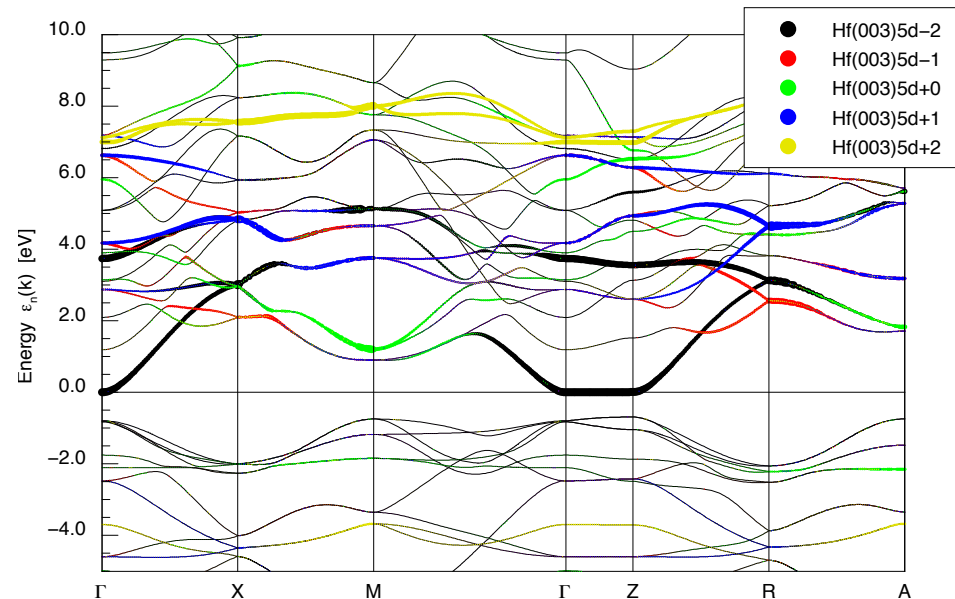


Expt.: D. H. Gregory et al. JSSC 137, 62 (1998)

Theory: A. Kaur et al., PRB 82, 155125 (2010)

- Structure: ionic-covalent square  $\text{Hf}_2\text{N}_2$  layer, cladded by BaN on each side-->neutral slabs
- ? Intercalate with Li, Na, ... to get **superconductivity?** as in ZrNCl.

BaHfN2 bandstructure



Features of doped ionic 2D band insulators  
Relevant for the effective el-el interaction  
[Examples:  $A_x\text{ZrNCl}$  and isovalent:  $T_c = 15\text{--}25\text{K}$ ]

- Low density 2DEG (one carrier for each  $\sim 4 \times 4$  supercell)
  - static lattice: weak screening behavior at small distance
    - study  $\epsilon^{-1}(r, r'; \omega)$  in a material-dependent way, look for attractive interaction in certain regions of  $q, \omega$
  - for 2D plasmons,  $\omega_p(q) \sim q^{1/2}$  implying potential low energy dynamics and screening (Andreas Bill et al.)
  - dynamic lattice: calculate dynamic ionic polarizability, intermediate range interaction not well screened out
  - dynamic lattice: separated electrons sloshing in a sea of vibrating highly charged ions  $\text{Zr}^{4+}$ ,  $\text{N}^{3-}$ , etc.
  - include dynamic electronic+charged lattice polarizability simultaneously (taking care of likely non-adiabatic effects, polaronic behavior, etc)



# Total dielectric function: Algebraic sign, electron-lattice response, and superconductivity

P B Allen, M L Cohen, D R Penn, PRB 1988

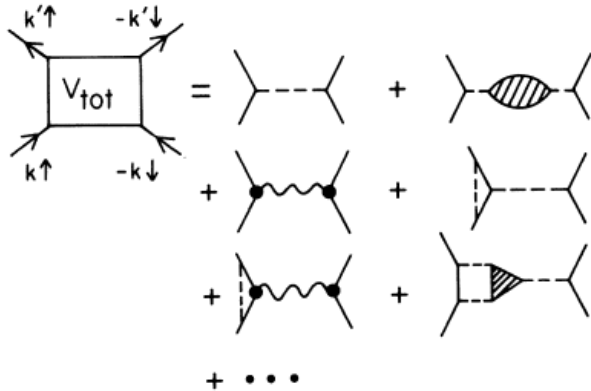


FIG. 2. Feynman graphs for the effective electron-electron interaction which binds a Cooper pair in the BCS theory.

Effective el-el coupling:  
screened el-el interaction  
+ el-lattice-el interaction

Total effective interaction  
in mean field approximation

$$V_{12, \text{MF}} = v_{1/2} \epsilon_{s, \text{MF}}^{-1} v_{1/2} ,$$

$$\epsilon_{s, \text{MF}} = 1 - v_{1/2} \chi_0 v_{1/2} .$$

$$V_{12, \text{MF}} = V_{\text{el}} + \sum_{\alpha, \beta} \phi_{\alpha}(\mathbf{Q} + \mathbf{G}) D_{\alpha\beta}(\mathbf{Q}\omega) \phi_{\beta}(\mathbf{Q} + \mathbf{G}') , \quad (67)$$

where the first term is the purely electronic part of the screened interaction

$$V_{\text{el}} = \epsilon_{\text{MF}, \text{el}}^{-1}(\mathbf{Q} + \mathbf{G}, \mathbf{Q} + \mathbf{G}', \omega) \frac{4\pi e^2}{\Omega(\mathbf{Q} + \mathbf{G})^2} . \quad (68)$$

Dynamic lattice response

$$D_{\alpha\beta}^{-1} = M(\omega^2 - \Omega_0^2) \delta_{\alpha\beta} - \sum_{\mathbf{G}_1, \mathbf{G}_2} f_{\alpha}(\mathbf{Q} + \mathbf{G}_1) [\delta_{\mathbf{G}_1, \mathbf{G}_2} + v_{1/2}(\mathbf{Q} + \mathbf{G}_1) \chi_{\text{MF}, \text{el}}(\mathbf{Q} + \mathbf{G}_1, \mathbf{Q} + \mathbf{G}_2, \omega) v_{1/2}(\mathbf{Q} + \mathbf{G}_2)] f_{\beta}(\mathbf{Q} + \mathbf{G}_2) ,$$



# Doped Band Insulating Oxides: a comparison to ponder

Two superconducting doped, highly ionic insulators

BaBiO<sub>3</sub>

$\text{Ba}^{2+}_{1-x}\text{K}^+_x\text{Bi}^{4+}(\text{O}^{2-})_3$  (BKBO):  $T_c = 30+$  K at  $x=0.35-0.40$   
wide band perovskite, sp electron carrier system  
discussed as a negative-U, or valence skipping, system

SrTiO<sub>3</sub>

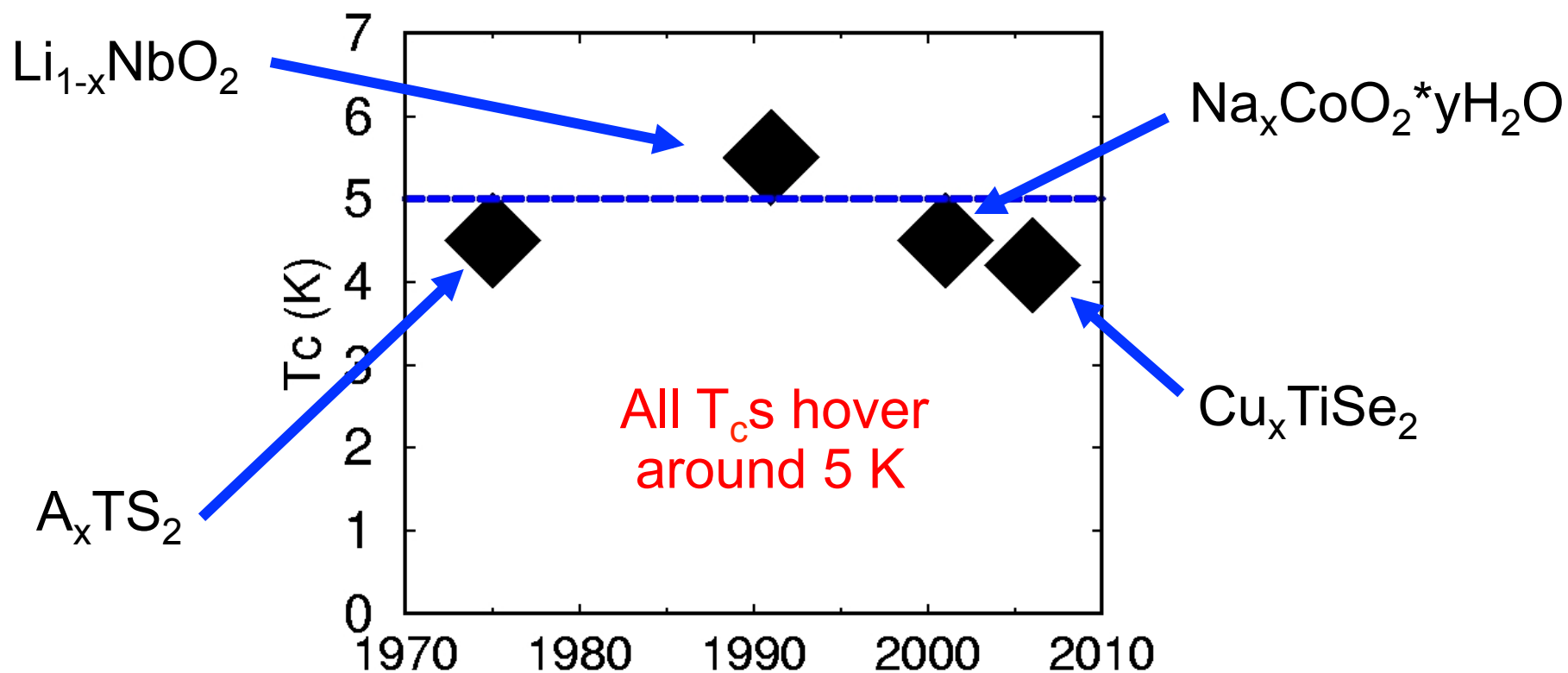
$\text{Sr}^{2+}_{1-x}\text{La}^{3+}_x\text{Ti}^{4+}(\text{O}^{2-})_3$  (SLTO):  $T_c < 0.7$  K at  $x < 0.001$   
wide band perovskite, s-p electron carrier system  
van der Marel, van Mechelen, Mazin, arXiv:1109.3050:  
an anti-adiabatic electron-phonon system

Why (1) are these two doped insulators so different?

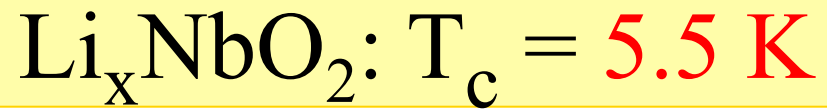
Why (2) do most doped perovskites not superconduct at all?

# Synopsis: $T_c$ in 2D Triangular Oxides/Chalcogenides

## Triangle Lattice Transition Metal Chalcogenides



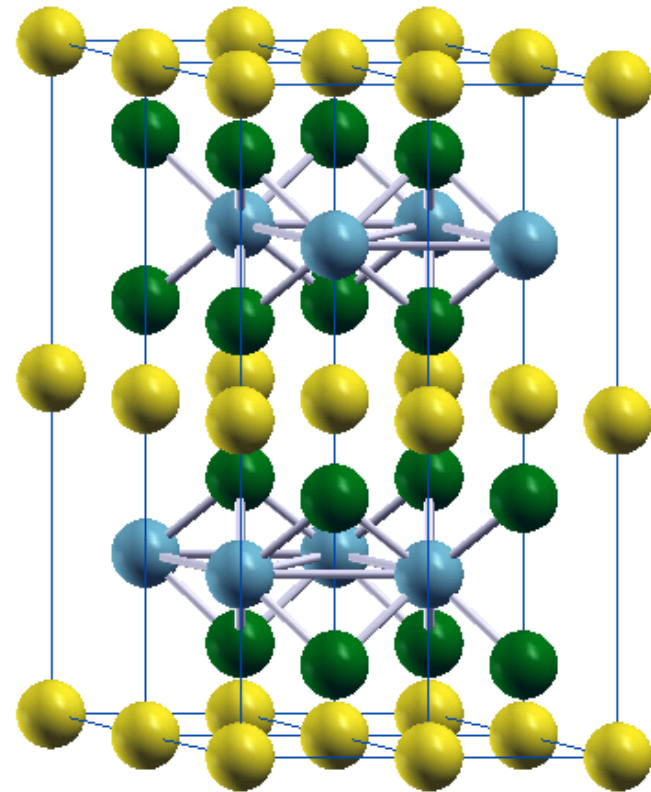
A doped triangular lattice TM oxide:  
 $\text{Li}_x\text{NbO}_2$



3d<sup>1+x</sup> system

- ◇  $a = 2.90 \text{ \AA}$
- ◇  $c = 10.46 \text{ \AA}$
- ◇ Triangular Nb lattice
- ◇ Double layer unit cell
- ◇ Strongly layered structure
- ◇  $T_c$  independent of  $x$
- ◇ Li can be de-intercalated
- ◇ Nb **trigonal prismatically** coordinated\*

[A. Stacy group (Berkeley, 1991-92)]



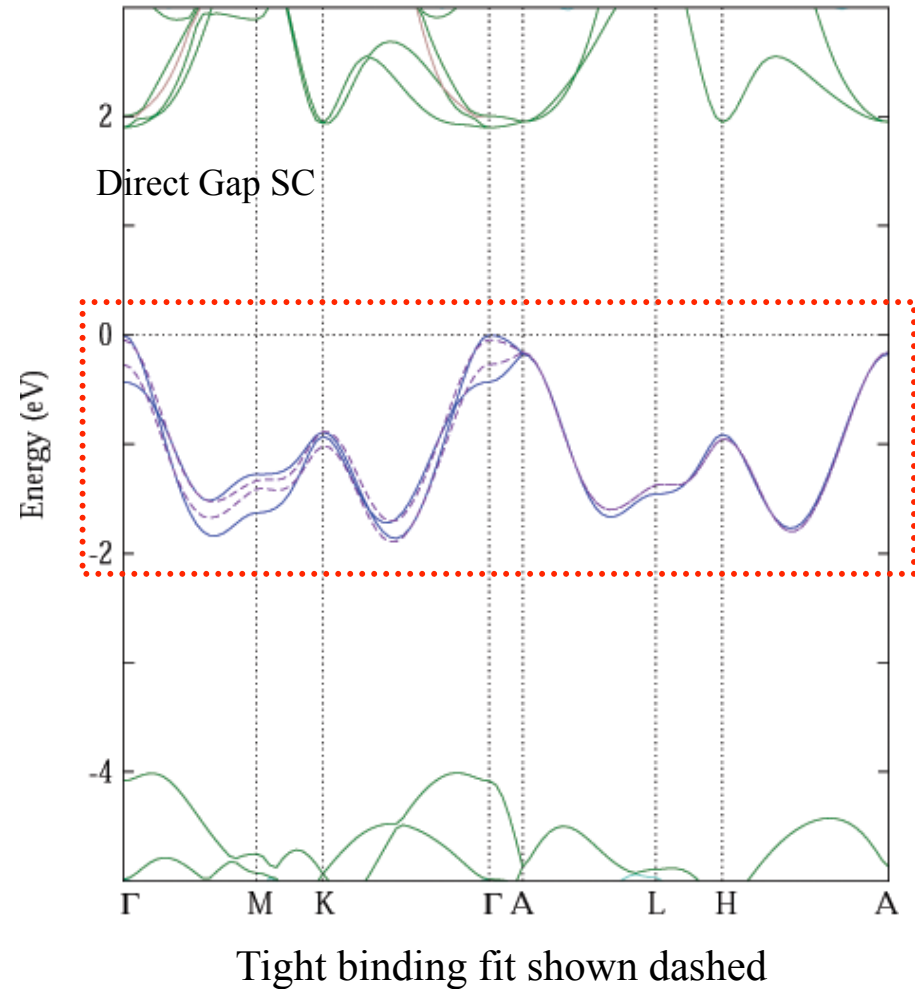
*Experimental structure from  
Meyer and Hoppe,  
Angew. Chem. Int. 13, 744 (1974)*

# A Single Band System: The LiNbO<sub>2</sub> Band Structure

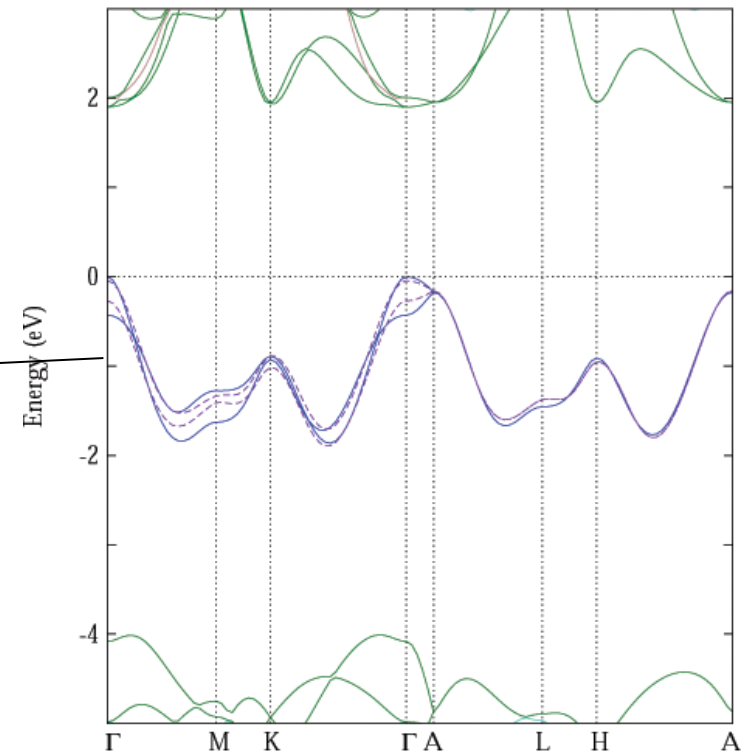
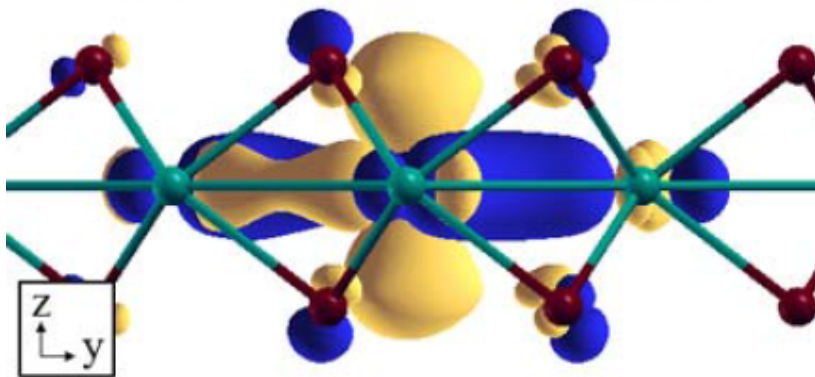
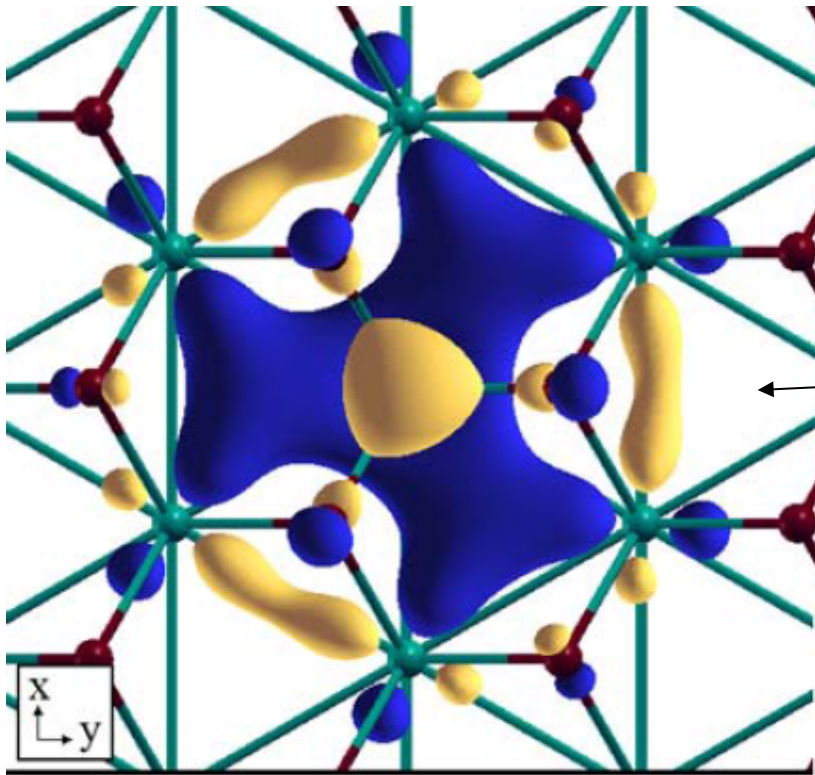
[E. Ylvisaker and WEP]

- Nb<sup>4+</sup> d<sup>1</sup> configuration
- Crystal field splitting gives
- **single d(z<sup>2</sup>) band** (per Nb)
- Bandwidth 1.8 eV
- Large **second** neighbor
- Substantial el-ph coupling

$z_0$	$t_1$	$t_2$	$t_3$	$t_z$
0.117	24	<b>107</b>	17	231
<b>0.126</b>	<b>73</b>	<b>104</b>	<b>43</b>	<b>27</b>
0.136	<b>136</b>	97	68	28

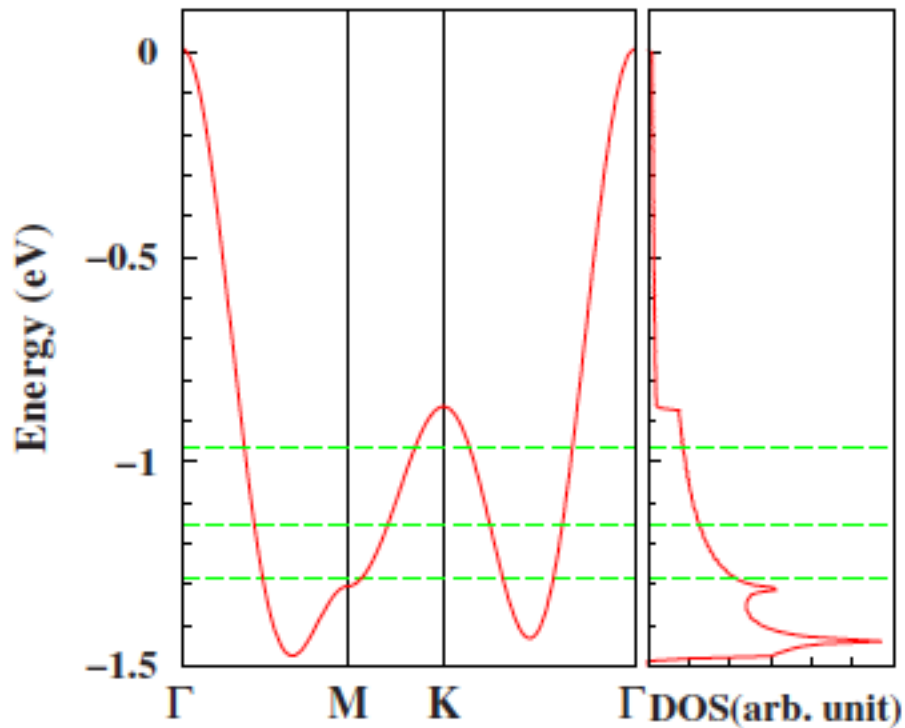


# A Single Band System: The $\text{LiNbO}_2$ Wannier Function

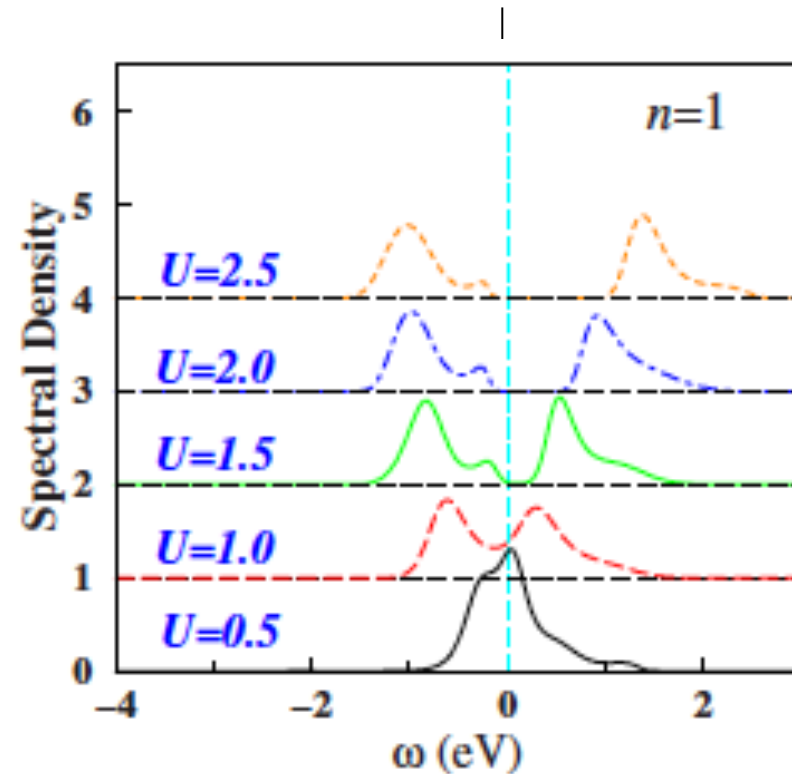


# A Single Band System $\text{LiNbO}_2$ : a DMFT study

Lee, Kunes, Scalettar, WEP, PRB 2007



Band and DOS



Spectral density vs  $U$   
at half filling

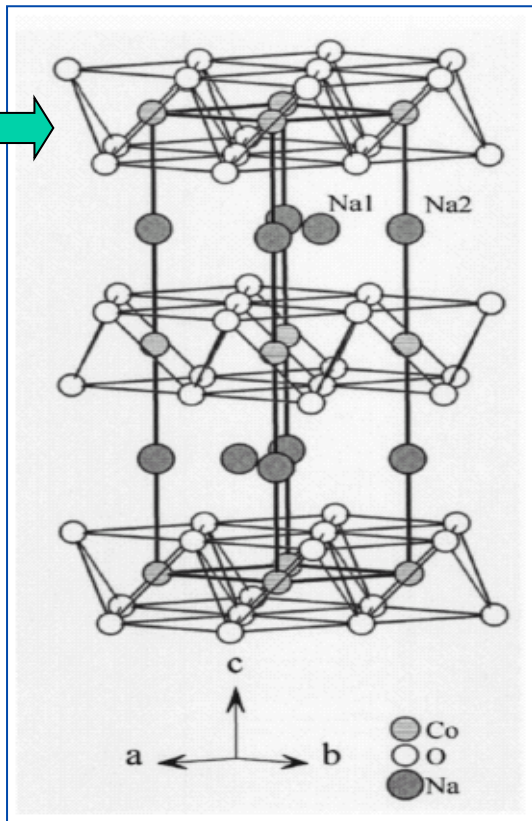
Likely  $\text{Li}_{1/2}\text{NbO}_2$  is a correlated electron system. However,  $U$  for the Wannier function is not known. The origin of pairing is a mystery at present.

A doped triangular lattice TM oxide:  
 $\text{Na}_x\text{CoO}_2 \cdot n\text{H}_2\text{O}$



# $\text{Na}_x\text{CoO}_2$ , the Dehydrated Superconductor Just add water!

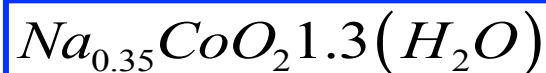
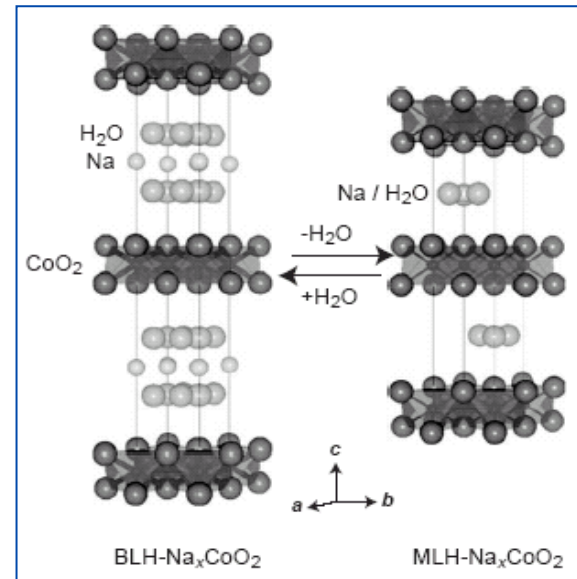
Edge-sharing  
 $\text{CoO}_6$   
Octahedra



By the  
trigonal  
symmetry of  
the Co site:

$t_{2g}: e_g' + a_g$

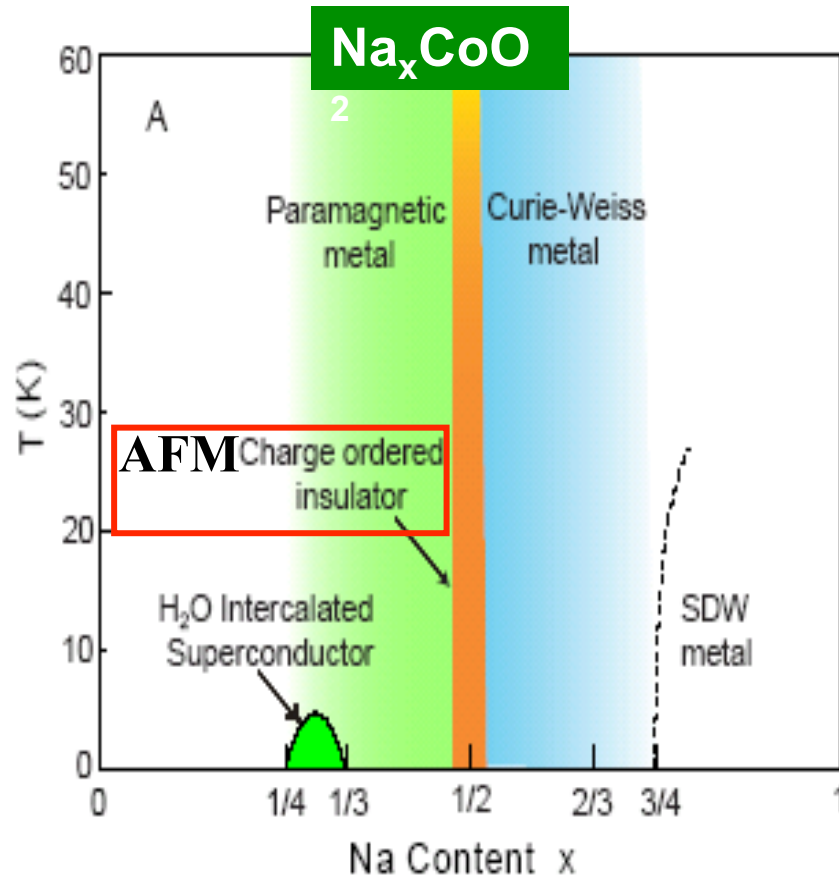
Water



J.D. Jorgensen et al. (ANL)  
Phys. Rev. B **68**, 214517 (2003)

K. Takada et al., Nature **422**, 53 (2003);  
Adv. Mater. **16**, 1901 (2004) [NIMS, Tsukuba]

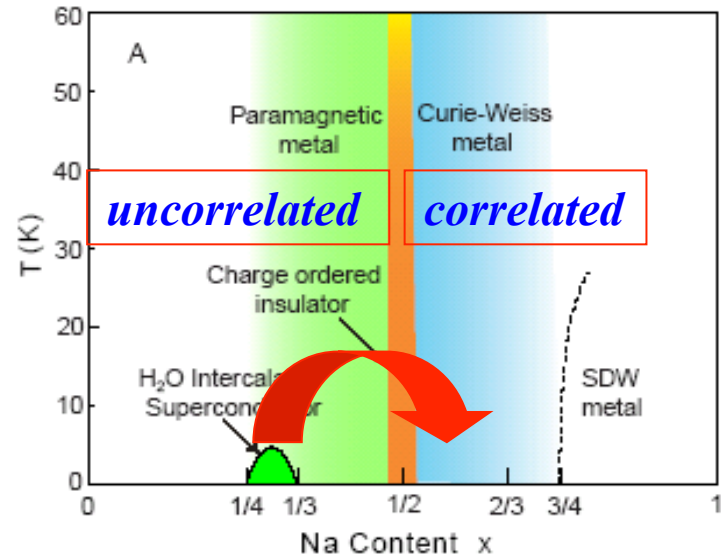
# Phase Diagram of $\text{Na}_x\text{CoO}_2$



M.L. Foo et al. (Princeton), Phys. Rev. Lett. 92, 247001 (2004)

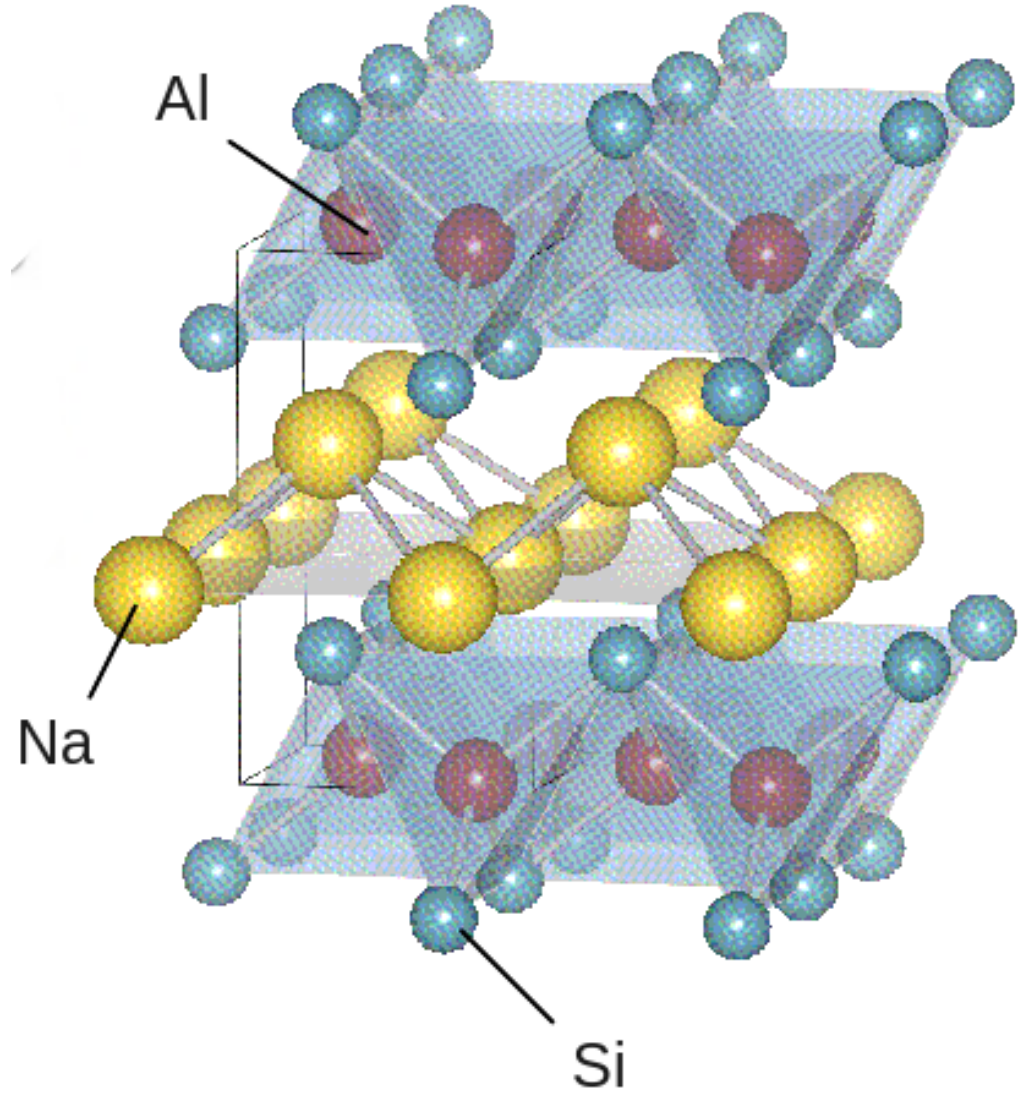
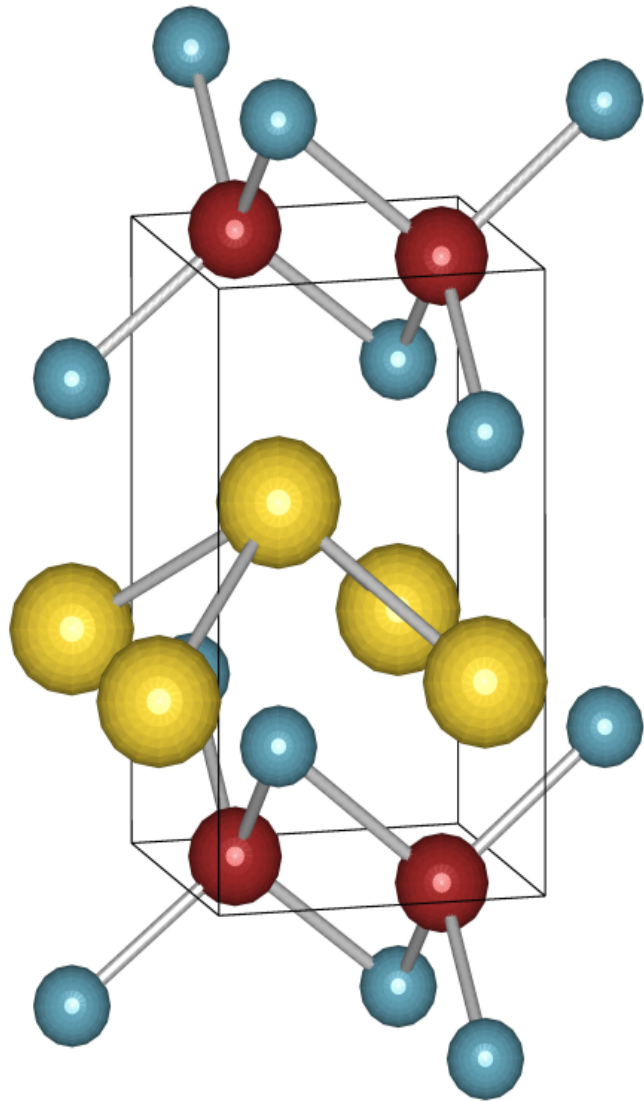
Several reports of  $\text{H}_2\text{O}$ -to-oxonium ( $\text{H}_3\text{O}^+$ ) conversion in sample; shift in Co mean valence

- Takada et al., J Mater Chem 14, 1448 (2004)
- Karpinnen et al., Chem Mater 16, 1693 (2004)
- Milne et al., Phys Rev Lett 93, 247007 (2004)
- Takada et al., Adv. Mater. 16, 1901 (2004)
- Chen et al., cond-mat/0501181



Another self-doped s-p 2D superconductor  
besides  $\text{MgB}_2$ :  
 $\text{NaAlSi}$

NaAlSi



Structural similarities to the Fe pnictide superconductors.  
Completely different electronic properties.

# NaAlSi: self-doped semimetallic superconductor with Al free electrons and covalent Si holes

[NaAlSi: Z=11:13:14]

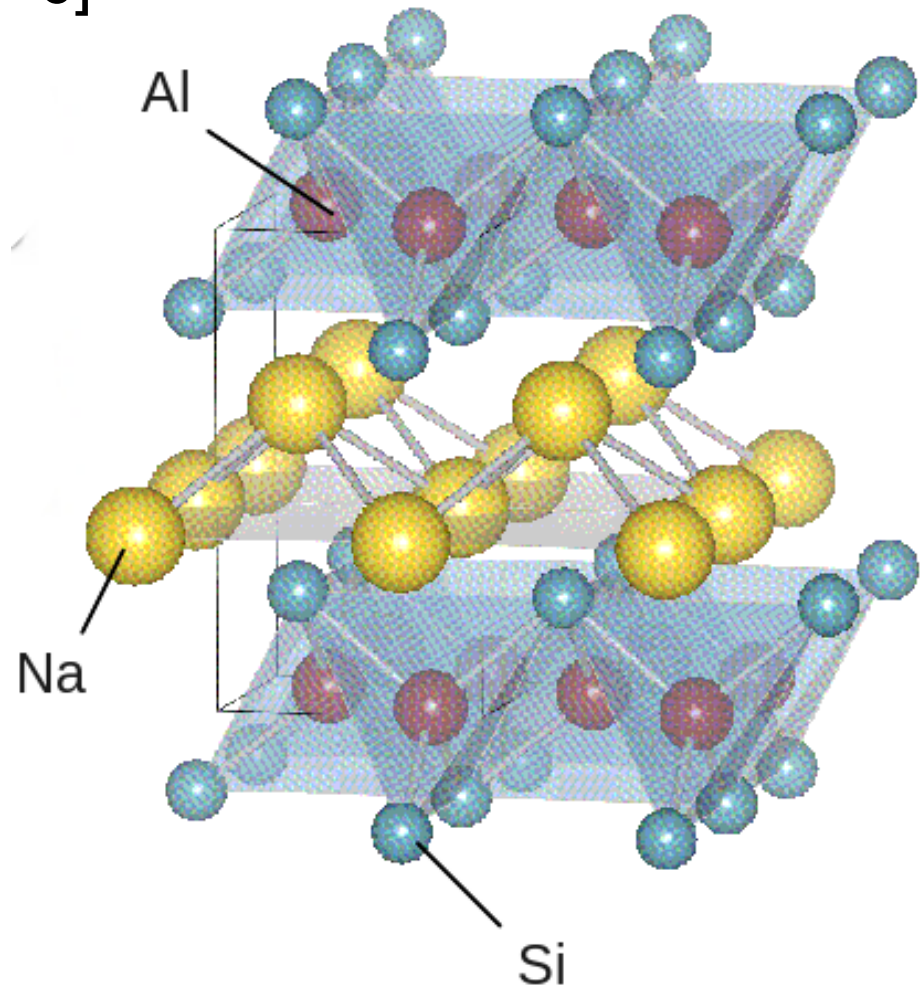
[LiBC: Z= 3: 5: 6]

*sp* electron superconductor, semimetal

$T_c = 7 \text{ K}$  [Kuroiwa *et al.*, Physica C (2007)]

Related systems

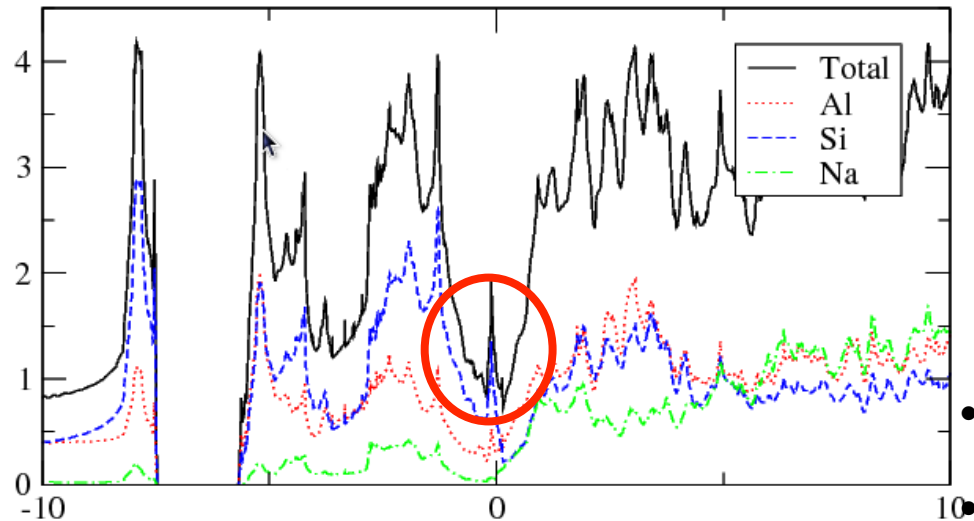
- Isovalent to LiBC and (sort of)  $\text{MgB}_2$
- More closely related to CaAlSi
- $\text{Al}_2\text{Si}_2$  layer structure like Fe-pnictides:  
Si-Al-Si layer  $\leftrightarrow$  As-Fe-As layer



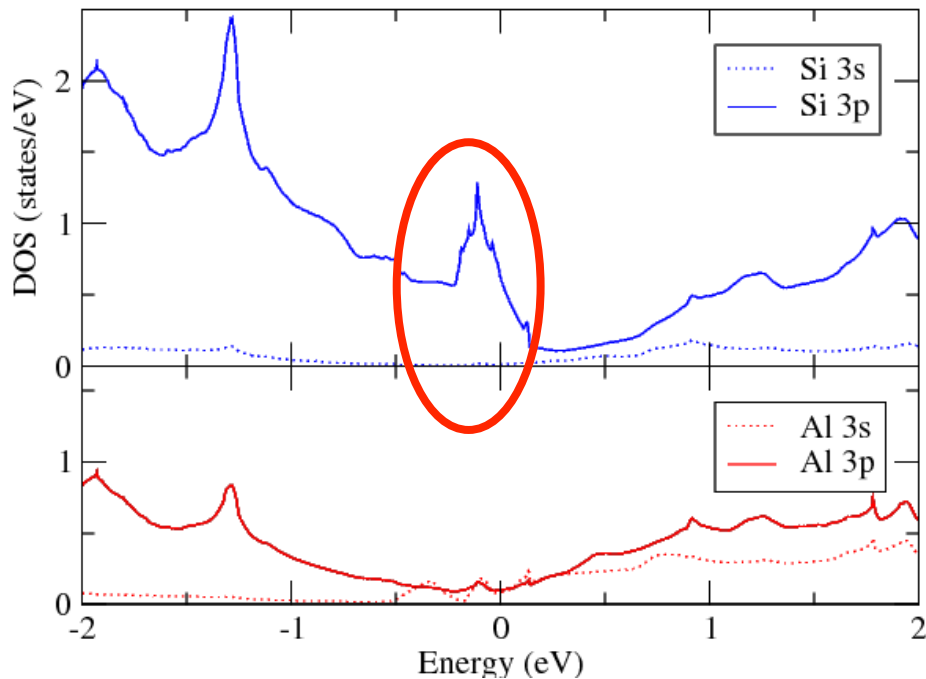
Rhee, Banerjee, Ylvisaker, WEP,  
Phys Rev B (2010)

# NaAlSi

unusual semimetallic  
quasi-two-dimensional  
superconductor



- Sharp narrow peak within deep pseudogap
- Peak is Si 3p character
- Steep slope means high thermopower:



Thermoelectric Application?

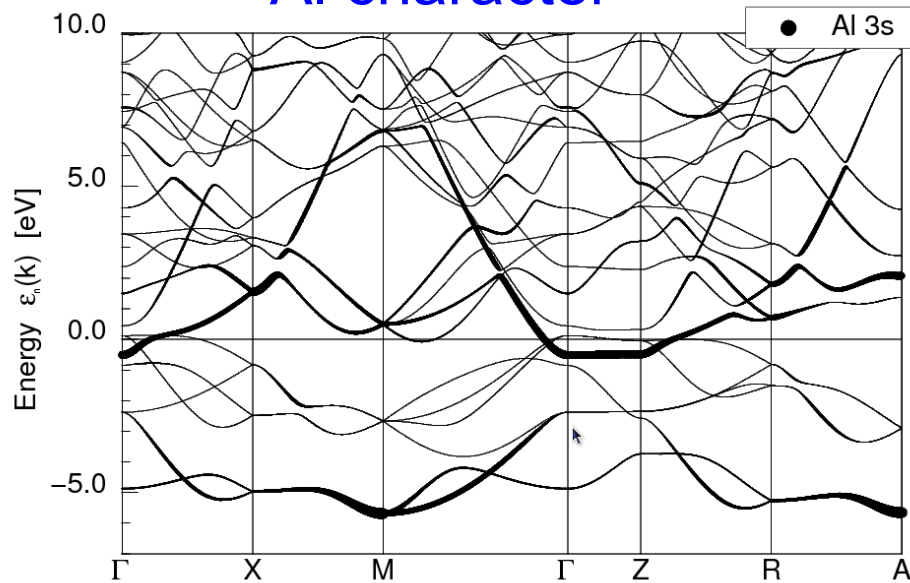
$$S(T) \rightarrow -\frac{\pi^2 k_B}{3e} \left. \frac{d \ln \sigma(E)}{dE} \right|_{\varepsilon_F} k_B T$$

$$\sigma(E) = 4\pi e^2 \langle \vec{v}(E) \vec{v}(E) \rangle N(E) \tau(E)$$

$$d \ln N(E) / dE |_{\varepsilon_F} = -4.0 \text{ eV}^{-1}$$

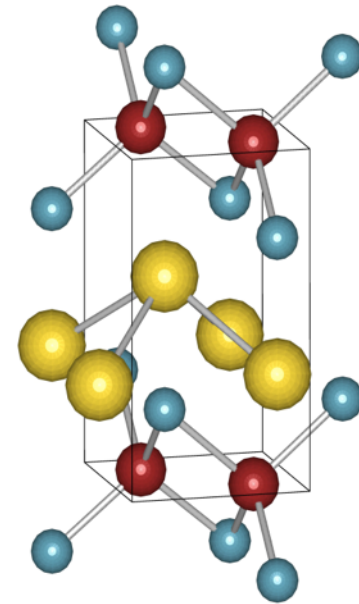


## Al character



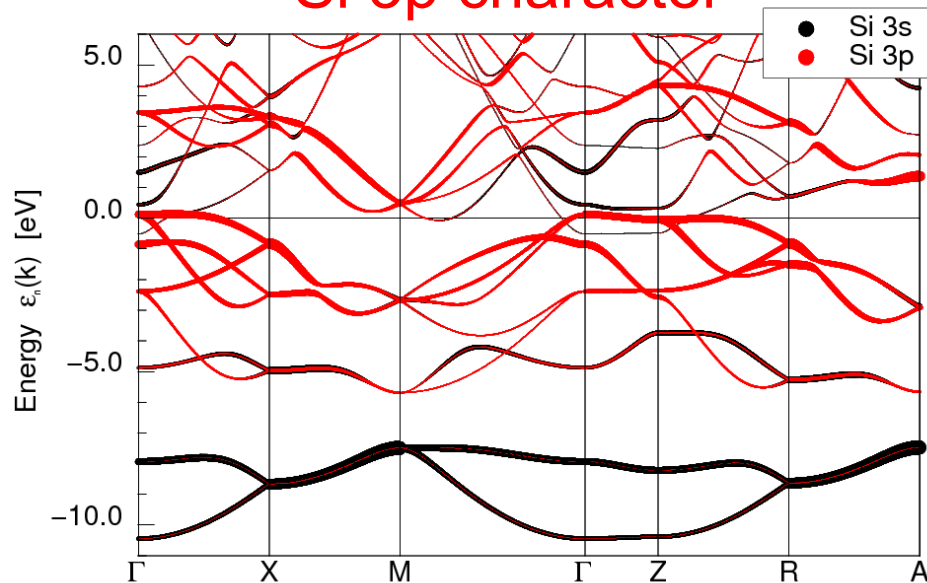
NaAlSi

$T_c = 7\text{K}$



Al 3s: nearly free electrons, semimetallic

## Si 3p character



- Al electron pockets, Si hole pockets
- Ionic Na, covalent (and ionic) Al-Si
- Free-electron-like Al 3s bands
- Valence states Si-derived, 2D
- s-p electrons, no magnetism

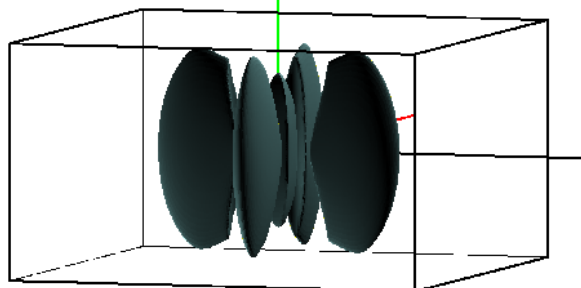
Si 3s,3p holes: covalent-like bands

# NaAlSi Fermi Surfaces

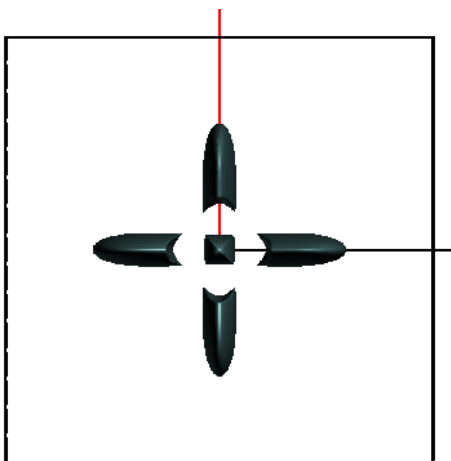
Black: holes. Yellow: electrons.

Side view

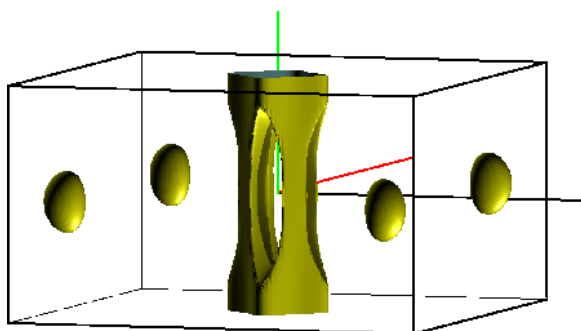
Paddlewheel



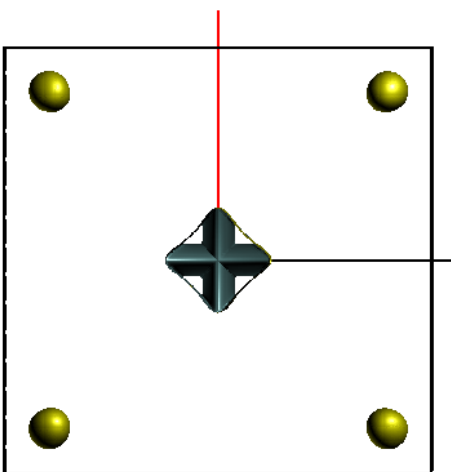
Top view



Side view



Top view



What is  $\lambda$  &  $T_c$   
(calculated)?  
No answer  
yet to this question.

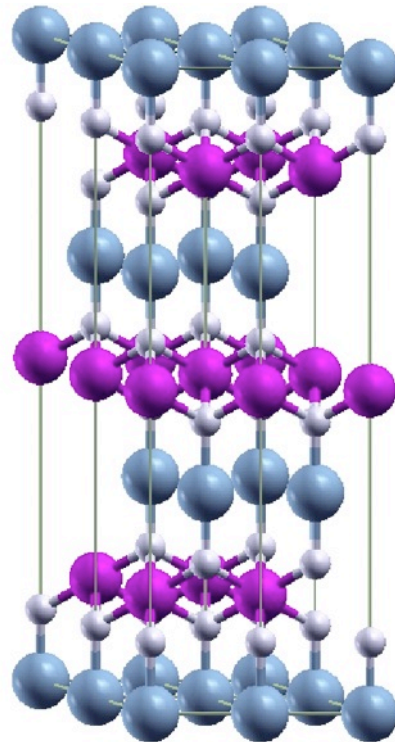
DFT linear response  
el-ph coupling calcs?

Accurate integrated  $\lambda$   
value is difficult to obtain  
due to small Fermi surfaces  
and code architectures

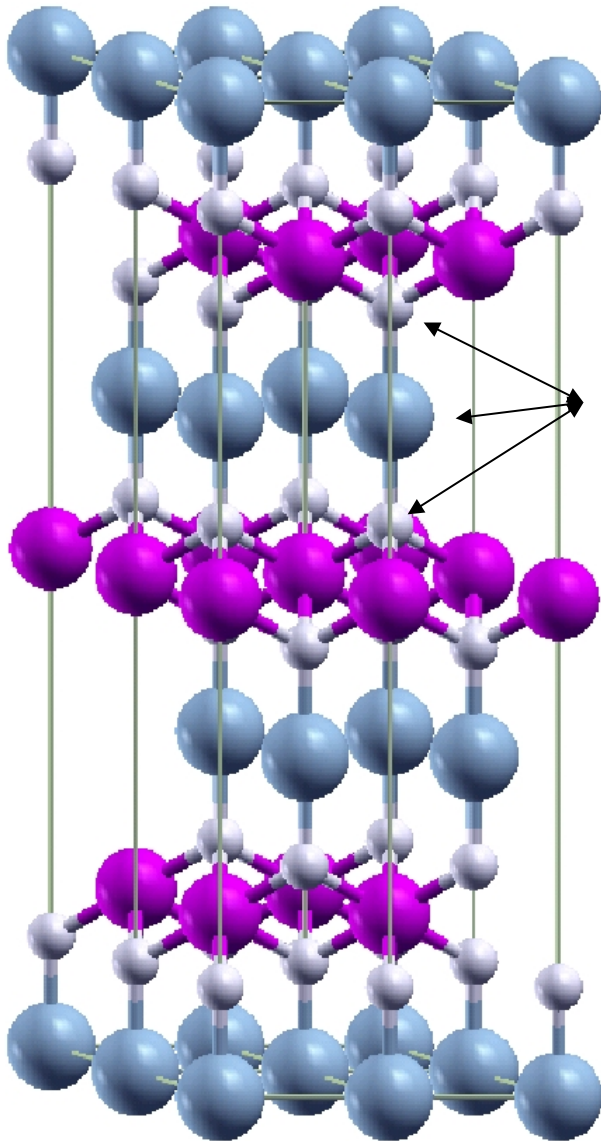


An unusual new cuprate superconductor;  
a high temperature superconductor?

hole-doped delafossite  $\text{CuAlO}_2$



# Triangular lattice delafossite $\text{CuAlO}_2$



Linear  
O-Cu-O  
trimers

Layered structure,  
rhombo stacking

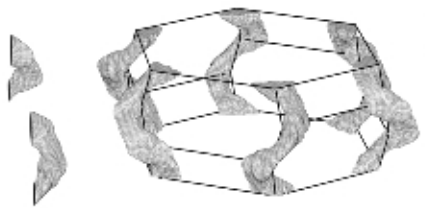
$\text{AlO}_4$  units  
(hexagonal)

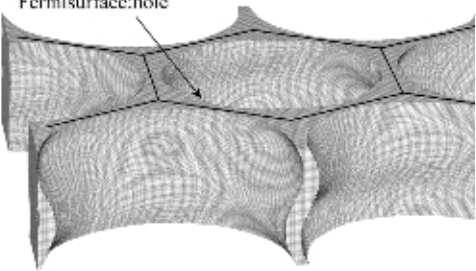
Formally:  $\text{Cu}^{1+}\text{Al}^{3+}(\text{O}^{2-})_2$   
~3 eV gap insulator

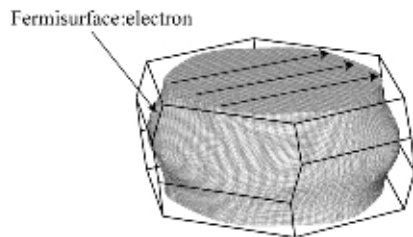
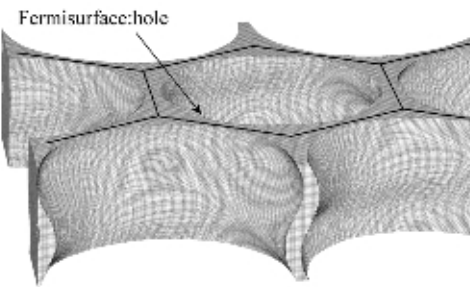
Mostly studied as a  
transparent metal  
& thermoelectric  
(is naturally a slightly  
hole-doped conductor)

# CuAlO<sub>2</sub> Electronic structure

## Rigid band Fermi surfaces

(a)  $E_f = 0.372$  Ry. (lightly p-type doped CuAlO) 

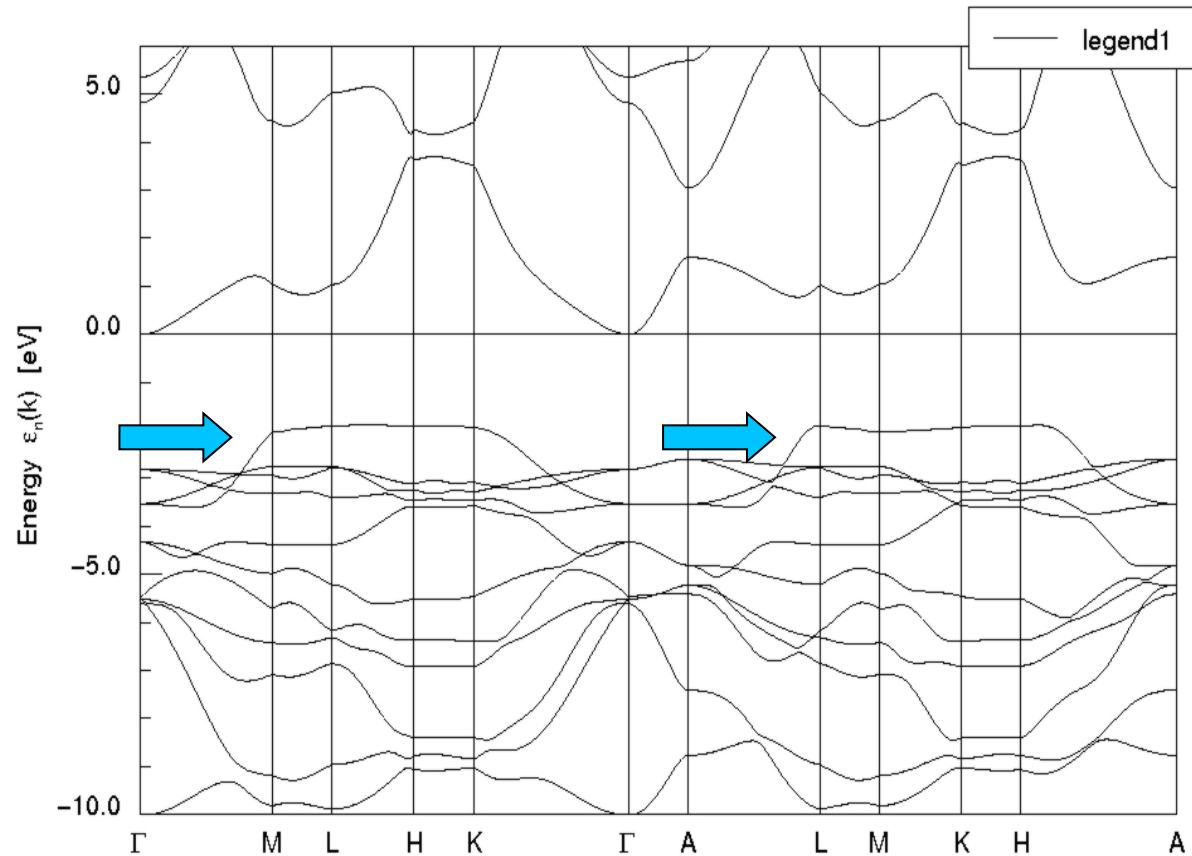
(b)  $E_f = 0.340$  Ry. (heavily p-type doped CuAlO<sub>2</sub>) 



Nakanishi &  
Katayama-Yoshida

## Band plot along hexagonal lines in BZ

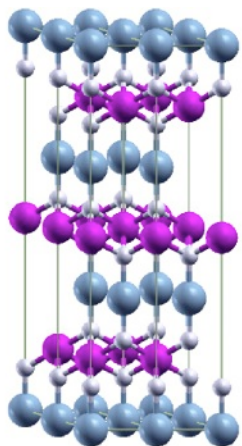
CuAlO<sub>2</sub> Hexagonal Points



Flat band all around sides of “hexagonal BZ”

E. R. Ylvisaker & WEP (2002)

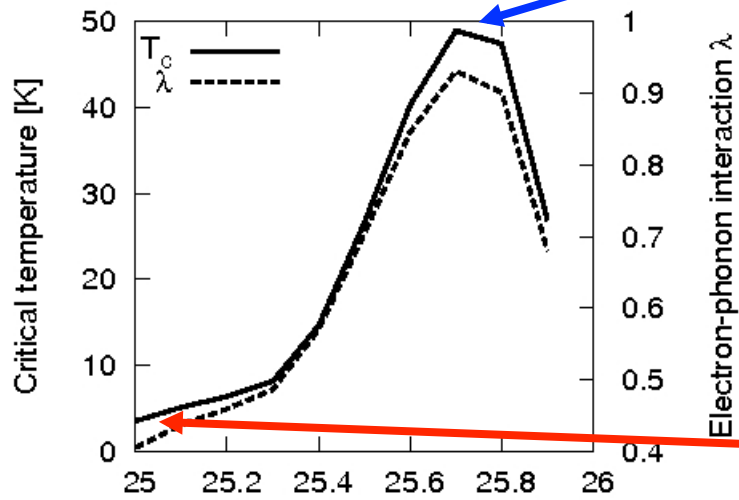
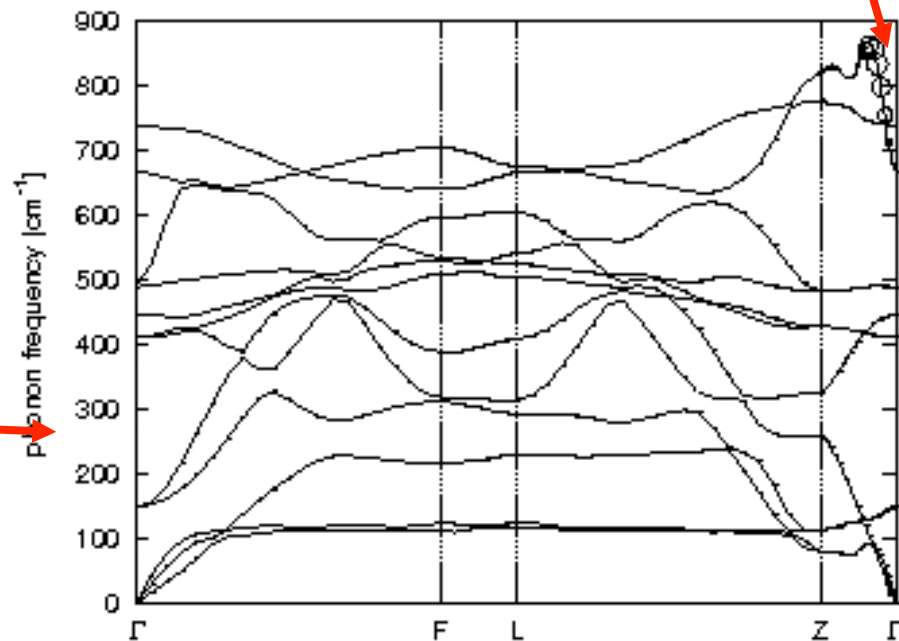
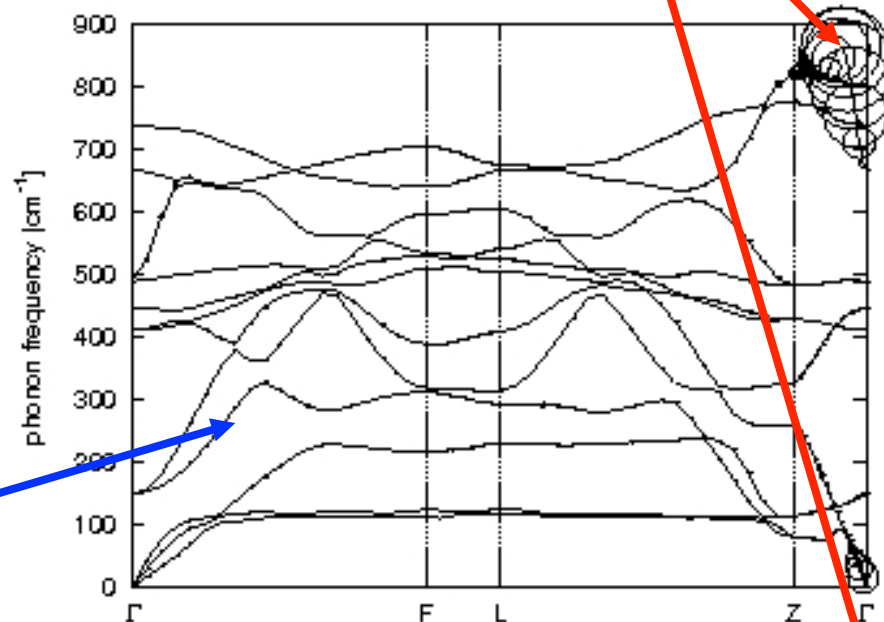
Nakanishi & Katayama-Yoshida  
 PREDICTION (SSC 2012)  
 Superconductivity up to  
 50K in hole-doped  
 delafossite  $\text{CuAlO}_2$



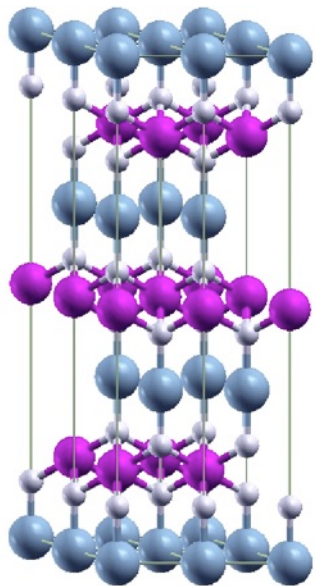
Rigid band doping;  
 matrix elements from  
 undoped material.

Very weak el-ph coupling almost  
 everywhere, except ...

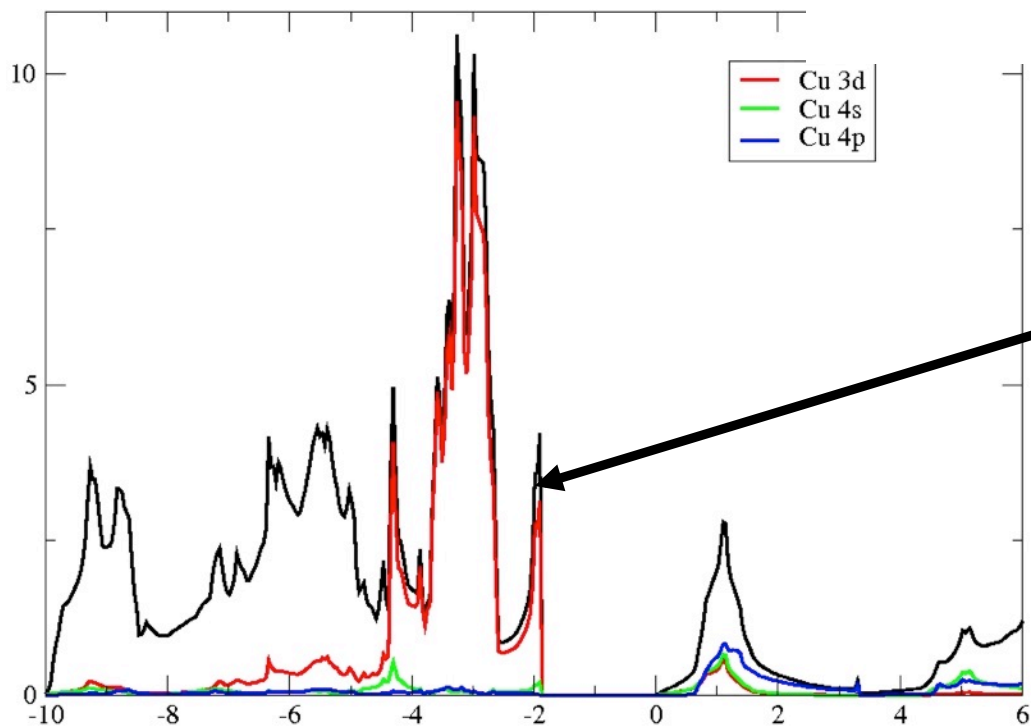
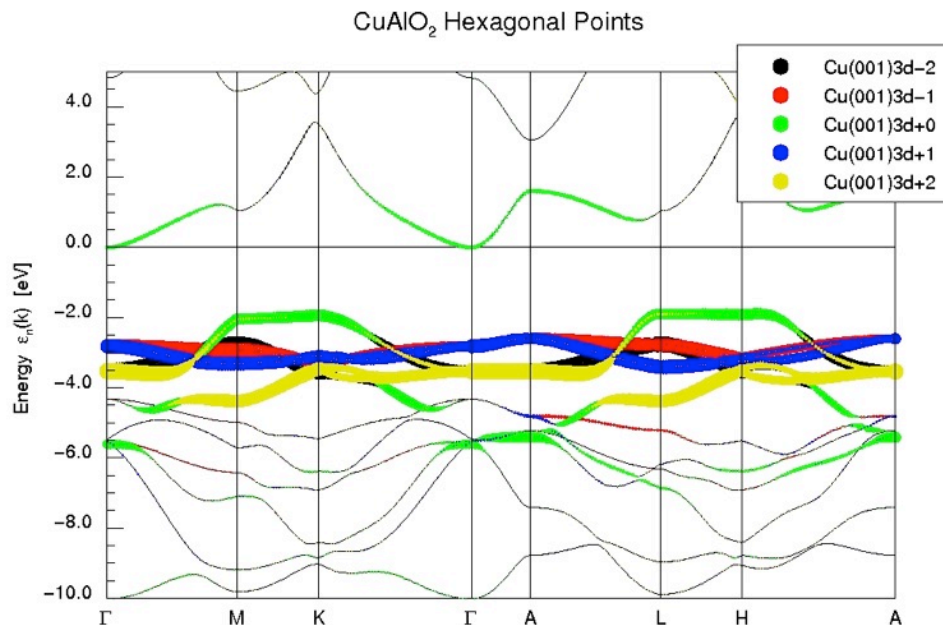
$$Q = (\varepsilon, \varepsilon', q), q < 0.5\pi$$



$\text{CuAlO}_2$  el-ph coupling  $\lambda$ ,  $T_c$



CuAlO<sub>2</sub>  
electronic  
character



Cu really is Cu<sup>1+</sup>: d<sup>10</sup>

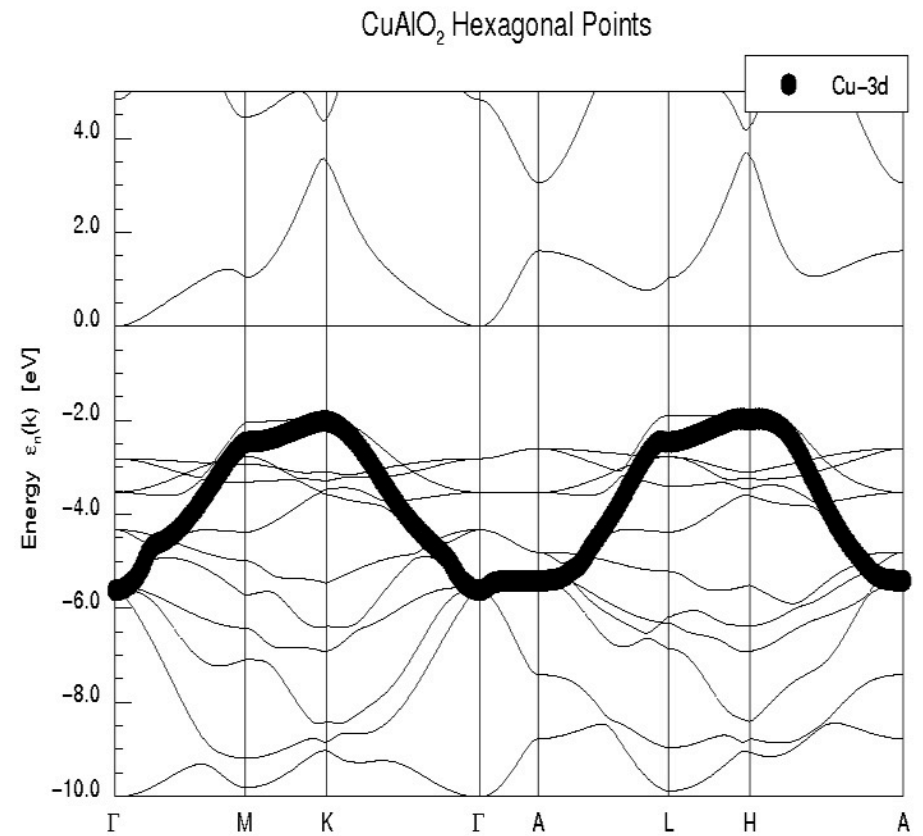
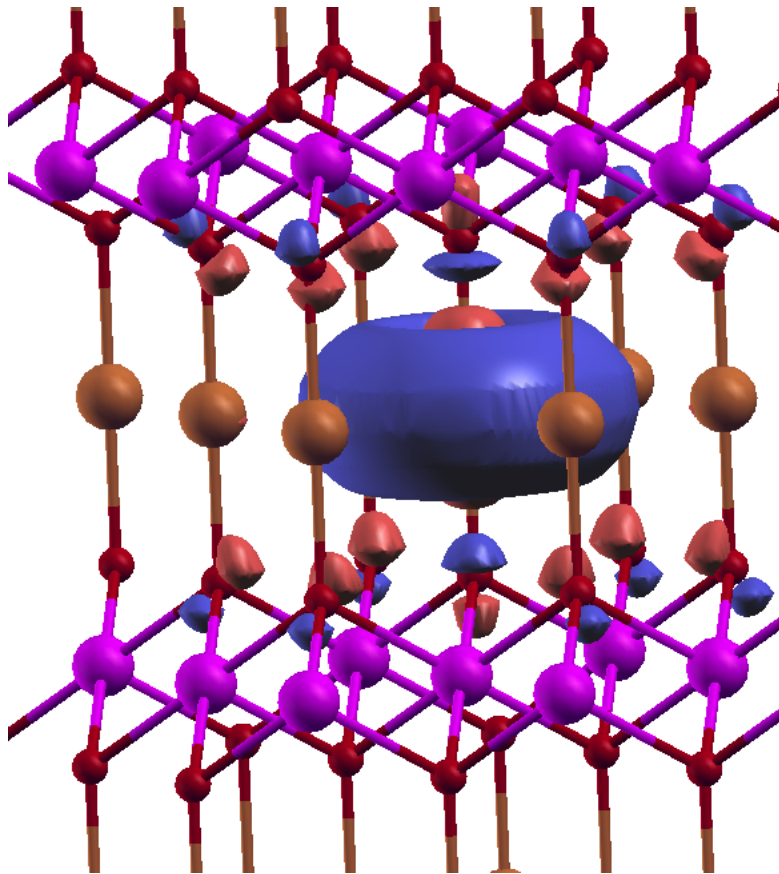
Hole-doping is into Cu d(z<sup>2</sup>) - O p<sub>z</sub>  
antibonding states on triangular lattice

Flat valence band max around edge  
of hexagonal BZ gives "1D" DOS edge

Not near half-filling, but "narrow band."  
Polaronic? Or some quasi-1D physics?

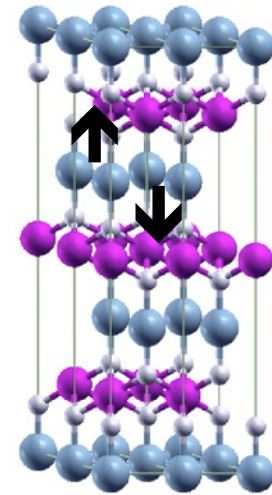
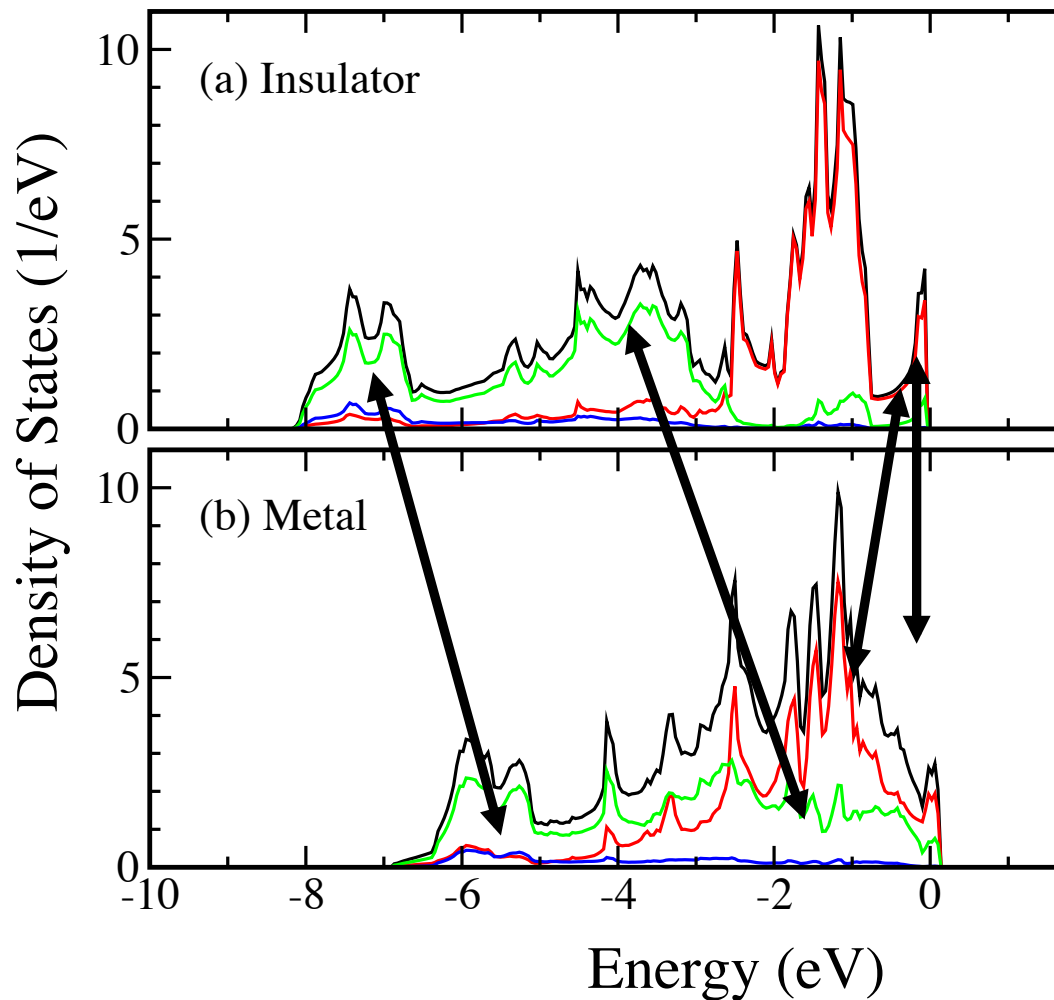


CuAlO<sub>2</sub>: the active Cu  $d(z^2)$  - O  $p_z$  Wannier function (valence band maximum):  
unexpected character



# Unusual spectral density shift: doping 0.3 holes into $\text{Cu}(\text{Mg},\text{Al})\text{O}_2$

Not rigid band at all.



Doping holes

- from Al layer to Cu layer
- introduces dipole layer (recall polar catastrophe?!)
- large  $\text{O } 2p \leftrightarrow \text{Cu } 3d$  shift

Effects:

- changes character around  $E_F$
- screens the EP matrix elements

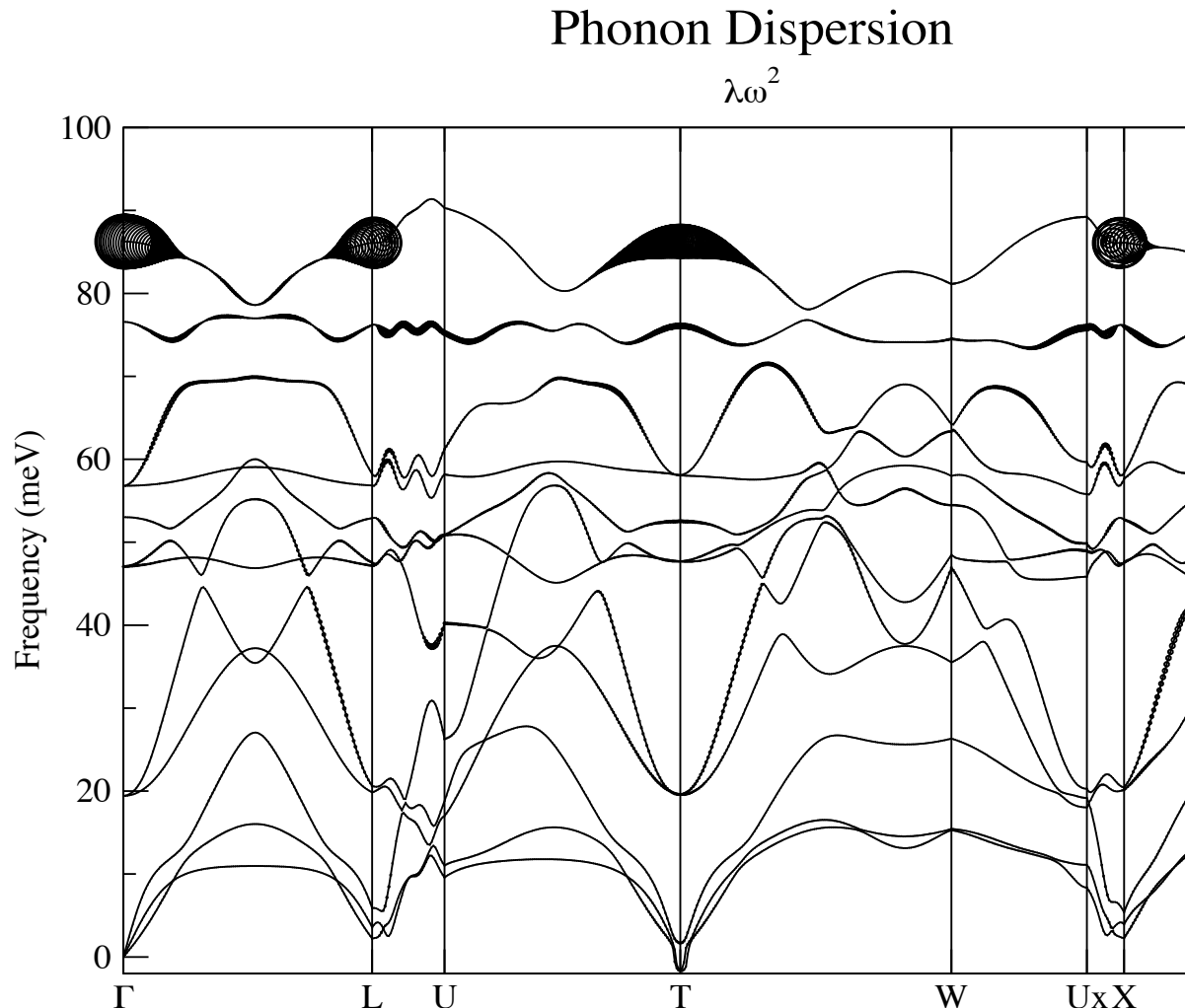
This is a **2D** physics aspect:

- dipole layer / potential shift

# CuAl<sub>0.7</sub>Mg<sub>0.3</sub>O<sub>2</sub> phonon dispersion, weighted by $\lambda\omega^2$

Full DFT linear response for metallic phase

E. R. Ylvisaker and WEP, EPL 2013



[24x24x24 mesh]

Calculated  $\lambda \sim 0.2$ ;  
so  $T_C \sim 0$   
rather than  $T_C \sim 50\text{K}$

Not superconducting  
even at optimal doping.

Full self-consistent  
calculation for the  
doped system is  
necessary for  
definitive results.



## Final Comments

Most interesting new sc'ors in last 25 years are

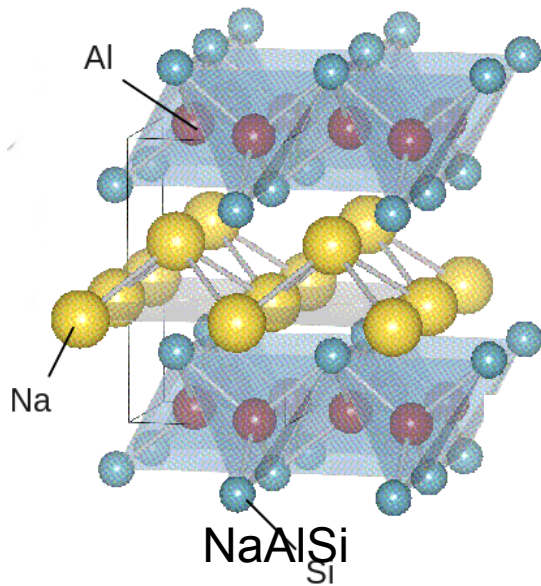
- 2D structurally and electronically, both the magnetic and nonmagnetic examples
- doped insulators (some hole, some electron)

These two features hold for both

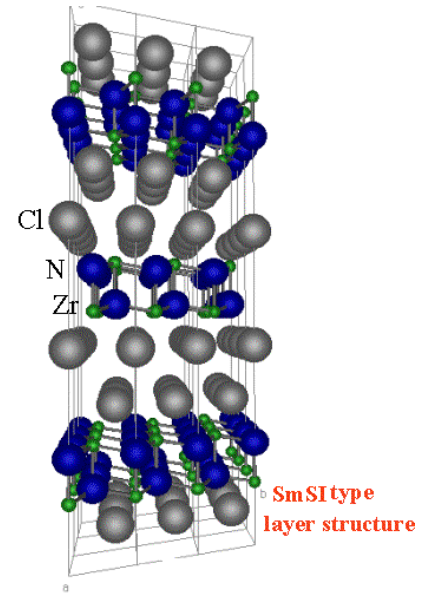
- Mott insulating parent phases (viz. cuprates)
- Fe-based pnictide system
- $\text{MgB}_2$  is intrinsically self-doped
- band ionic insulator parent phases (viz.  $\text{HfNCl}$ )

There surely are some very general characteristics favorable for superconductivity that we do not yet fathom (many new doped 2D insulators do ~~not~~ superconduct).

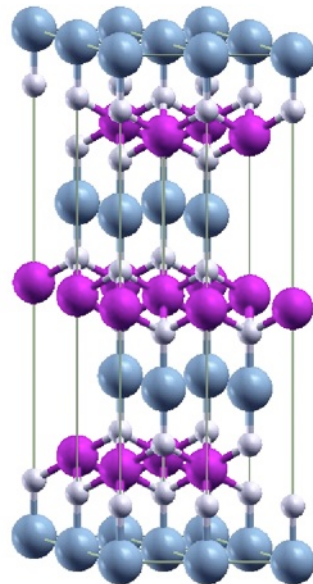
# Superconductivity and two dimensionality: experimental facts, theoretical issues, novel materials



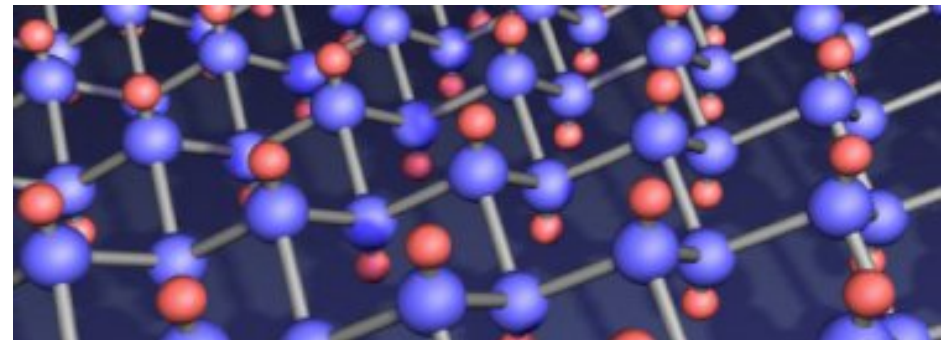
An area for new discoveries  
in materials physics



$\text{CuAlO}_2$



Graphane CH



# Acknowledgments

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