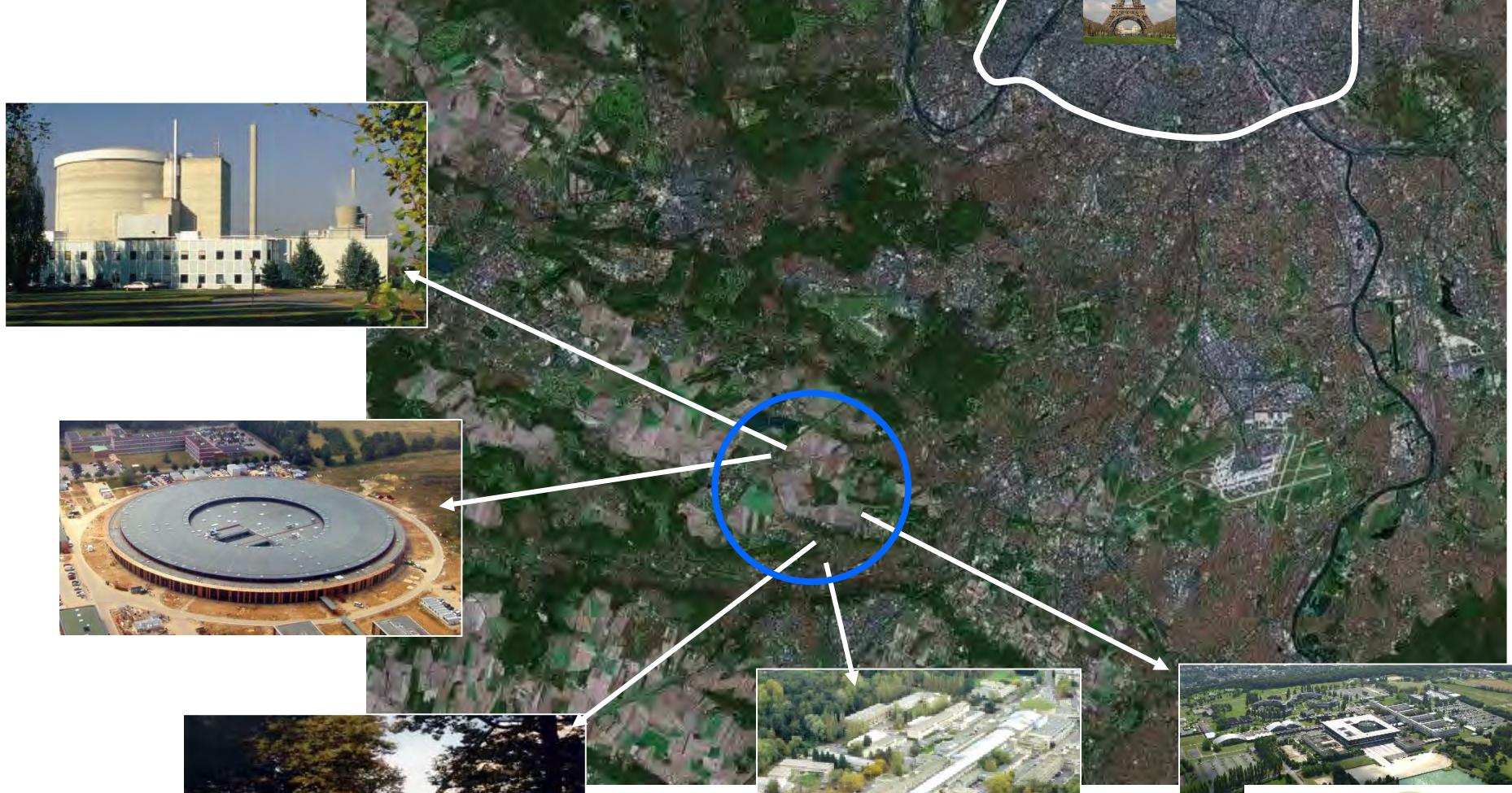




An Advanced Research Cluster in Physics  
at Palaiseau, Orsay and Saclay, south of Paris



H. Alloul, Autumn School on correlated electrons

Julich ,15 /09 /2016



# New States of matter probed by NMR, $\mu$ SR, ARPES

Laboratoire de  
Physique des  
Solides



UMR 8502 - Université Paris-Sud, Bât. 510 - 91405 Orsay cedex



## The LPS research themes



UNIVERSITÉ  
PARIS-SUD 11



### Nouveaux états électroniques de la matière

L'existence de fortes corrélations entre électrons fait apparaître de nouveaux états de la matière, originaux et inattendus.



### Phénomènes physiques aux dimensions réduites

Phénomènes physiques propres aux objets de dimensions réduites : surfaces, nano-objets, molécules et atomes.



### Matière molle et interface physique-biologie

Polymères, polymères composites, fluides complexes, fibres biologiques, propriétés de l'ADN et de la chromatine

# NMR studies of correlated electron systems

From the Mott insulator to superconductivity through the cuprate pseudogap



P. Mendels



J. Bobroff



V. Brouet



P. Wzietek



F. Rullier-Albenque

RMN-  $\mu$ SR

ARPES

High  
pressures

Transport  
(SPEC /CEA Saclay)

G. Collin , N. Blanchard, D. Colson, A. Forget  
(material synthesis and structures)

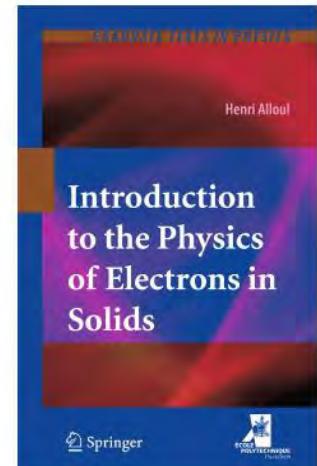
Deceased march 20th 2016

University of Kazan (Russia) : I. Mukhamedshin, A. Dooglav

University of Parma (Italy) : D. Pontiroli, M. Ricco

U. Fukuoka and Hyogo (Japan) : Y. Ihara , T. Mito

# Observation reveals Amazing Phenomena

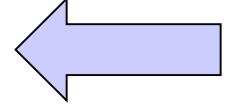


*Introduction to the Physics of Electrons in Solids,  
Editions de l'Ecole polytechnique (2007)  
English edition , Springer (january 2011)*



# NMR in correlated electron systems

## Illustration in the case of the cuprates

- *Introduction to Magnetic resonance (NMR and ESR)*   
Hyperfine couplings , NMR shifts
- *Magnetic spin susceptibilities in NMR*  
Metals and superconductors: Singlet spin pairing  
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Spin echoes and  $T_2$ : NMR applications

**Conclusion: NMR is a powerful tool in Solid State Physics**

# ESR and NMR : SPIN RESONANCE IN THE PARAMAGNETIC REGIME

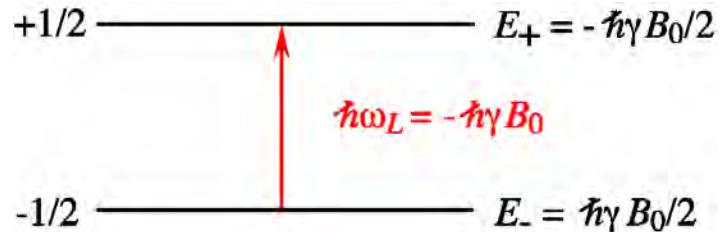
|               | Angular momentum | magnetic moment               |
|---------------|------------------|-------------------------------|
| Nuclear spin  |                  |                               |
| Electron spin |                  | $\mu_n \approx 10^{-3} \mu_e$ |

Zeeman Effect

$$H_Z = -\mu \cdot B_0 = -\hbar \gamma S \cdot B_0 = -\hbar \gamma S_z \cdot B_0$$

$$B_0 / \text{z}$$

$$I = 1/2$$



$$I > 1/2$$

$2I+1$  equidistant levels (  $h \gamma_n B_0$  )

Larmor frequency

$$\omega_L = -\gamma B_0$$

Absorption  
spectroscopy

Electron spins: ESR ~ 30 GHz/Tesla

microwave frequencies

Nuclear spins: NMR~ 10 MHz/Tesla

radiofrequencies



Spins I

$$^1H \quad I=1/2 \quad 42,57 \text{ MHz/Tesla}$$

$$^2H \quad I=1 \quad 6,53 \text{ MHz/Tesla}$$

$$^{63}\text{Cu} \quad I=3/2 \quad 11,28 \text{ MHz/Tesla}$$

$$^{65}\text{Cu} \quad I=3/2 \quad 12,08 \text{ MHz/Tesla}$$

Gyromagnetic ratios  $\gamma_n$

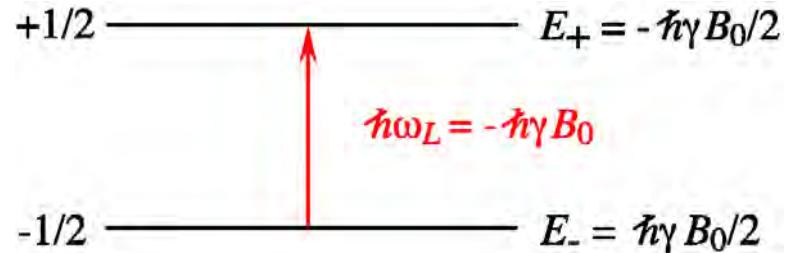
H. Alloul, Autumn School on correlated electrons

Julich ,15 /09 /2016

# ESR and NMR : SPIN RESONANCE IN THE PARAMAGNETIC REGIME

$$H_Z = -\mu \cdot B_0 = -\hbar \gamma S \cdot B_0 = -\hbar \gamma S_z \cdot B_0$$

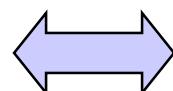
$$\mathbf{B}_0 // \mathbf{z}$$



Exciting ac field  
Act as a perturbation for  $H_Z$

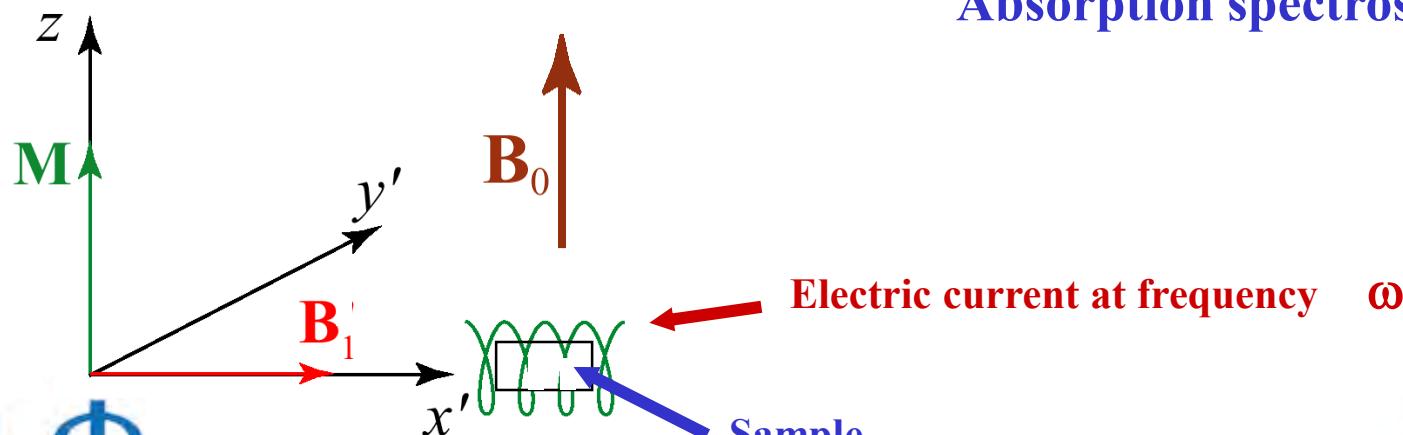
$$H_{rf} = -\hbar \gamma \mathbf{S} \cdot \mathbf{B}_1 \cos \omega_L t$$

transitions  $| -1/2 \rangle \rightarrow | 1/2 \rangle$   
if  $\langle 1/2 | H_{rf} | -1/2 \rangle \neq 0$



$$\mathbf{B}_1 \perp \mathbf{z}$$

Absorption spectroscopy

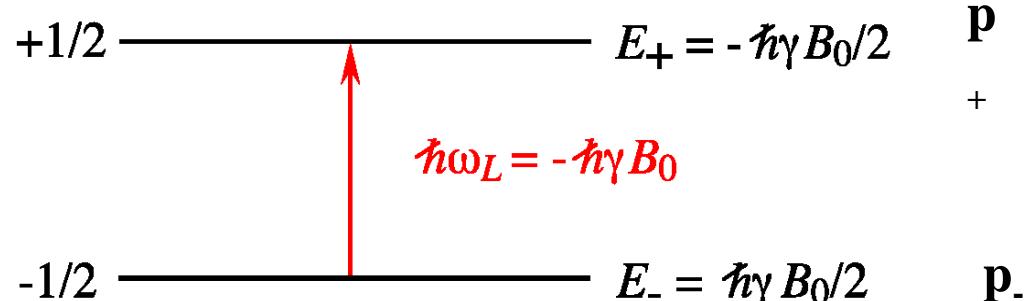


# NUCLEAR MAGNETISM

$$H_Z = -\hbar\gamma \mathbf{S} \cdot \mathbf{B}_0 = -\hbar\gamma S_z B_0$$

$$\omega_L = -\gamma B_0$$

**Thermodynamic ensemble**



$$\langle \mu_z \rangle = N \mu_B \tanh \frac{\hbar\gamma B_0}{2k_B T}$$

Curie susceptibility  $\chi = C / T$

$$\hbar\gamma_n B_0 \approx 10^{-6} \text{ K} \ll k_B T \quad \chi_n \approx 10^{-6} \chi_e \quad \text{Very weak } \chi$$

**Spectroscopic techniques : sensitivity**

**Radio Sources**  
**Radio frequencies**  
**(Electronic oscillators )**

- Stables ( $10^{-10}$ ) (monochromatic)
- coherent
- intense

**Radio, radar, television Technologies**

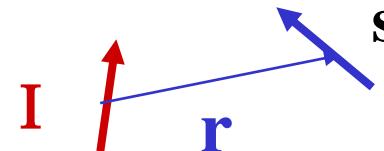
*H. Alloul, Autumn School on correlated electrons  
Julich, 15/09/2016*

# Hyperfine Interactions - NMR Frequency Shifts

Interactions between nuclear moments  $\vec{I}$  and electronic moments  $\vec{s}$  et  $\vec{l}$

Dipolar

$$H_{dd} = -\frac{\hbar^2 \gamma_n \gamma_e}{r^3} \left\{ \vec{I} \cdot \vec{s} - 3 \frac{(\vec{I} \cdot \vec{r})(\vec{s} \cdot \vec{r})}{r^2} \right\}$$



Orbital

$$H_{orb} = -\frac{\hbar^2 \gamma_n \gamma_e}{r^3} \vec{I} \cdot \vec{l}$$

- Filled atomic shells :

Contact

$$H_c = \frac{8\pi}{3} \hbar^2 \gamma_n \gamma_e \vec{I} \cdot \vec{s} \delta(\vec{r})$$

$$H_{orb} \equiv 0 ; H_{dd} \equiv 0$$

- Paramagnetic or diamagnetic compounds:

$$H_T = H_Z + H_{dd} + H_{orb} + H_c = -\hbar \gamma_n \vec{I} \cdot (\vec{B}_0 + \vec{B}_L)$$

$$\vec{B}_L = \langle \vec{B}_L \rangle + [\vec{B}_L - \langle \vec{B}_L \rangle]$$



Relaxation time

Mean field  
Linear  
response

$$\langle \vec{B}_L \rangle \propto \chi B_0$$



Frequency shift

Local measurement of the  
electronic susceptibility

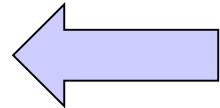
Insulators  $H_{orb}$   
Chemical shift  
(orbital currents)

metals  $\chi_{\text{Pauli}}$   
Knight shift  
(unpaired electrons)

# NMR in correlated electron systems

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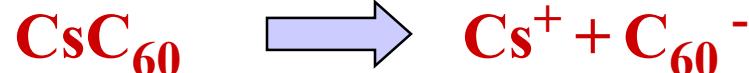
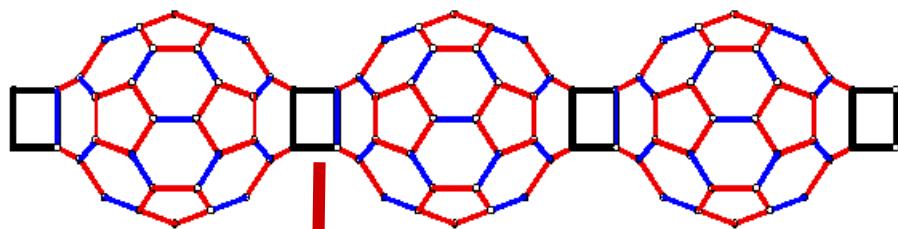
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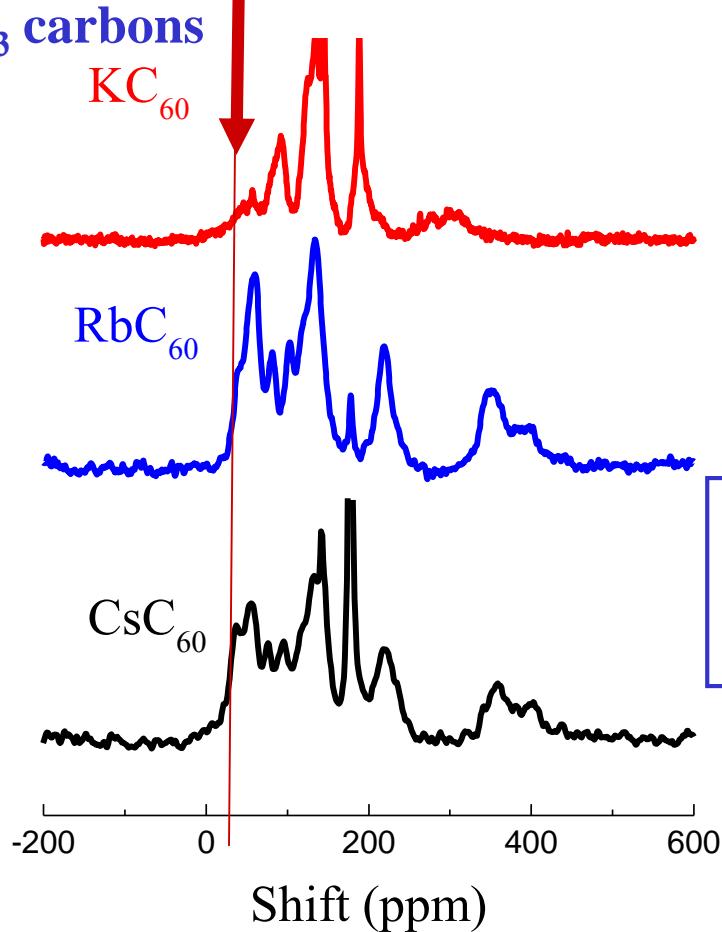
**Conclusion: NMR is a powerful tool in Solid State Physics**

# POLYMERIZED AC<sub>60</sub> PHASES

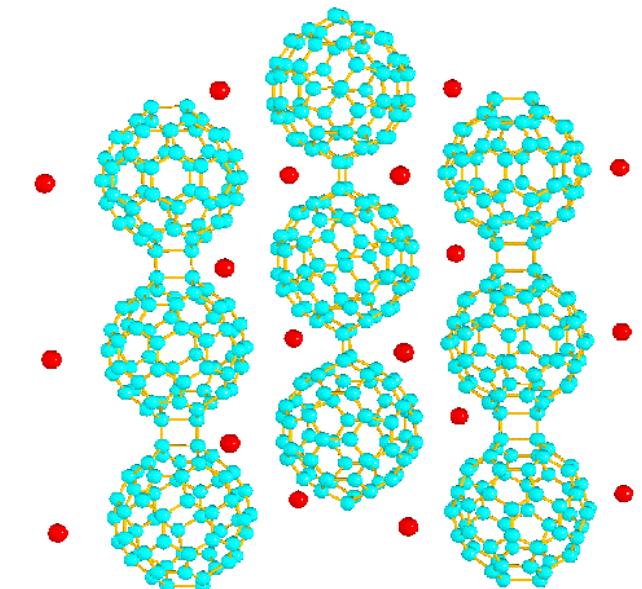
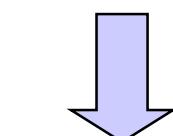
1D metal



The  $^{13}\text{C}$  site differentiation evidences the polymerization



Different 3D ordering of the polymer chains



## KNIGHT SHIFT IN METALS

**Contact hamiltonian for the spin  $I$**   
**(for all the electrons of the metallic band)**

$$H_c = \sum_i \frac{8\pi}{3} \hbar^2 \gamma_n \gamma_e \vec{I} \cdot \vec{s}_i \delta(\vec{r}_i)$$

$$H_c = \frac{8\pi}{3} \hbar^2 \gamma_n \gamma_e \vec{I} \cdot \sum_i \vec{s}_i \delta(\vec{r}_i) = \frac{8\pi}{3} \hbar \gamma_n \vec{I} \cdot \vec{M}(0)$$

**M(0) magnetization density operator at the nuclear site**

**Bloch electronic states**

$$|\vec{k}, \vec{s}\rangle = u_{\vec{k}}(\vec{r}) e^{i\vec{k}\cdot\vec{r}} \psi_{\vec{s}}$$

$$\vec{M}(0) = \left\langle |u_{\vec{k}}(\vec{r})|^2 \right\rangle_{k_F} \chi_P B_0 \quad \chi_P = \frac{1}{2} (\hbar \gamma_e)^2 n(E_F)$$

with  $A = \frac{8\pi}{3} \hbar^2 \gamma_n \gamma_e \left\langle |u_{\vec{k}}(\vec{r})|^2 \right\rangle_{k_F}$  one might write

**Pauli susceptibility  
of the electronic band**

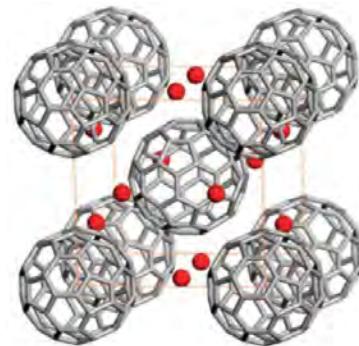
$$H_c = A \vec{I} \cdot \sum_i \vec{s}_i \delta(\vec{r}_i)$$

- The electrons are then considered as free electrons
- The hyperfine coupling  $A$  contains informations on the electronic band structure

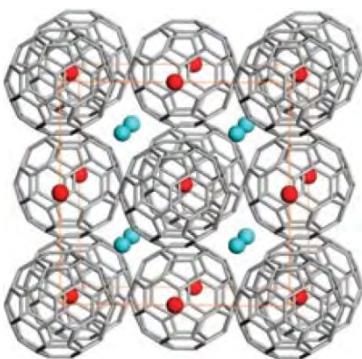
$$K = \frac{\langle B_L \rangle}{B_0} = \frac{A}{\hbar^2 \gamma_e \gamma_n} \chi_P$$

**Knight shift**

## Multiple phases in the samples: $^{133}\text{Cs}$ NMR is very helpful

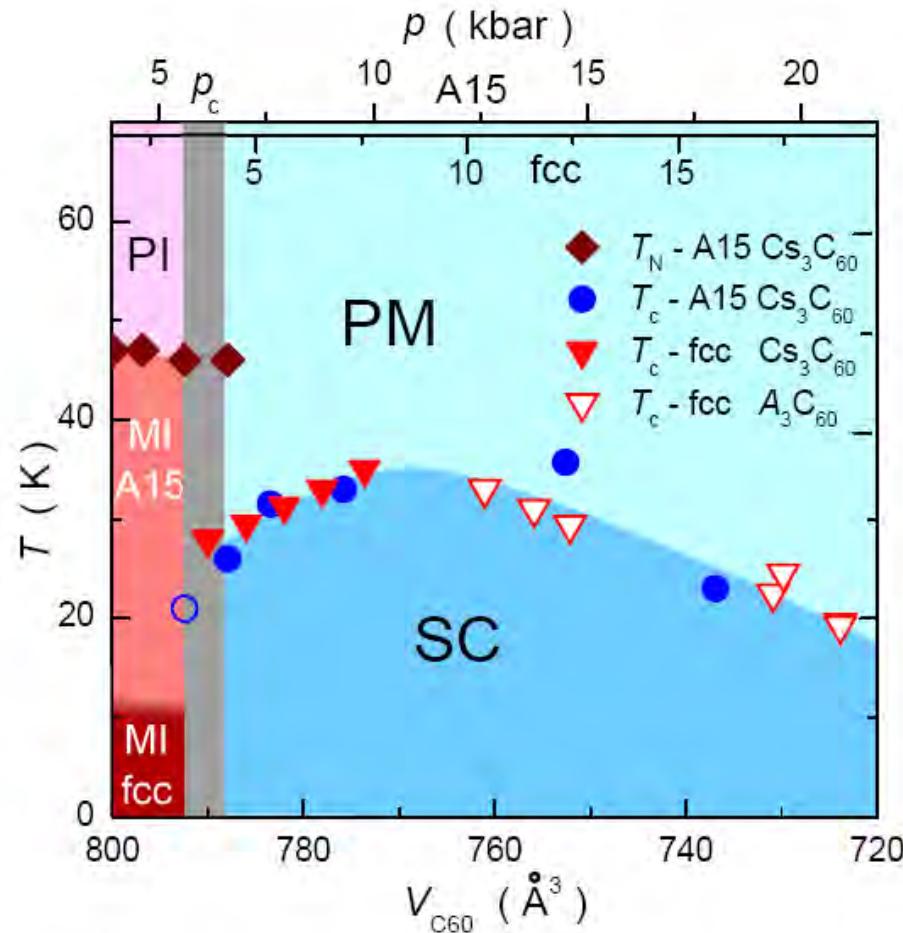


A15  $\text{Cs}_3\text{C}_{60}$



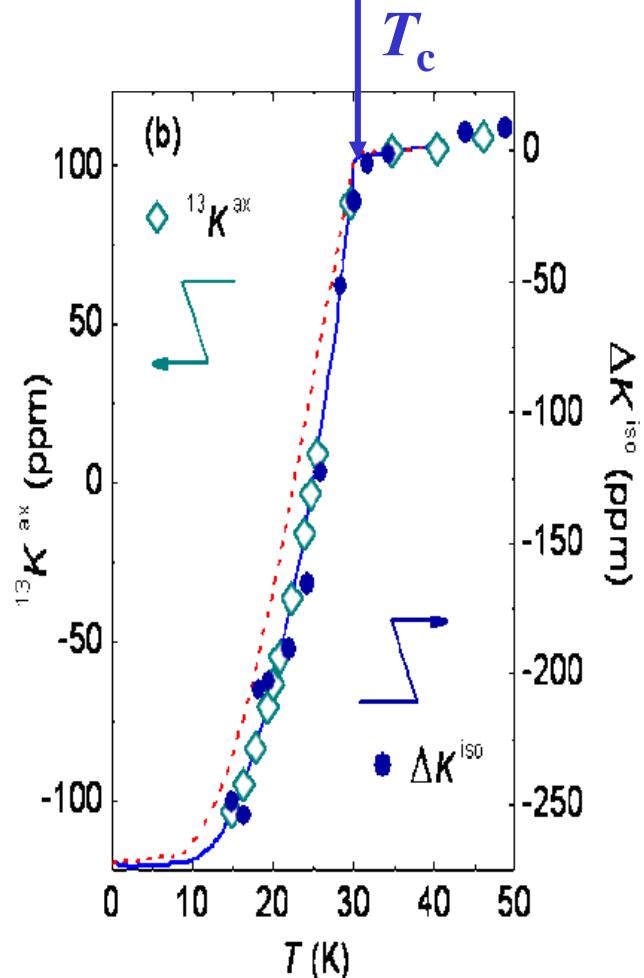
fcc  $\text{Cs}_3\text{C}_{60}$

Differences allow  
selective NMR experiments



Crystal structure  
has no incidence on the SC side

# Knight Shift in the SC state

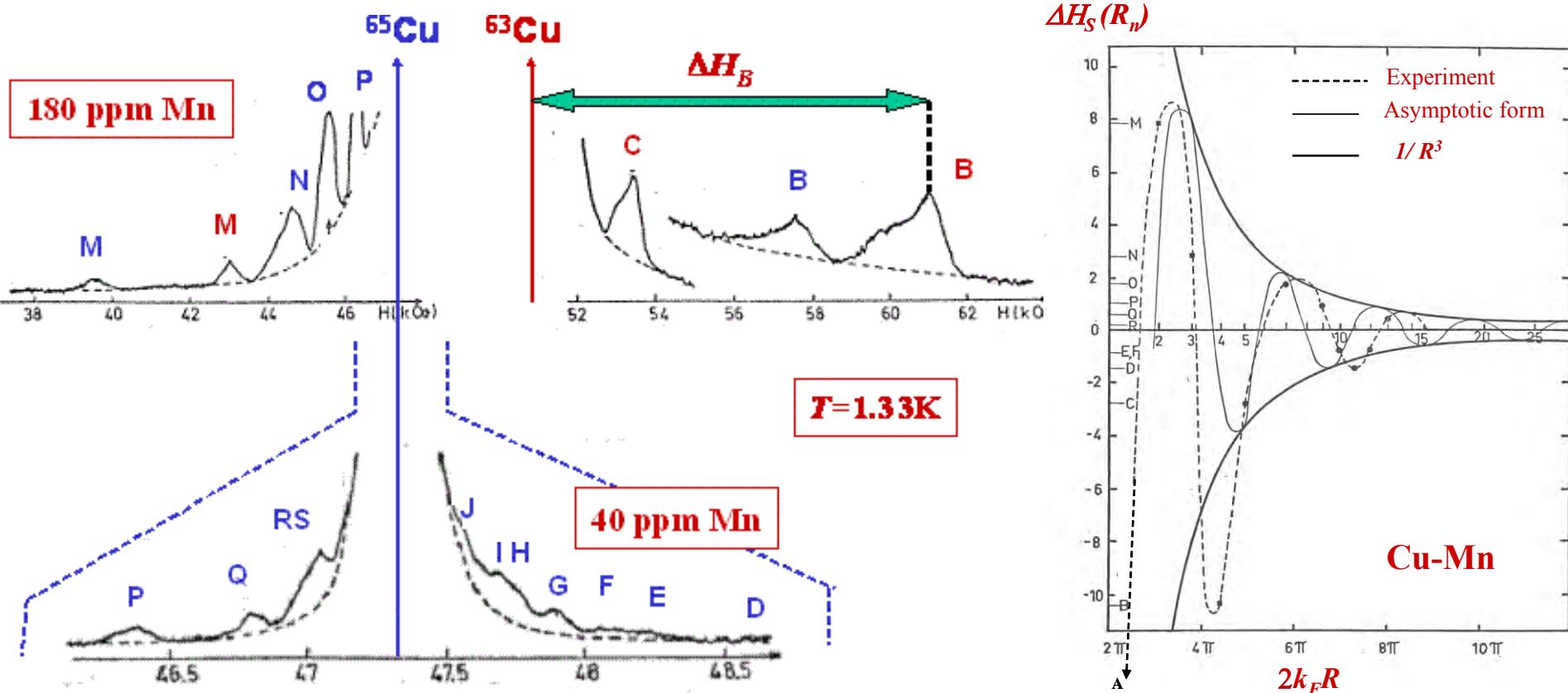


Vanishing of  $\chi_P$  in the superconducting state  
Cooper pairs are in a singlet state

P. Wzietek et al PRL 2014

# Magnetic Impurities in normal metals

## RKKY oscillations

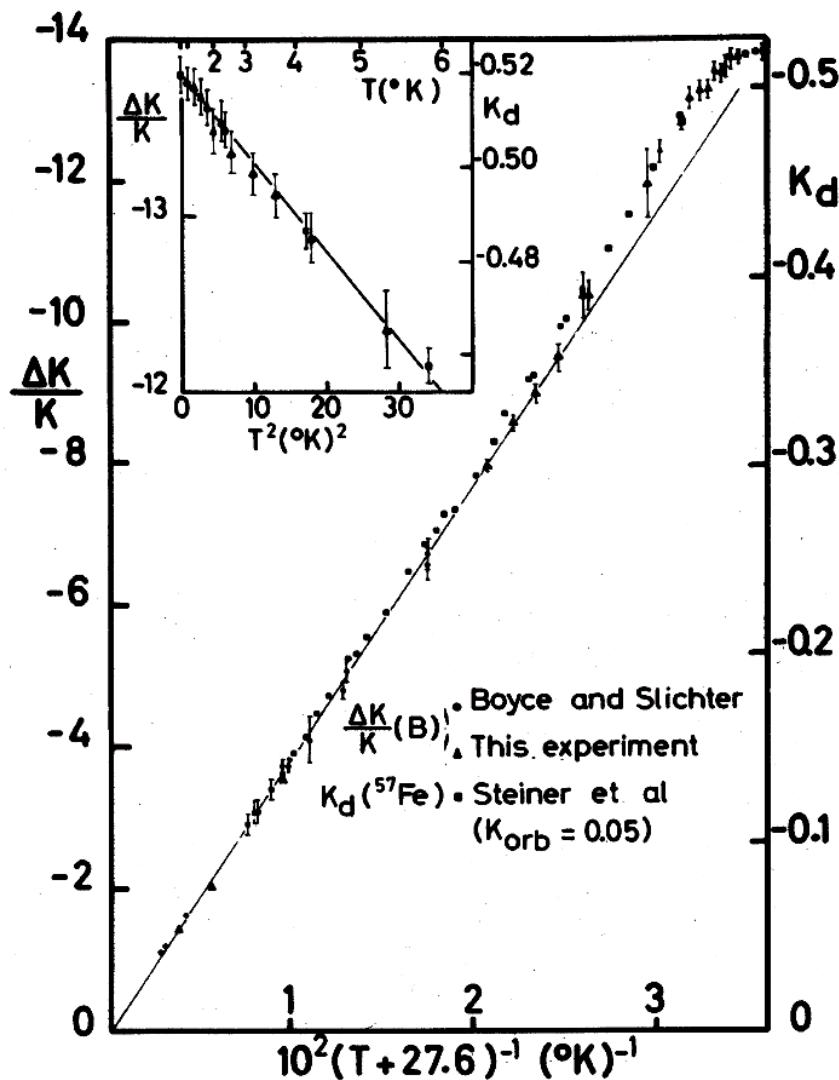


Transferred hyperfine couplings  
 Nuclear spins can sense the magnetism on  
 their neighbour atomic sites

H. Alloul, Autumn School on correlated electrons  
 Julich ,15 /09 /2016

# Magnetic Impurities in normal metals

## Kondo effect



H. Alloul, Autumn School on correlated electrons  
Julich, 15/09/2016

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Spin echoes and  $T_2$ : NMR applications

***Conclusion: NMR is a powerful tool in Solid State Physics***

# Magnetic susceptibilities

## SQUID Measures

**Ion cores**

**orbital**

**Spin**

**always**

$$\chi_{\alpha}^m(T) = \chi^{dia} + \chi_{\alpha}^{orb} + \chi_{\alpha}^s + (\chi^{imp})$$

$$= \chi^{dia} + \sum_i [\chi_{i,\alpha}^{orb} + \chi_{i,\alpha}^s(T)]$$

$\alpha = (x,y,z)$

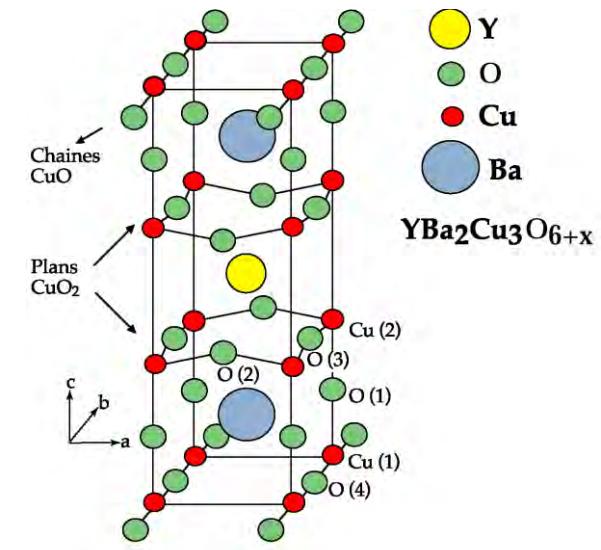
$i = \text{atomic sites}$

## NMR shift measures on each nuclear site i

$$K_{i,\alpha}(T) = K_i^{dia} + K_{i,\alpha}^{orb} + K_{i,\alpha}^s(T)$$

$$= K_i^{dia} + A_{i,\alpha}^{orb} \chi_{i,\alpha}^{orb} + A_{i,\alpha}^s \chi_{i,\alpha}^s(T)$$

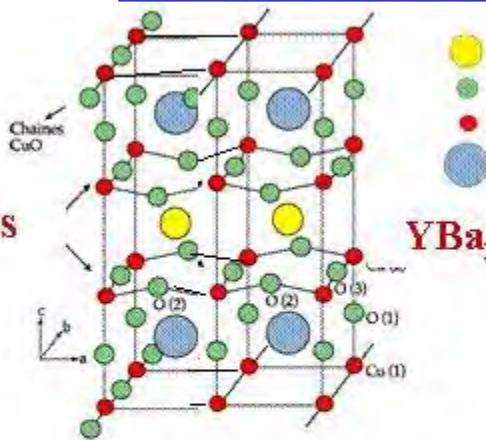
**Local magnetic measurement  
on each nuclear site i**



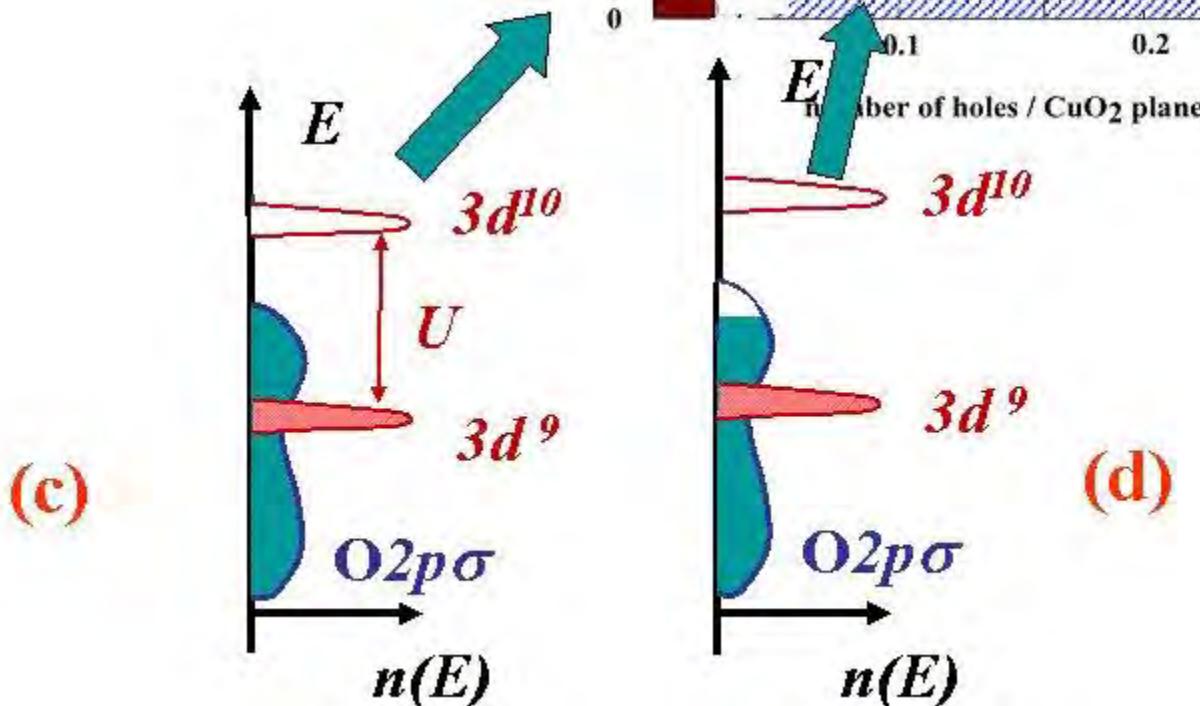
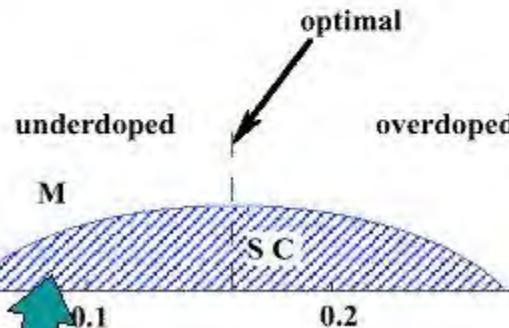
# Phase Diagram and Band Structure

(a)

CuO<sub>2</sub> layers



(b)



# $^{89}\text{Y}$ NMR shift in the metallic state

*H.A , T. Ohno and P. Mendels, PRL 1989*

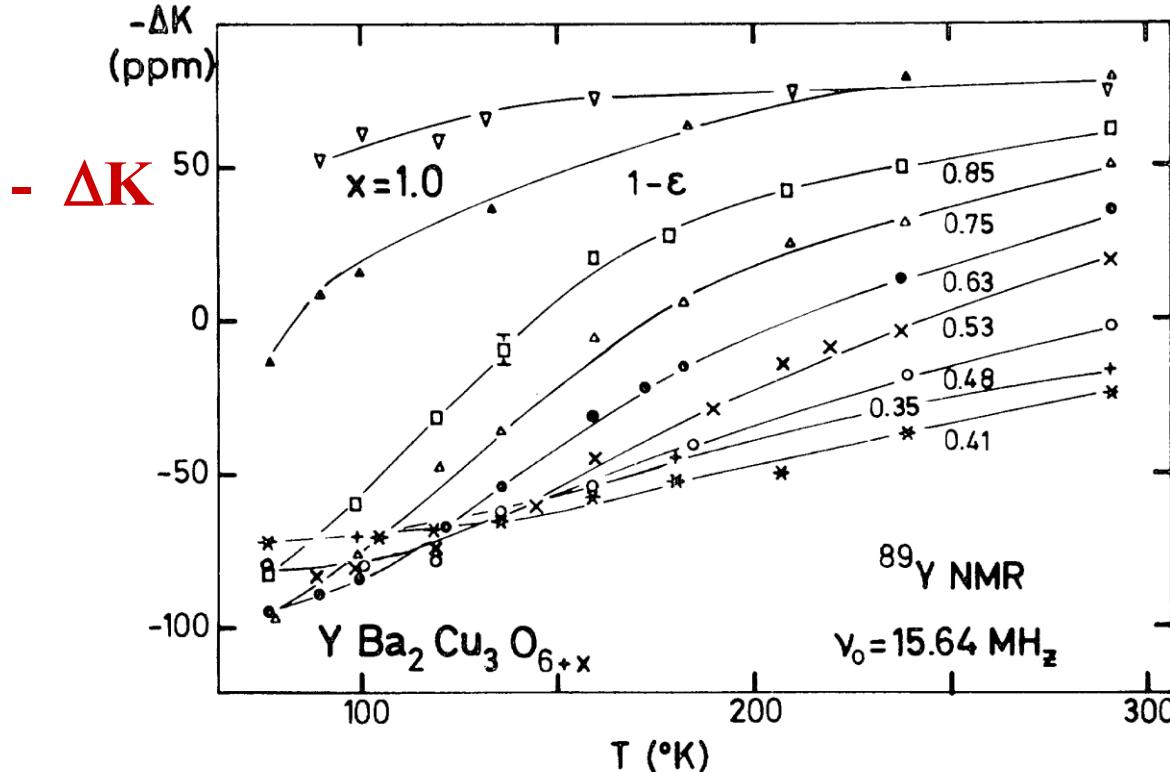
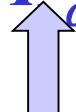


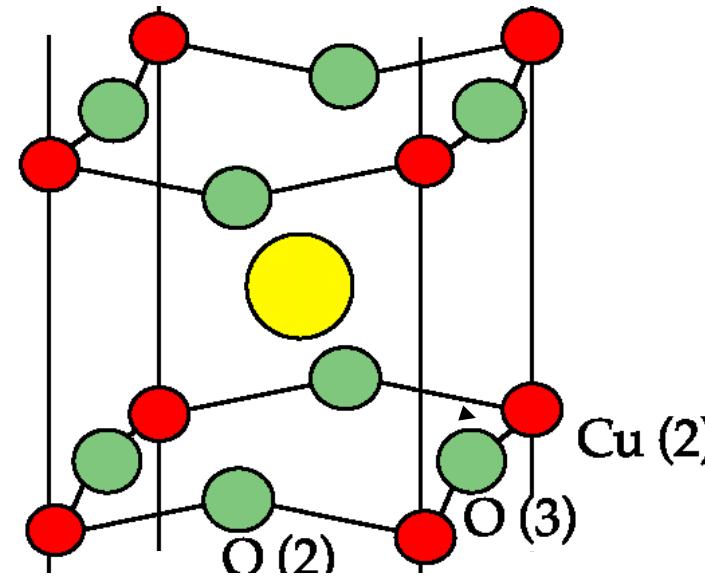
FIG. 1. The shift  $\Delta K$  of the  $^{89}\text{Y}$  line, referenced to  $\text{YCl}_3$  plotted vs  $T$ , from 77 to 300 K. The lines are guides to the eye.

$$K_{i,\alpha}(T) = K_i^{\text{dia}} + A_{i,\alpha}^{\text{orb}} \chi_{i,\alpha}^{\text{orb}} + A_{i,\alpha}^s \chi_{i,\alpha}^s(T)$$



Local magnetic measurement

$\text{CuO}_2$  plane



$\text{CuO}_2$  plane

But transferred hyperfine couplings

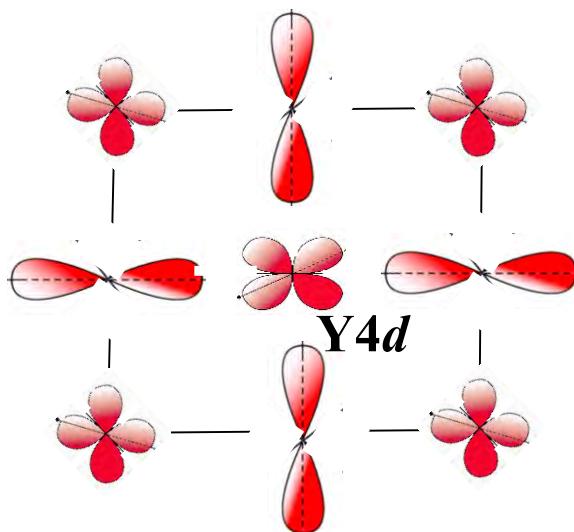
*H. Alloul, Autumn School on correlated electrons*

*Julich, 15/09/2016*

## Sign of $^{89}\text{Y}$ NMR shift

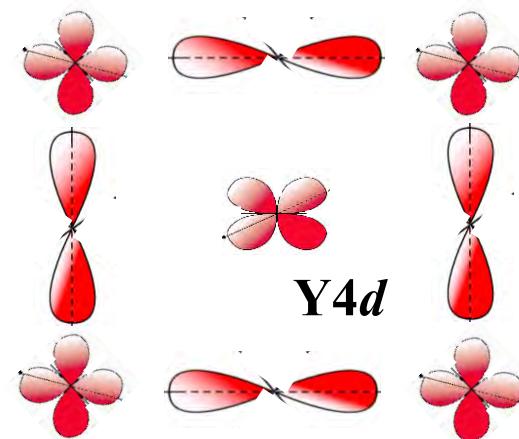
Negative sign comes from  $\text{Y}4d$  orbitals: core polarization

$p\pi \text{ Y}4d$



Very weak

$p\sigma \text{ Y}4d$

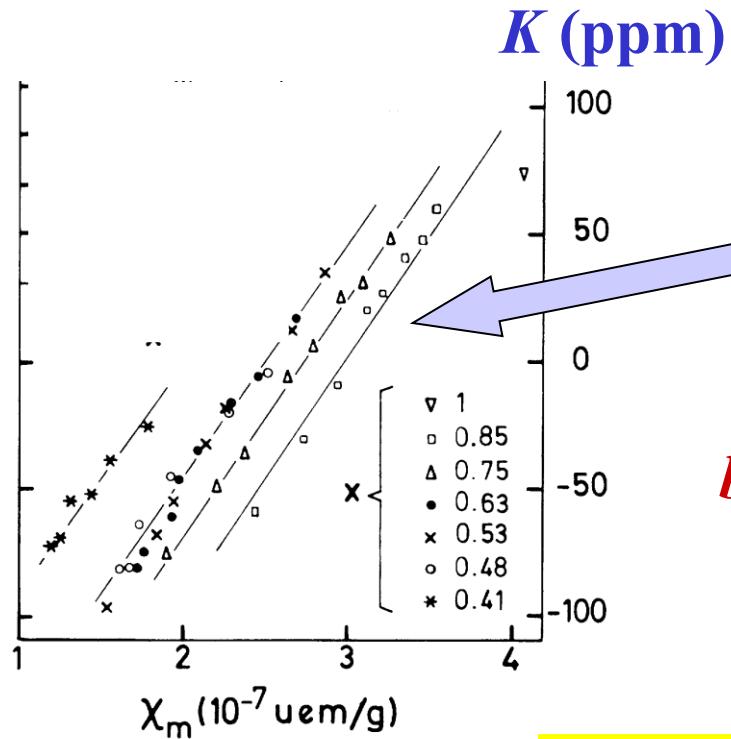


OK

So negative sign comes from  $p\sigma - \text{Y}4d$  hybridization

# Is there an independent oxygen band at the Fermi level?

H.A. , T. Ohno and P. Mendels, PRL 1989



$$K^s(T) = A \chi^s(T)$$

does not change with hole doping

*A is driven  
by ( $Y4d$ - $O2p\sigma$ )– $Cu(3d)$  covalency*

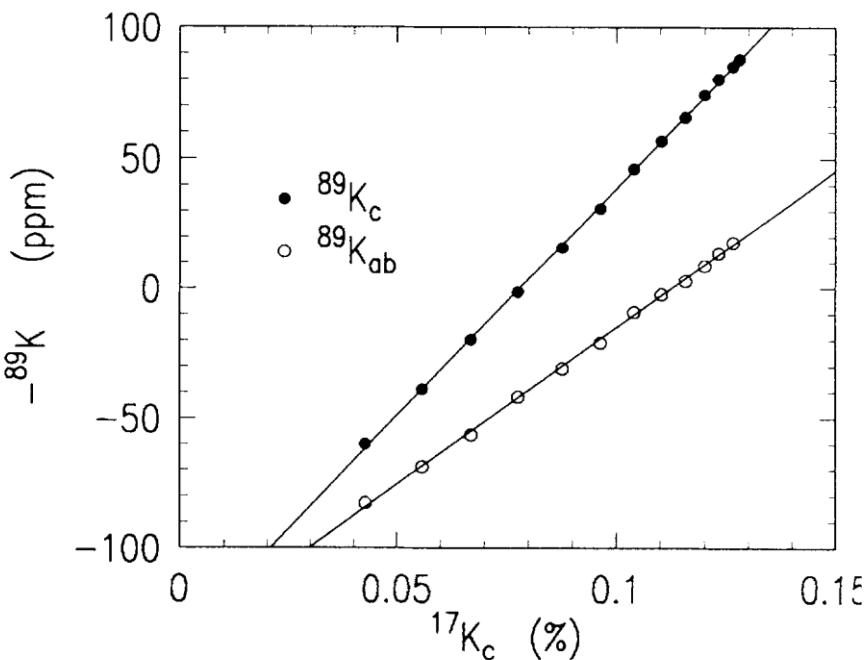
*So there is no independent oxygen spin  
degree of freedom at  $E_F$*

# Single spin fluid behaviour

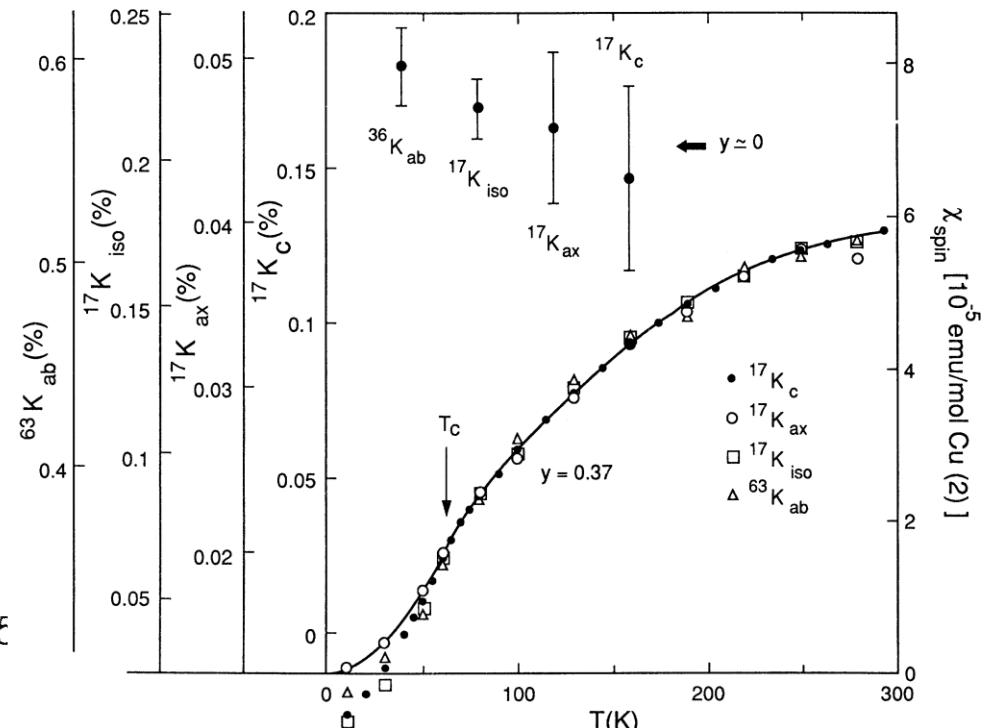
**YBCO<sub>6.63</sub>**

$$K_{i,\alpha}(T) = K_i^{dia} + A_{i,\alpha}^{orb} \chi_{i,\alpha}^{orb} + A_{i,\alpha}^s \chi_{i,\alpha}^s(T)$$

*M. Takigawa et al 1991, 1993*



**<sup>89</sup>Y versus <sup>17</sup>O**



**<sup>63</sup>Cu versus <sup>17</sup>O**

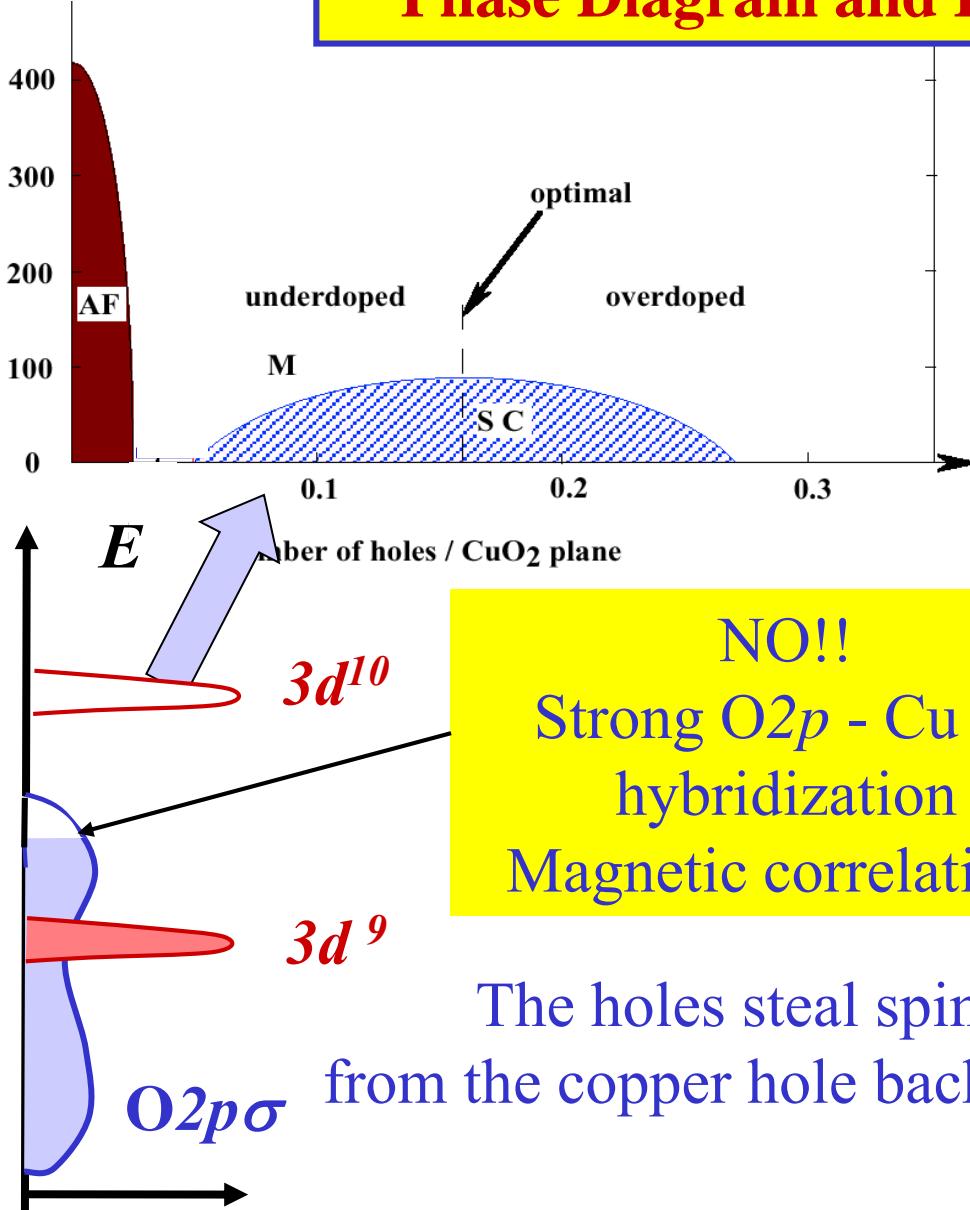
*A single T dependence for  $K_{i,a}(T)$ : due to  $\chi_{Cu}(T)$*

Notice: this allows determinations of  
the shift references for all nuclei

*H. Alloul, Autumn School on correlated electrons*

*Julich, 15/09/2016*

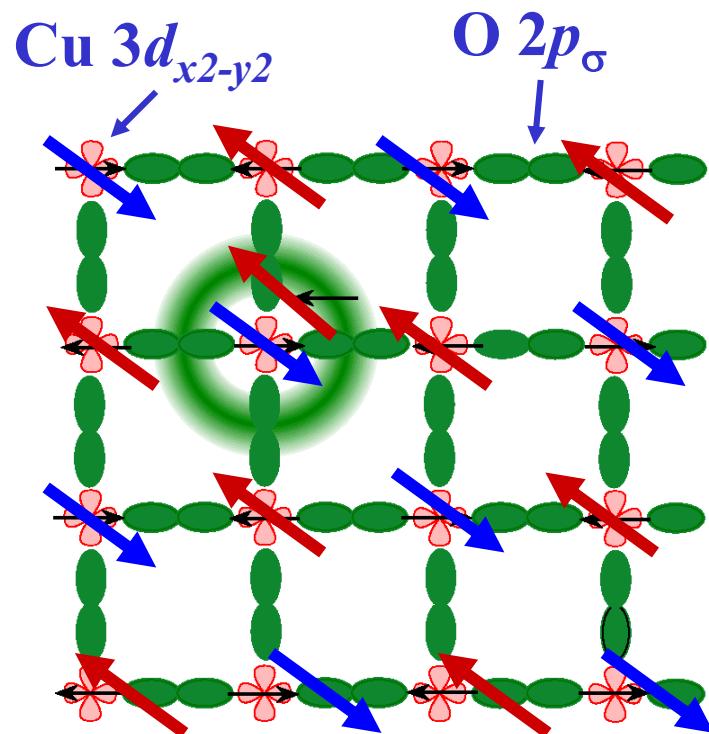
# Phase Diagram and Band Structure



NO!!  
Strong  $O2p - \text{Cu } 3d$   
hybridization  
Magnetic correlations

The holes steal spins  
from the copper hole background

Two types of holes ?



***There is a single spin fluid***  
Zhang Rice spin singlets  
 $\text{Cu}3d - O2p\sigma$

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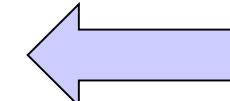
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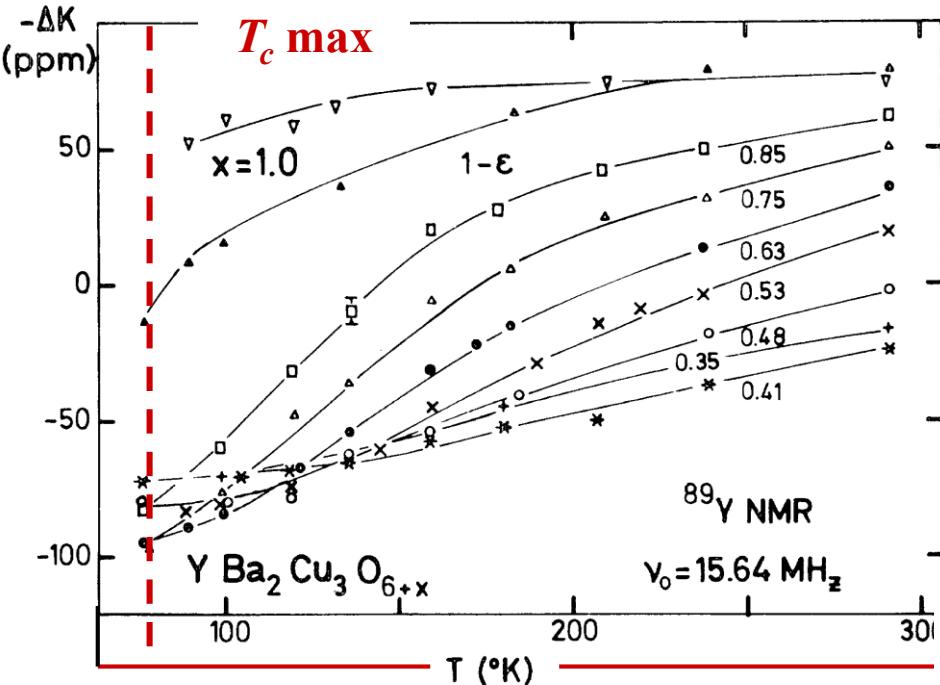
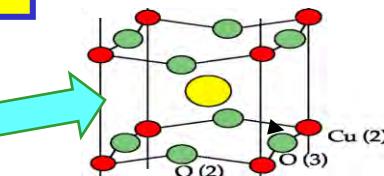
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**Conclusion: NMR is a powerful tool in Solid State Physics**

# What about the origin of this large decrease at low T?



$^{89}\text{Y}$  NMR shift



$$K = \sigma + A \chi_s(T)$$

In usual metals  $\chi_s$  is  $T$  independent (as for  $x=1$ ) and vanishes at  $T=0$  in the SC

Large decrease  
(nearly full loss)  
of  $\chi^s(T)$  above  $T_c$

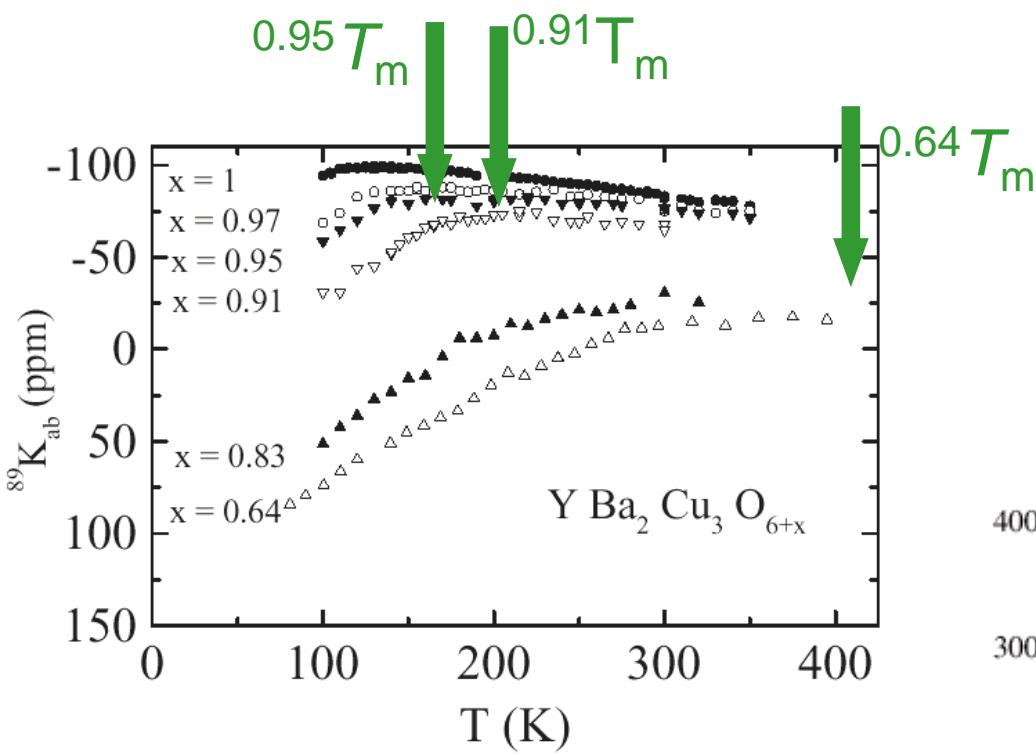
Pseudogap in the electronic excitations

H.A, T. Ohno and P. Mendels, PRL 1989

tuations on the Cu than on the Y or O, which are symmetric sites for the AF lattice of the  $\text{O}_6$  compound.<sup>7</sup> In the band picture, AF correlations might induce a **pseudogap**, as suggested by Friedel,<sup>24</sup> which could explain the reduction of  $\chi_s$  at low  $T$ . However, it is less clear whether this approach is compatible with the smooth variation of  $\chi_s$  and  $K_s$  from the metal to the insulating state.

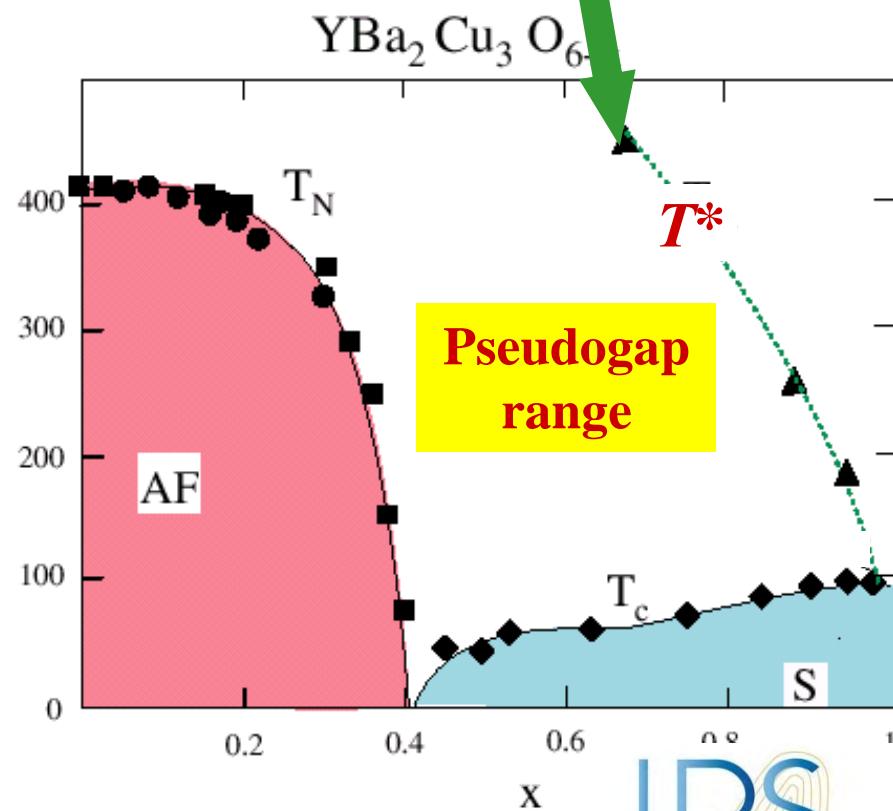
# Phase Diagram and Pseudogap

$^{89}\text{Y}$  NMR shift



Alloul et al

Low  $T$  decrease of the susceptibility:  
opening of the pseudogap

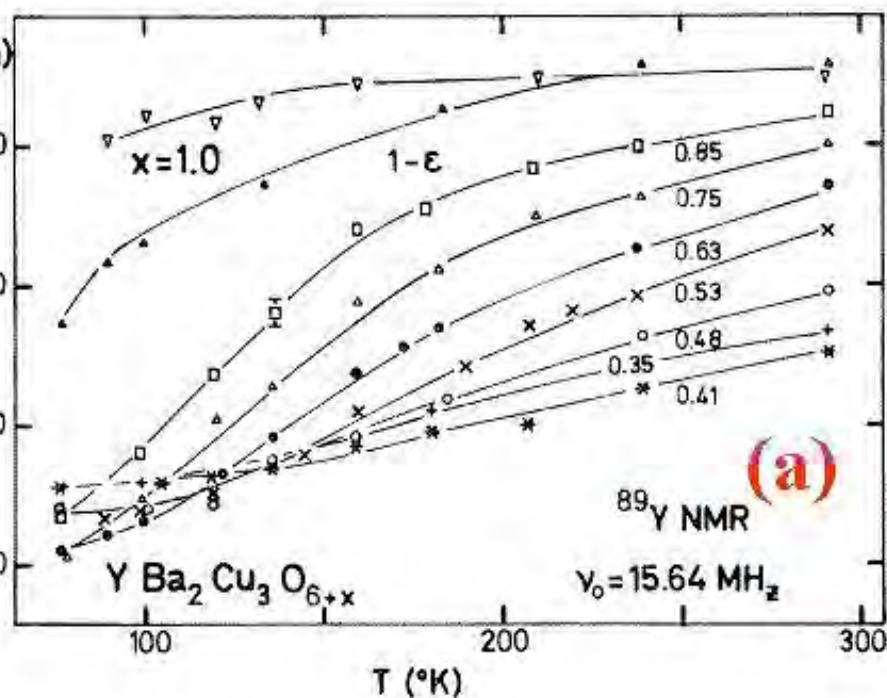


H. Alloul, Autumn School on correlated electrons  
H. Alloul, ITAP, Turung, 09/07/2011  
Jülich, 15/09/2016

# The pseudogap is generic in the various families of cuprates

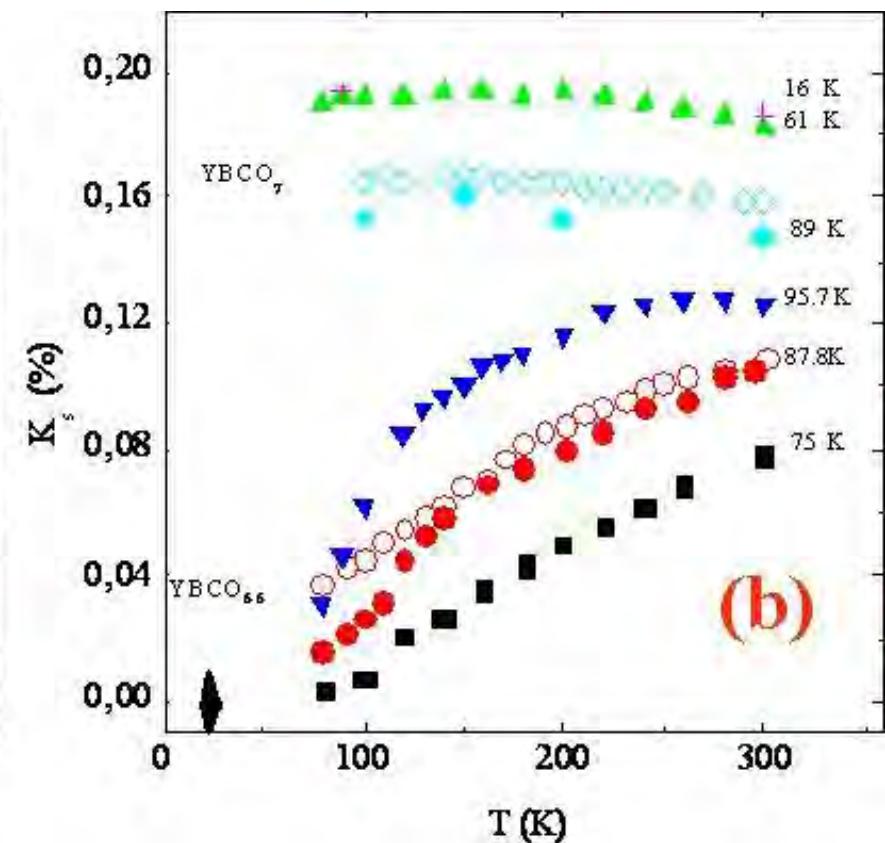
## YBCO : two CuO<sub>2</sub> layer

H.Alloul , T. Ohno and P. Mendels, PRL 1989



## Hg1201 : one CuO<sub>2</sub> layer

J. Bobroff, H.A,... PRL 1997

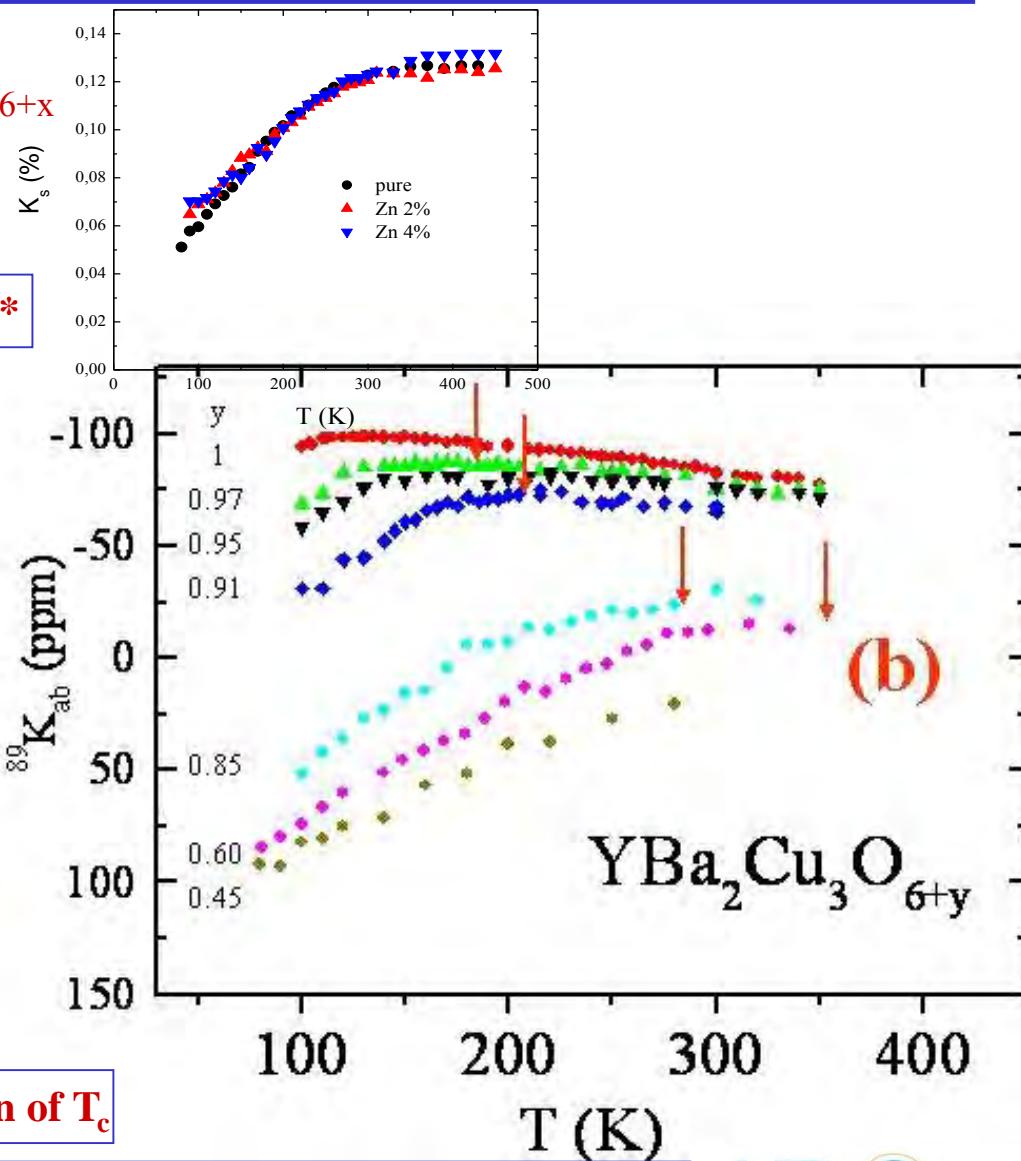
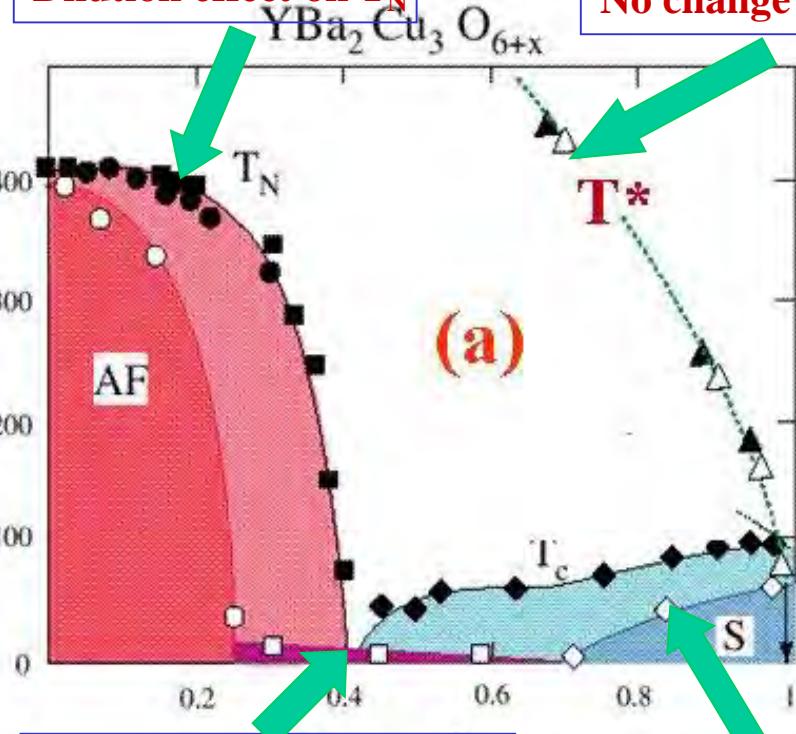


# Incidence of non magnetic impurities on the SC and pseudogap



H. Alloul et al , PRL 67, 3140 (1991)

Dilution effect on  $T_N$       No change of  $T^*$



Increase of the disordered magnetism range

Large depression of  $T_c$

The pseudogap is robust and insensitive to disorder

H. Alloul, Autumn School on correlated electrons

Julich ,15/09/2016

## Correlations between Magnetic and Superconducting Properties of Zn-Substituted $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$

H. Alloul,<sup>(1)</sup> P. Mendels,<sup>(1)</sup> H. Casalta,<sup>(1)</sup> J. F. Marucco,<sup>(2)</sup> and J. Arabski<sup>(1)</sup>

<sup>(1)</sup>*Laboratoire de Physique des Solides, Université Paris-Sud, 91405 Orsay, France*

<sup>(2)</sup>*Laboratoire des Composés Non Stoechiométriques, Université Paris-Sud, 91405, Orsay, France*

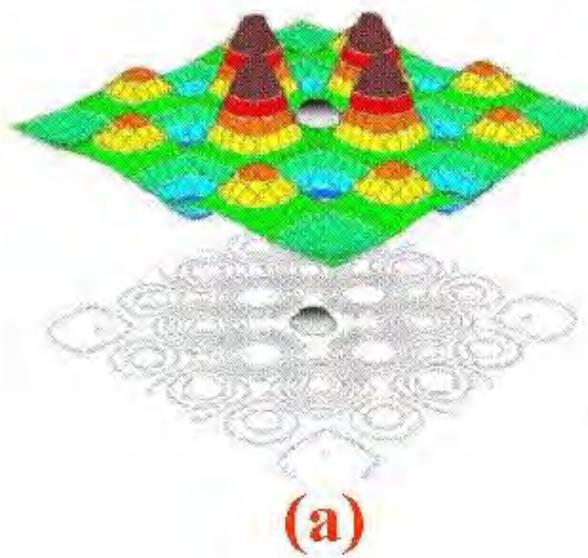
(Received 8 August 1991)

$T_c$  and  $T_N$  (Néel) have been measured for a series of  $\text{YBa}_2(\text{Cu}_{0.96}\text{Zn}_{0.04})_3\text{O}_{6+x}$  samples. The  $T$  variations of the homogeneous susceptibility  $\chi_s$  of the  $\text{CuO}_2$  planes, given by the shift of the  $^{89}\text{Y}$  NMR line, are found to be nearly unchanged with respect to pure samples for  $x > 0.5$ , which implies that the charge transfer is negligibly modified by Zn, and that the magnetic pseudogap is not associated with superconducting pairing. Detection of an unusual Curie contribution to the  $^{89}\text{Y}$  NMR width for  $x \geq 0.5$  provides evidence that Zn induces magnetic moments in the  $\text{CuO}_2$  planes, which play a role in the depression of  $T_c$ .

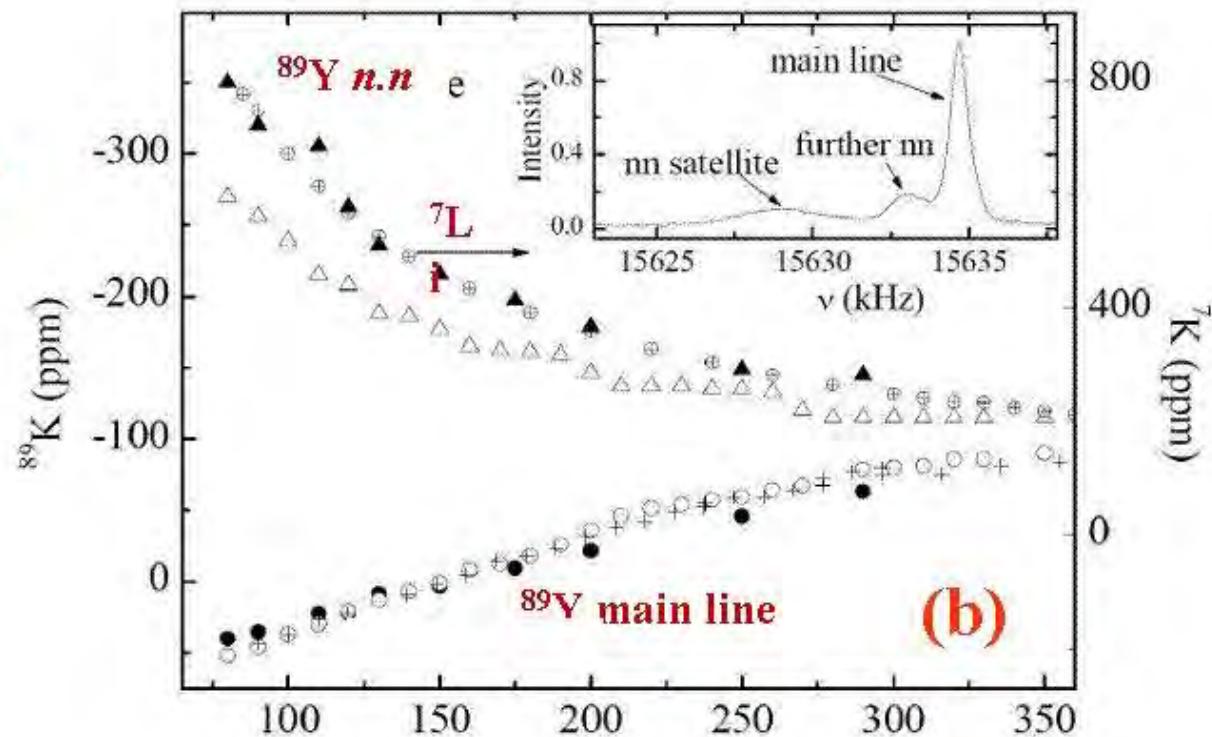
PACS numbers: 74.70.Hk, 75.20.Hr, 75.30.Kz, 76.60.Cq

**Pseudogap and correlations : Mott physics?**

## Local magnetic response induced by non magnetic Zn



(a)

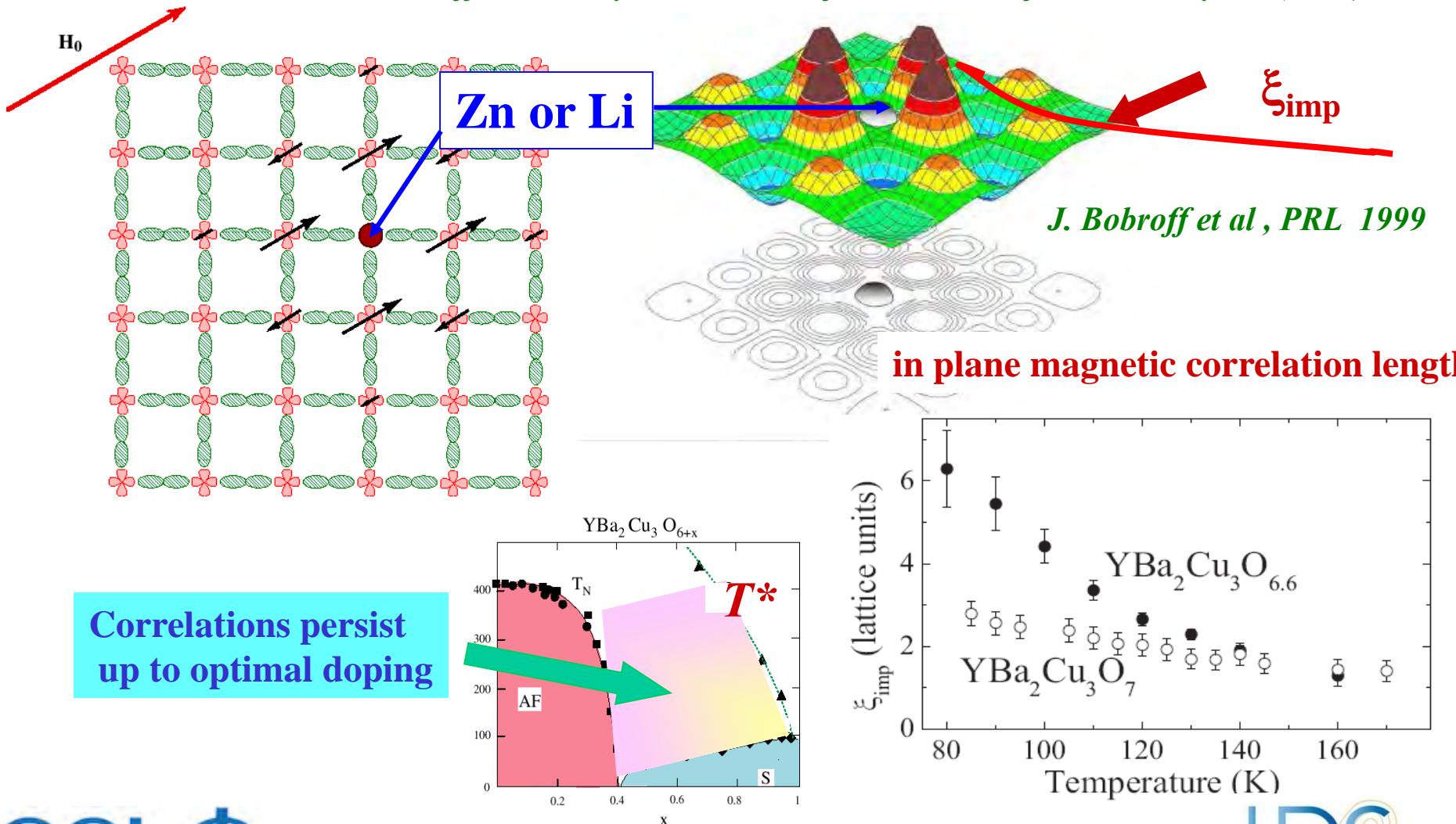


(b)

# Importance of correlations : magnetism induced by non magnetic substitutions

H. Alloul, P. Mendels et al PRL 1991; A. Mahajan, H. A et al PRL 1993

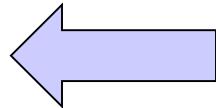
Review: H.A, J. Bobroff, M. Gabay and P. Hirschfeld. Review of Modern Physics (2009).



# NMR in correlated electron systems

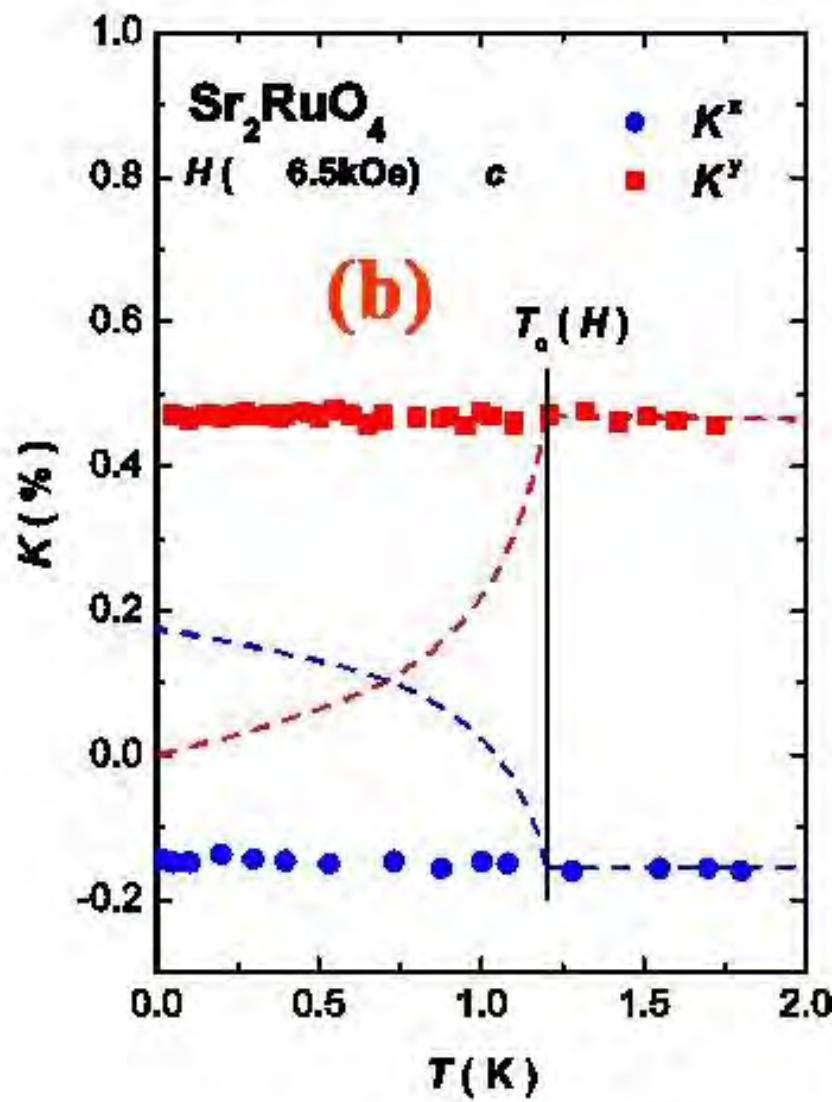
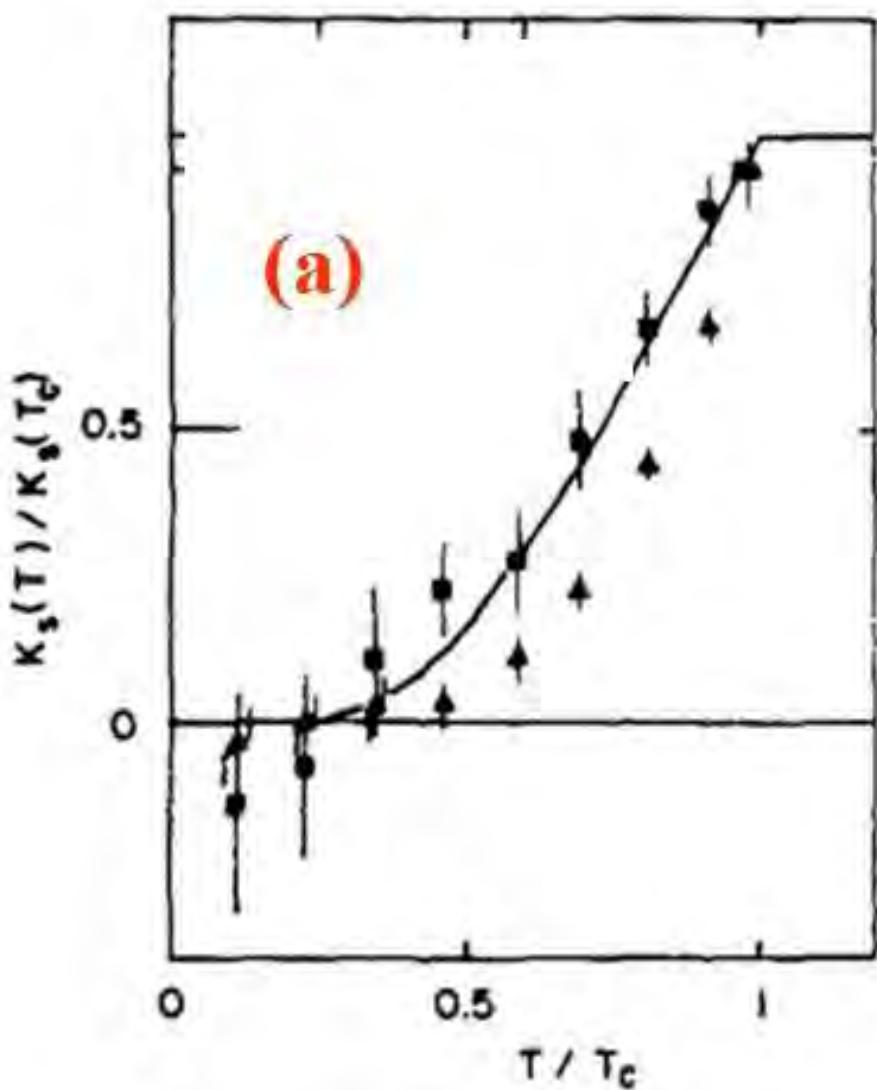
## Illustration in the case of the cuprates

- *Introduction to Magnetic resonance (NMR and ESR)*  
Hyperfine couplings , NMR shifts
- *Magnetic spin susceptibilities in NMR*  
Metals and superconductors: Singlet spin pairing  
Impurity magnetism , RKKY , Transferred hyperfine couplings
- *Magnetic spin susceptibilities in NMR : the cuprate case*  
Electronic structure . Single spin fluid in the normal state
- *The pseudogap and the phase diagram*  
Pseudogap and disorder. Impurities reveal the magnetic correlations
- *Spin lattice relaxation  $T_1$  and transverse relaxation  $T_2$*   
Dynamic susceptibilities and spin lattice relaxation  
d- wave SC , magnetic correlations in the cuprate phase diagram  
Spin echoes and  $T_2$ : NMR applications



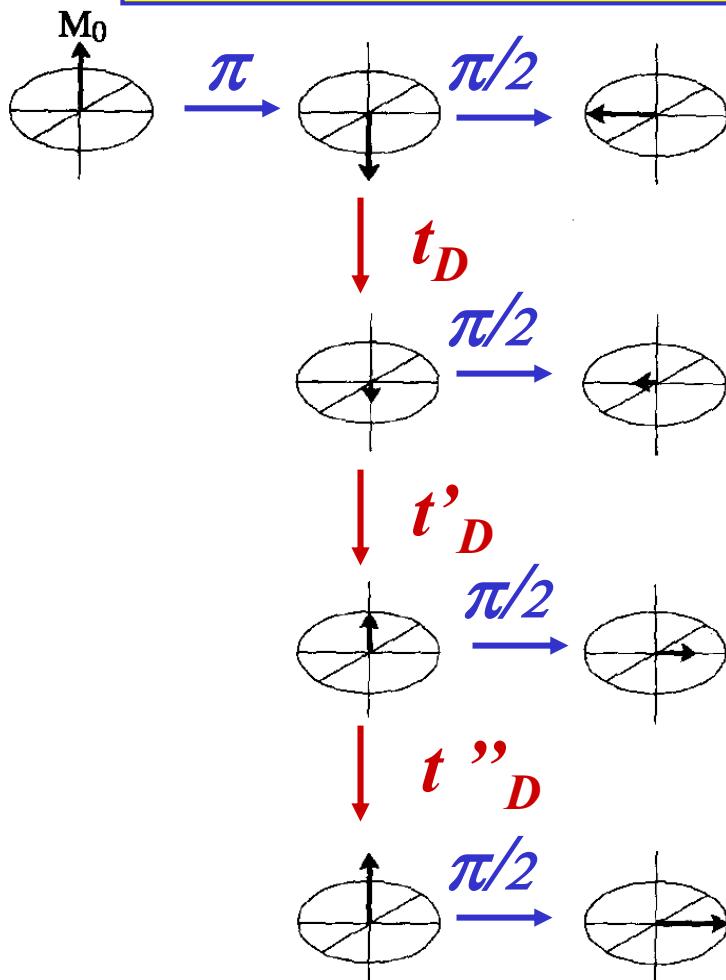
**Conclusion: NMR is a powerful tool in Solid State Physics**

## Exotic superconductivities; singlet or triplet?

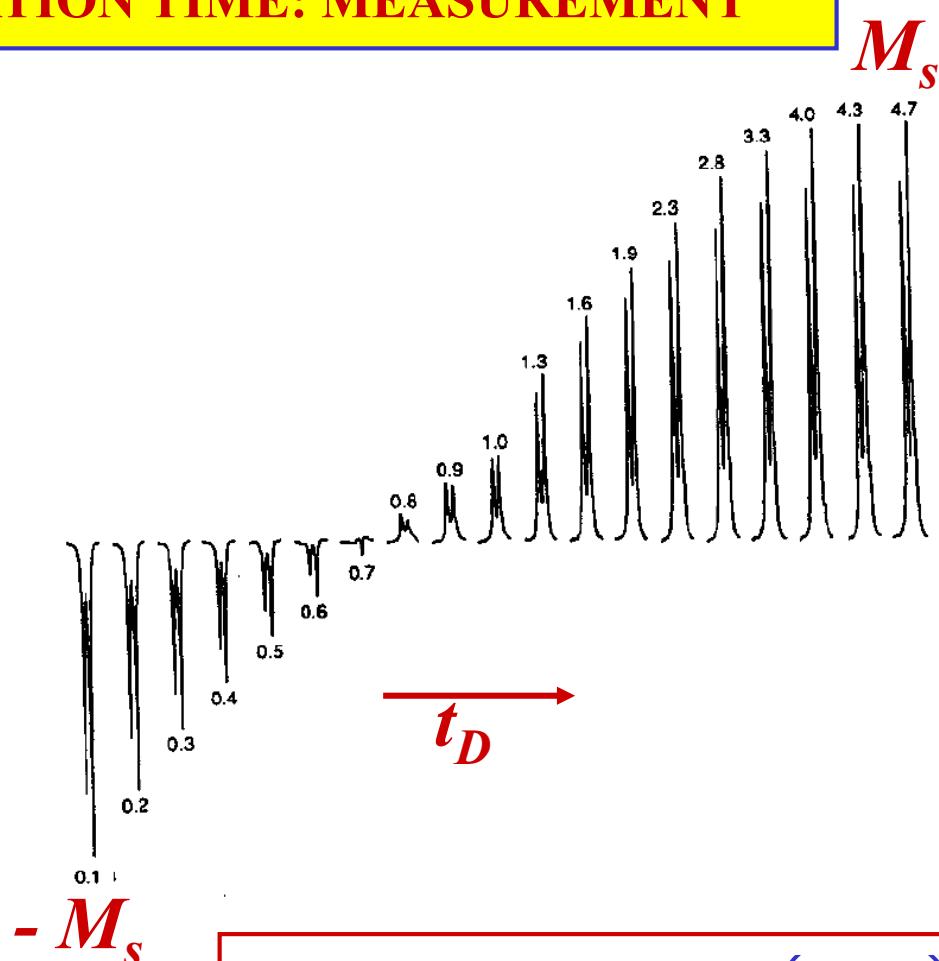


# SPIN LATTICE RELAXATION TIME: MEASUREMENT

$M_s$



$(\pi - t_D - \pi/2)$  sequence



$$M(t_D) = M_s - 2M_s \exp\left(-\frac{t}{T_1}\right)$$

# Physical Origin of the Spin Lattice Relaxation

$$+1/2 \quad E_+ = -\hbar\gamma B_0/2$$

$$H_Z = -\hbar\gamma S_z \cdot B_0$$

$$\hbar\omega_L = -\hbar\gamma B_0$$

$$B_0 // z$$

$$-1/2 \quad E_- = \hbar\gamma B_0/2$$

rf exciting field  
perturbation for  $H_Z$

$$H_{rf} = -\hbar\gamma \mathbf{S} \cdot \mathbf{B}_1 \cos\omega_L t$$

$$\vec{B}_L = \langle \vec{B}_L \rangle + [ \vec{B}_L - \langle \vec{B}_L \rangle ]$$


transitions  $| -1/2 \rangle \rightarrow | 1/2 \rangle$

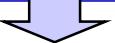
if  $\langle 1/2 | H_{rf} | -1/2 \rangle \neq 0$



$$B_1 \perp z$$

**Relaxation:** transverse components of the  
fluctuating field at the Larmor frequency

Transition probability

$$\frac{1}{T_1} = \gamma_n^2 \int_{-\infty}^{\infty} \langle B_L^+(t) B_L^-(0) \rangle \exp(-i\omega_n t) dt$$


Correlation function of the local field

$T_1$  results from the coupling with the equilibrium  
fluctuations of the electron spins degrees of freedom

## T<sub>1</sub> IN A METAL: KORRINGA LAW

$$H = H_Z + H_c = -\hbar \gamma_n \vec{I} \cdot \vec{B}_0 + A \cdot \vec{I} \cdot \sum_i \vec{s}_i \delta(\vec{r}_i) \quad \vec{B}_L = -\frac{A}{\hbar \gamma_n} \sum_i \vec{s}_i \delta(\vec{r}_i) = -\frac{A}{\hbar^2 \gamma_e \gamma_n} \vec{M}(0)$$

$$\frac{1}{T_1} = \gamma_n^2 \int_{-\infty}^{\infty} \langle B_L^+(t) B_L^-(0) \rangle \exp(-i\omega_n t) dt \quad \frac{1}{T_1} = \frac{A^2}{\hbar^4 \gamma_e^2} \int_{-\infty}^{\infty} \langle \vec{M}^+(t) \vec{M}^-(0) \rangle \exp(-i\omega_n t) dt$$

**Fluctuation-dissipation theorem** (Transverse dynamic  $\chi$  of the electron gas)

$$\chi_T'(\omega_n) = \frac{1}{2\hbar} (1 - \exp \frac{\hbar \omega_n}{k_B T}) \int_{-\infty}^{\infty} \langle \vec{M}^+(t) \vec{M}^-(0) \rangle \exp(-i\omega_n t) dt$$

with  $\hbar \omega_n \ll k_B T$

$$\frac{1}{T_1} = \frac{2A^2}{\hbar^2 \gamma_e^2} k_B T \frac{\chi_T'(\omega_n)}{\omega_n}$$

$$\text{For a fermion gas} \quad \chi_T(\omega) = \frac{1}{2} \hbar^2 \gamma_e^2 \left\{ n(E_F) + i\pi \hbar \omega n^2(E_F) \right\}$$

$$\frac{1}{T_1} = \frac{\pi}{\hbar} A^2 n^2(E_F) k_B T$$

$$K = \frac{A}{\hbar^2 \gamma_e \gamma_n} \quad \chi_P = \frac{A \gamma_e}{2 \gamma_n} n(E_F)$$

$$T_1 T K^2 = \frac{\hbar}{4\pi k_B} \left( \frac{\gamma_e}{\gamma_n} \right)^2$$

### Korringa law for a metal

H. Alloul, Autumn School on correlated electrons

Julich, 15/09/2016

# Spin lattice relaxation in a free electron metal

$$\frac{1}{T_1} = \frac{2A^2}{\hbar^2 \gamma_e^2} k_B T \frac{\chi_T''(\omega_n)}{\omega_n}$$

$$\chi_T''(\omega_n) = \sum_q \chi_T''(q, \omega_n)$$

For a free electron gas  $\chi''(q, \omega_n)$  is q independent

$$\chi_T(\omega) = \frac{1}{2} \hbar^2 \gamma_e^2 \left\{ n(E_F) + i\pi\hbar\omega n^2(E_F) \right\}$$

$$\frac{1}{T_1} = \frac{\pi}{\hbar} A^2 n^2(E_F) k_B T$$

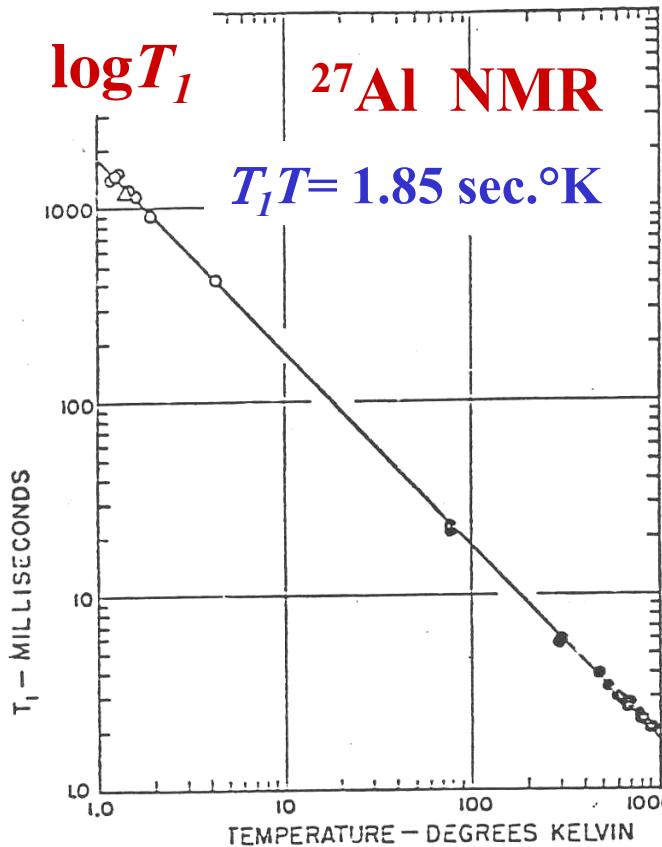
$$K = \frac{A}{\hbar^2 \gamma_e \gamma_n} \quad \chi_P = \frac{A \gamma_e}{2 \gamma_n} \quad n(E_F)$$

$$T_1 T K^2 = \frac{\hbar}{4\pi k_B} \left( \frac{\gamma_e}{\gamma_n} \right)^2 = S_0$$

Korringa law for a metal

Thermometry

Al metal

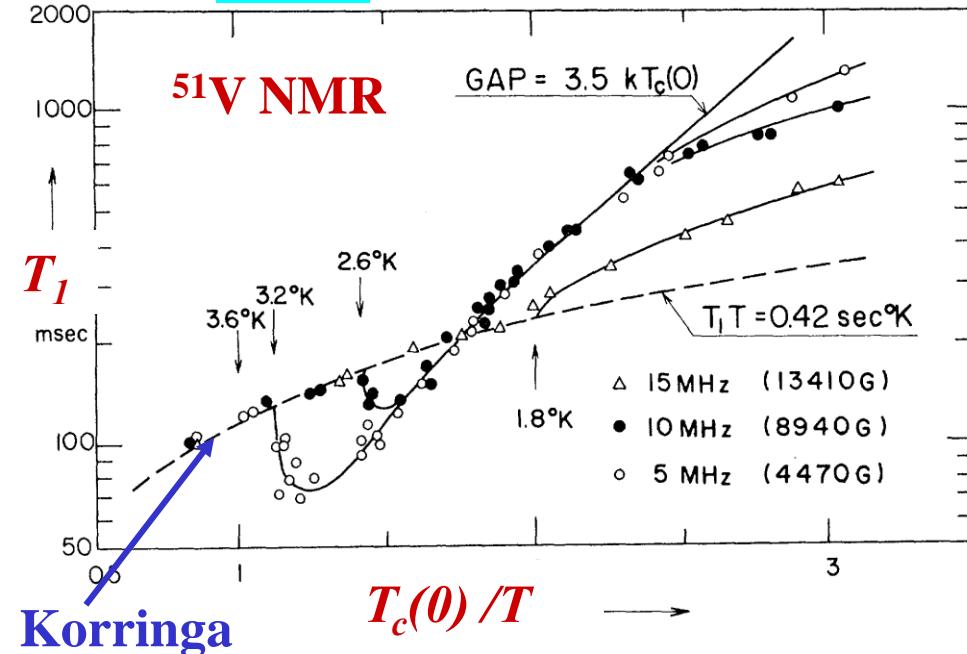


$\log(1/T)$

V<sub>3</sub>Sn

# T<sub>1</sub> in the superconducting state

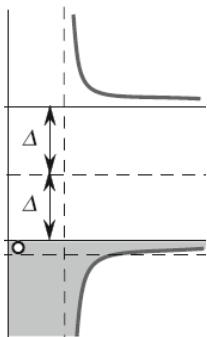
Cuprates



Korringa

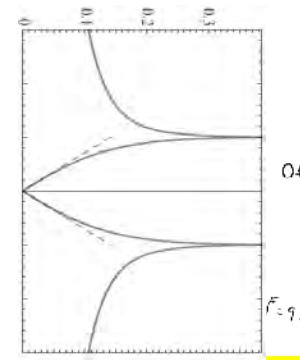
s wave superconductor

$$(T_1 T)^{-1} \sim \exp(-\Delta/k_B T) \quad \text{for } T \ll T_c$$

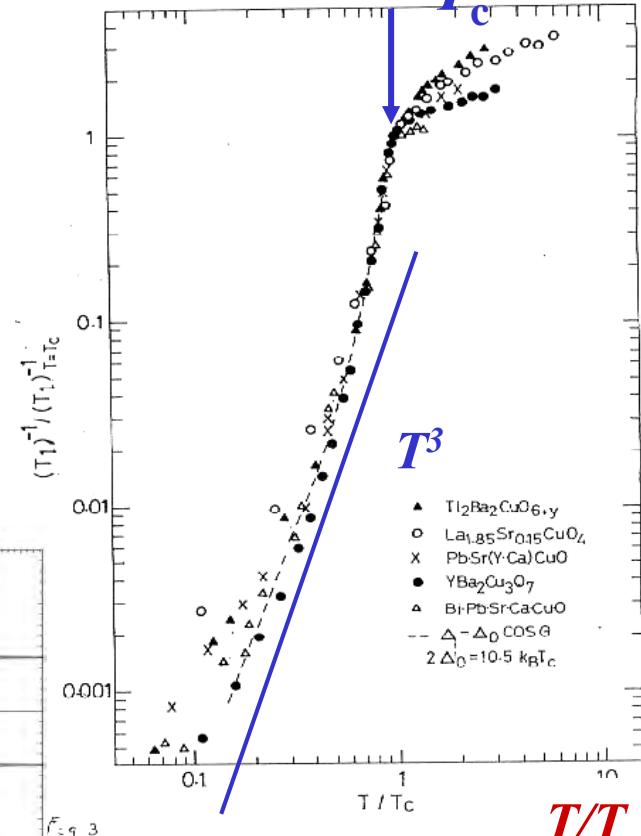


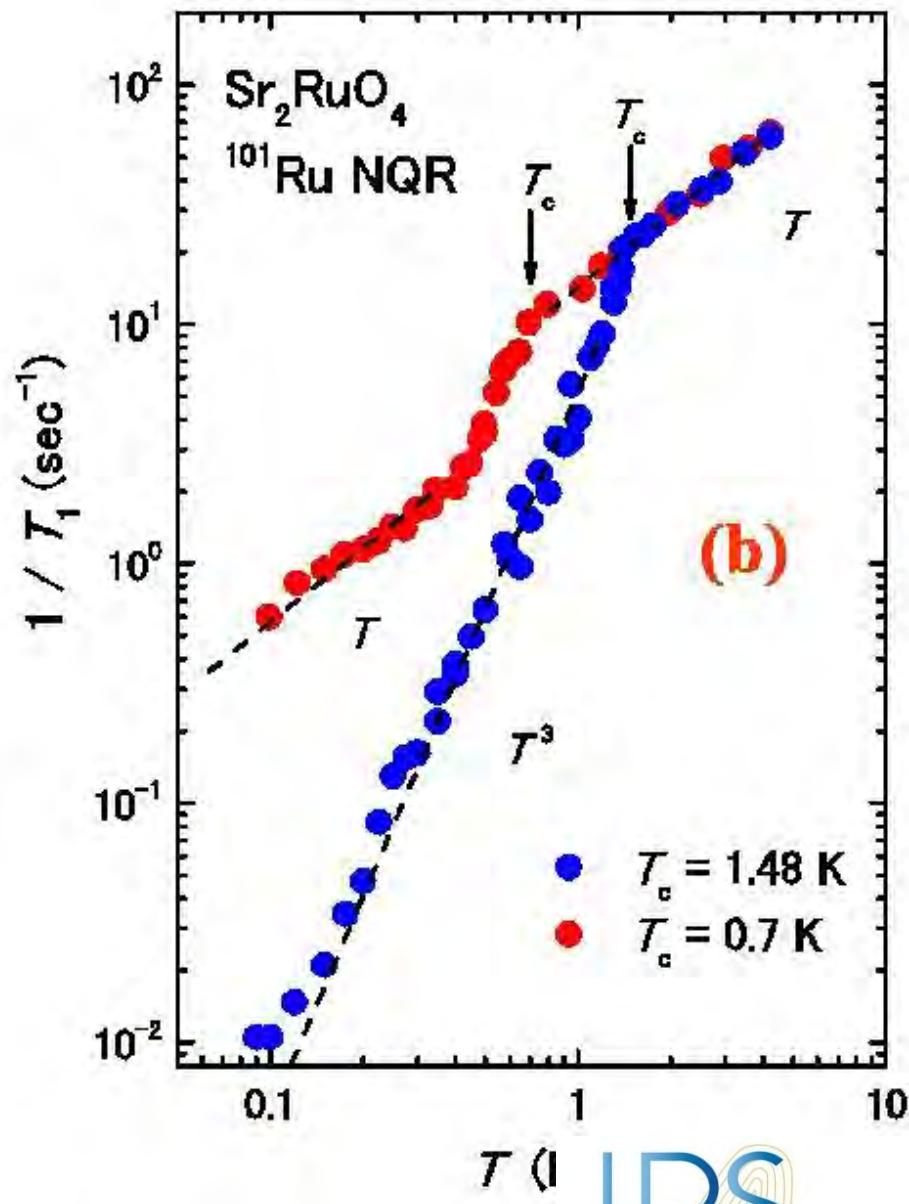
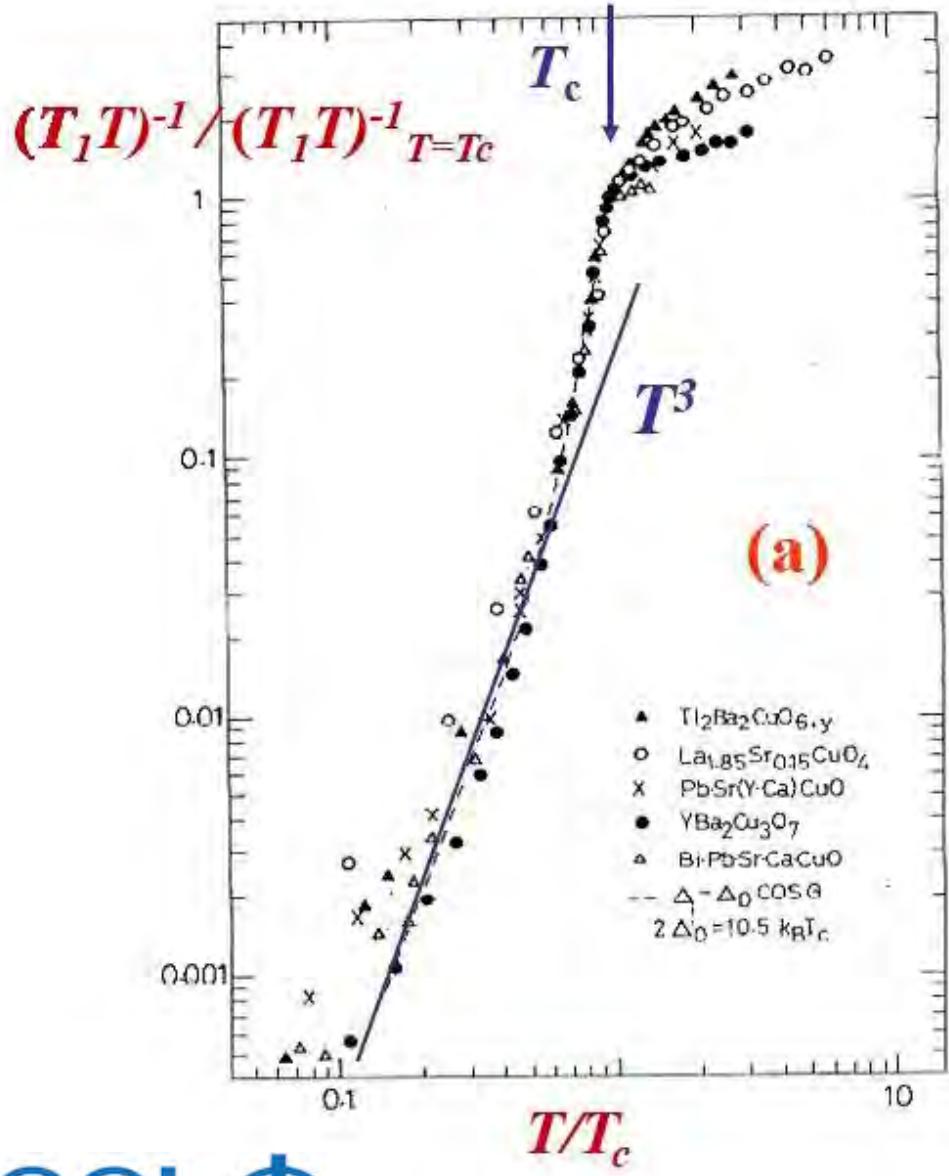
$T_1$  minimum below T<sub>c</sub>  
(Hebel-Slichter peak in  $1/T_1$ )

$$(T_1 T)^{-1}/(T_1 T)^{-1}_{T=T_c}$$



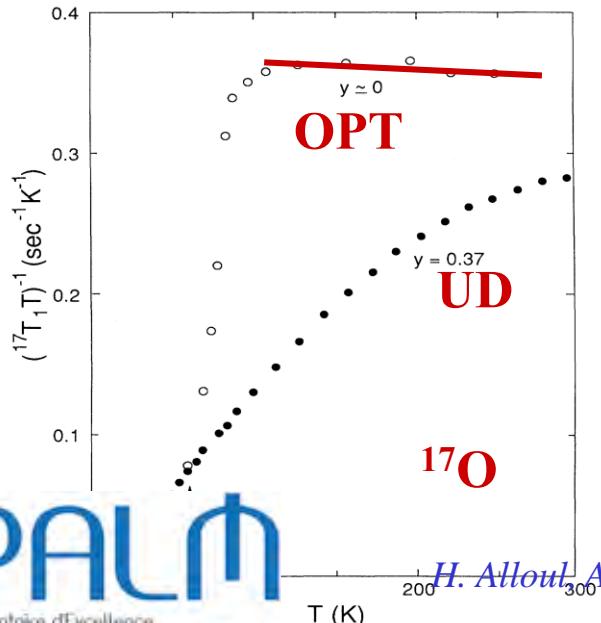
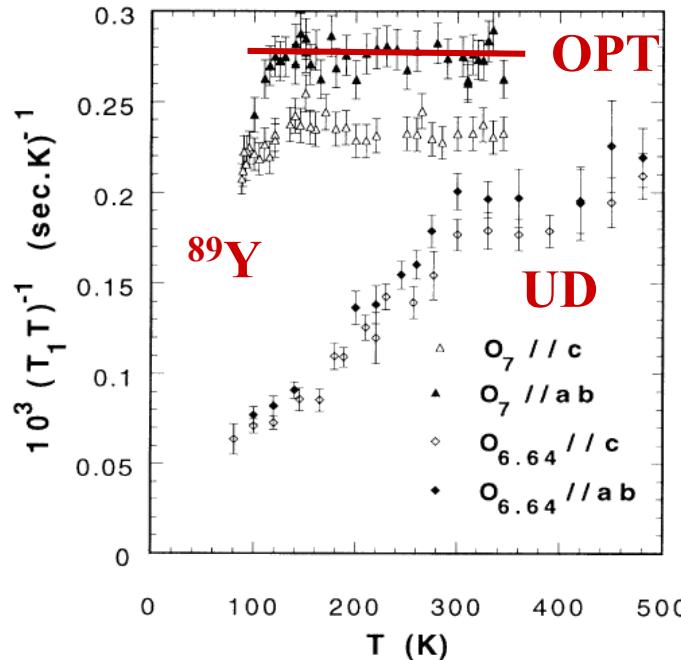
d wave superconductivity  
 $T^3$  variation for  $T \ll T_c$



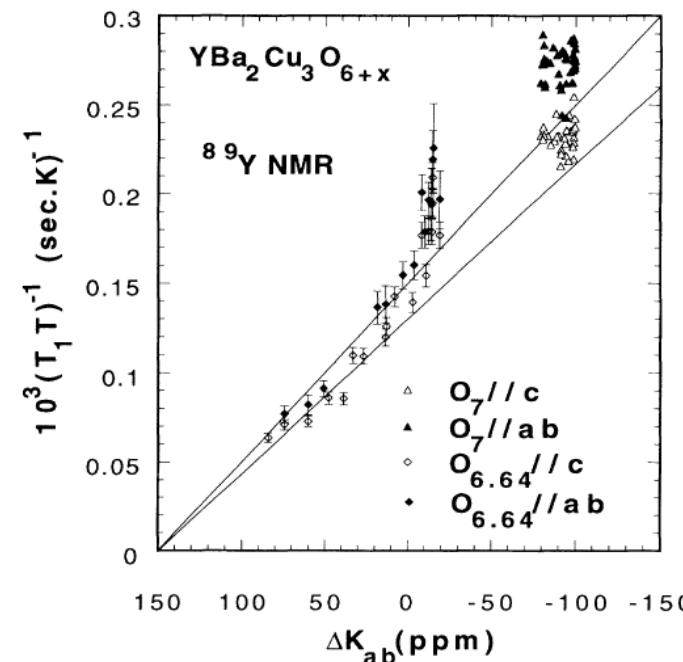


# Comparison of $(T_1 T)^{-1}$ on $^{89}\text{Y}$ and $^{17}\text{O}$ above Tc

$(T_1 T)^{-1}$



In YBCO<sub>7</sub>,  $T_1 T$  is nearly constant on  $^{17}\text{O}$  and  $^{89}\text{Y}$   
Like in a free electron metal



In YBCO<sub>6.6</sub>

$$\frac{1}{T_1 T} \propto K$$

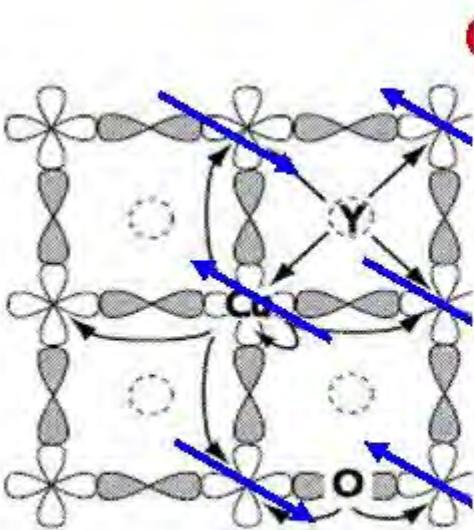
Metallic like component

**$^{89}\text{Y}$  NMR Evidence for a Fermi-Liquid Behavior in  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$**

H. Alloul, T. Ohno,<sup>(a)</sup> and P. Mendels

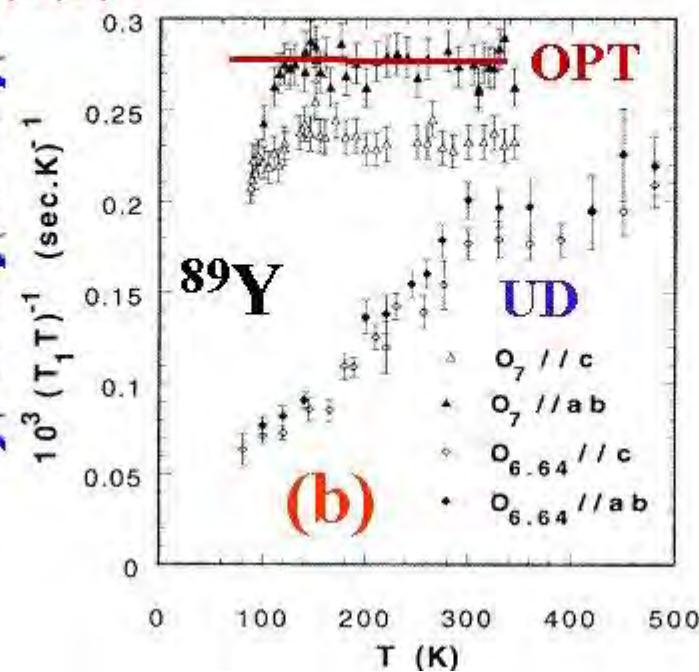
Physique des Solides, Université de Paris-Sud, 91405 Orsay, France  
(Received 15 May 1989)

# Distinct behaviour of $(T_1 T)^{-1}$ on the Cu site: AF correlations

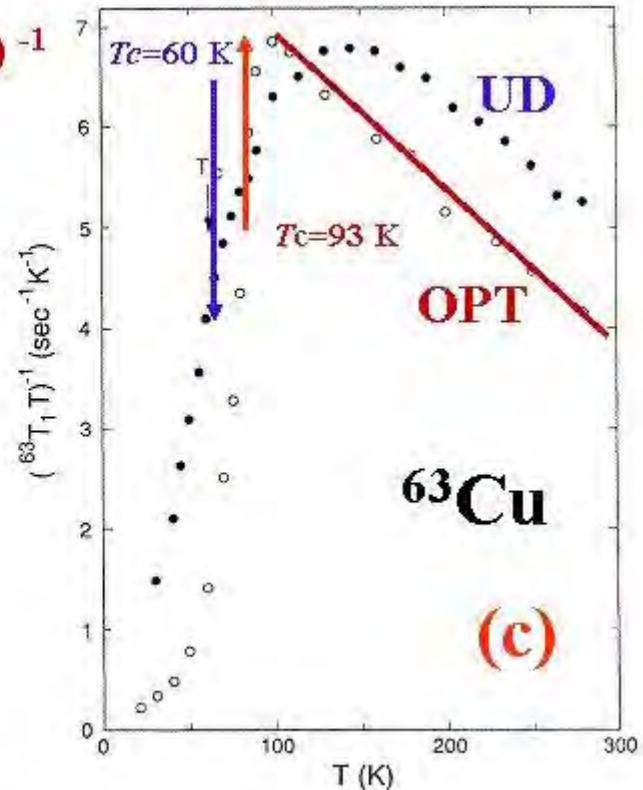


(a)

$(T_1 T)^{-1}$



$(T_1 T)^{-1}$

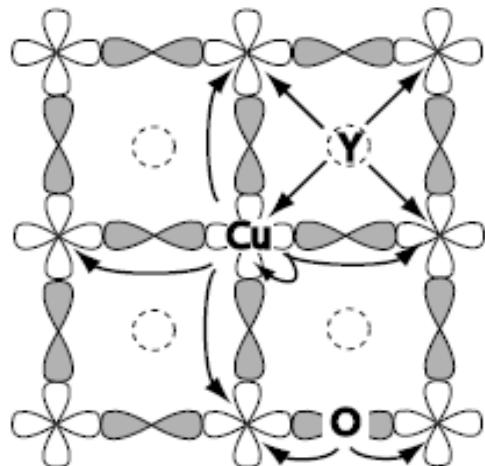


In  $\text{YBCO}_7$ ,  $T_1 T$  is nearly constant on  $^{89}\text{Y}$  and  $^{17}\text{O}$  but increases at low  $T$  for  $^{63}\text{Cu}$   
O and Y are insensitive to AF correlations while Cu probes them fully

Increase of AF correlations at low  $T$   
Even more for the underdoped case

# $T_1$ for nuclei coupled to neighbouring sites

Non local hyperfine coupling  
q dependence of the HF coupling



$$\frac{1}{T_1} = \frac{2k_B T}{\hbar^2 \gamma_e^2} \sum_{\mathbf{q}} A^2(\mathbf{q}) \chi_{\perp}''(\mathbf{q}, \omega_n) / \omega_n$$

$\xi(T)$  is the AF correlation length probed by  $^{63}\text{Cu}$  NMR

$$\omega \rightarrow 0$$

$$A_{O,\alpha}^s(\mathbf{q}) = A_{O,\alpha}^s \sum_{\mathbf{r}_i} \exp(i \mathbf{q} \cdot \mathbf{r}_i)$$

$$^{89}\text{Y} \quad A_{Y,\alpha}^s(q) = 8D_\alpha (\cos q_x a/2 \cos q_y a/2)$$

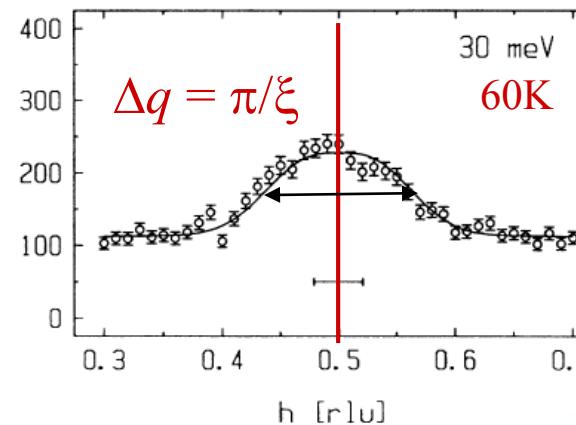
$$^{17}\text{O} \quad A_{O,\alpha}^s(q) = 2C_\alpha \cos q_x a/2$$

$$^{63}\text{Cu} \quad A_{Cu,\alpha}^s(q) = A_\alpha + 2B_\alpha (\cos q_x a + \cos q_y a)$$

For Y and O,  $A(\mathbf{q})$  vanishes for  $\mathbf{q}_{AF} = (\pi/a, \pi/a)$   
The AF fluctuations are filtered out by  $A(\mathbf{q})$

$$\chi_{\perp}''(\mathbf{q}, \omega_n)$$

YBCO<sub>6.6</sub>



$$(\pi/a, \pi/a)$$

## Some conclusions

- *Magnetic spin susceptibilities in NMR :*
  - Singlet spin pairing
  - Single spin fluid in the normal state
  - The pseudogap is generic and robust to disorder
- *Dynamic susceptibilities and spin lattice relaxation :*
  - Magnetic correlations up to the Optimal doping
  - Metallic like at  $q=0$ , AF correlations for  $q=(\pi/a, \pi/a)$
  - d- wave SC
- *The pseudogap and questions on the phase diagram*
  - Importance of disorder in the phase diagram
  - MIT and SG phases governed by disorder
- *SC Fluctuations and pseudogap ( Conf: Florence Rullier Albenque)*
  - SC Fluctuations follow Tc versus hole doping , remain with disorder
  - A preformed pair scenario does not apply
  - Pseudogap is intimately linked with magnetism (competing order?)
  - NMR will be helpful to check possible models

## More

- *Totally missing in my talk:*

Quadrupolar effects

coupling of the nuclear spin with the charge distribution:  
charge order, structural transitions etc

- *Other examples of NMR studies :*

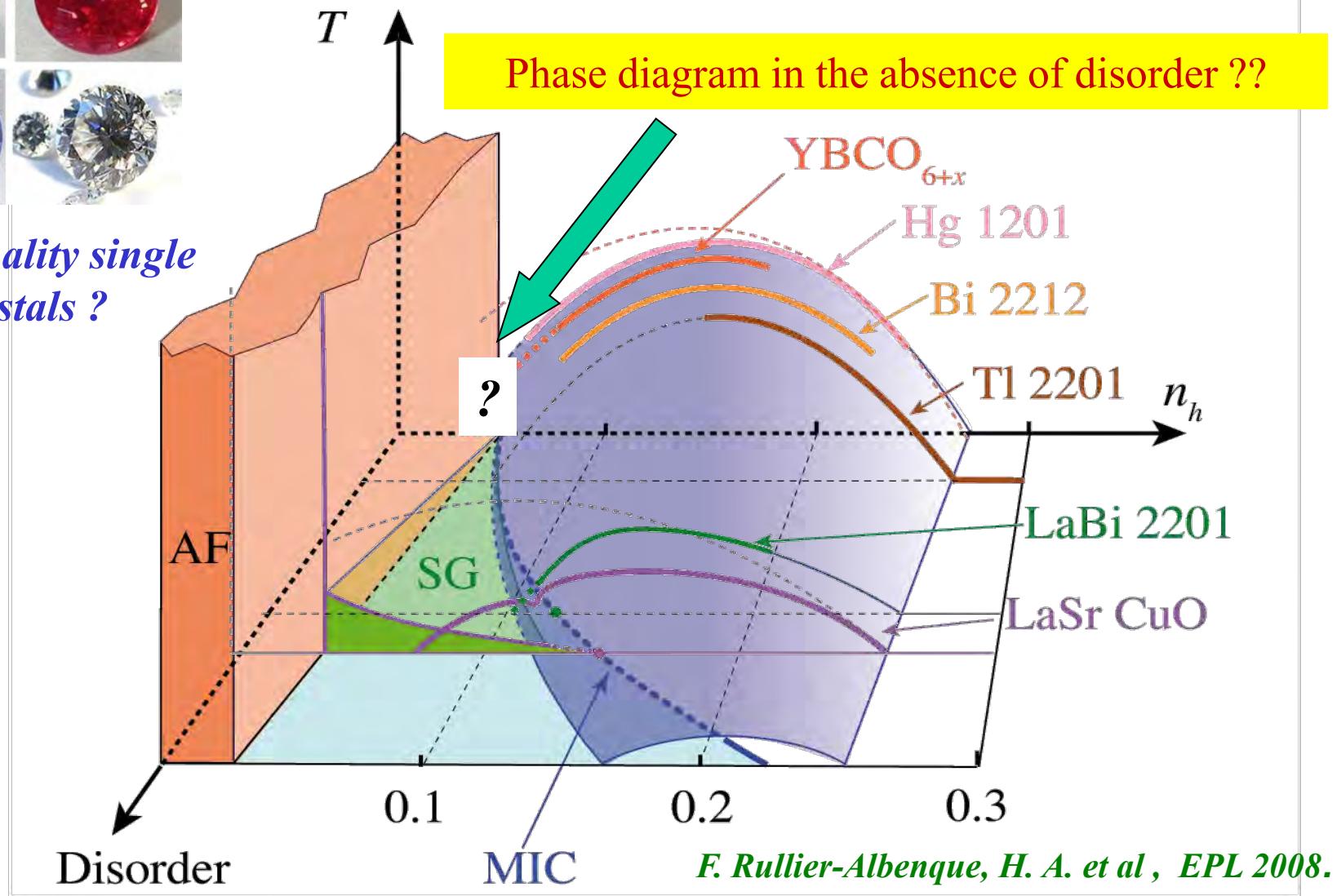
H. Alloul, “NMR studies of electronic properties of solids”, Scholarpedia, 9(9):32069 (2014)

H. Alloul, “NMR in strongly correlated materials”, Scholarpedia, 10(1):30632 (2015)

# The various cuprate families



High quality single crystals ?

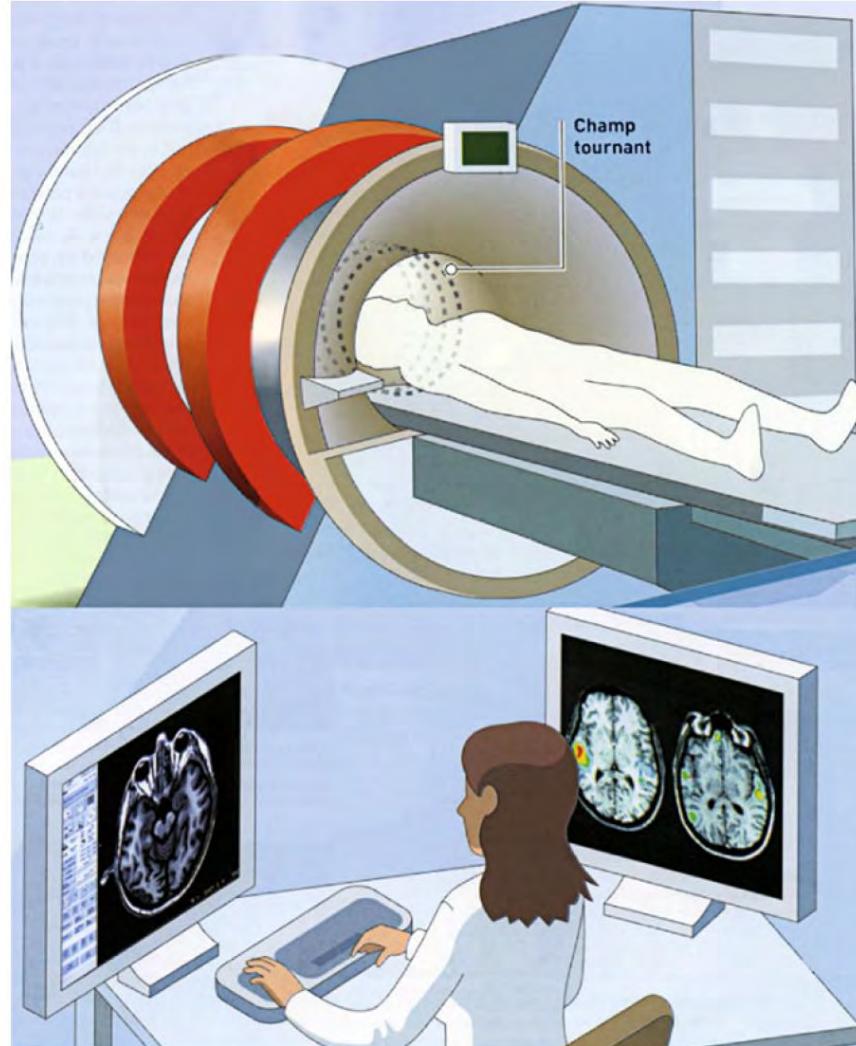


SG and MIT are determined by disorder

*H. Alloul, Autumn School on correlated electrons  
Julich ,15 /09 /2016*

# Magnetic Resonance Imaging (MRI)

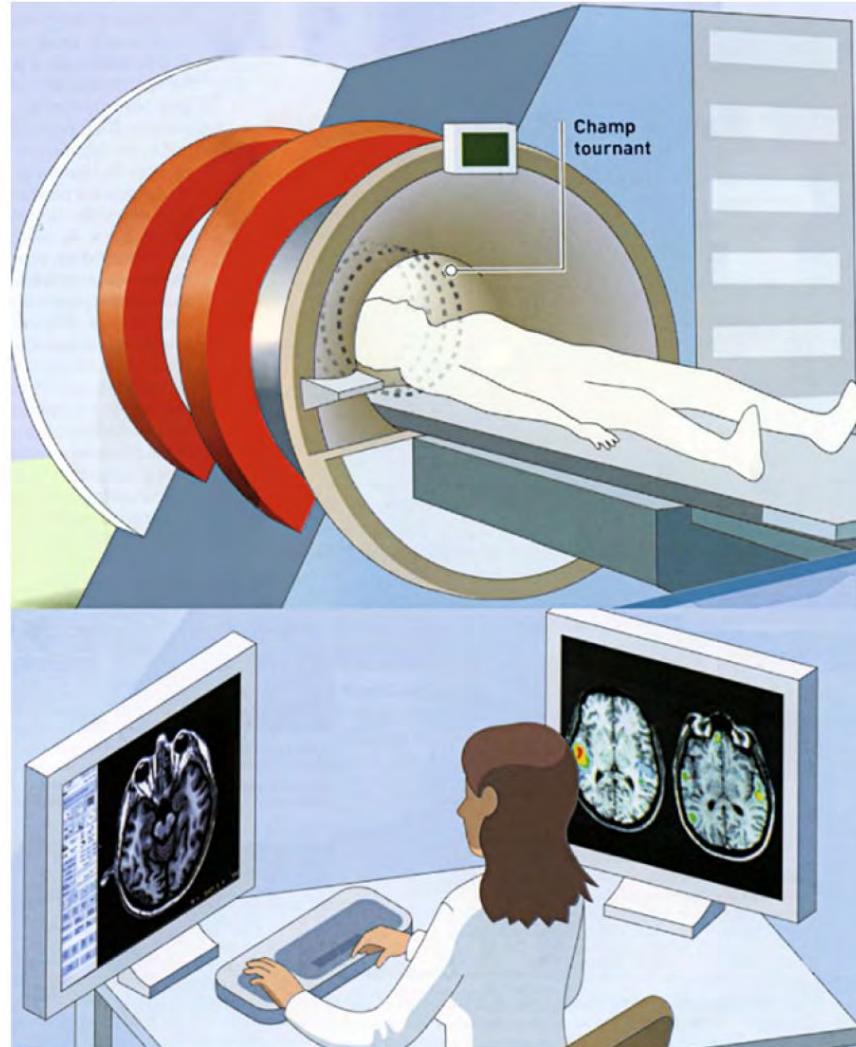
*What is imaged here?*



# Magnetic Resonance Imaging (MRI)

*What is imaged here?*

$^1\text{H}$  proton  
NMR  
Intensity,  
But also  $T_2$



# SPIN ECHOES

$\delta\omega$

Distribution of Larmor frequencies

