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Non-equilibrium dynamical mean-field theory

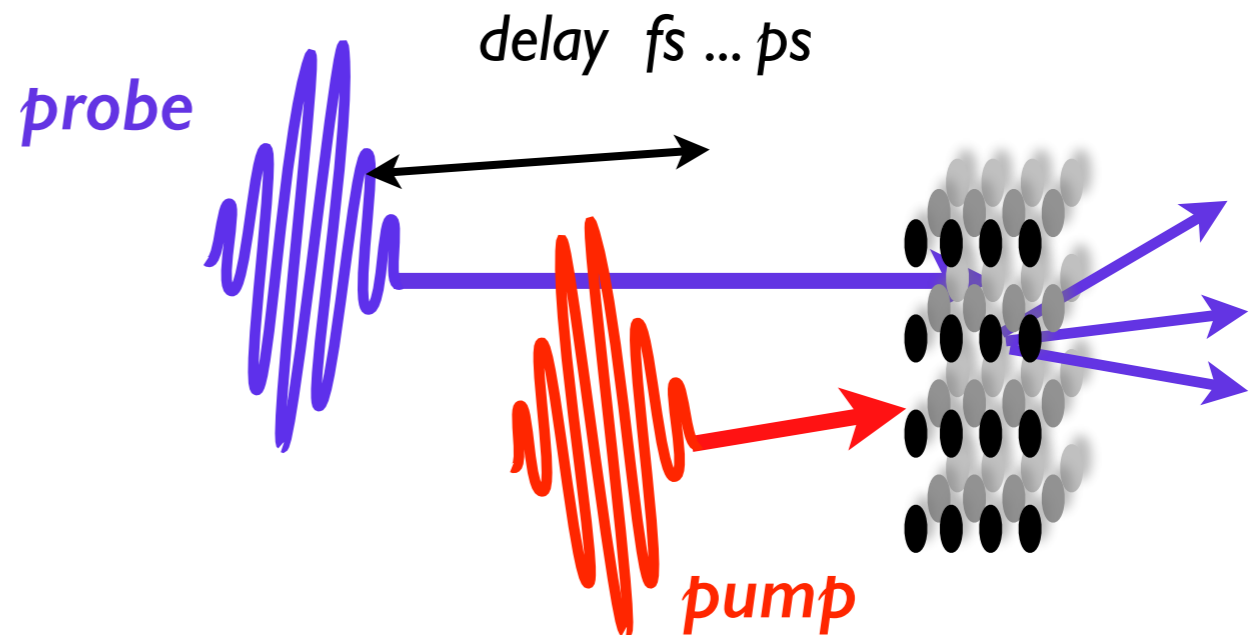
Martin Eckstein

Jülich, September 20, 2018

Pump-probe experiments on correlated solids

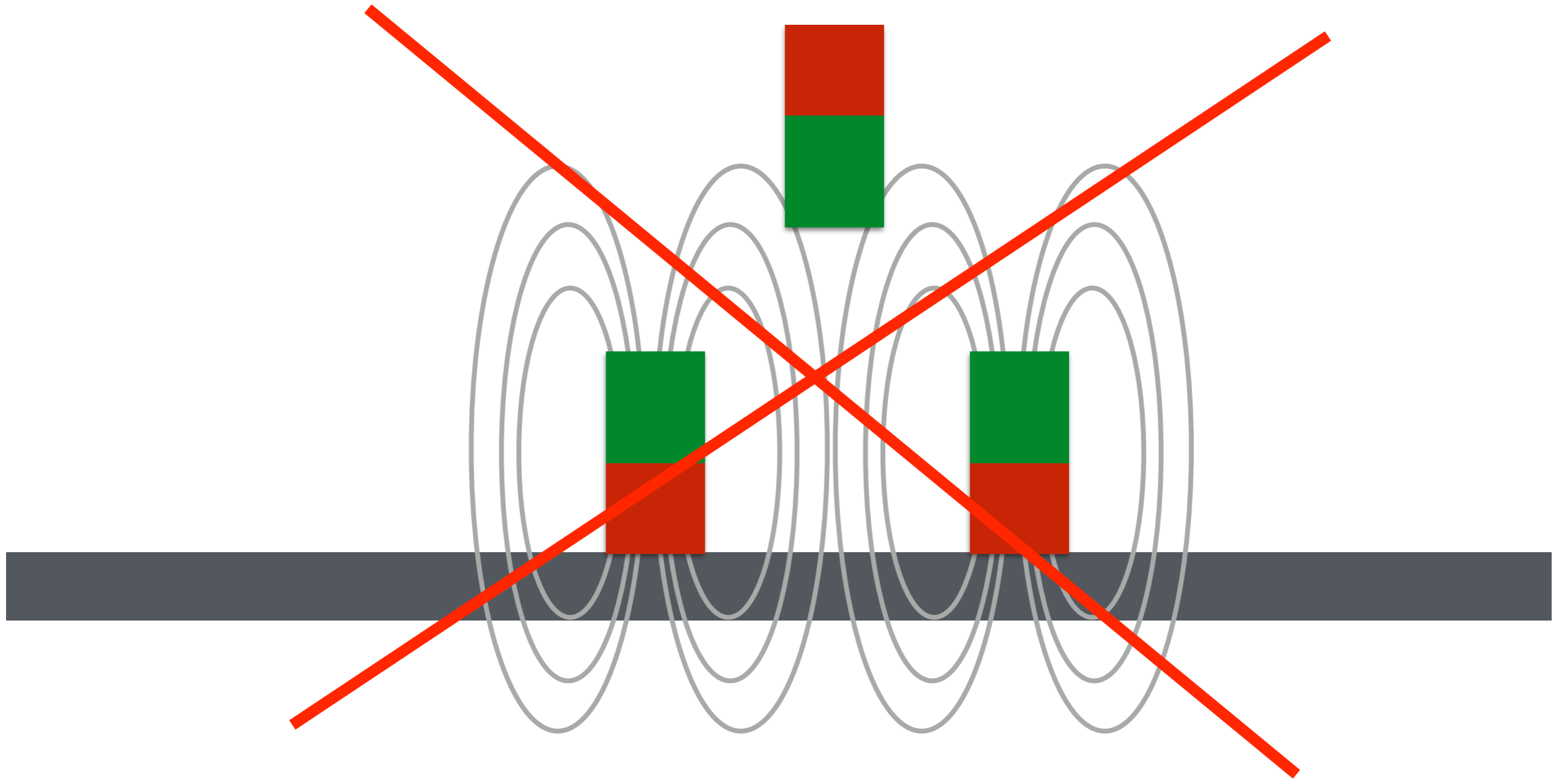
Pump-probe experiments

Time resolved THz, optics,
UV (photoemission), Xray, ...



- ⇒ Real-time experiments to understand origin of complex states
- ⇒ Dynamical stabilization, ultra-fast switching to “novel” phases

Dynamic stabilization



Magnetic top



Example: control of the magnetic exchange interaction

one electron per site

$$H = -t \sum_{\langle ij \rangle, \sigma = \uparrow, \downarrow} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \xrightarrow{U \gg t} H = \frac{2t^2}{U} \sum_{\langle ij \rangle} \vec{S}_i \vec{S}_j$$

+ electric field $E_0 \cos(\omega t) \xrightarrow{\quad} J_{ex}(E_0, \omega)$



LETTER

doi:10.1038/nature25135

Enhancement and sign change of magnetic correlations in a driven quantum many-body system

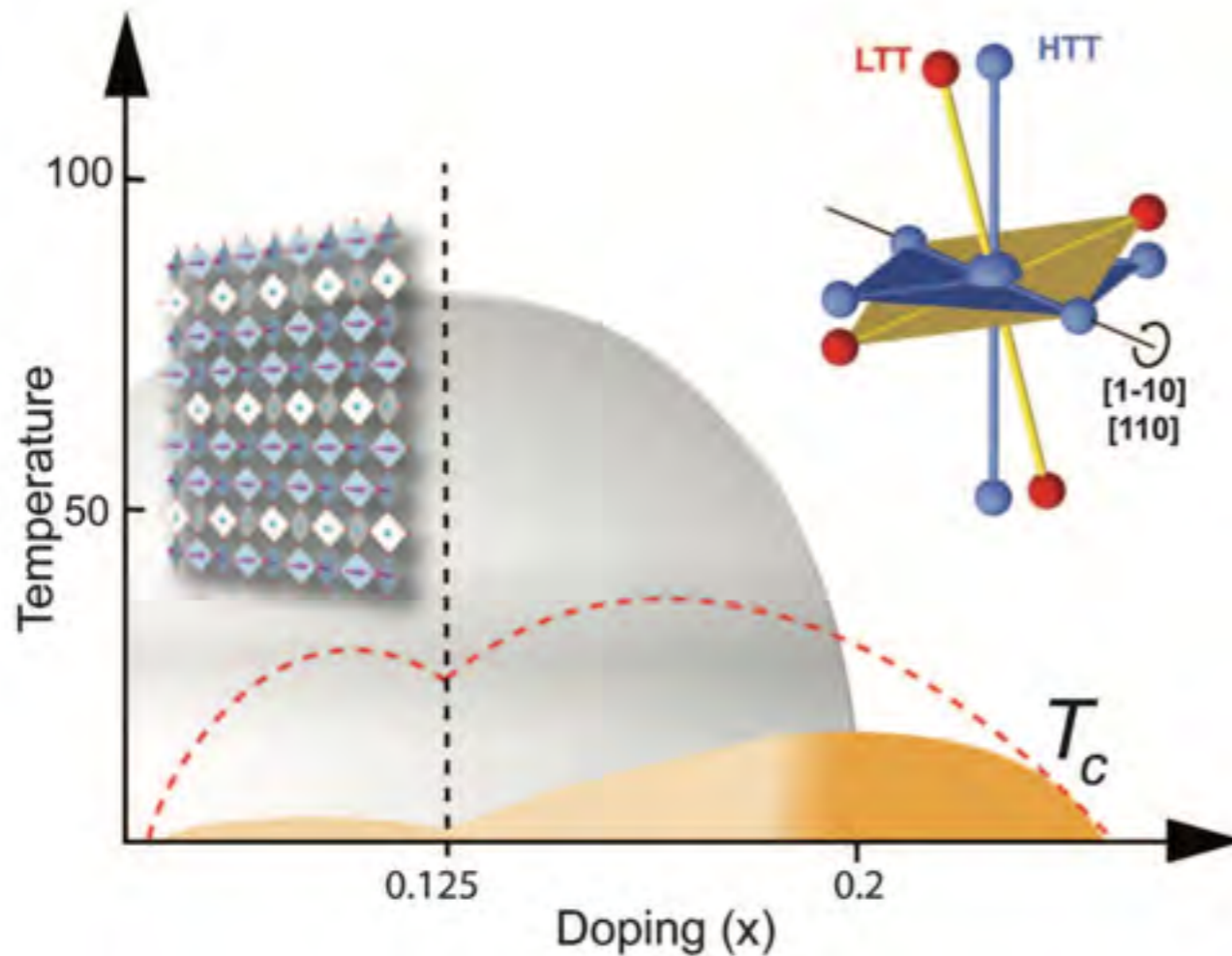
Frederik Görg¹, Michael Messer¹, Kilian Sandholzer¹, Gregor Jotzu^{1,2}, Rémi Desbuquois¹ & Tilman Esslinger¹

Mentink, Balzer, and Eckstein, Nature Comm 6 (2015)

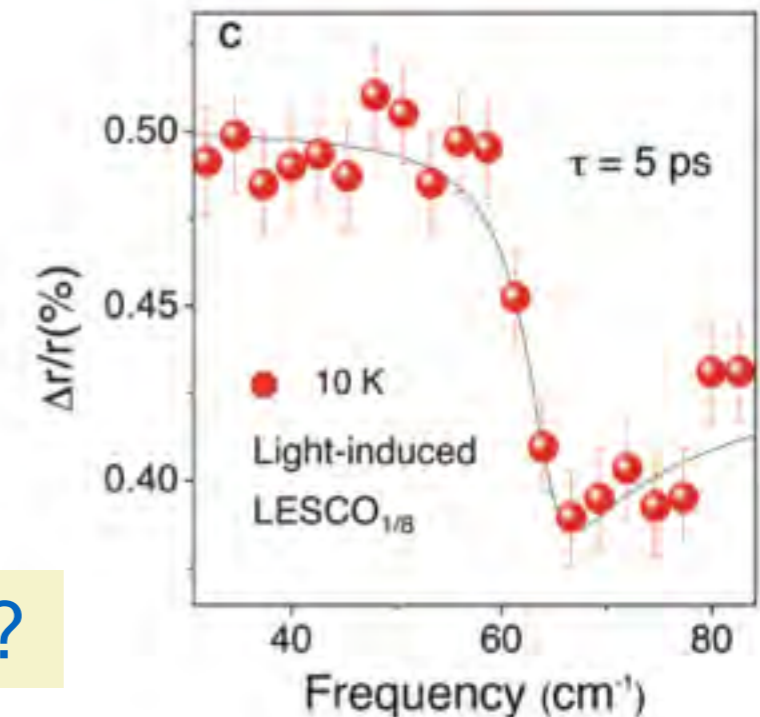
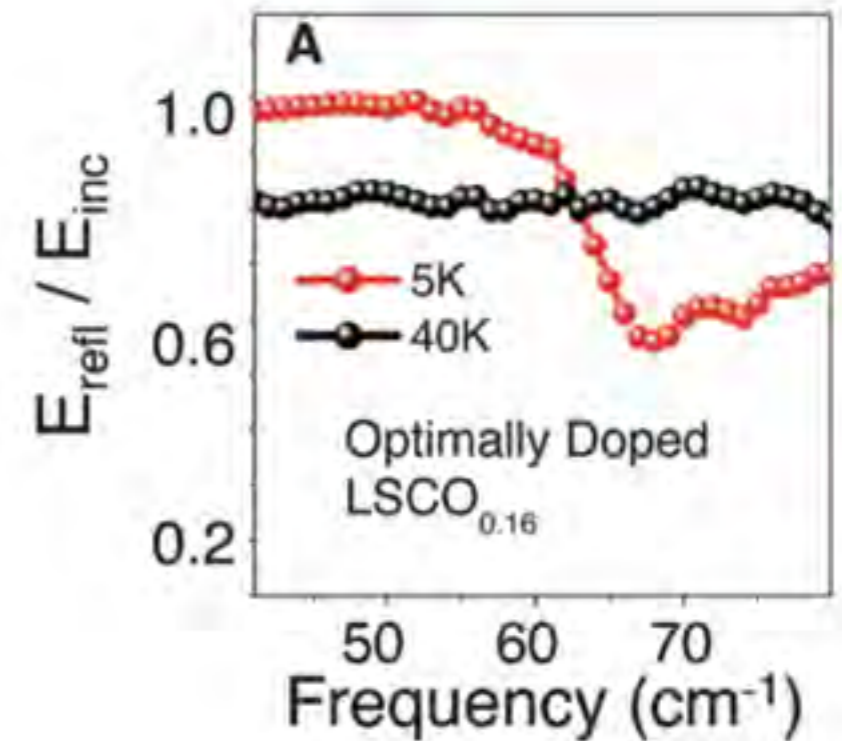
Non-equilibrium phases of matter

Light-Induced Superconductivity in a Stripe-Ordered Cuprate

D. Fausti,^{1,2*}† R. I. Tobey,²†§ N. Dean,^{1,2} S. Kaiser,¹ A. Dienst,² M. C. Hoffmann,¹ S. Pyon,³ T. Takayama,³ H. Takagi,^{3,4} A. Cavalleri^{1,2*}



Josephson-Plasma Resonance



Laser-induced suppression of competing phase?

Outline

⇒ Keldysh formalism & non-equilibrium Green's functions

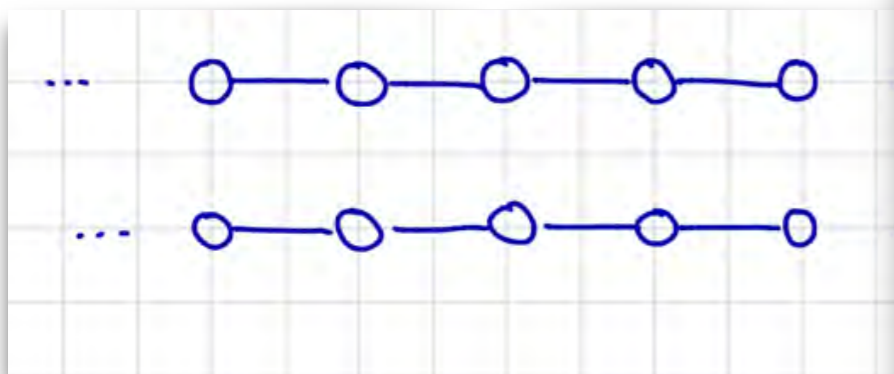
Goal: electronic structure of correlated systems out of equilibrium

⇒ Non-equilibrium DMFT

⇒ Photo-excited Mott & charge transfer insulators

Why Green's function?

Effective single-particle descriptions



$$H = \sum_{\mathbf{k}} \sum_{\alpha=1,2} \epsilon_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha} + U \sum_j \underbrace{c_{j1}^{\dagger} c_{j1} c_{j2}^{\dagger} c_{j2}}_{\otimes}$$

mean-field decoupling:

$$\otimes \Rightarrow -c_{j1}^{\dagger} c_{j2} \langle c_{j2}^{\dagger} c_{j1} \rangle - \langle c_{j1}^{\dagger} c_{j2} \rangle c_{j2}^{\dagger} c_{j1}$$

Fock exchange self energy

$$H = \sum_{\mathbf{k}} \begin{pmatrix} c_{\mathbf{k}1}^{\dagger} & c_{\mathbf{k}2}^{\dagger} \end{pmatrix} \underbrace{\begin{pmatrix} \epsilon_{\mathbf{k}1} & U \bar{\Phi} \\ U \bar{\Phi}^* & \epsilon_{\mathbf{k}2} \end{pmatrix}}_{h_{\mathbf{k}}} \underbrace{\begin{pmatrix} c_{\mathbf{k}1} \\ c_{\mathbf{k}2} \end{pmatrix}}_{\psi_{\mathbf{k}}}$$

$$\bar{\Phi} = -\frac{1}{N} \sum_{\mathbf{k}} \langle c_{\mathbf{k}1}^{\dagger} c_{\mathbf{k}2} \rangle$$

Effective single-particle descriptions

$$H = \sum_{\mathbf{k}} \begin{pmatrix} c_{\mathbf{k}1}^\dagger & c_{\mathbf{k}2}^\dagger \end{pmatrix} \begin{pmatrix} \epsilon_{\mathbf{k}1} & U\bar{\Phi} \\ U\bar{\Phi}^* & \epsilon_{\mathbf{k}2} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}1} \\ c_{\mathbf{k}2} \end{pmatrix} \quad \bar{\Phi} = -\frac{1}{N} \sum_{\mathbf{k}} \langle c_{\mathbf{k}1}^\dagger c_{\mathbf{k}2} \rangle$$

$h_{\mathbf{k}}$
 $\psi_{\mathbf{k}}$

Fock exchange self energy

Coherent evolution of single-particle states:

$$H = \sum_{\mathbf{k}} c_{\mathbf{k},n}^\dagger h_{\mathbf{k}} [\rho]_{nm} c_{\mathbf{k},m} \quad (\rho_{\mathbf{k}})_{mn} = \langle c_{\mathbf{k},n}^\dagger c_{\mathbf{k},m} \rangle$$

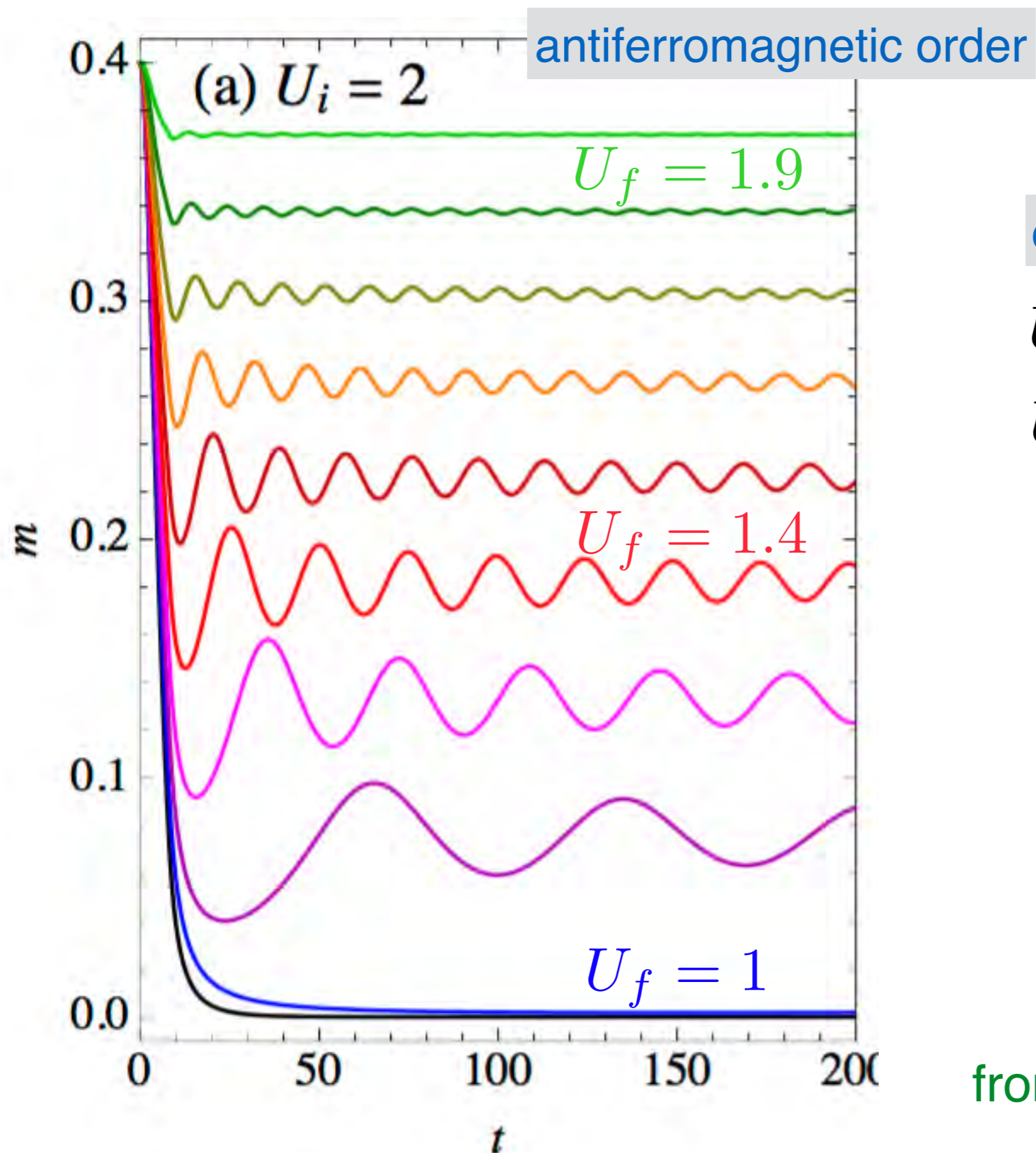
Single-particle Hamiltonian:
Instantaneous function of
 one-particle density matrix

$$\frac{d}{dt} \rho_{\mathbf{k}} = i[h_{\mathbf{k}}, \rho_{\mathbf{k}}]$$

⇒ Time-dependent BCS theory, ...

⇒ Time-dependent density-functional theory (adiabatic approximation)

Effective single-particle descriptions



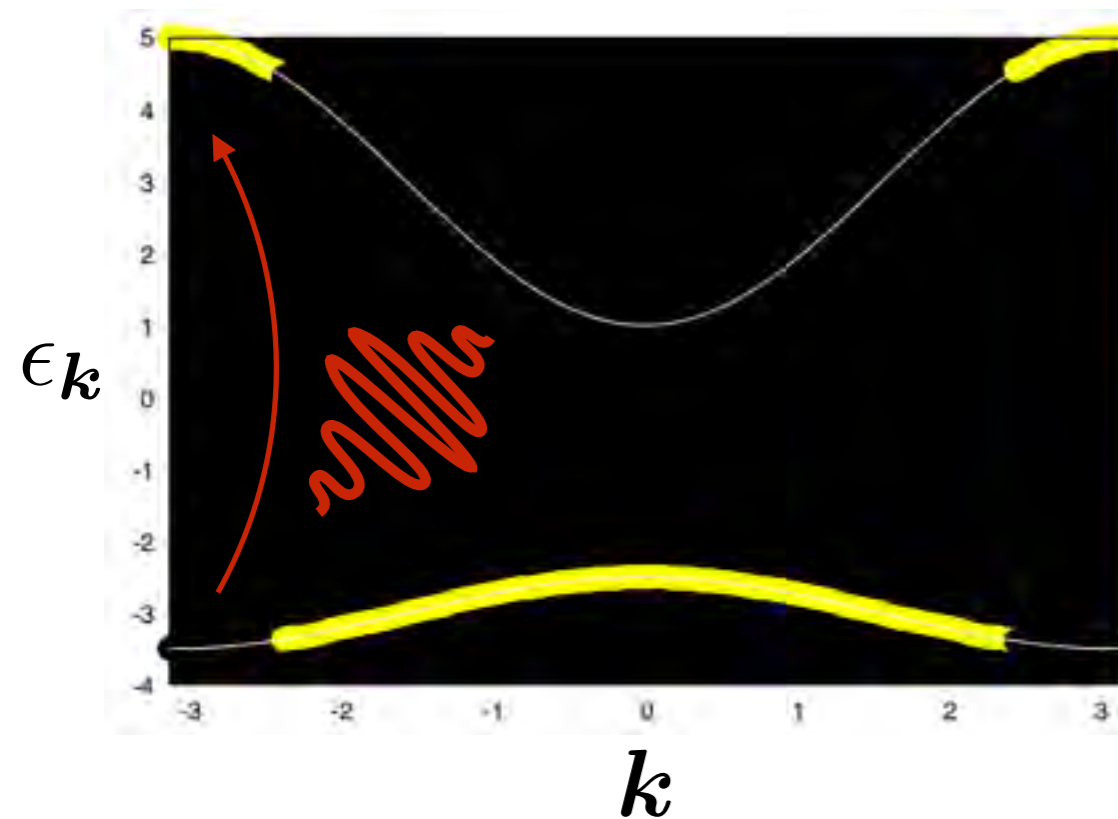
quench in the Hubbard model

$$U(t) = U_i = 2 \text{ for } t < 0$$

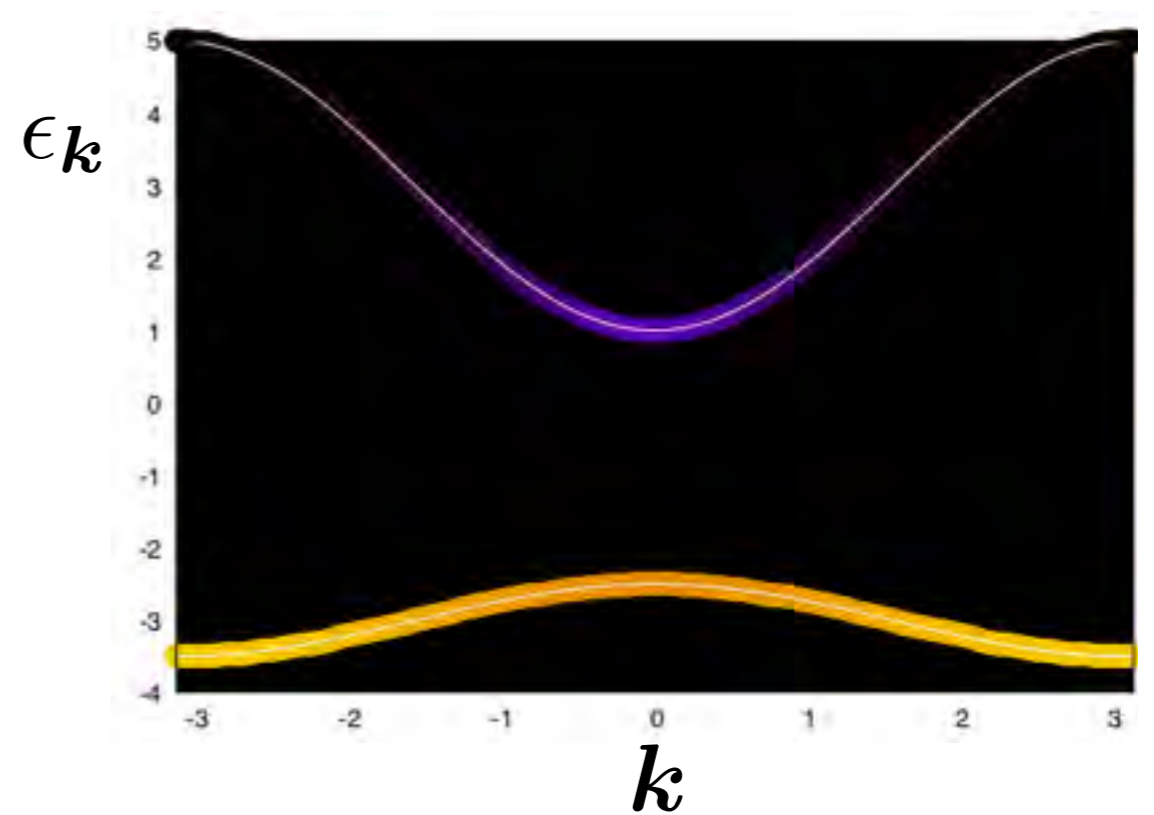
$$U(t) = U_f \text{ for } t > 0$$

from Tsuji, ME, Werner, (2012)

Effective single-particle descriptions: Thermalization?



⇒ ?



Thermal state: $\text{tr} \rho_{\mathbf{k}} = \sum_n \langle c_{\mathbf{k},n}^\dagger c_{\mathbf{k},n} \rangle$ at small k increased for higher temperature

Coherent evolution of single-particle states:

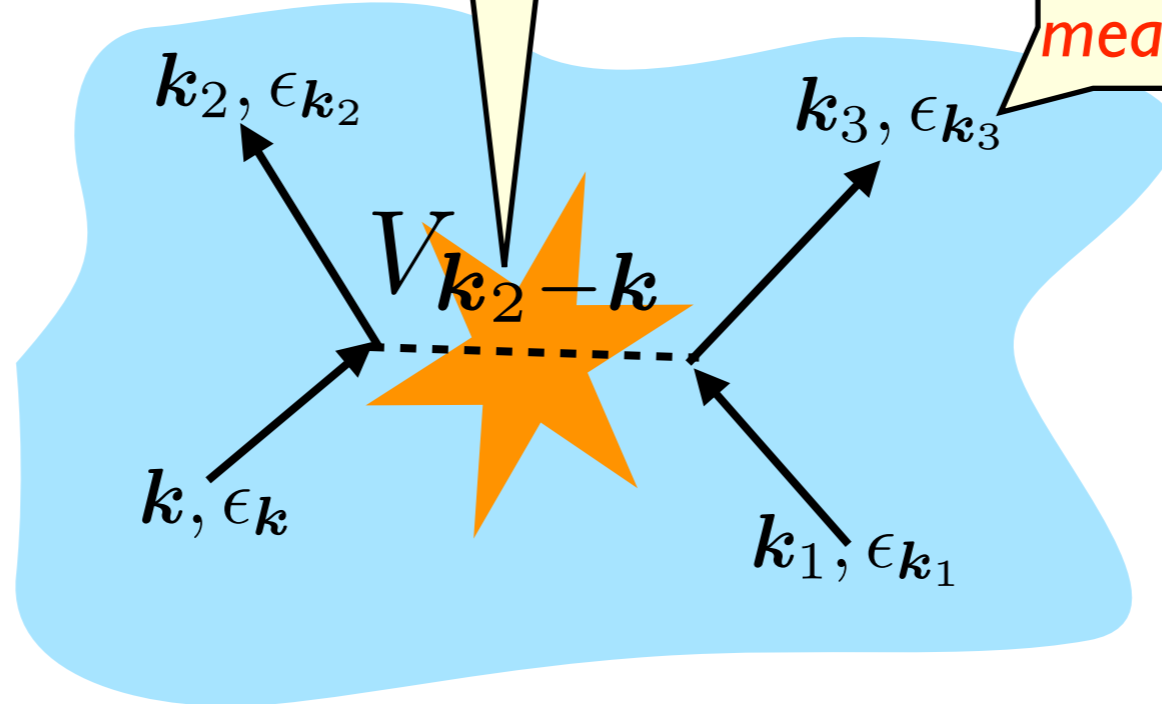
$$\frac{d}{dt} \rho_{\mathbf{k}} = i[h_{\mathbf{k}}, \rho_{\mathbf{k}}] \Rightarrow \text{tr} \rho_{\mathbf{k}} \text{ conserved if } H \text{ is translationally invariant}$$

Cannot describe electron thermalization (lack of incoherent scattering)

Kinetic equations

Matrix element derived from
(dynamically screened)
Coulomb interaction

band energies
(time-dependent e.g.
mean-field shifts)



occupation of states

$$\partial_t n_{\mathbf{k}}(t) = \sum_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3} V_{\mathbf{k}_2 - \mathbf{k}} \delta(\epsilon_{\mathbf{k}} + \epsilon_{\mathbf{k}_1} - \epsilon_{\mathbf{k}_2} - \epsilon_{\mathbf{k}_3}) \times \\ \times [n_{\mathbf{k}} n_{\mathbf{k}_1} (1 - n_{\mathbf{k}_2}) (1 - n_{\mathbf{k}_3}) - (1 - n_{\mathbf{k}}) (1 - n_{\mathbf{k}_1}) n_{\mathbf{k}_2} n_{\mathbf{k}_3}]$$

Occupation of states, spectrum, quasiparticle lifetime, screening of Coulomb interaction, etc. evolve on comparable timescales

Non-equilibrium Green's functions

Non-equilibrium Green's functions

$$H = H_0 + \phi_q(t) \rho_{-q}(t)$$

$$\delta \rho_q = \int_{-\infty}^{\pm} \chi_q(t, t') \phi(t') dt' \quad \chi_q = -i \Theta(t, t') \langle [\rho_q(t), \rho_{-q}(t')] \rangle$$

Kubo

$$\frac{1}{\pi} \text{Im} \chi_q(\omega) \equiv A_q(\omega) \quad \rightarrow \text{phonon spectrum}$$

$|\Psi\rangle$

Non-equilibrium Green's functions

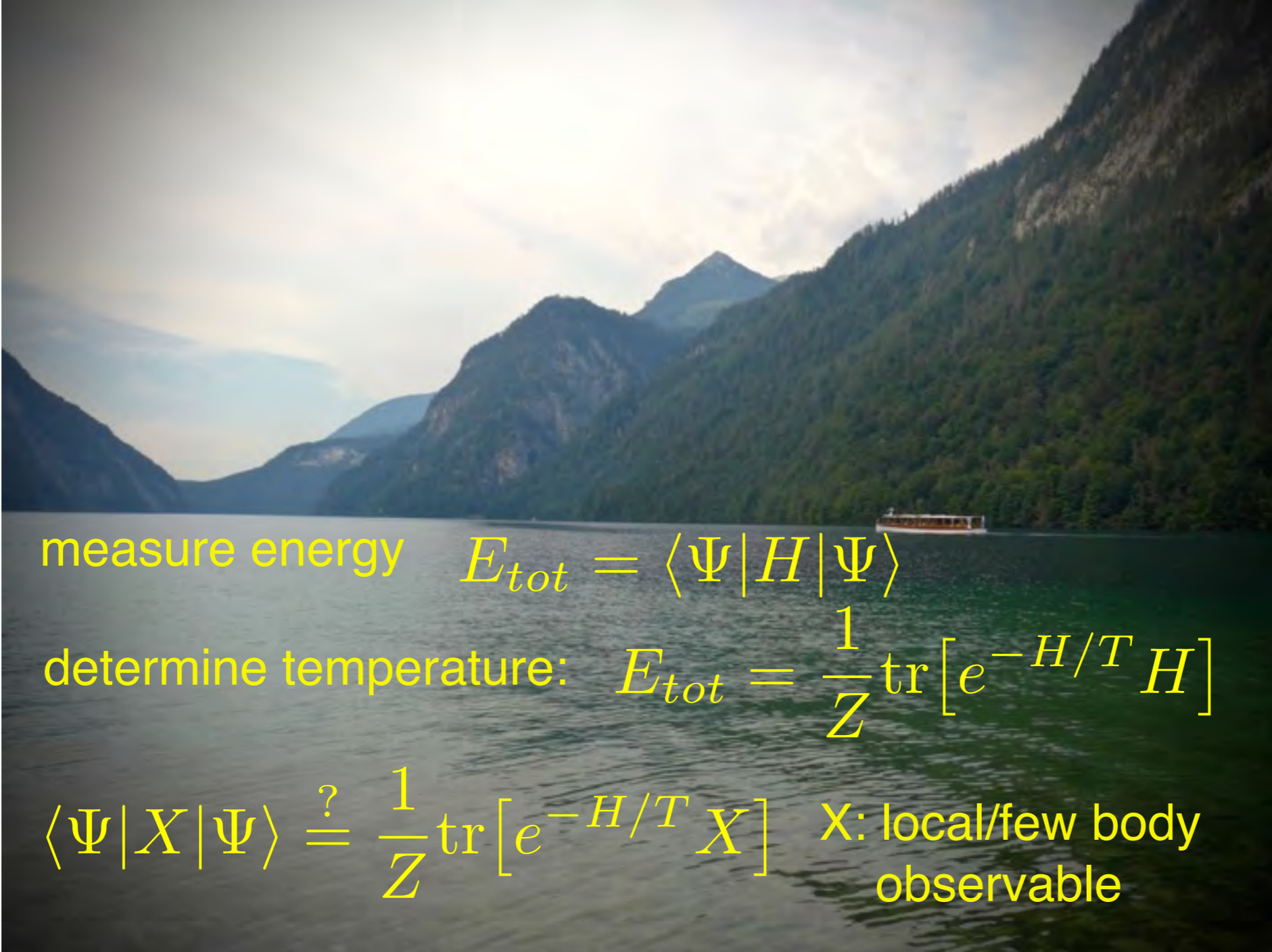
How to decide whether a given system is “in equilibrium”?



$|\Psi\rangle$

Non-equilibrium Green's functions

How to decide whether a given system is “in equilibrium”?



measure energy $E_{tot} = \langle \Psi | H | \Psi \rangle$

determine temperature: $E_{tot} = \frac{1}{Z} \text{tr} [e^{-H/T} H]$

$\langle \Psi | X | \Psi \rangle \stackrel{?}{=} \frac{1}{Z} \text{tr} [e^{-H/T} X]$ X: local/few body observable

Requires knowledge of the Hamiltonian.

Non-equilibrium Green's functions

How to decide whether a given system is “in equilibrium”?

fluctuations:

$\hat{=}$

occupation function

$$C_q(t, t') = \langle \rho_{-q}(t') \rho_q(t) \rangle$$

$$\rightarrow C(t-t') \rightarrow C(\omega)$$

$$C(\omega) = \frac{1}{e^{\beta\omega} - 1} A_q(\omega)$$

$|\Psi\rangle$

Fluctuation dissipation theorem:
Universal relation between density
of states and occupation

Non-equilibrium Green's functions

Equilibrium:

Formulate many body theory in terms of propagators which describe **spectrum** of collective and quasiparticle excitations; **time-translational invariance**

(mathematically, this is formulated in imaginary time)

Non-equilibrium:

Response and correlation function (**spectrum and occupation function**) are independent dynamic variables in many-body theory; **time-translational invariance can be lost**

Non-equilibrium Green's functions

hole propagator $G_{jj'}^{>}(t, t') = -i \langle c_j(t) c_{j'}^\dagger(t') \rangle$

electron propagator $G_{jj'}^{<}(t, t') = i \langle c_{j'}^\dagger(t') c_j(t) \rangle$

$$G_k^r(t, t') = -i \Theta(t, t') \langle [c_k(t), c_k^\dagger(t')]_+ \rangle$$

$$A_k(\omega) = -\frac{1}{\pi} \text{Im} G_k^r(\omega + i0) \quad \underline{\text{Spectrum}}$$

$$G_k^<(t, t') = \langle c_k^\dagger(t') c_k(t) \rangle$$

$$G_k^<(\omega) = f(\omega) A_k(\omega) (2\pi i) \quad \underline{\text{occupied density of states}}$$

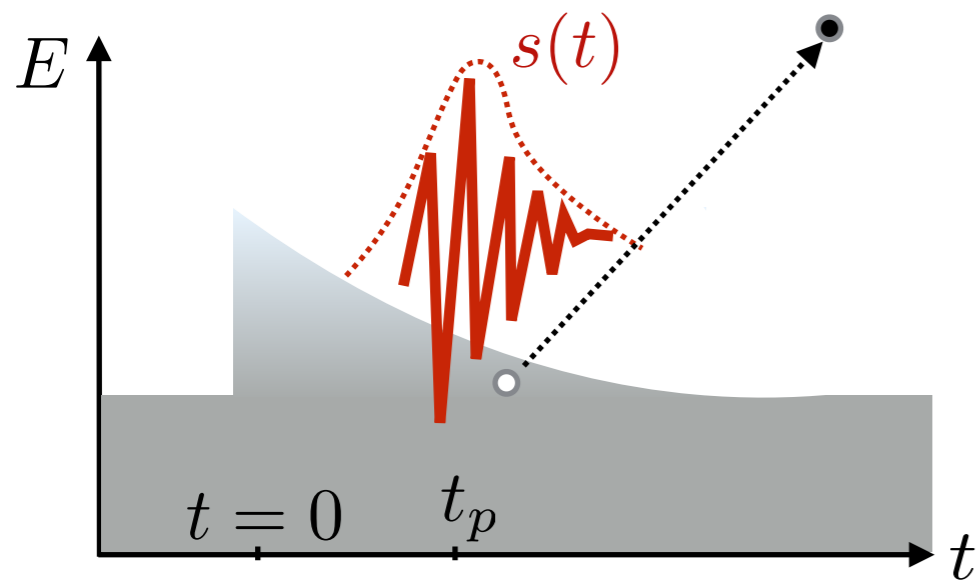
Kadanoff/Baym

Quantum
Statistical
Mechanics

ABP

Photoemission spectroscopy:

electron with asymptotic momentum k , energy E ,



classical field $A^+(t) = e^{i\Omega t} s(t)$

$$H_{int} = \sum_{k,\alpha} M_{k,\alpha} c_k^\dagger A^+(t) c_\alpha + h.c.$$

orbital in the solid

Outgoing electron with asymptotic momentum k

Sudden approximation: No interaction between outgoing electron states and electrons in solid

$$I_k(E, t_p) = -i \int dt dt' e^{iE(t-t')} \sum_{\alpha,\alpha'} M_{k,\alpha} M_{k,\alpha'} G_{\alpha,\alpha'}^<(t_p + t, t_p + t') S(t) S(t')$$

Green-function of solid only

Pulse Autocorrelation function

Freericks, Krishnamurthy & Pruschke, Phys. Rev. Lett. **102**, 136401 (2009).

Eckstein & Kollar, Phys. Rev. B, **78**, 245113 (2008).

Mixed time-frequency representation

Mixed time-frequency representation:

Gaussian pulse envelope

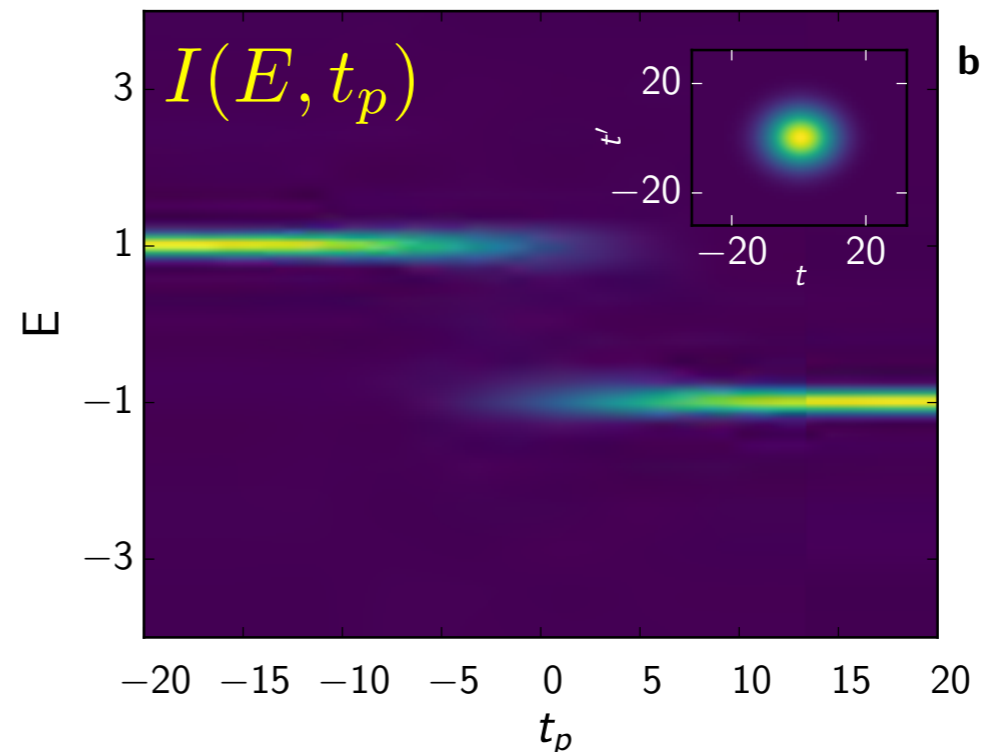
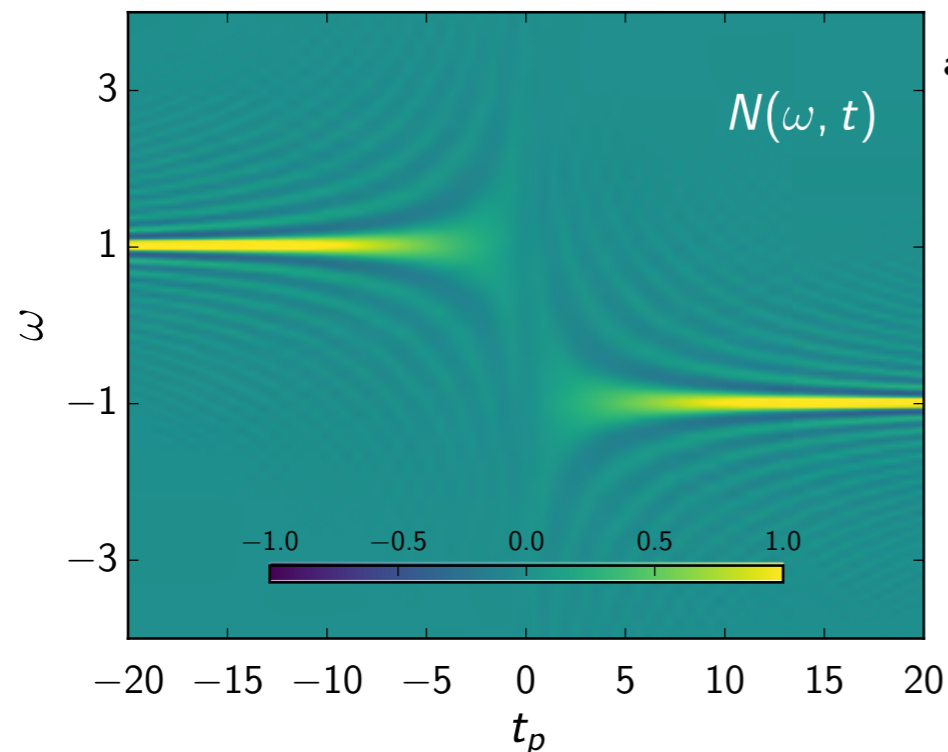
$$s(t) \propto e^{-(t/\delta t)^2}$$

$$H(t) = \begin{cases} \epsilon c^\dagger c & t < 0 \\ -\epsilon c^\dagger c & t > 0 \end{cases}$$

Wigner transform (time-dependent band structure)

$$I(E, t_p) \propto \int d\omega dt e^{-\frac{t^2}{\Delta t^2}} e^{-\omega^2 \Delta t^2} N(E + \omega, t_p + t)$$

filter constrained by energy-time uncertainty



with Francesco Randi, Daniele Fausti (Trieste)

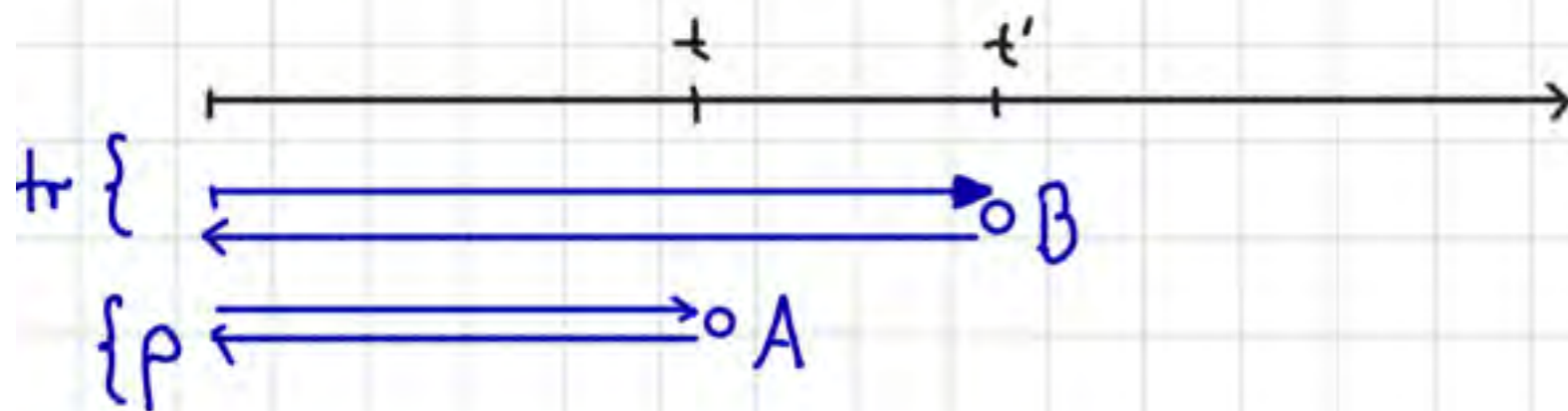
Many body theory with non-equilibrium Green's functions

Green's functions: Book-keeping trick (Keldysh)

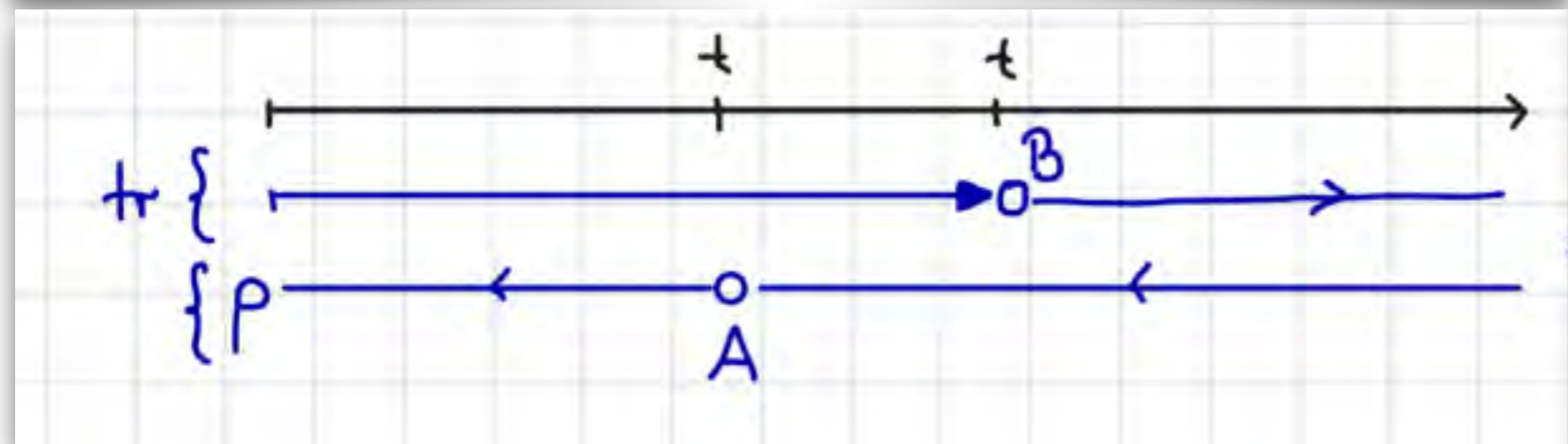
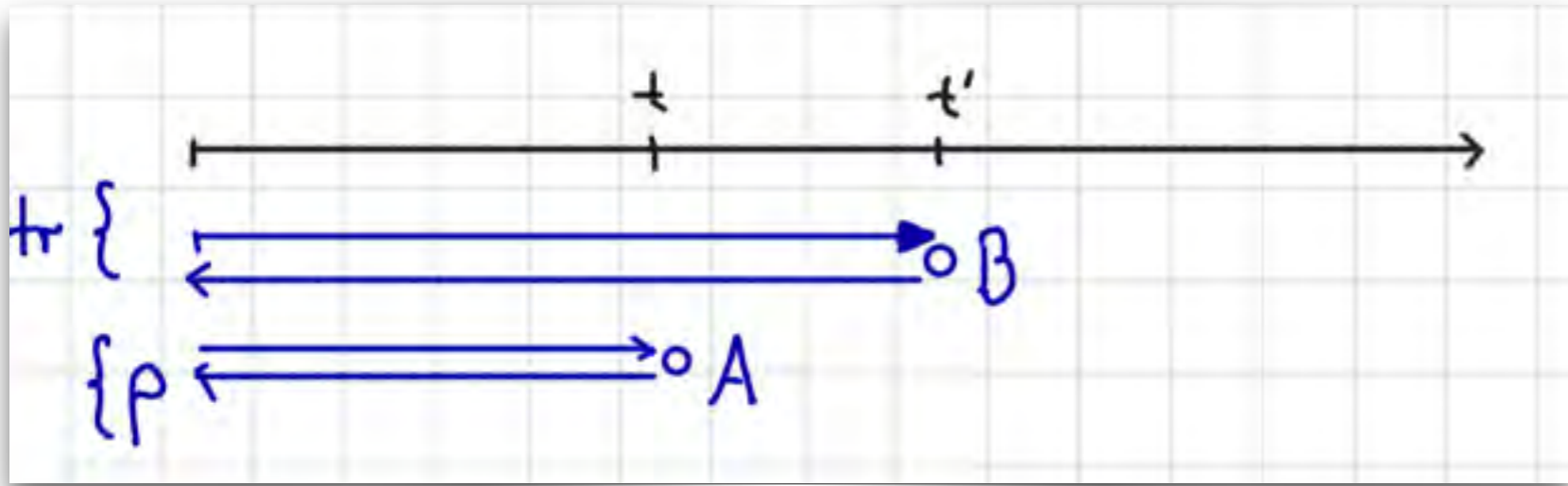
$$\langle A_H(t) B_H(t') \rangle \quad A_H(t) = e^{+iHt} A e^{iHt} \quad \langle \dots \rangle = \text{tr}(\rho_0 \dots)$$

$$t_1 \xrightarrow{\quad} t_2 = e^{-iH(t_2-t_1)} \equiv T_t e^{-i \int_{t_1}^{t_2} d\bar{t} H(\bar{t})}$$

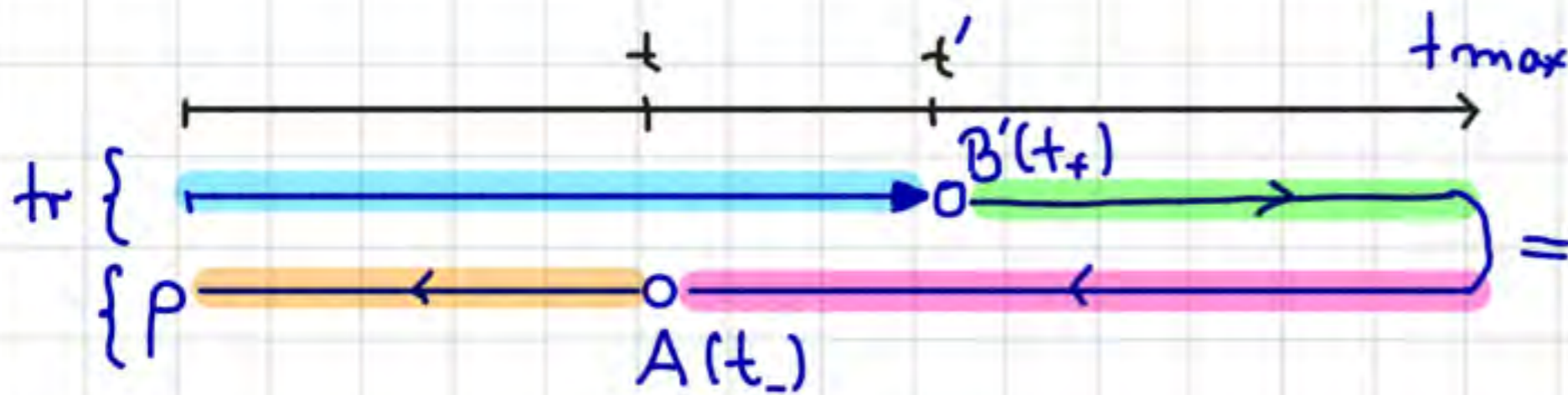
$$t_1 \xleftarrow{\quad} t_2 = e^{iH(t_2-t_1)} \equiv T_{\bar{t}} e^{-i \int_{t_2}^{t_1} d\bar{t} H(\bar{t})}$$



Green's functions: Book-keeping trick (Keldysh)



Green's functions: Book-keeping trick (Keldysh)

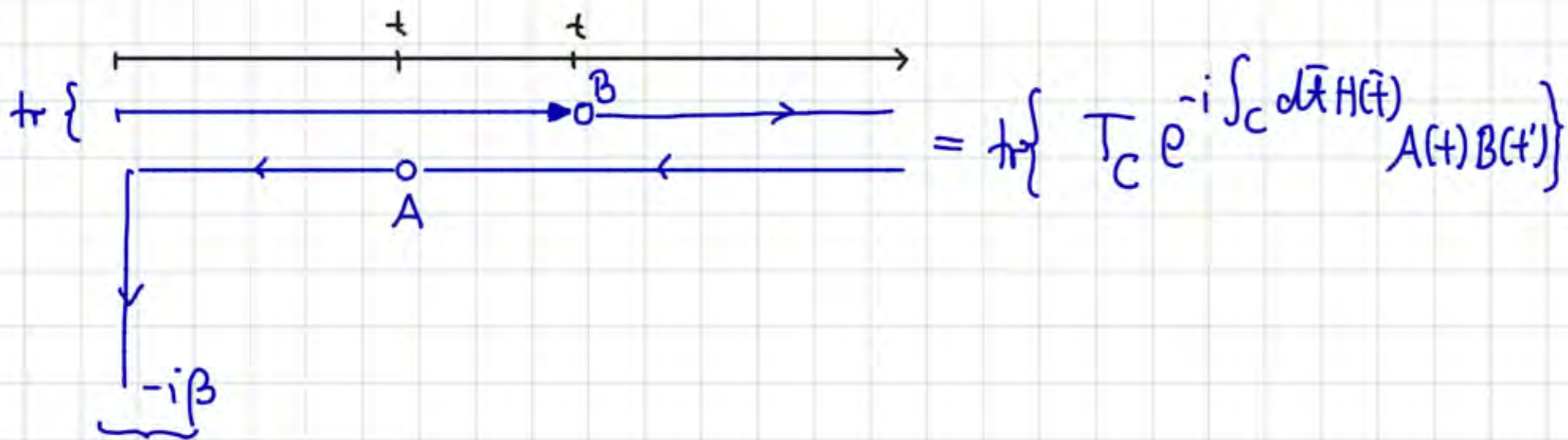
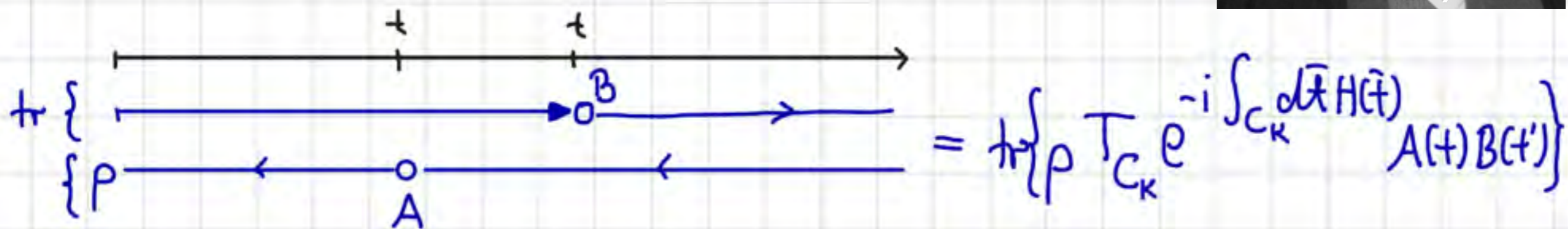
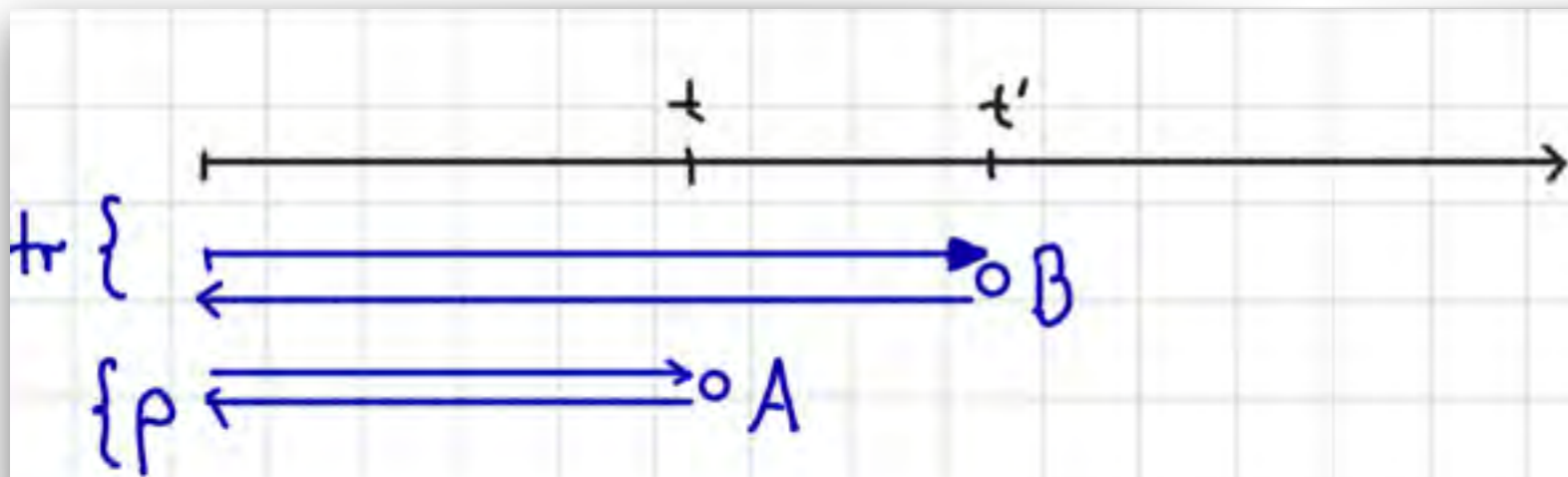
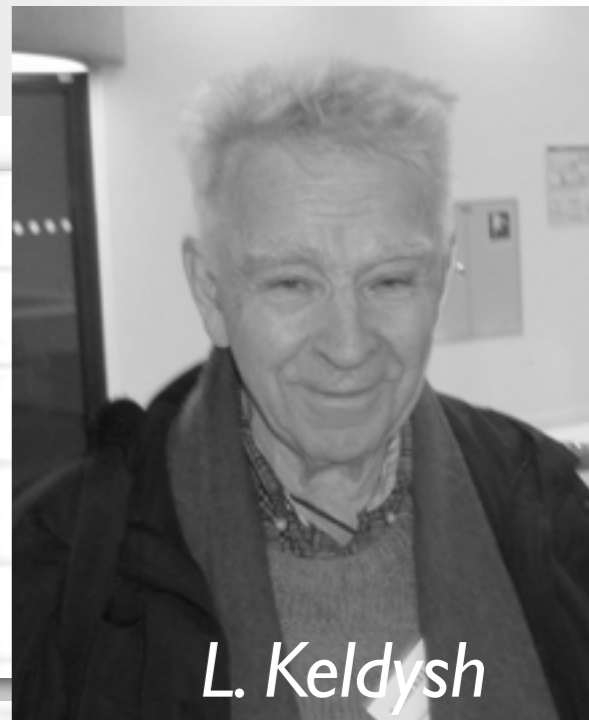


$$\text{tr} \{ \rho A_H(t) B_H(t') \} =$$

$$= \text{tr} \left\{ \rho \left[T_{\tilde{t}} e^{-i \int_t^0 d\tilde{t} H(\tilde{t})} \right] A \left[T_{\tilde{t}} e^{-i \int_{t_{max}}^+ d\tilde{t} H(\tilde{t})} \right] \right. \\ \left. \times \left[T_t e^{-i \int_{t'}^{+t_{max}} d\tilde{t} H(\tilde{t})} \right] B \left[T_t e^{-i \int_0^+ d\tilde{t} H(\tilde{t})} \right] \right\}$$

$$= \text{tr} \left\{ \rho T_{C_K} e^{-i \int_{C_K} d\tilde{t} H(\tilde{t})} A(t_-) B(t'_+) \right\}$$

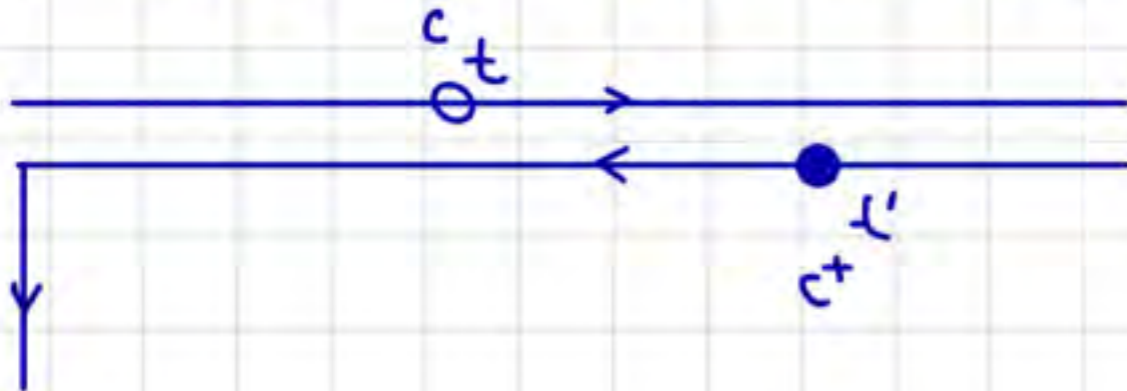
Green's functions: Book-keeping trick (Keldysh)



Green's functions: Book-keeping trick (Keldysh)

$$G(t, t') = -i \langle T_C c(t) c^\dagger(t') \rangle$$

$$\approx G^<(t, t') = i \langle c^\dagger(t') c(t) \rangle \equiv G(t_+, t'_-)$$



Contour-ordered correlation functions

$$\approx \text{path integrals: } \langle T_C A(t) B(t') \rangle_S = \int \mathcal{D}(c^*, c) e^{iS} A(t) B(t')$$

$c(0^+) = -c(-i\beta)$

$$S = \int_C dt \{ c^*(t) i \partial_t c(t) - H[c^*, c] \}$$

Green's functions: Book-keeping trick (Keldysh)

Feynman diagrams:

Obtained from S like in equilibrium!

Example:

$$\overleftarrow{G} = \overleftarrow{G_0} + \underbrace{\overleftarrow{G} \circlearrowleft \Sigma \overleftarrow{G}}_{\int dt_1 dt_2 G_0(t, t_1) \Sigma(t_1, t_2) G(t_2, t')}$$

$$\int dt_1 dt_2 G_0(t, t_1) \Sigma(t_1, t_2) G(t_2, t')$$



$$\Sigma(t, t') = U^2 G(t, t') G(t, t') G(t', t)$$



integral equation on C

Kadanoff-Baym equations

Dyson equation

→ equation of motion for Green's functions
with nonlinear Memory kernel $\Sigma[G]$

time-dependent mean-field

$$\left(i\partial_t - H_{MF}(t)\right)G(t, t') - \int_{\text{previous time}} ds \Sigma[G](t, s)G(s, t') = \delta(t, t')$$

„Kadanoff-Baym equations“

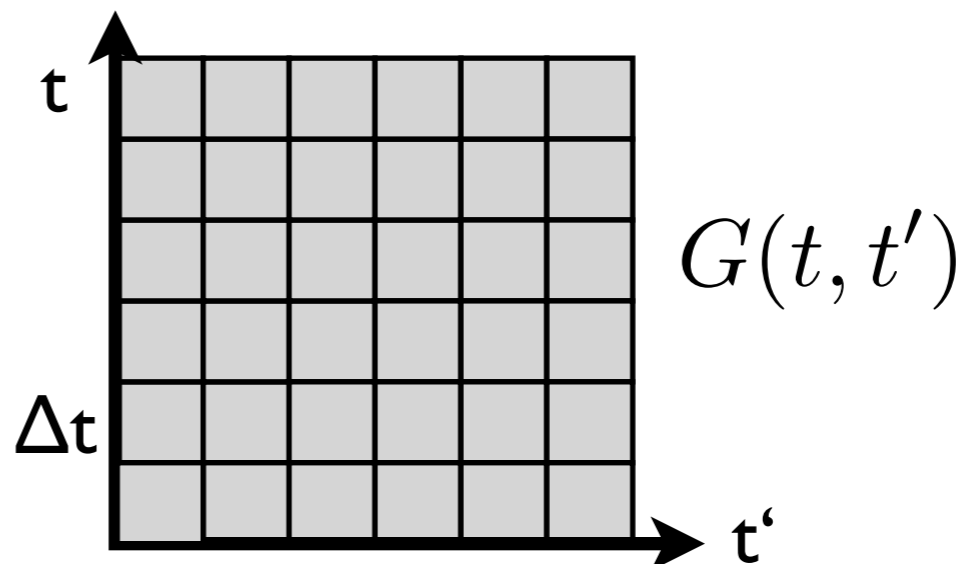
Kadanoff & Baym; Bonitz 2000; Bonitz
and Semkat 2003, Balzer & Bonitz 2012

„causal“ time-propagation:

Memory: $\sim N^2 L^2$

Compute time: $\sim N^3 L^3$

(N: #timesteps, L#orbitals)



Dynamical mean-field theory

Georges, Rozenberg, Krauth & Kotliar, RMP 1996
Aoki, Tsuji, ME, Kollar, Oka, Werner, RMP 2014

$$\overleftrightarrow{G}_k = \overleftrightarrow{G}_{0,k} + \overleftrightarrow{G}_{0,k} \circlearrowleft \Sigma \overleftrightarrow{G}_k$$

$$G_{0,k} = -i \langle T_c c_k(t) c_k^\dagger(t') \rangle_{H_0}$$

no interaction
 \Rightarrow solve $c_k(t)$
 by equations of
 motion
 (with external fields!)

$$\Sigma[G] \equiv \sum_{\text{loc}} [G_{\text{loc}}] \quad \text{all local diagrams}$$



$$S = S_{\text{loc}} + \int dt dt' c^*(t) \Delta(t, t') c(t')$$

$$G_{\text{imp}} = G_{\Delta} + G_{\Delta} \Sigma G_{\text{imp}}$$

$$= \frac{1}{Z} \int \mathcal{D}(c^*c) e^{iS} c(t) c^*(t')$$

Impurity problem:
 Many-body problem:
 QMC, IPT, NCA, ED,
 DMRG,

Formulation in Keldysh framework: Schmid & Monien (2002) Freericks, Turkowski, Zlatic 2006

The quantum impurity problem

Solution of effective impurity problem:

CT-QMC: numerically exact, phase problem
bold-line version work by E. Gull et al.

Strong-coupling expansion: NCA, OCA

ok for Mott phase;

- ✓ multi-orbital extension,
- ✓ bosonic systems,
- ✓ phonons (Hubbard Holstein)

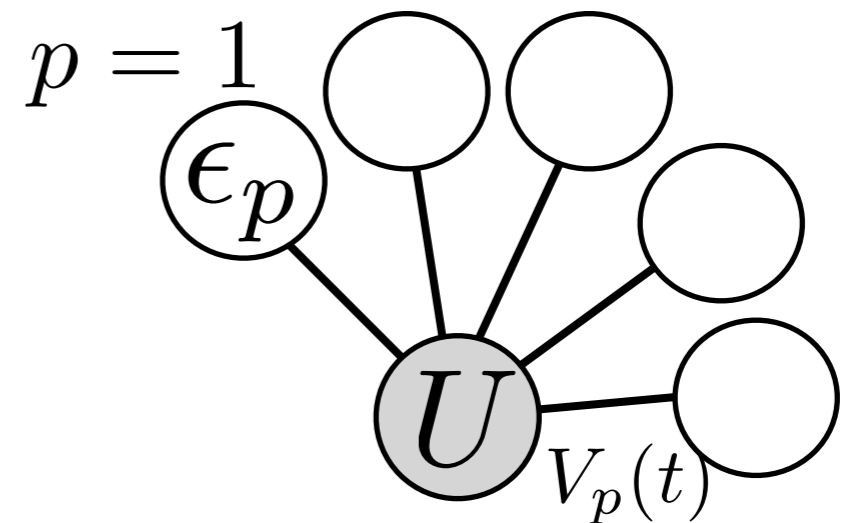
Weak-coupling perturbation theory

Finite representation of the bath

Gramsch, Balzer, ME, Kollar PRB 2013,

$$\Delta(t, t') \leftrightarrow \sum_p V_p(t) (i\partial_t - \epsilon_p)^{-1} V_p(t')^*$$

$p = 2 \quad \dots$



$$H_{imp} = U n_{\uparrow} n_{\downarrow} + \sum_{p\sigma} \epsilon_p a_{p\sigma}^{\dagger} a_{p\sigma} + \sum_{p\sigma} (V_p(t) c_{\sigma}^{\dagger} a_{p\sigma} + h.c.)$$

- Krylov
- Matrix-product states (td-DMRG)

Strong-coupling expansion

Strong-coupling expansion

Expansion in the coupling to the bath?

$$\langle \mathcal{O}(t) \rangle = \frac{1}{Z} \text{tr} \left[T_{\mathcal{C}} e^{-i \int_{\mathcal{C}} dt' H_{loc}(t')} e^{-i \int_{\mathcal{C}} dt_1 dt_2 c^\dagger(t_1) \Delta(t_1, t_2) c(t_2)} \mathcal{O}(t) \right]$$

0th order is interacting Hamiltonian $H_{loc} \Rightarrow$ very general starting point

But: no Wick's theorem \leadsto Resolvent expansions:

- Perturbative: “non-crossing approximation”, first developed for the Kondo model

Keiter & Kimball '71; Kuramoto '83; Grewe '83; Pruschke & Grewe 89
Bickers, Cox & Wilkins '87; Coleman '83; Haule, Kirchner, Kroha & Wölfle '01

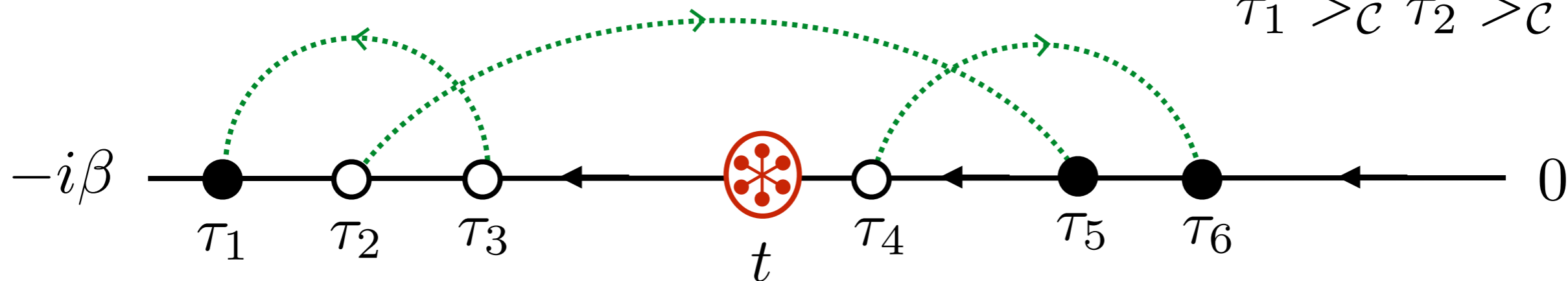
- Hybridization Quantum Monte Carlo: *Werner et al., 2006*
Stochastic resummation of perturbation series

Strong-coupling expansion

$$\begin{aligned}
 \langle \mathcal{O}(t) \rangle &= \frac{1}{Z} \text{tr} \left[T_C e^{-i \int_C dt' H_{loc}(t')} e^{-i \int_C dt_1 dt_2 c^\dagger(t_1) \Delta(t_1, t_2) c(t_2)} \mathcal{O}(t) \right] \\
 &= \frac{1}{Z} \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_C dt_1 dt'_1 \cdots dt_n dt'_n \Delta(t_1, t'_1) \cdots \Delta(t_n, t'_n) \times \\
 &\quad \times \text{tr} \left[T_C e^{-i \int_C dt' H_{loc}(t')} c^\dagger(t_1) c(t'_1) \cdots c^\dagger(t_n) c(t'_n) \mathcal{O}(t) \right]
 \end{aligned}$$

Sum of all possible contributions like this:

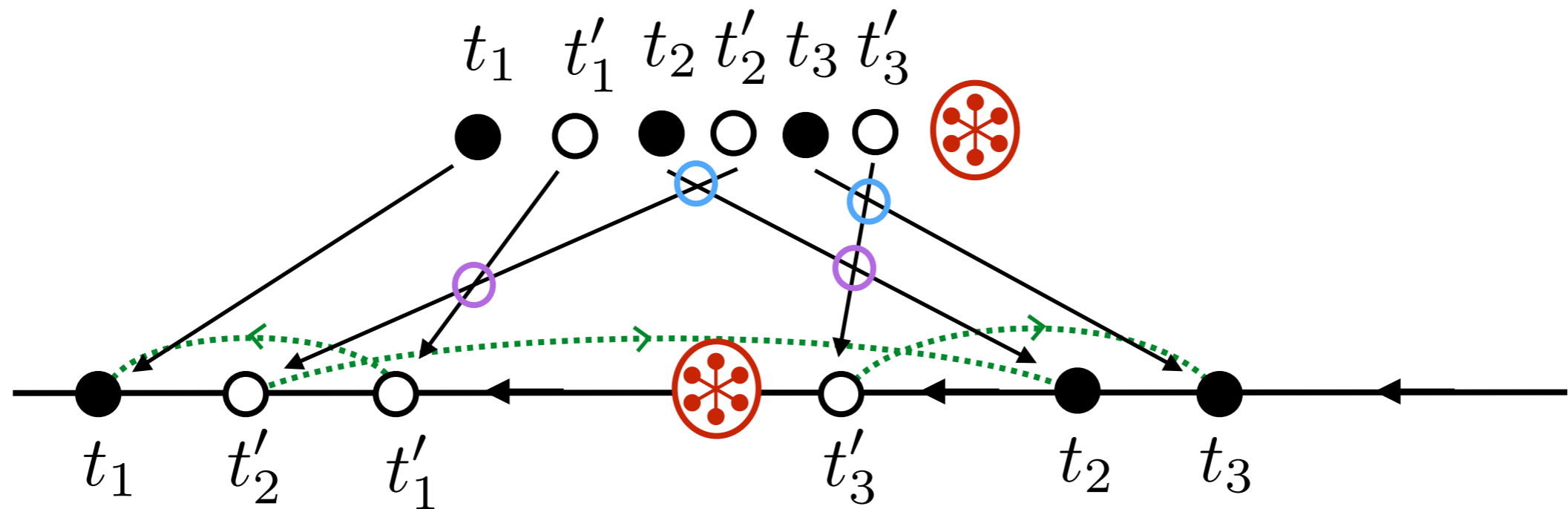
contour ordered
 $\tau_1 >_C \tau_2 >_C \dots$



$$\left. \begin{aligned}
 \text{⊗} &= \mathcal{O} & \bullet &= c^\dagger & \circ &= c & t' \cdots \rightarrow t &= \Delta(t, t') \\
 t' \longrightarrow t &= g(t, t') = T_C e^{-i \int_{t'}^t ds H_{loc}(s)} & & & & & &
 \end{aligned} \right\} \begin{array}{l} \text{matrices in local} \\ \text{many-body basis} \\ \mathcal{O}_{nm} \equiv \langle n | \mathcal{O} | m \rangle \\ \text{etc.} \end{array}$$

Strong-coupling expansion

sign of the diagram (from reordering operators under contour ordering)



$$(-1)^{\text{crossing of } \Delta \text{ lines}} \times (-1)^{\# \text{ of } \Delta \text{ lines with reversed direction}}$$

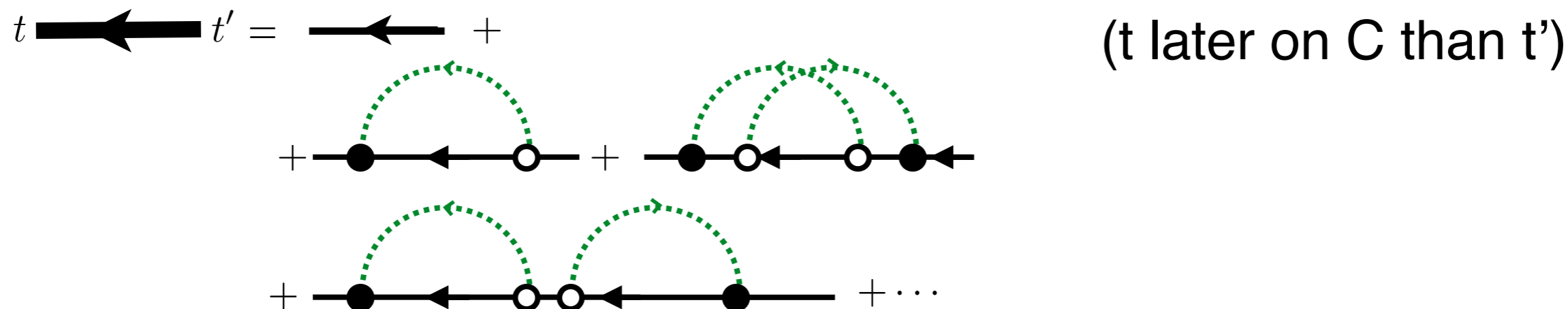
count every topology once removes $n!$ factor

Strong-coupling expansion

No symmetry factors for diagrams
(count every topology once removes $n!$ factor)

\Rightarrow

Define re-summed propagators

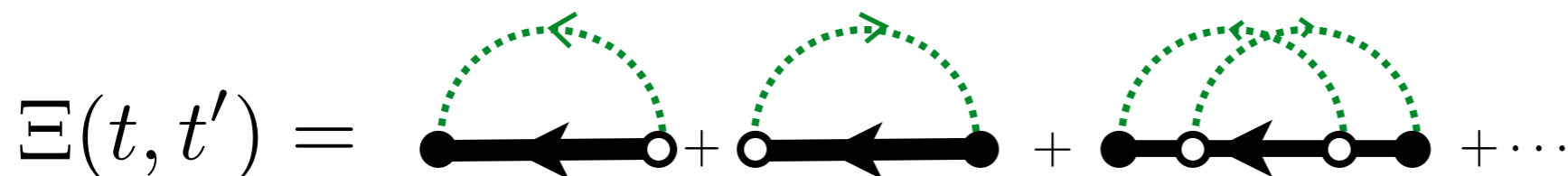


Dyson equation:

$$\mathcal{G}(t, t') = g(t, t') + \int_c^{t'} dt_1 dt_2 g(t, t_1) \Xi(t_1, t_2) \mathcal{G}(t_2, t')$$

$$t >_c t_1 >_c t_2 >_c t'$$

Skeleton expansion



Strong-coupling expansion

$$\overleftarrow{-i\beta} \quad 0^+ = ?$$

Strong-coupling expansion

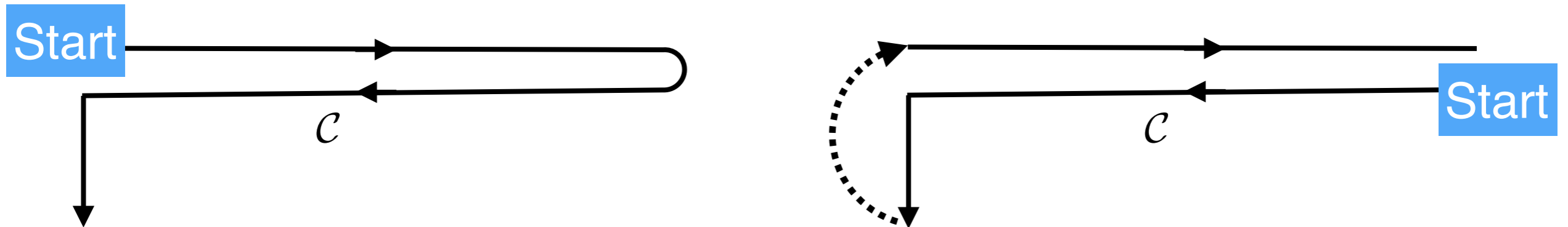
Cyclic permutation under the trace:

$$\text{tr} \left[\leftarrow \bigcirc \text{---} \text{---} \text{---} \bullet \leftarrow \right] = \text{tr} \left[\text{---} \text{---} \text{---} \bigcirc \leftarrow \text{---} \text{---} \text{---} \diamond \right]$$

$t^+ \quad 0^+ - i\beta \quad t^- \quad \xi$

Just redefine starting point on the contour

.... and an operator ξ whenever the propagator winds around 0



contour-ordered integrals \rightarrow cyclically ordered integrals, e.g. in Dyson

Strong-coupling expansion

Summary:

$$\langle \mathcal{O}(t) \rangle = \frac{1}{Z} \text{tr} [\mathcal{G}(t^+, t^-) \mathcal{O} \xi] \quad Z = \text{tr} [\mathcal{G}(t^+, t^-) \xi]$$

$$t \longleftarrow t' = \longleftarrow + \longleftarrow \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \longrightarrow t_2 \quad t, t_1, t_2, t' \text{ cyclic}$$

$$g(t, t') = T_c e^{-i \int_{t'}^t ds H_{loc}(s)}$$

$$g(t, t') = g(t_+, 0^-) \xi g(-i\beta, t') \quad (\text{for } t <_c t')$$

$$\Xi(t, t') = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots$$

Strong-coupling expansion

$$\Xi(t, t') = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots$$

Conserving approximation (Skeleton)

First order: “non-crossing approximation”:
recovers Kondo physics (but with underestimation of T_K)

In DMFT: artefacts in the metallic phase at low temperatures,
second order can already be almost quantitatively correct, in
particular towards the Mott phase

Very general starting point:

In real time, NCA for

Bosons (Bose Hubbard model)

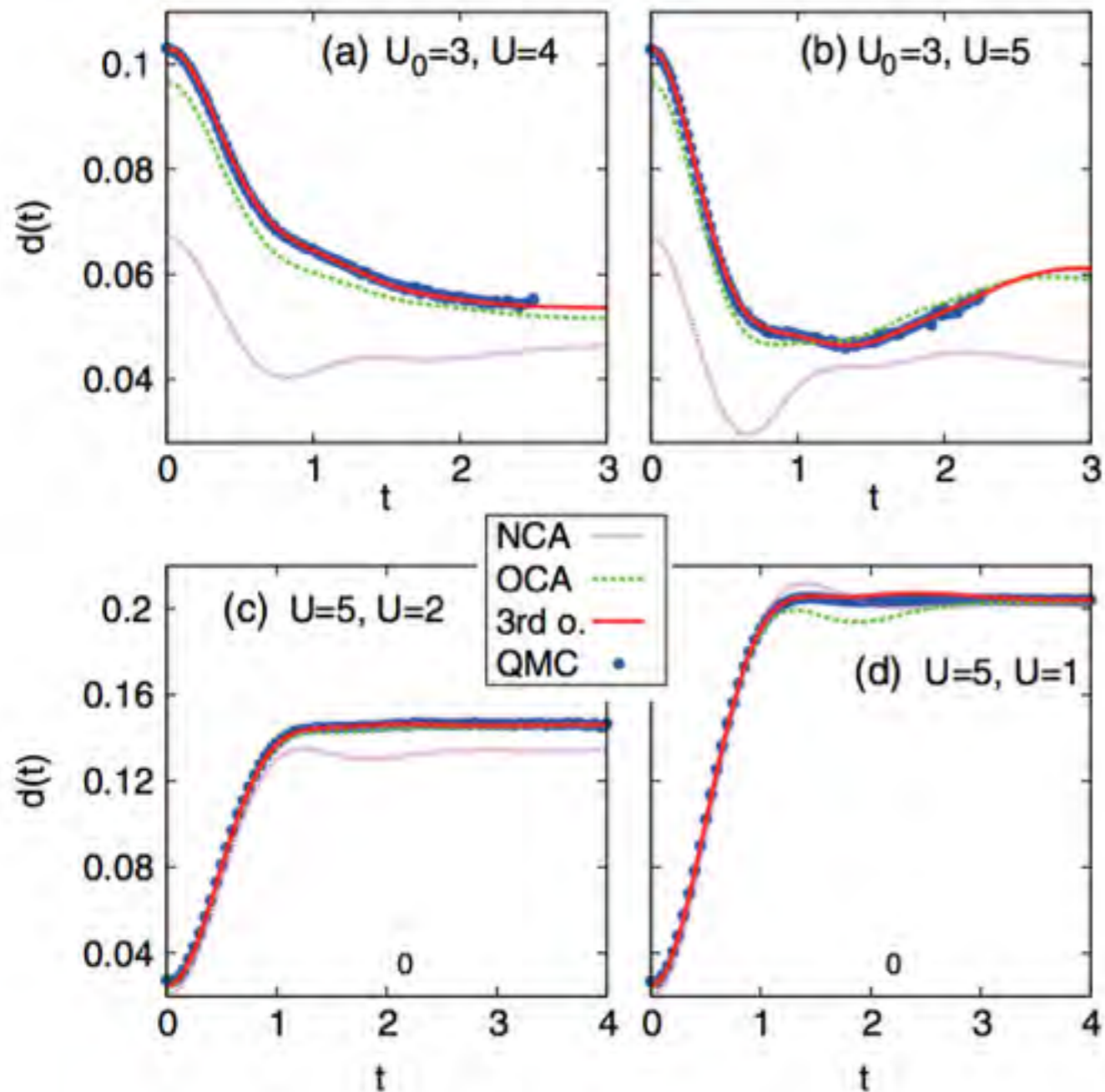
Hubbard-Holstein model

Cluster impurity model (larger local Hilbert space)

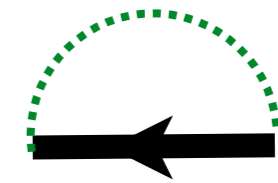
Strong-coupling expansion

Example: Hubbard model, quench U_0 to U

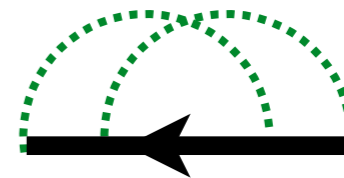
double occupancy $\langle n_{\uparrow} n_{i\downarrow} \rangle$:



NCA



OCA



3rd order

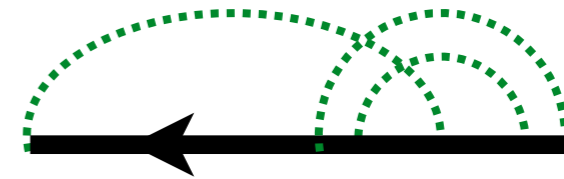
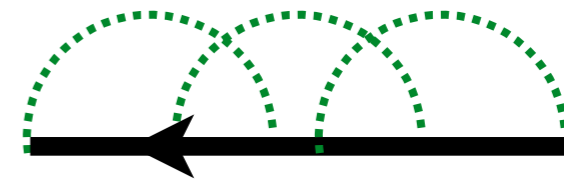
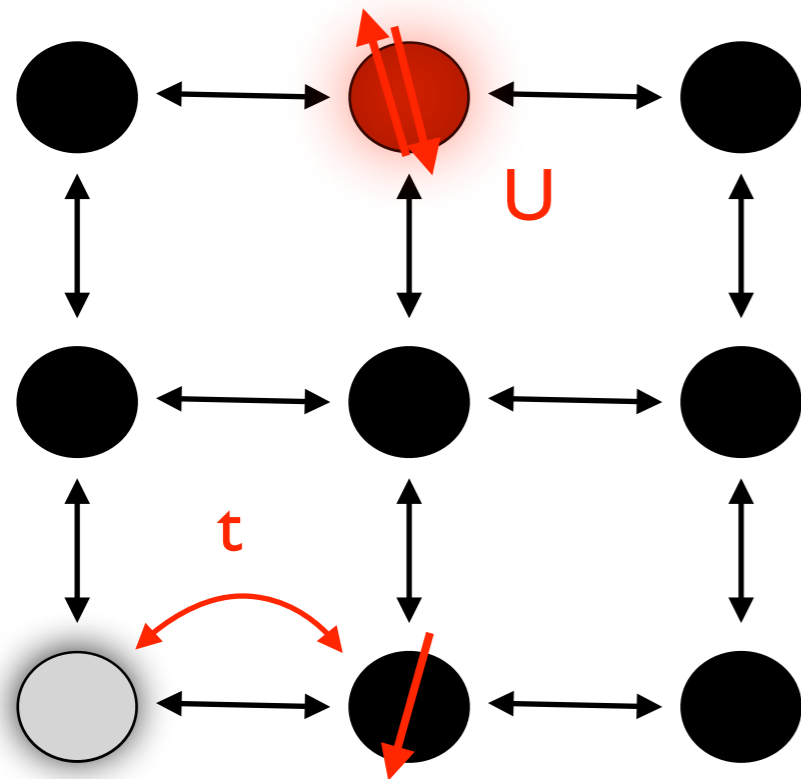


Photo-doped states in the single-band Mott insulator

Hubbard model and Peierls substitution

Hubbard model



$$H = -t \sum_{\langle ij \rangle, \sigma = \uparrow, \downarrow} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

+ electric fields (Peierls substitution)

$$t_{ij} \rightarrow t_{ij} e^{i\phi_{ij}} \quad \phi_{ij} = e\vec{A}(t)(\vec{r}_j - \vec{r}_i)$$

$$\epsilon(\mathbf{k}) \rightarrow \epsilon(\mathbf{k} - \mathbf{A})$$

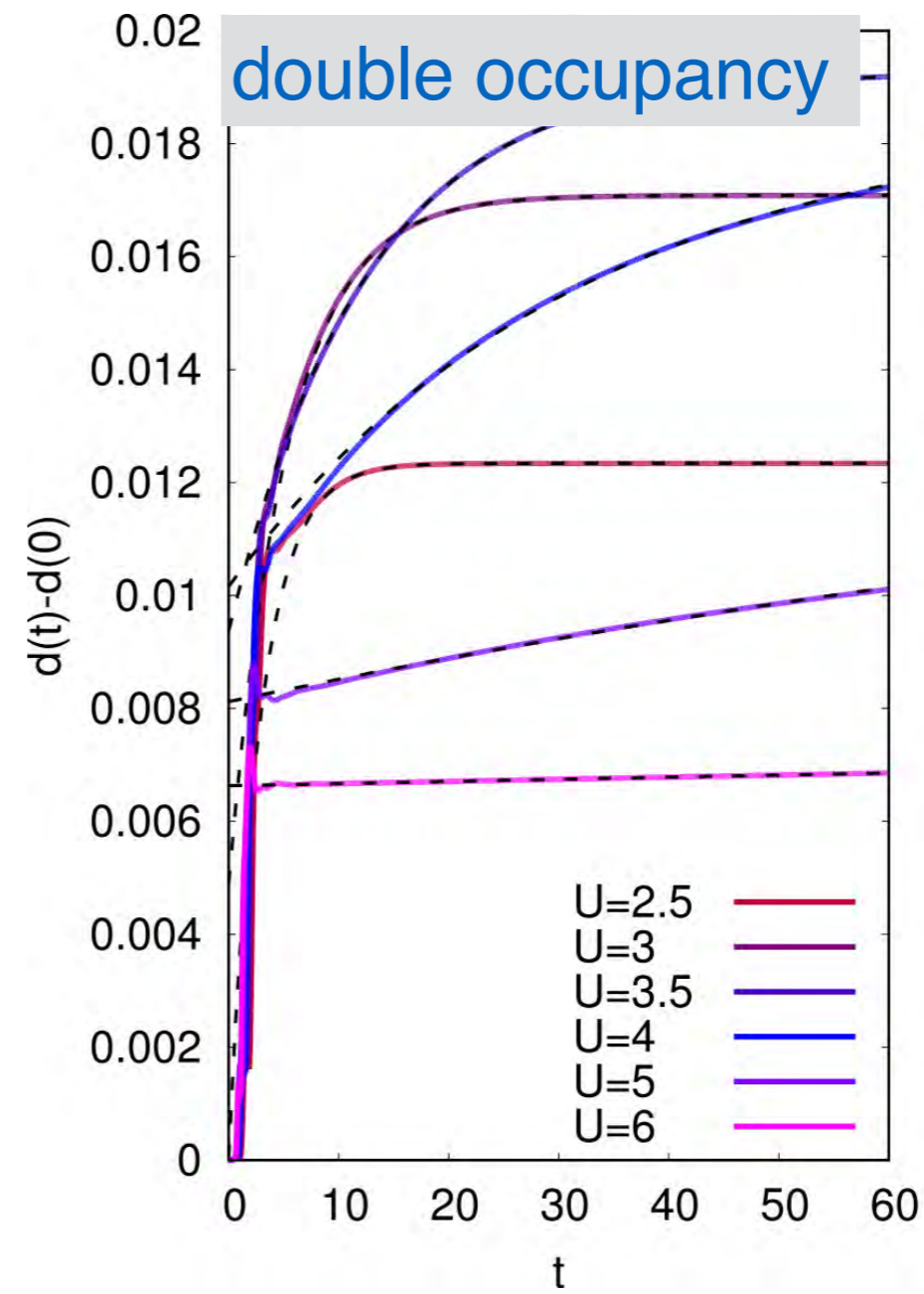
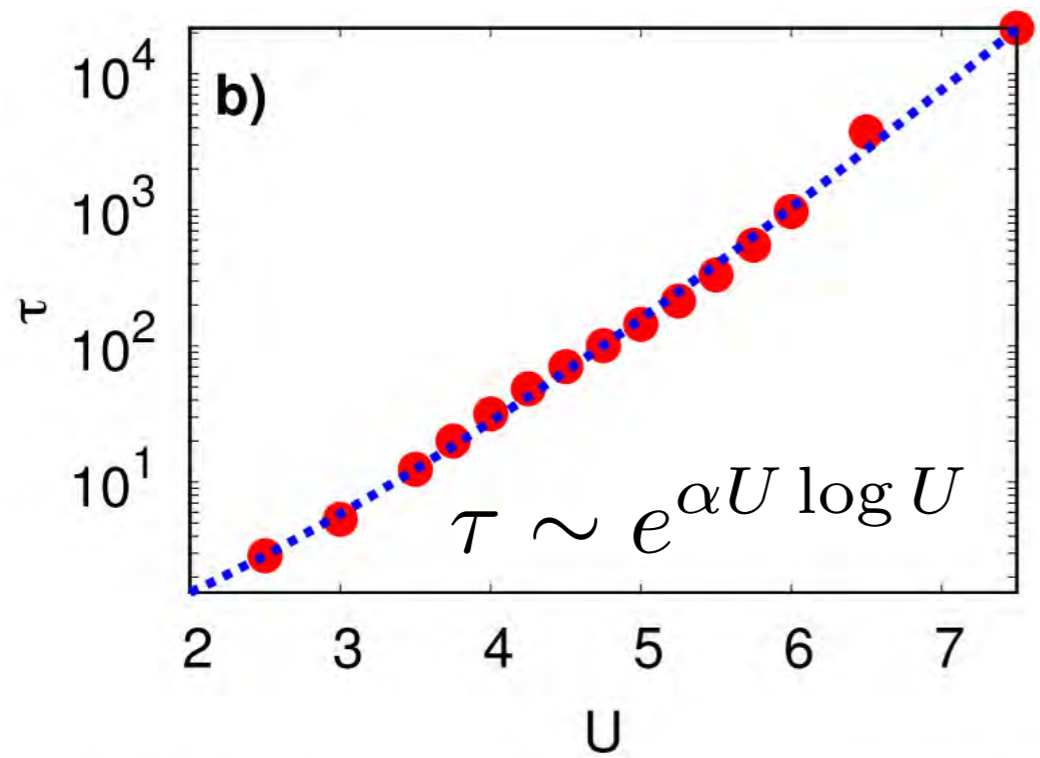
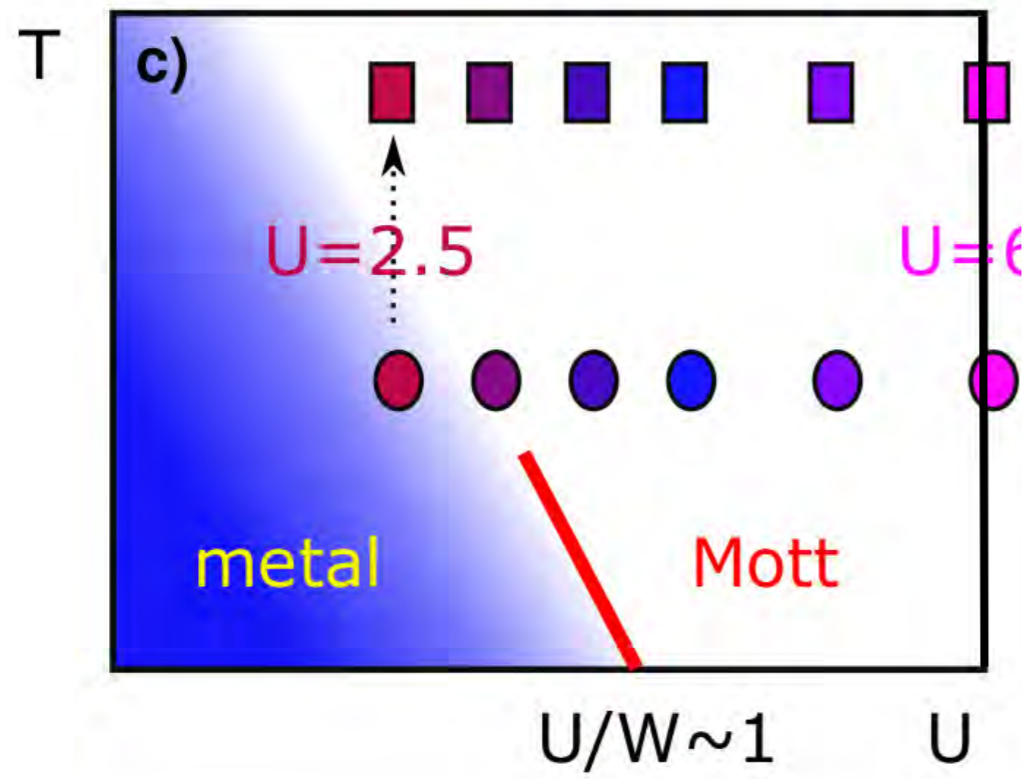
Units:

Energy: hopping $\sim 1\text{eV}$

Time ($\hbar = 1$): $\hbar/1\text{eV} = 1\text{fs}$

Field: hopping / lattice constant

bandwidth=4 (Bethe lattice) (in NCA: $U_c \sim 3.4$)



fit: $d(t) = d(T_{\text{eff}}) + Ae^{-t/\tau}$

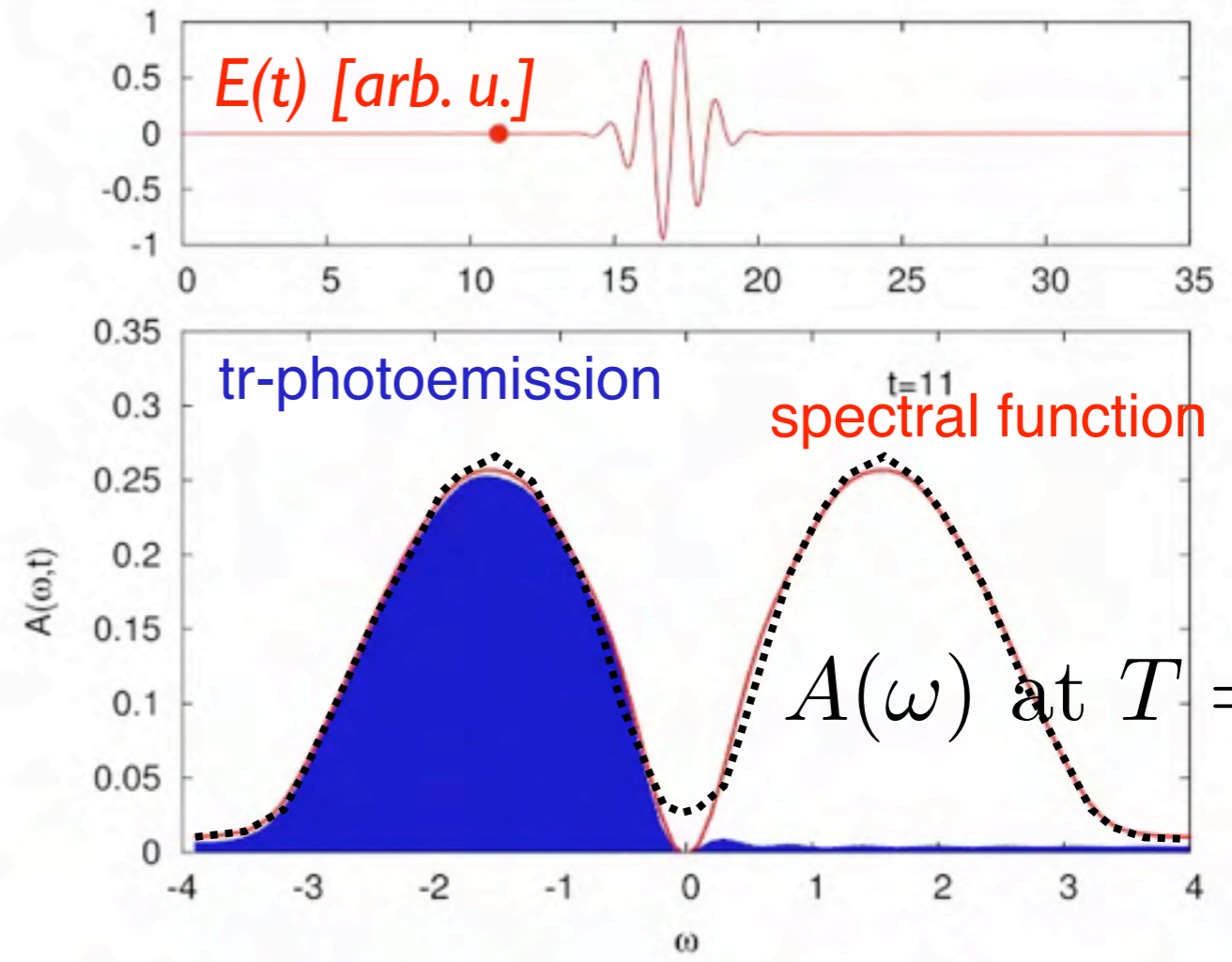
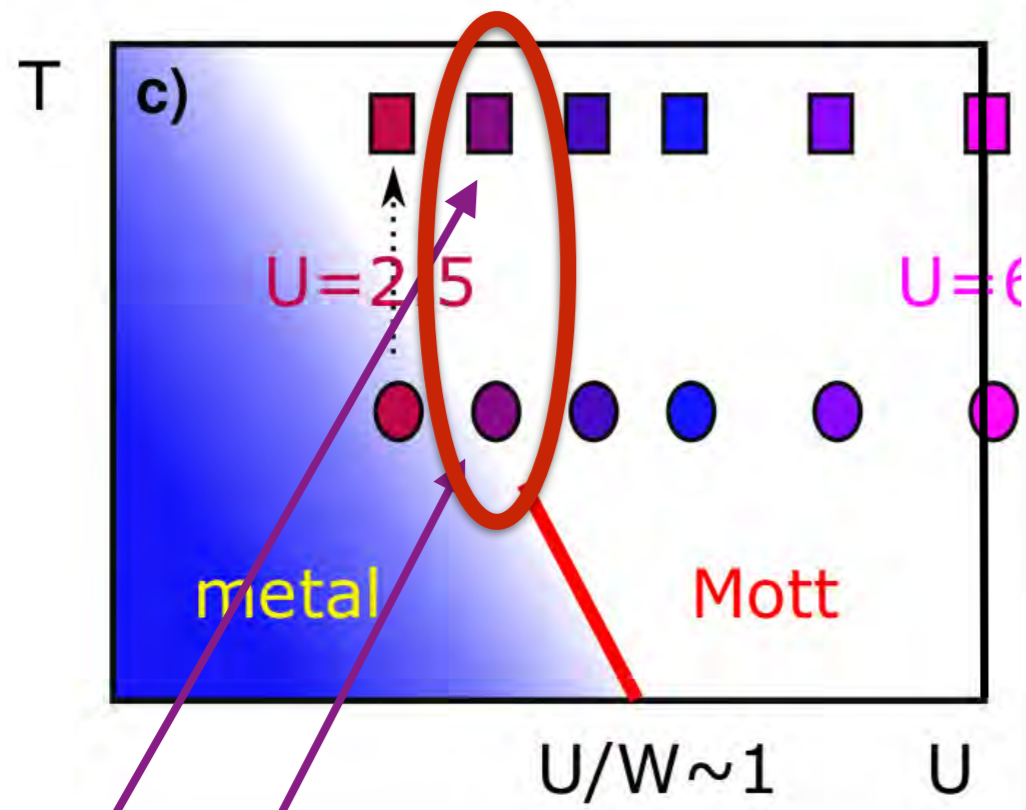
$U >$ bandwidth: slow thermalization

Rosch et al. PRL 2008; Strohmaier et al. PRL 2010

Photo-doping: correlated metal

Eckstein & Werner, PRB (2012)

bandwidth=4 (Bethe lattice) (in NCA: $U_c \sim 3.4$)

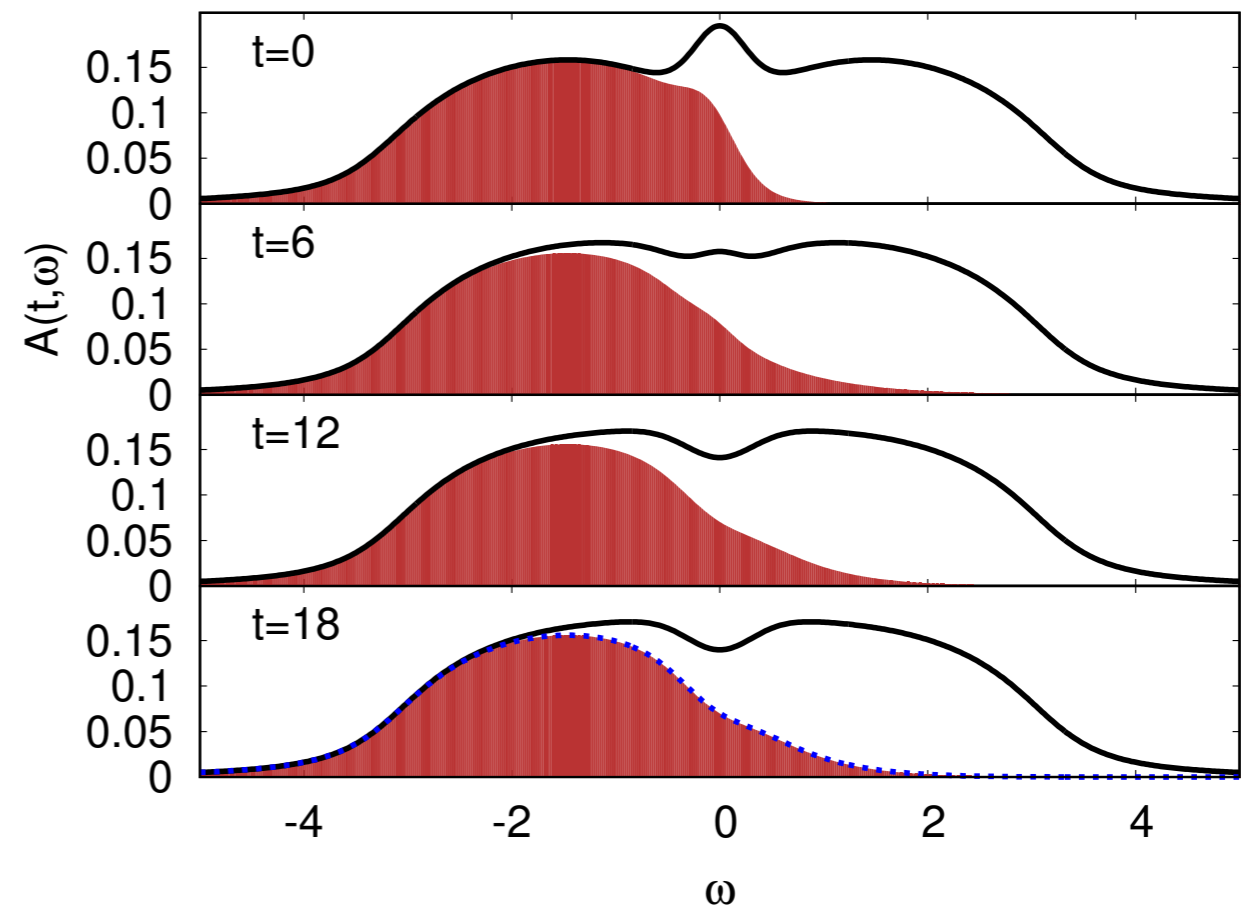
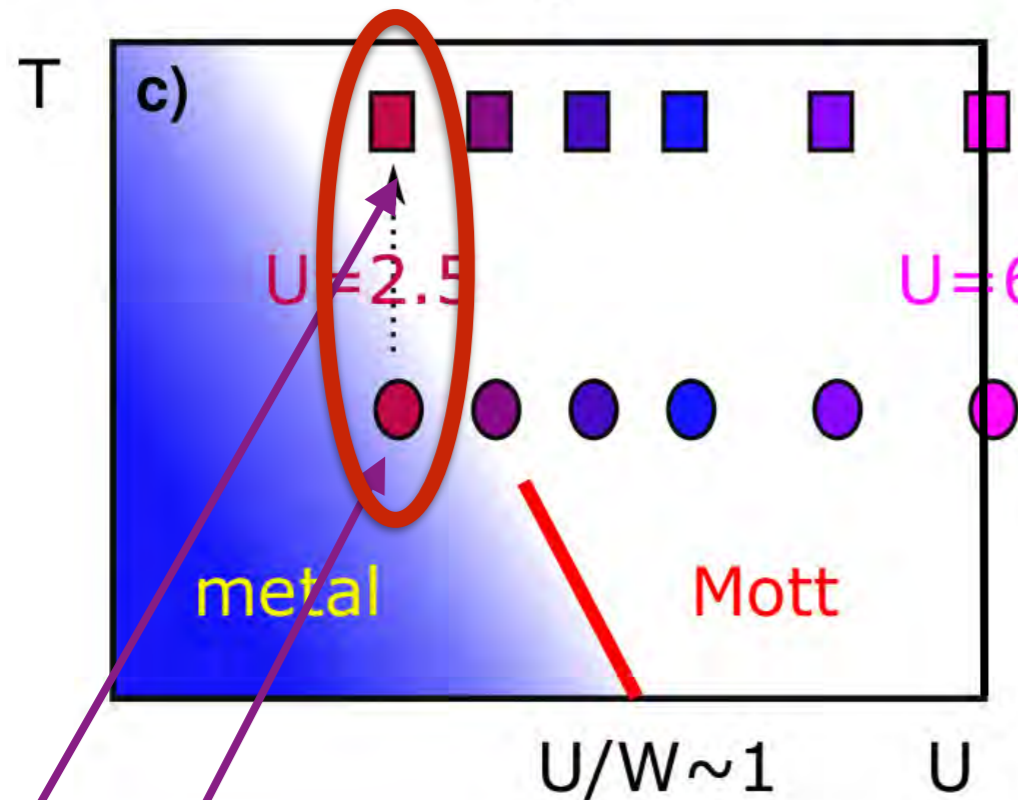


Initial state ($T=0.2$)

after (possible) thermalization

Ultra-fast electron thermalization (without well-defined quasi-particles!)

bandwidth=4 (Bethe lattice) (in NCA: $U_c \sim 3.4$)



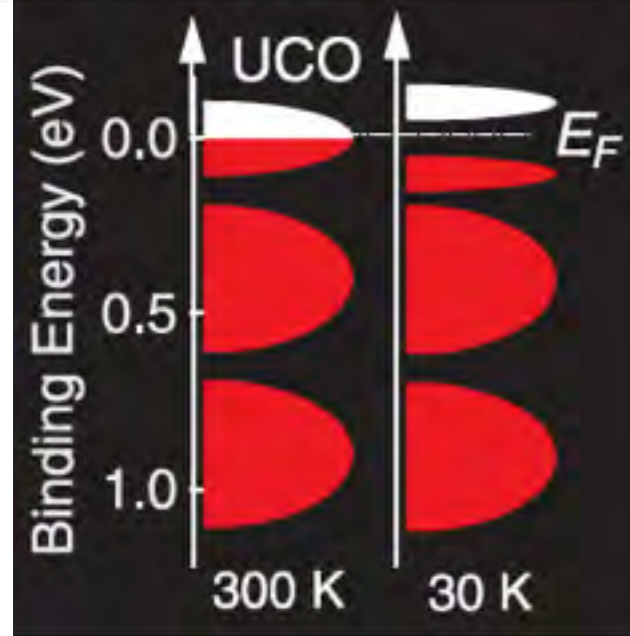
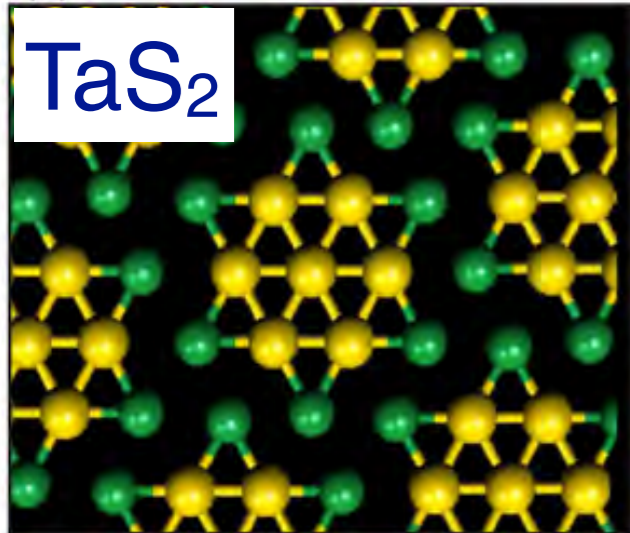
Initial state ($T=0.2$)

after (possible) thermalization

Ultra-fast electron thermalization (without well-defined quasi-particles!)

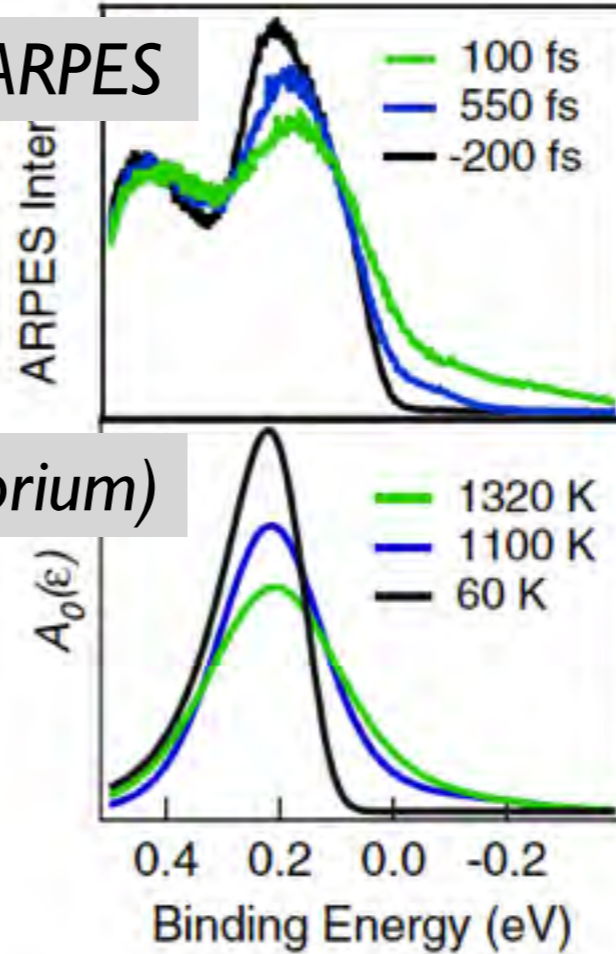
Experiment: Manuel Ligges et al. arXiv:1702.05300 (PRL)

Ultra-fast electron thermalization:

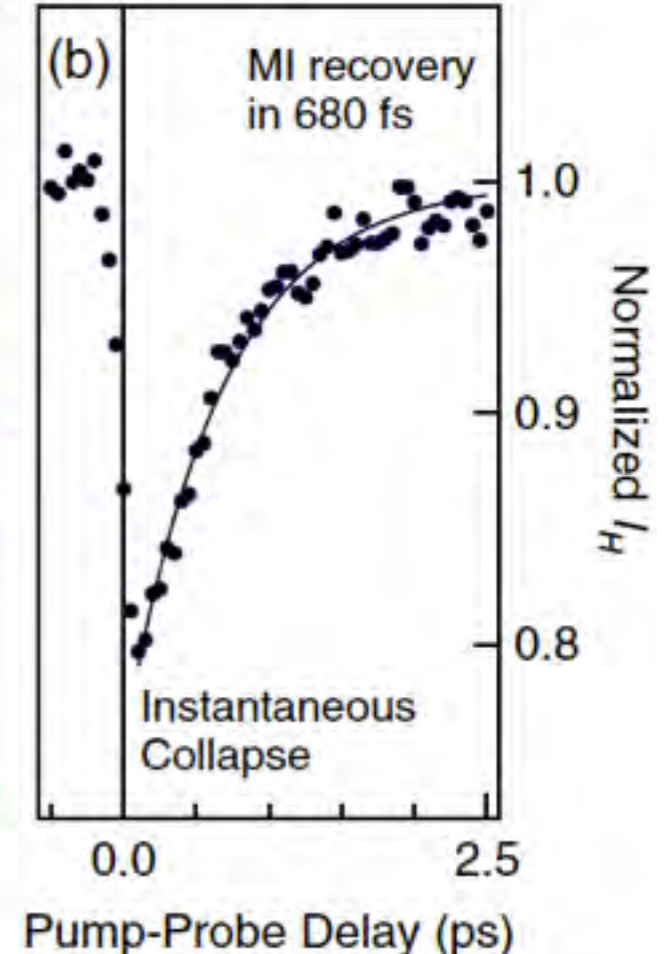


Perfetti et al., Phys. Rev. Lett. **97**, 067402 (2007)

time-resolved ARPES



DMFT (equilibrium)



⇒ consistent with photo-induced transition to bad metallic state at high electronic temperature

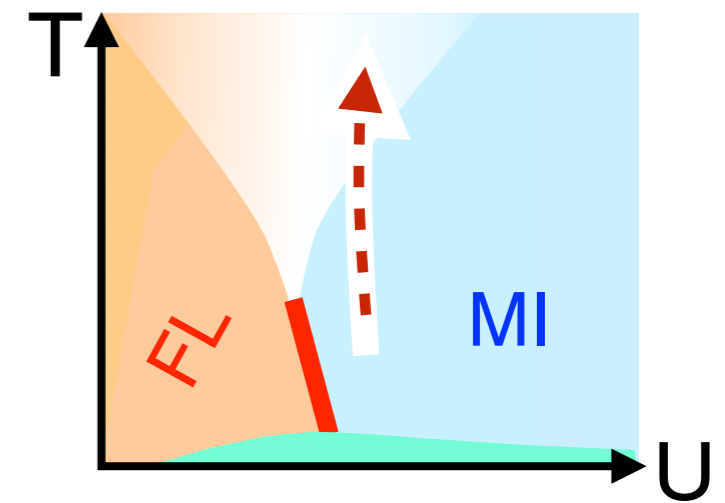
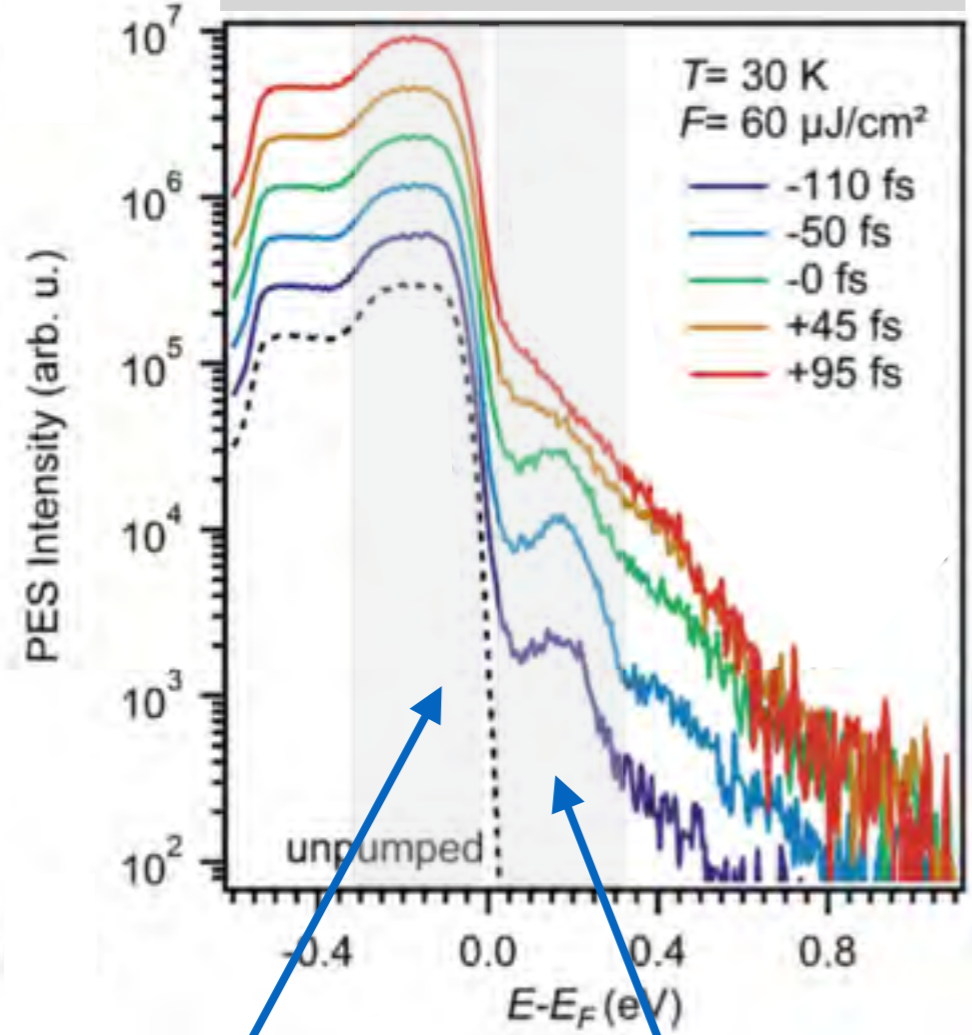


Photo-doping: correlated metal

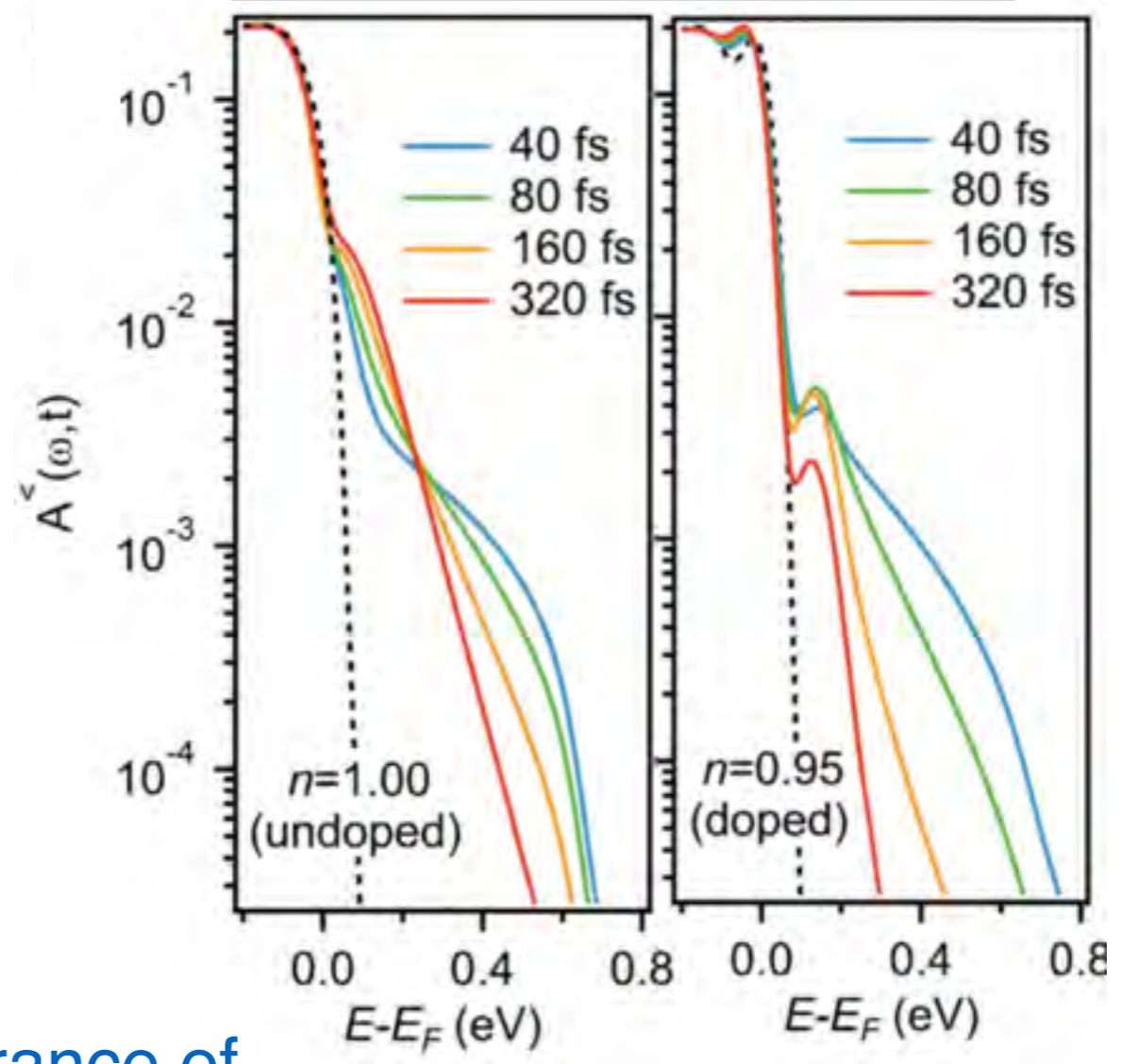
New measurement on TaS₂

Ligges et al., arXiv 2017 (to appear in PRL)

time-resolved ARPES



DMFT (non-equilibrium)

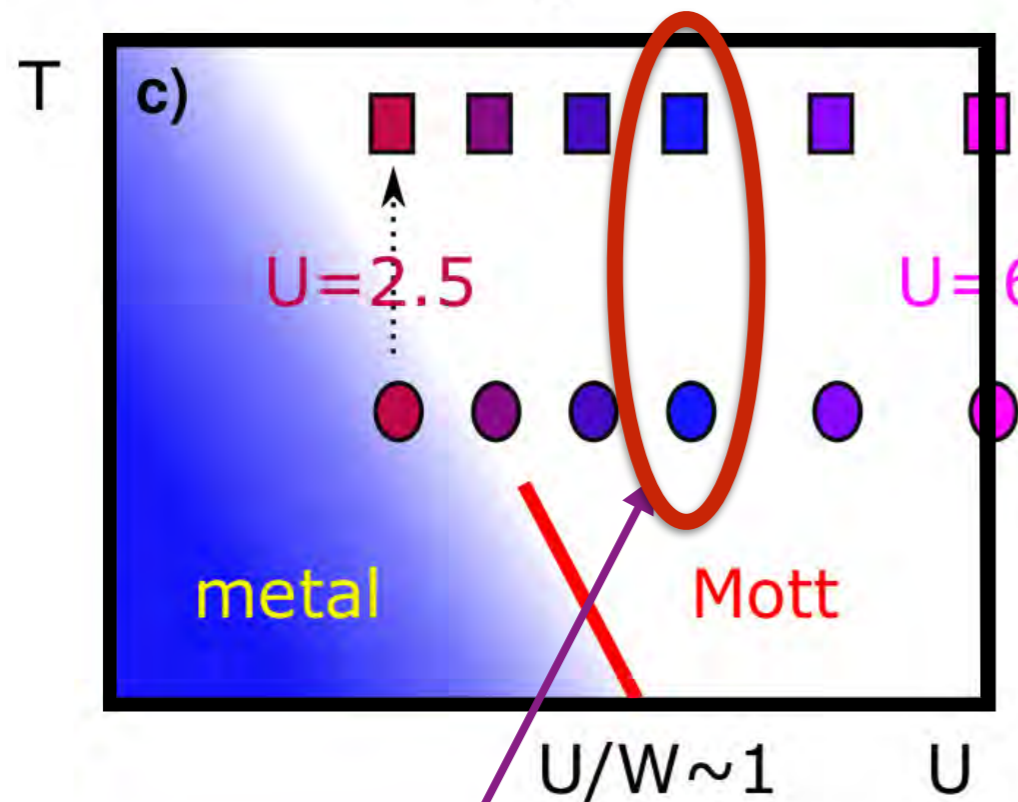


lower Hubbard band transient appearance of occupied upper HB decay less than 50fs

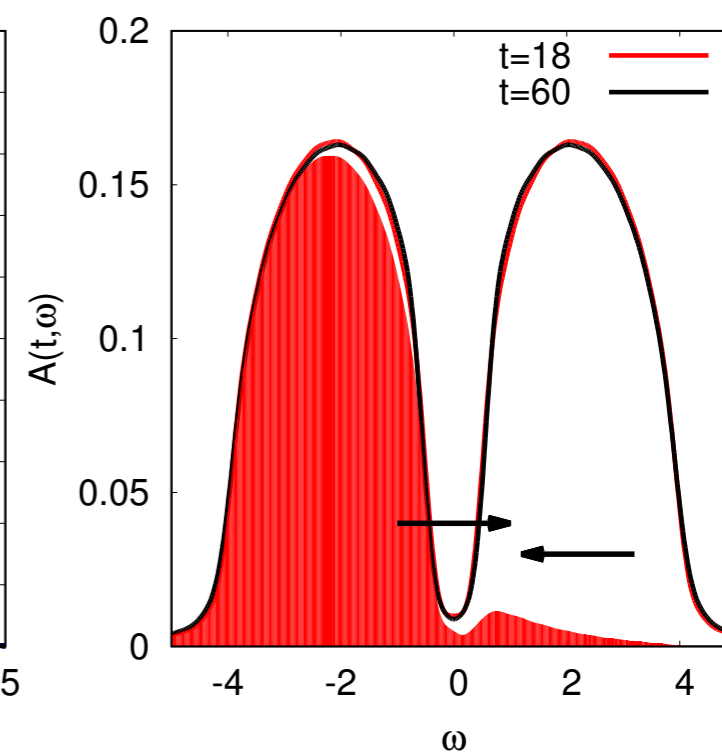
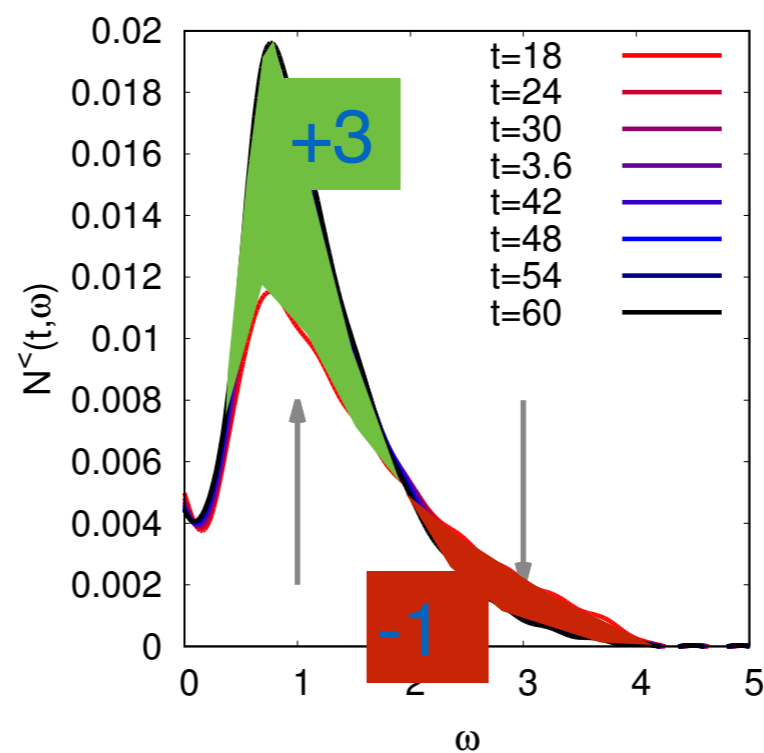
⇒ consistent with electronic thermalization in slightly doped regime

Photo-doping: correlated metal

bandwidth=4 (Bethe lattice) (in NCA: $U_c \sim 3.4$)



Initial state ($T=0.2$)



Thermalization involves “impact ionization”

Werner, Eckstein, Held, PRB (2014)