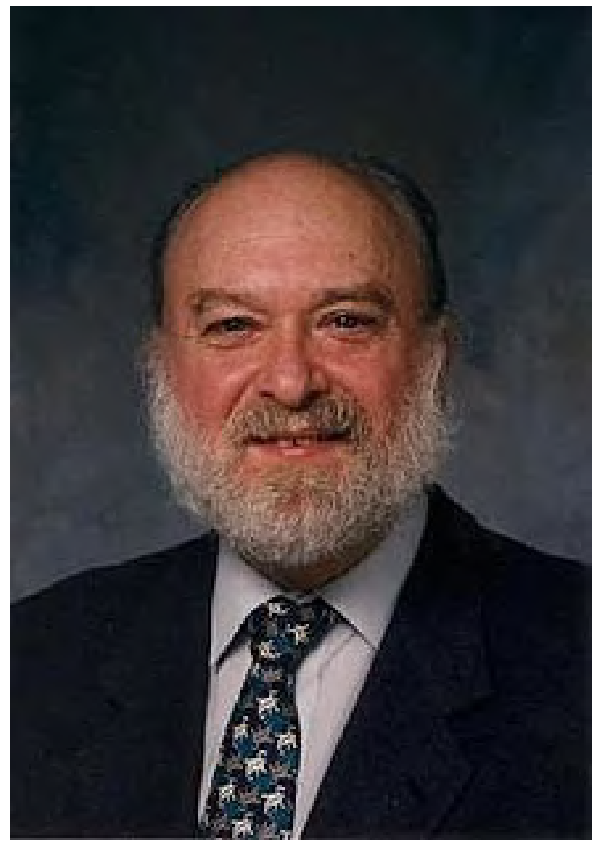
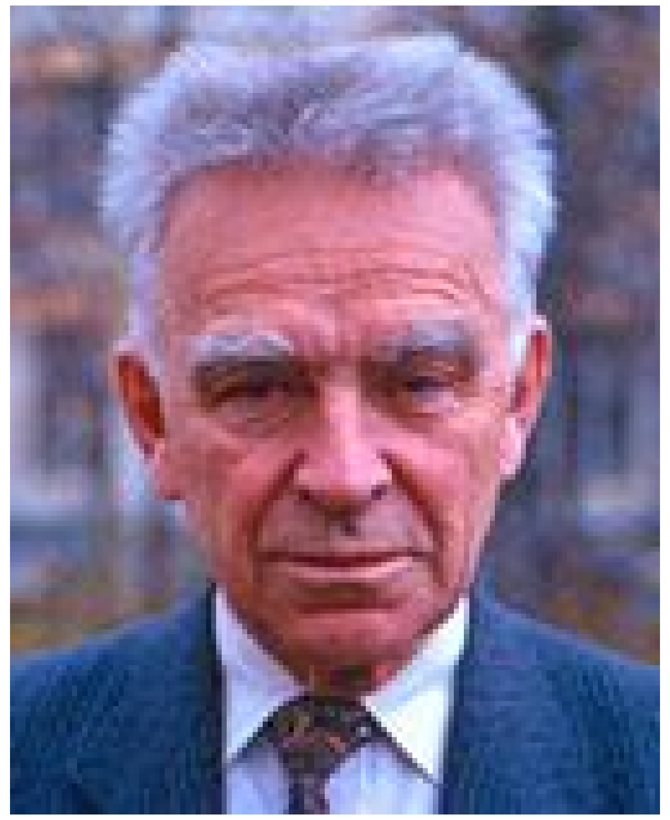
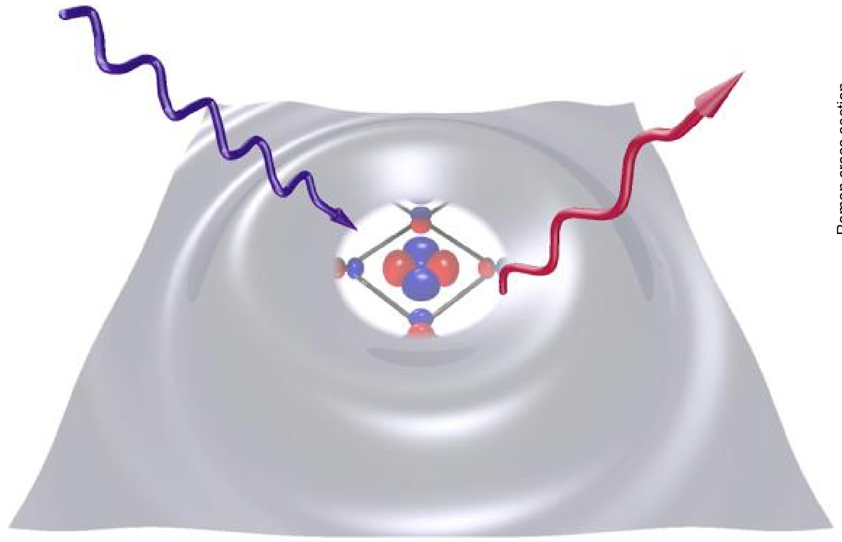
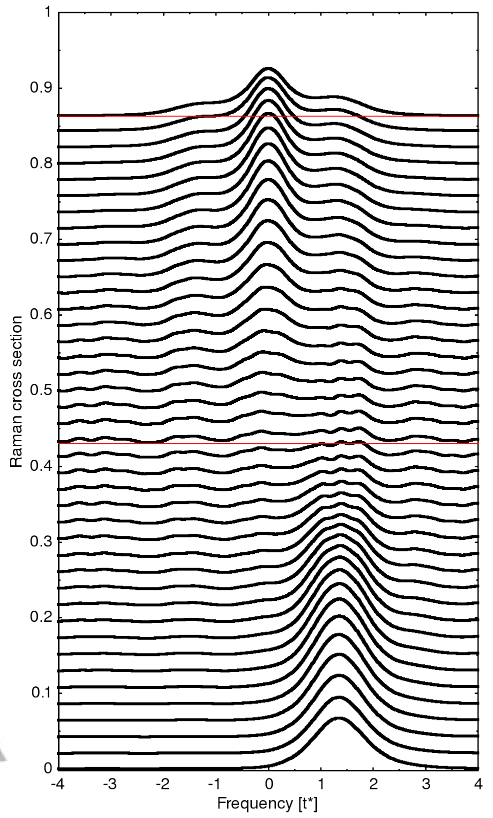
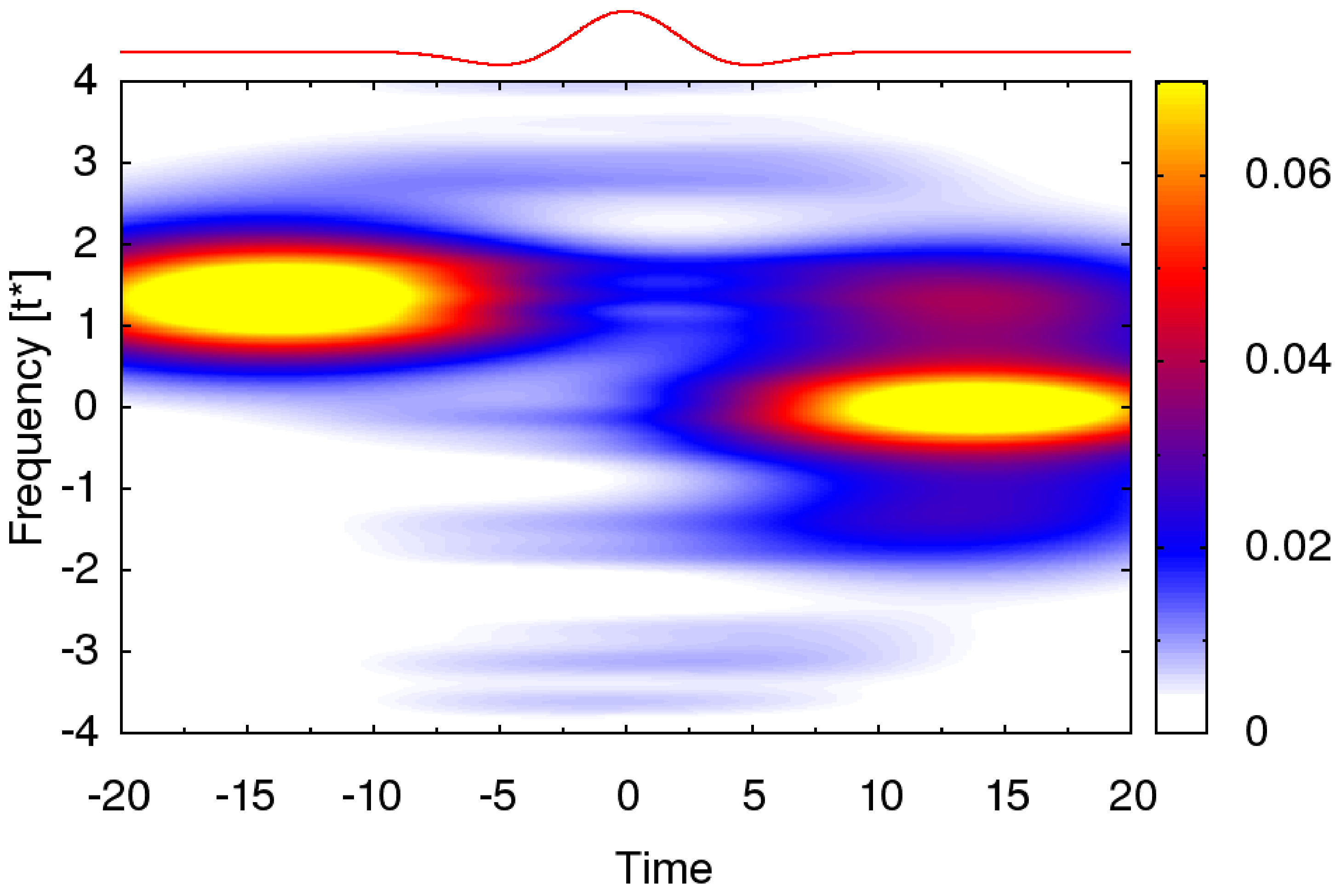
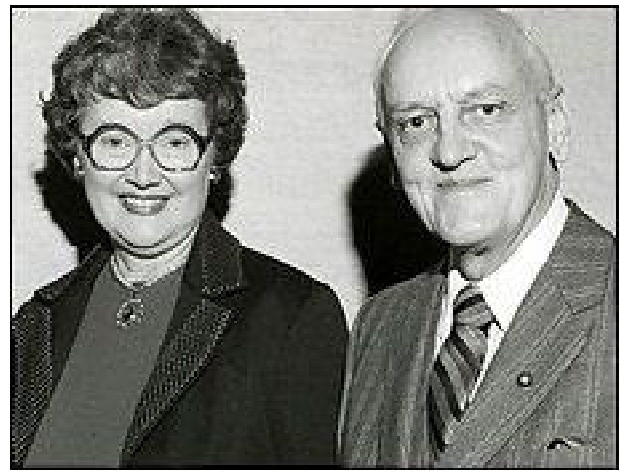
Introduction to Nonequilibrium Green's Functions



Jim Freericks Georgetown University



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Mark Jarrell (1960-2019)

I am dedicating this talk to Mark Jarrell, friend, mentor, collaborator, and physicist.

Mark passed away this summer after a long bout with kidney cancer.

What can we do with Green's functions?

Use of Green's functions

$$G^{>}_{ij\sigma}(t,t') = -\frac{i}{Z}\mathrm{Tr}\left\{e^{-\beta H}c_{i\sigma}(t)c^{\dagger}_{j\sigma}(t')\right\}$$

Greater Green's function

$$G^{R}_{ij\sigma}(t,t') = -\frac{i}{Z}\theta(t-t')\mathrm{Tr}\left\{e^{-\beta H}\left[c_{i\sigma}(t),c^{\dagger}_{j\sigma}(t')\right]_{+}\right\}$$

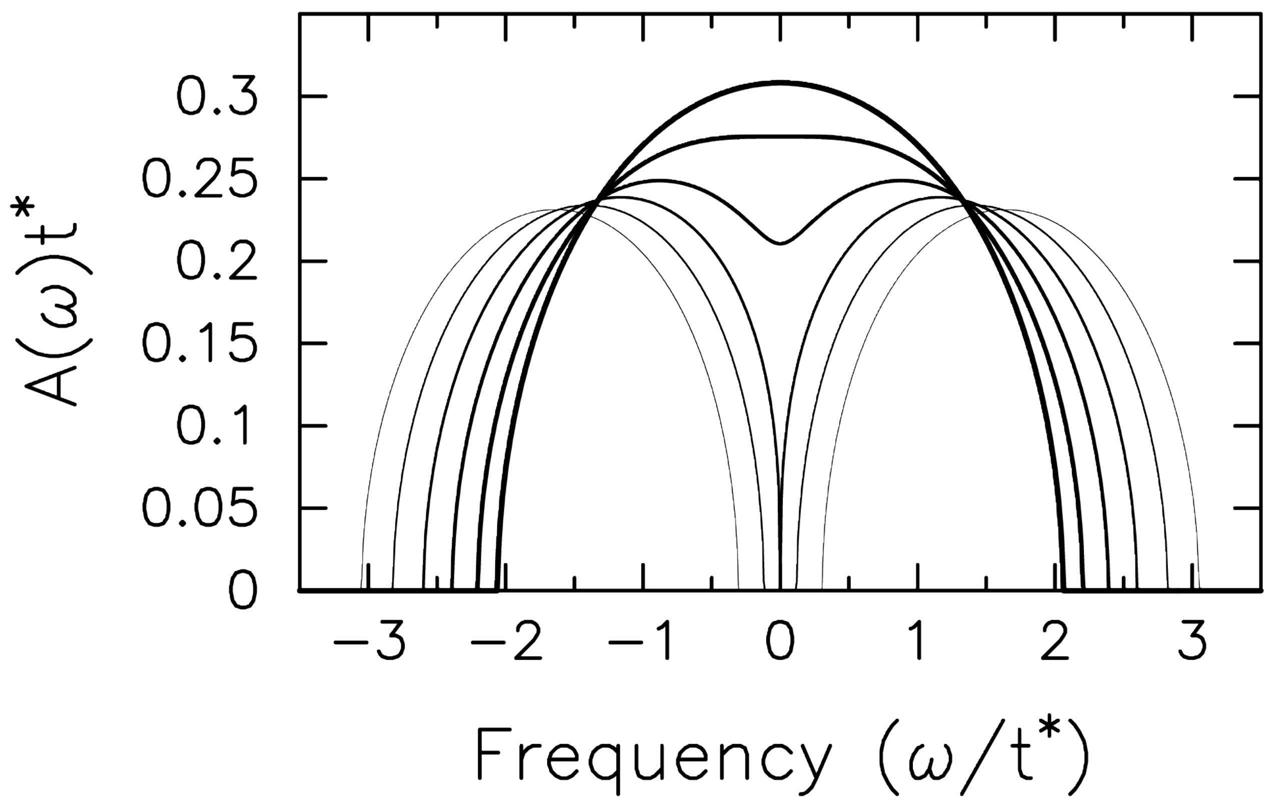
Retarded Green's function

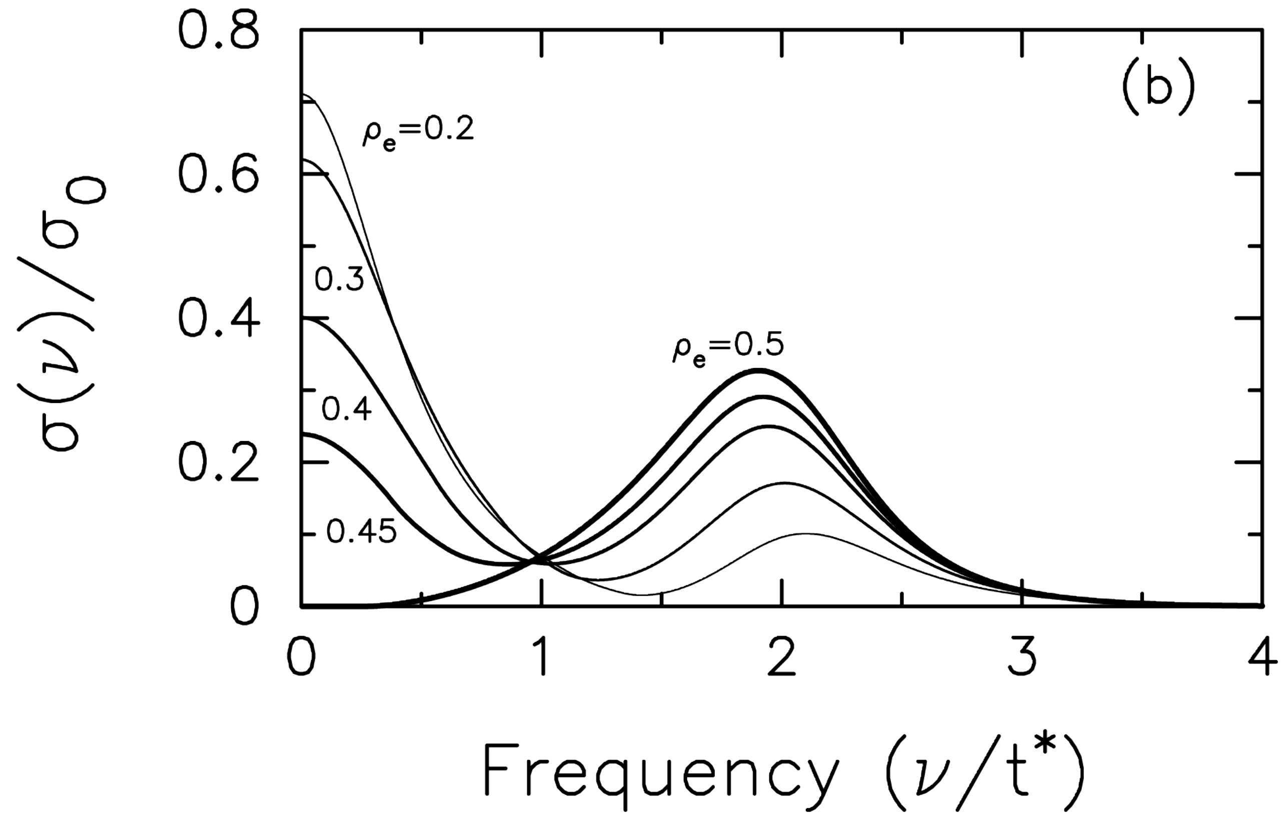
$$G_{ij\sigma}(\omega) = \int dt\, e^{-i\omega(t-t')}G_{ij\sigma}(t,t')$$

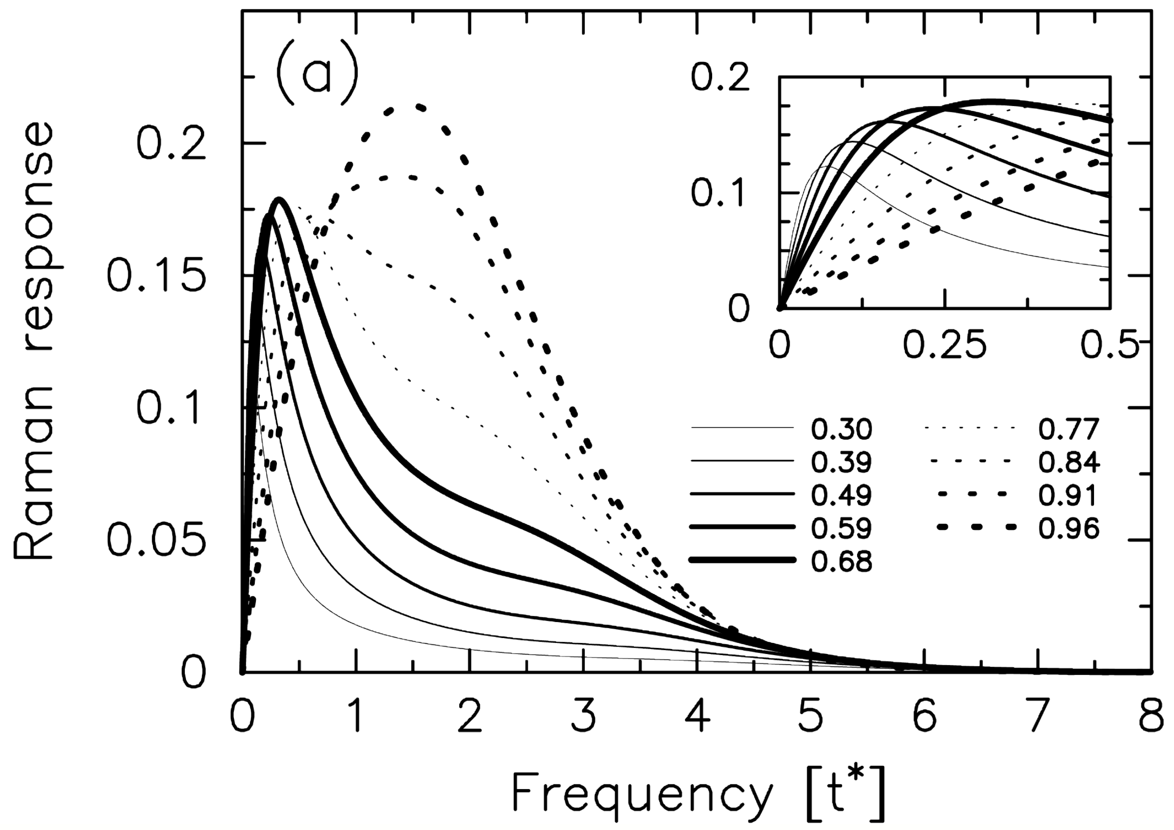
Frequency-dependent Green's function

$$A_{\sigma}(\omega) = -\frac{1}{\pi}\mathrm{Im}\, G^{R}_{ii\sigma}(\omega)$$

Local density of states

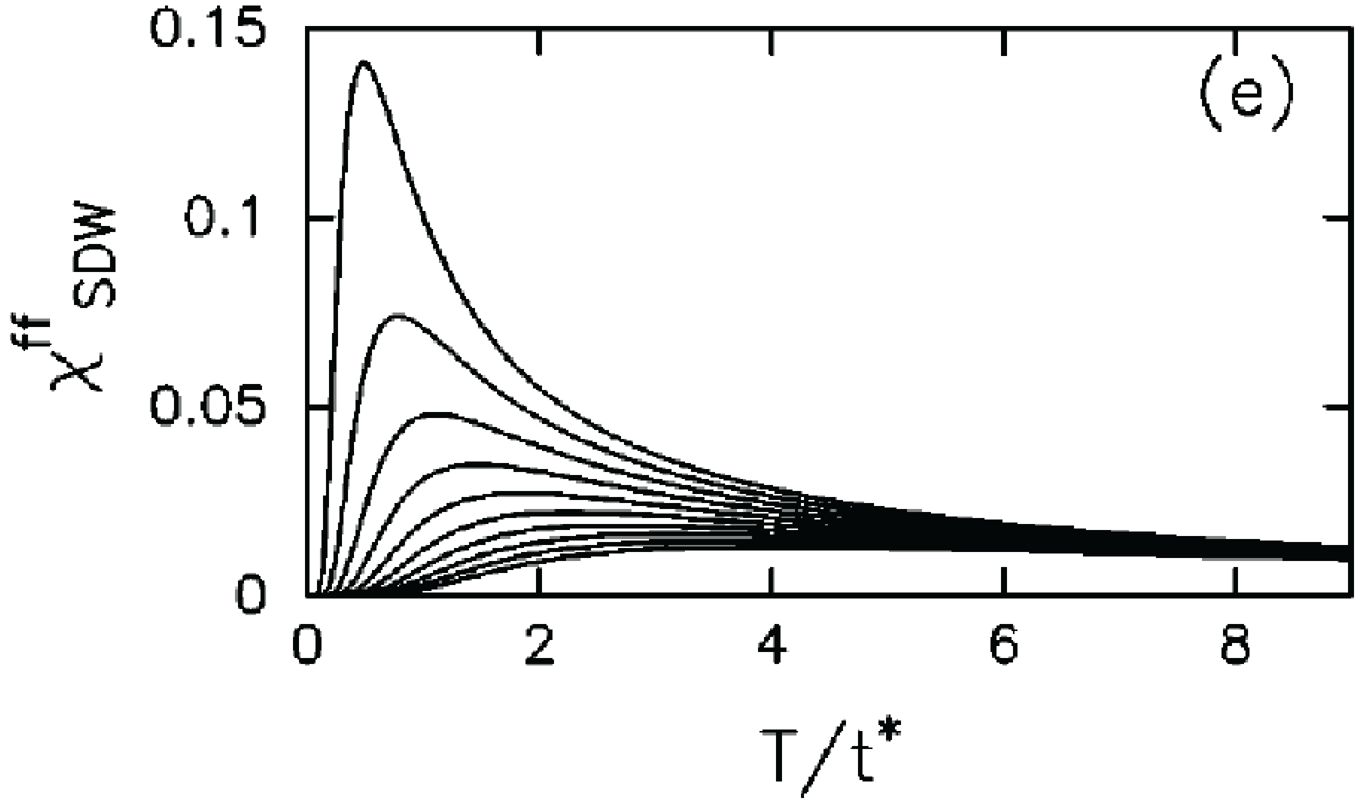




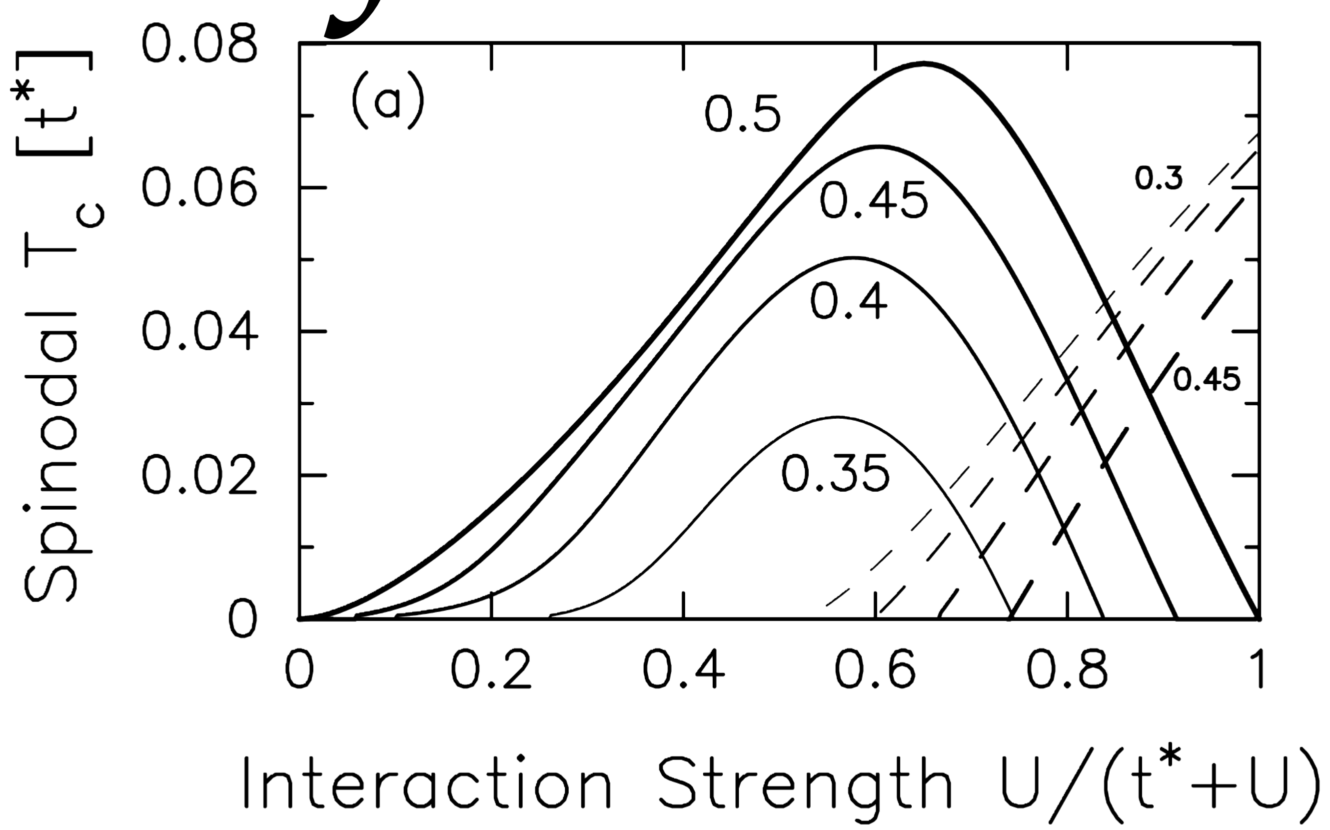


Freericks, et al, Phys. Rev. B, Fig. 3(a)

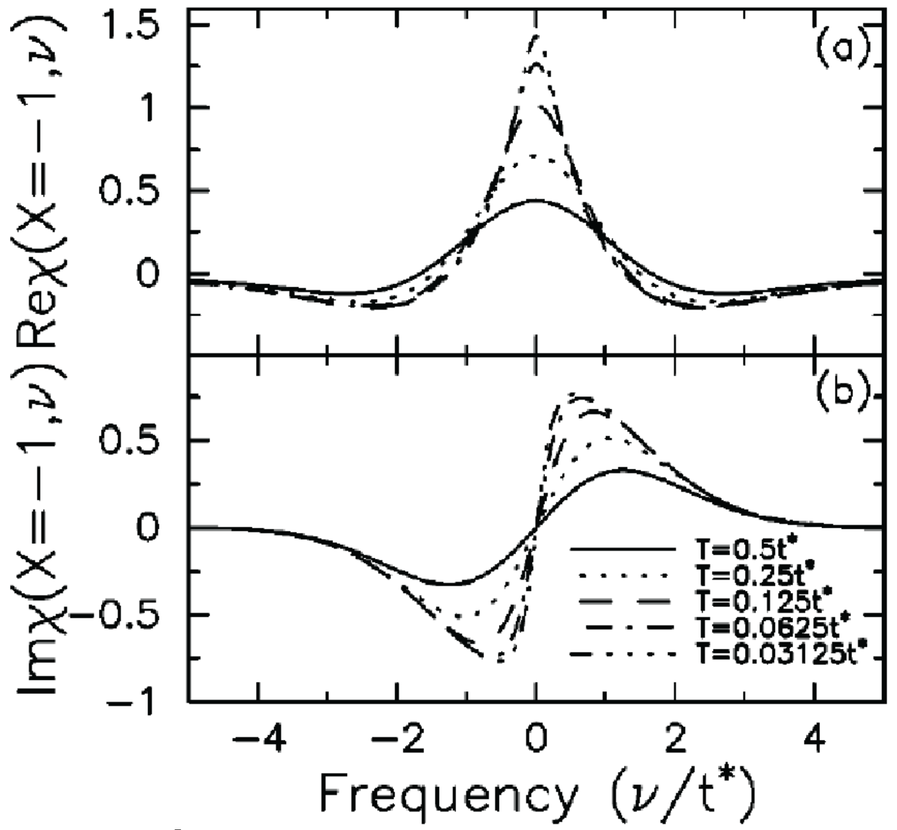
Multiparticle Green's functions



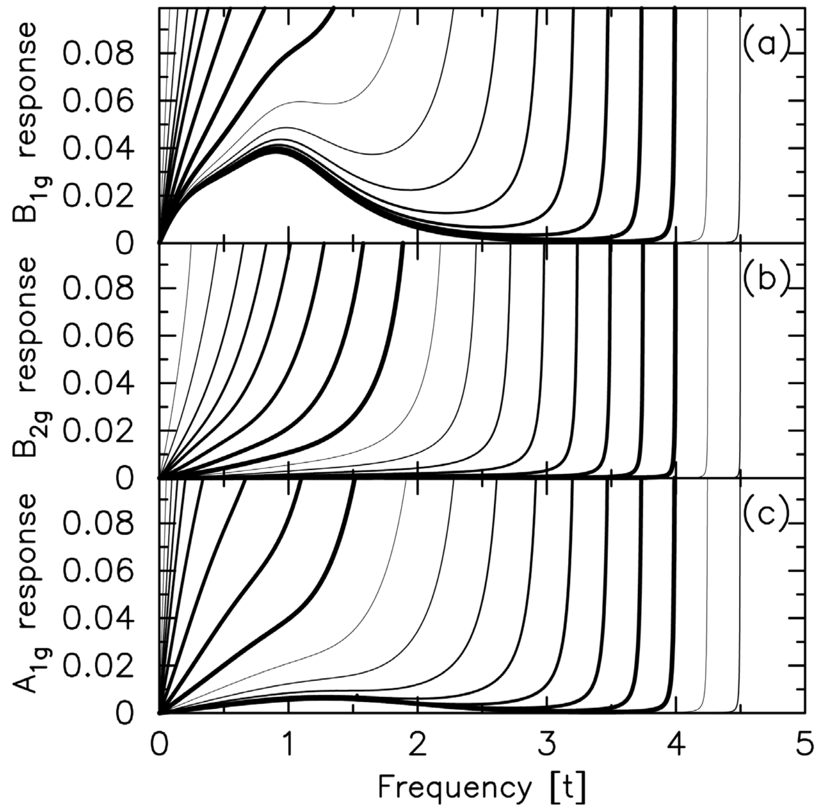
Magnetic susceptibility



Transition temperature



Dynamic charge susceptibility



Resonant Raman scattering

Why do we need Green's functions?

Easier to use than wavefunctions
Can directly calculate thermal averages
No easier method is known

Equilibrium

Time translation invariance Lesser Green's function

$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z} Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}(t') c_{i\sigma}(t)$$

$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z} Tr\, e^{-\beta H} e^{iHt'} c^{\dagger}_{j\sigma}\, e^{-iHt}\, e^{iHt} c_{i\sigma}\, e^{-iHt}$$

$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z} Tr\, e^{iHt'} e^{-\beta H} c^{\dagger}_{j\sigma}\, e^{-iHt'} e^{iHt} c_{i\sigma}\, e^{-iHt}$$

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Time translation invariance
Lesser Green's function

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$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z} Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}\, e^{iH(t-t')} c_{i\sigma}\, e^{-iH(t-t')}$$

Time translation invariance Lesser Green's function

$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z} Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')\, c_{i\sigma}(t)$$

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$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z} Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}\, c_{i\sigma}(t-t')$$

Time translation invariance
Lesser Green's function

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$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z} Tr\, e^{iHt'} e^{-\beta H} c^{\dagger}_{j\sigma}\, e^{-iHt'} e^{iHt} c_{i\sigma}\, e^{-iHt}$$

$$G^{<}_{ij\sigma}(t,t') = G^{<}_{ij\sigma}(t-t')$$

The equilibrium Green's function is time-translation invariant!

Lehmann Representation
Lesser Green's function (i=j)

$$G^{<}_{jj\sigma}(t,t') = \quad \frac{i}{Z}\, Tr\; e^{-\beta H} c^{\dagger}_{j\sigma}(t')\; c_{j\sigma}(t)$$

$$G^{<}_{jj\sigma}(t,t') = \quad \frac{i}{Z} \sum_{m} \langle m|\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')\, c_{j\sigma}(t) |m\rangle$$

Lehmann Representation
Lesser Green's function (i=j)

$$G^{<}_{jj\sigma}(t,t') = \frac{i}{Z}\, Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')\, c_{j\sigma}(t)$$

$$G^{<}_{jj\sigma}(t,t') = \frac{i}{Z}\sum_{mn} \langle m|\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')|n\rangle\langle n|c_{i\sigma}(t)|m\rangle$$

Lehmann Representation Lesser Green's function (i=j)

$$G^{<}_{jj\sigma}(t,t') = \frac{i}{Z}\, Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')\, c_{j\sigma}(t)$$

$$G^{<}_{jj\sigma}(t,t') = \frac{i}{Z}\sum_{mn}\langle m|\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')|n\rangle\langle n|c_{i\sigma}(t)|m\rangle$$

$$G^{<}_{jj\sigma}(t,t') = \frac{i}{Z}\sum_{mn} e^{-\beta E_m - iE_m(t-t')} e^{iE_n(t-t')}\langle m|\, c^{\dagger}_{j\sigma}|n\rangle\langle n|c_{j\sigma}|m\rangle$$

Lehmann Representation
Lesser Green's function (i=j)

$$G^{<}_{jj\sigma}(t,t') = \frac{i}{Z}\, Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')\, c_{j\sigma}(t)$$

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$$G^{<}_{jj\sigma}(t,t') = \frac{i}{Z}\sum_{mn} e^{-\beta E_m - iE_m(t-t')} e^{iE_n(t-t')} |\langle m|\, c^{\dagger}_{j\sigma}|n\rangle|^2$$

Lehmann Representation
Lesser Green's function (i=j)

$$G^{<}_{jj\sigma}(t,t') = \quad \frac{i}{Z}\, Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')\, c_{j\sigma}(t)$$

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$$G^{<}_{jj\sigma}(t,t') = \quad \frac{i}{Z}\sum_{mn} e^{-\beta E_m - i(E_m - E_n)(t-t')}|\langle m|\, c^{\dagger}_{j\sigma}|n\rangle|^2$$

Lehmann Representation
Lesser Green's function (i=j)

$$G^{<}_{jj\sigma}(t,t') = \frac{i}{Z} Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')\, c_{j\sigma}(t)$$

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$$G^{<}_{jj\sigma}(t,t') = \frac{i}{Z} \sum_{mn} e^{-\beta E_m - i(E_m - E_n)(t-t')} |\langle m|\, c^{\dagger}_{j\sigma}|n\rangle|^2$$

$$G^{<}_{jj\sigma}(\omega\,) = \int_{-\infty}^{\infty} d(t-t')e^{-i\omega t}\ \frac{i}{Z} \sum_{mn} e^{-\beta E_m - i(E_m - E_n)(t-t')} |\langle m|\, c^{\dagger}_{j\sigma}|n\rangle|^2$$

Lehmann Representation
Lesser Green's function (i=j)

$$G^{<}_{jj\sigma}(t,t') = \quad \frac{i}{Z}\, Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')\, c_{j\sigma}(t)$$

$$G^{<}_{jj\sigma}(t,t') = \quad \frac{i}{Z} \sum_{mn} \langle m|\, e^{-\beta H} c^{\dagger}_{j\sigma}(t')|n\rangle\langle n|c_{i\sigma}(t)|m\rangle$$

$$G^{<}_{jj\sigma}(t,t') = \quad \frac{i}{Z} \sum_{mn} e^{-\beta E_m - i(E_m - E_n)(t-t')} |\langle m|\, c^{\dagger}_{j\sigma}|n\rangle|^2$$

$$G^{<}_{jj\sigma}(\omega\,) = \frac{2\pi i}{Z} \sum_{mn} e^{-\beta E_m}\, |\langle m|c^{\dagger}_{j\sigma}|n\rangle|^2 \delta(\omega + E_m - E_n)$$

Nonequilibrium

The lesser Green's function

$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z}\, Tr\, e^{-\beta H} c^{\dagger}_{j\sigma}(t') c_{i\sigma}(t)$$

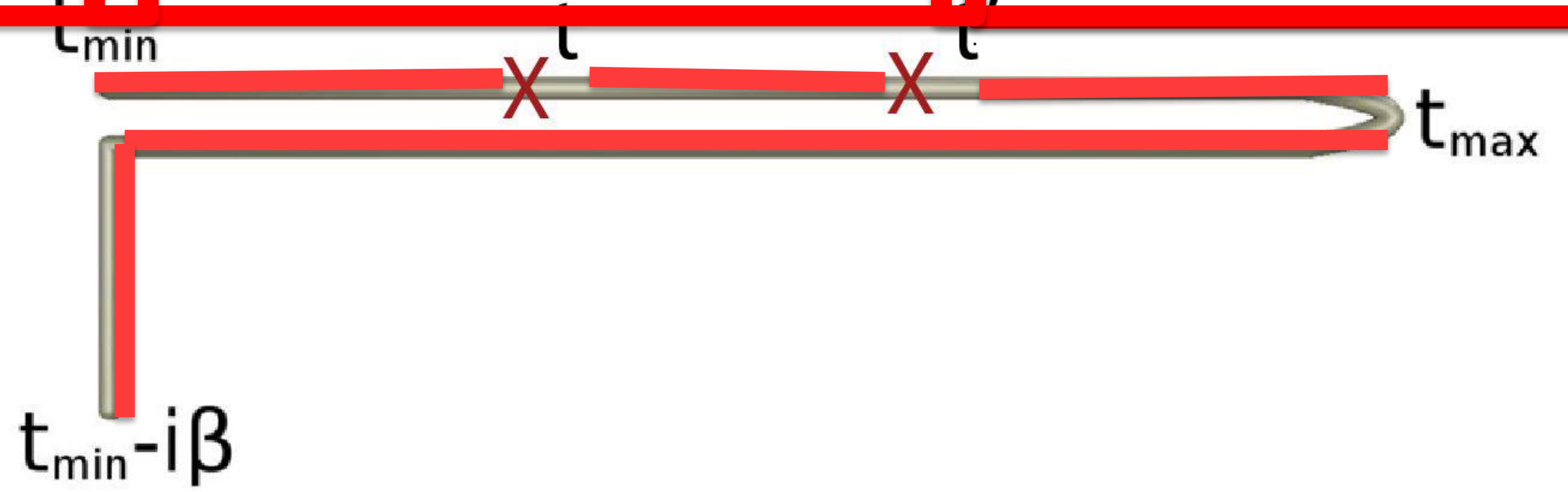
Assume t<t'

$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z}\, Tr\, e^{-\beta H} U^{\dagger}(t',t_{min}) c^{\dagger}_{j\sigma} U(t',t_{min}) U^{\dagger}(t,t_{min}) c_{i\sigma} U(t,t_{min})$$

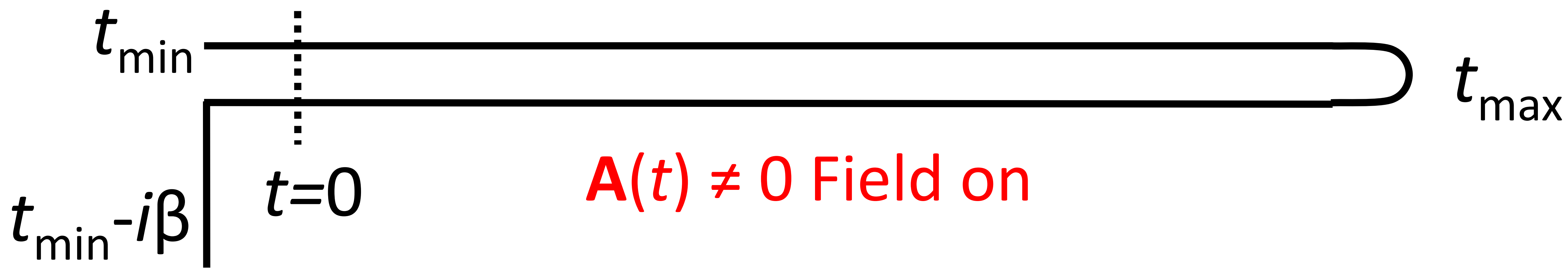
$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z}\, Tr\, e^{-\beta H} U^{\dagger}(t',t_{min}) c^{\dagger}_{j\sigma} U(t',t)\, c_{i\sigma} U(t,t_{min})$$

$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z}\, Tr\, e^{-\beta H} U^{\dagger}(t_{max},t_{min}) U(t_{max},t_{min}) U^{\dagger}(t',t_{min}) c^{\dagger}_{j\sigma} U(t',t)\, c_{i\sigma} U(t,t_{min})$$

$$G^{<}_{ij\sigma}(t,t') = \frac{i}{Z}\, Tr\, e^{-\beta H} U^{\dagger}(t_{max},t_{min}) U(t_{max},t') c^{\dagger}_{j\sigma} U(t',t)\, c_{i\sigma} U(t,t_{min})$$



The contour-ordered Green's function



A(t)=0

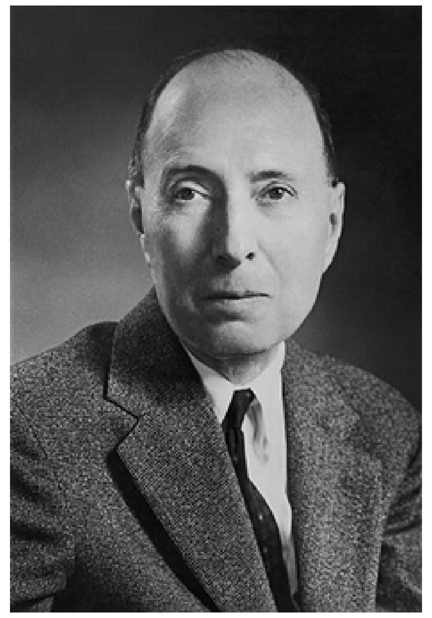
No field

$$G^{c}_{ij\sigma}(t,t') = -\frac{i}{Z}\,Tr\,e^{-\beta H(t_{min})}T_c c_{i\sigma}(t)c^{\dagger}_{j\sigma}(t')$$

Both times, t and t', lie on the contour. One can extract many different Green's functions from the contour-ordered Green's function.

The DOS at t_{ave}

$$G^R_{ij\sigma}(t,t') = -\frac{i}{Z}\theta(t-t')Tr\, e^{-\beta H(-\infty)}\left\{c_{i\sigma}(t), c^\dagger_{j\sigma}(t')\right\}_+$$



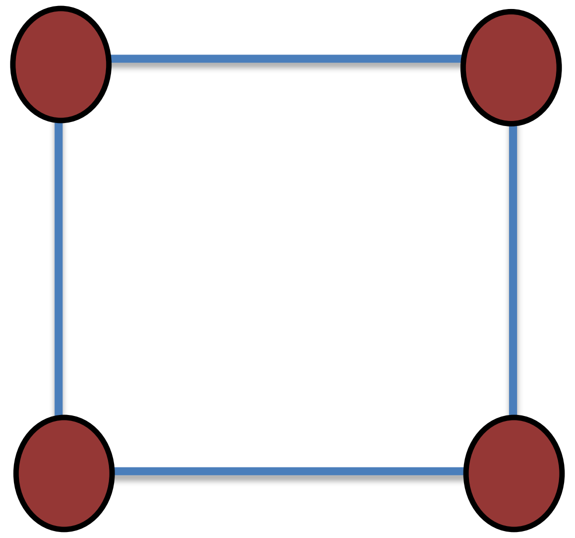
$$t_{ave} = \frac{t+t'}{2}\; ; \;\; t_{rel} = t - t'$$

$$\rho_{ii}(\omega, t_{ave}) = -\frac{1}{\pi} Im \int_0^\infty e^{i\omega t_{rel}} G^R_{ii\sigma}\left(t_{ave} + \frac{1}{2}t_{rel}, t_{ave} - \frac{1}{2}t_{rel}\right) dt_{rel}$$

We find the DOS by performing a Fourier transformation with respect to relative time, keeping the average time fixed.

Electric fields

The Peierls substitution



$$-t \rightarrow -te^{-\frac{ie}{\hbar c}\int_{R_i}^{R_j} \vec{A}(r,t)\cdot\overrightarrow{dr}}$$

We work in a vector potential only gauge. This produces a time-dependent phase on the hopping. If the field is uniform in space, we preserve translational invariance.

$$\epsilon(\boldsymbol{k}) \rightarrow \epsilon(\boldsymbol{k} - \boldsymbol{A}(t))$$

Noninteracting electrons

The noninteracting problem

$$H_S(t) = \sum_{k\sigma} \epsilon(\boldsymbol{k} - \boldsymbol{A}(t)) c_{k\sigma}^\dagger c_{k\sigma} \qquad [H_S(t), H_S(t')] = 0$$

$$c_{k\sigma}^\dagger(t) = e^{i\int_{-\infty}^{t} dt' [\epsilon(\boldsymbol{k}-\boldsymbol{A}(t'))-\mu]} c_{k\sigma}^\dagger, \quad c_{k\sigma}(t) = e^{-i\int_{-\infty}^{t} dt' [\epsilon(\boldsymbol{k}-\boldsymbol{A}(t'))-\mu]} c_{k\sigma}$$

$$G_{k\sigma}^R(t, t') = -i\theta(t - t') e^{-i\int_{t'}^{t} \{\epsilon(\boldsymbol{k}-\boldsymbol{A}(\bar{t}))-\mu\} d\bar{t}}$$

Using the EOM, we can immediately solve for the Green's function

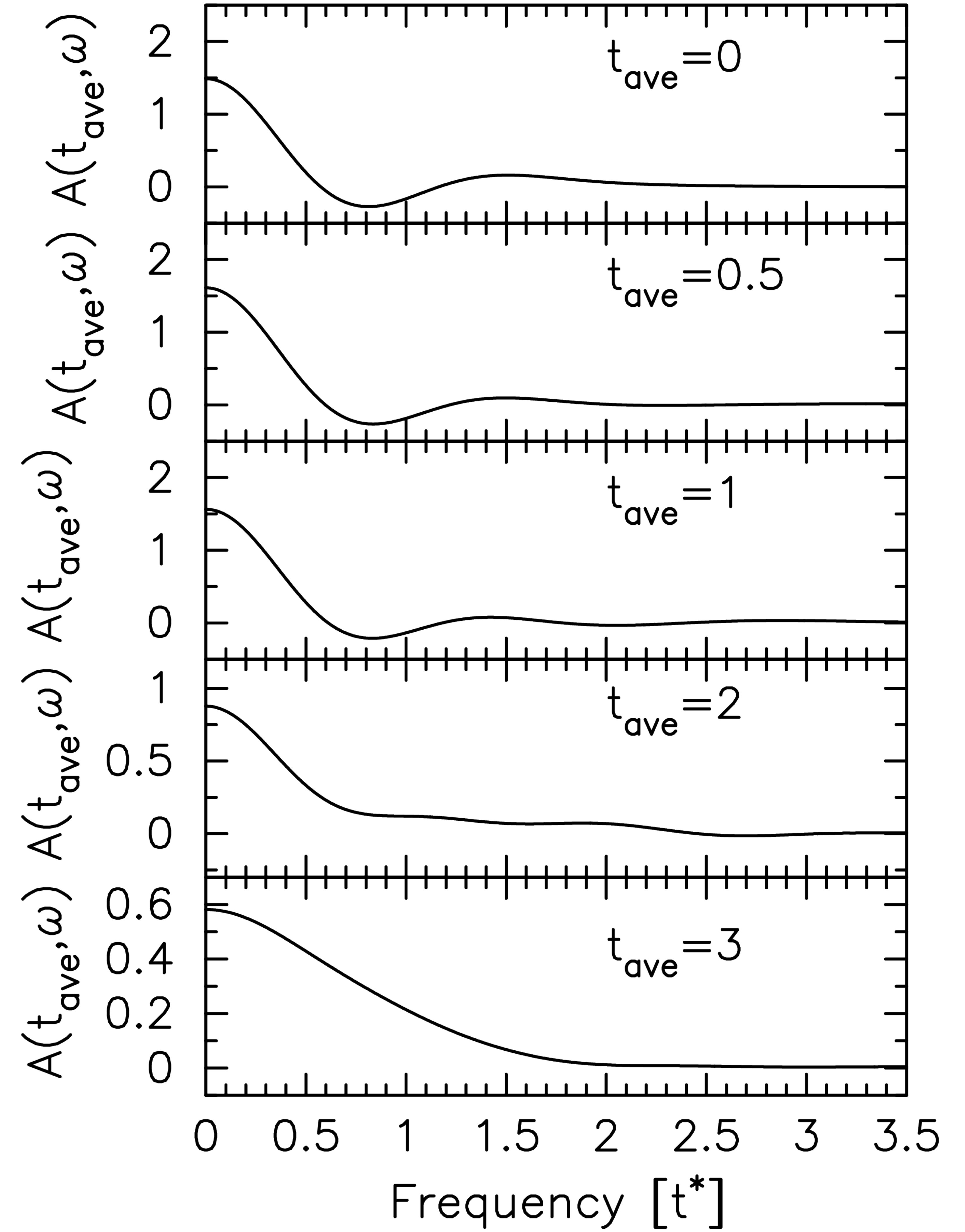
The noninteracting DOS

$$DOS = \rho(\omega, t_{ave}) = -\frac{1}{\pi} Im\, G^R_{ii\sigma}(\omega, t_{ave})$$

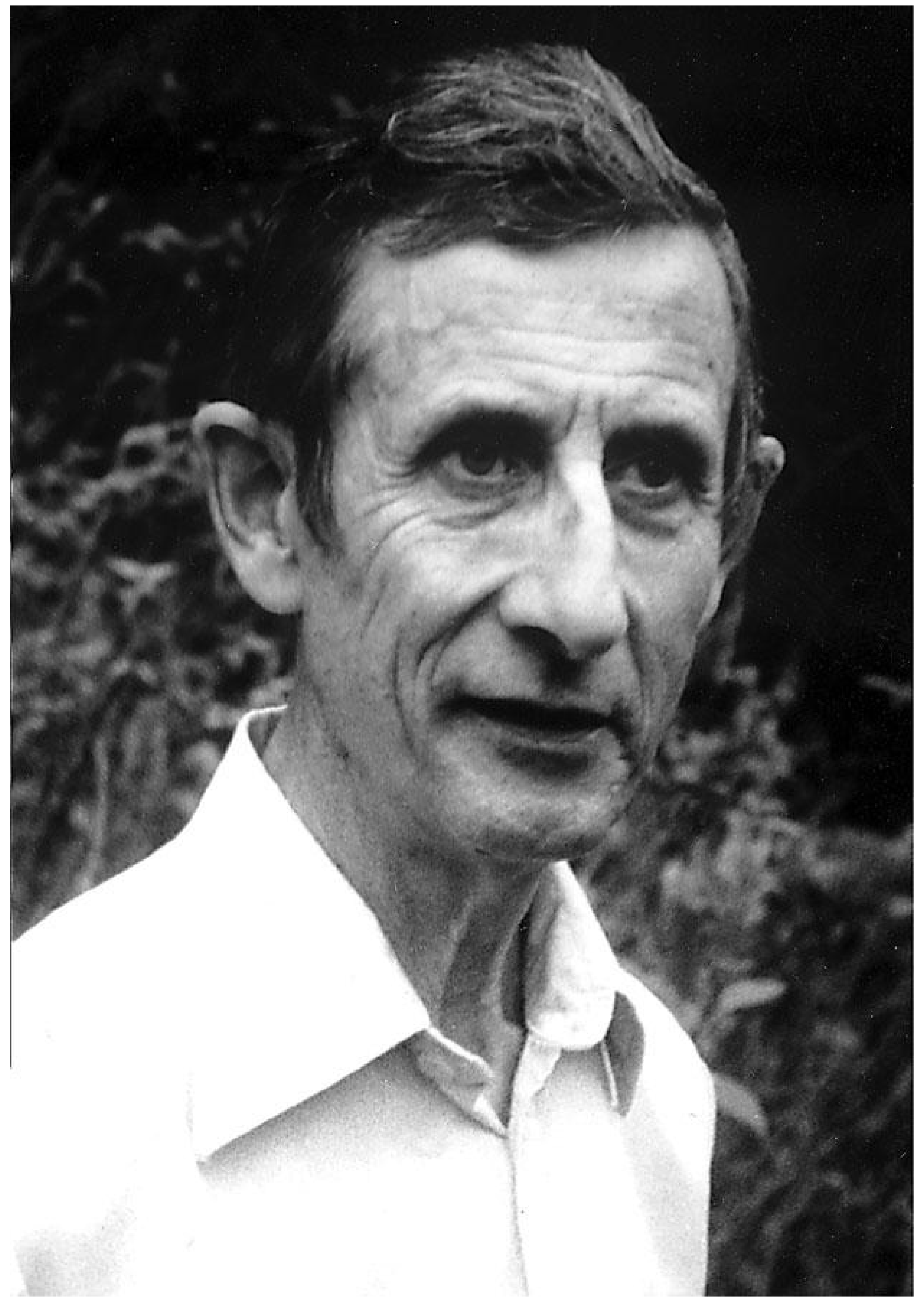
Gaussian pulse field

Note that the set of instantaneous eigenvalues does not depend on the field due to the Peierls substitution. $\{\epsilon(\boldsymbol{k})\} = \{\epsilon(\boldsymbol{k} - \boldsymbol{A}(t))\}$

But the density of states depends strongly on the field and on average time.



Interacting electrons

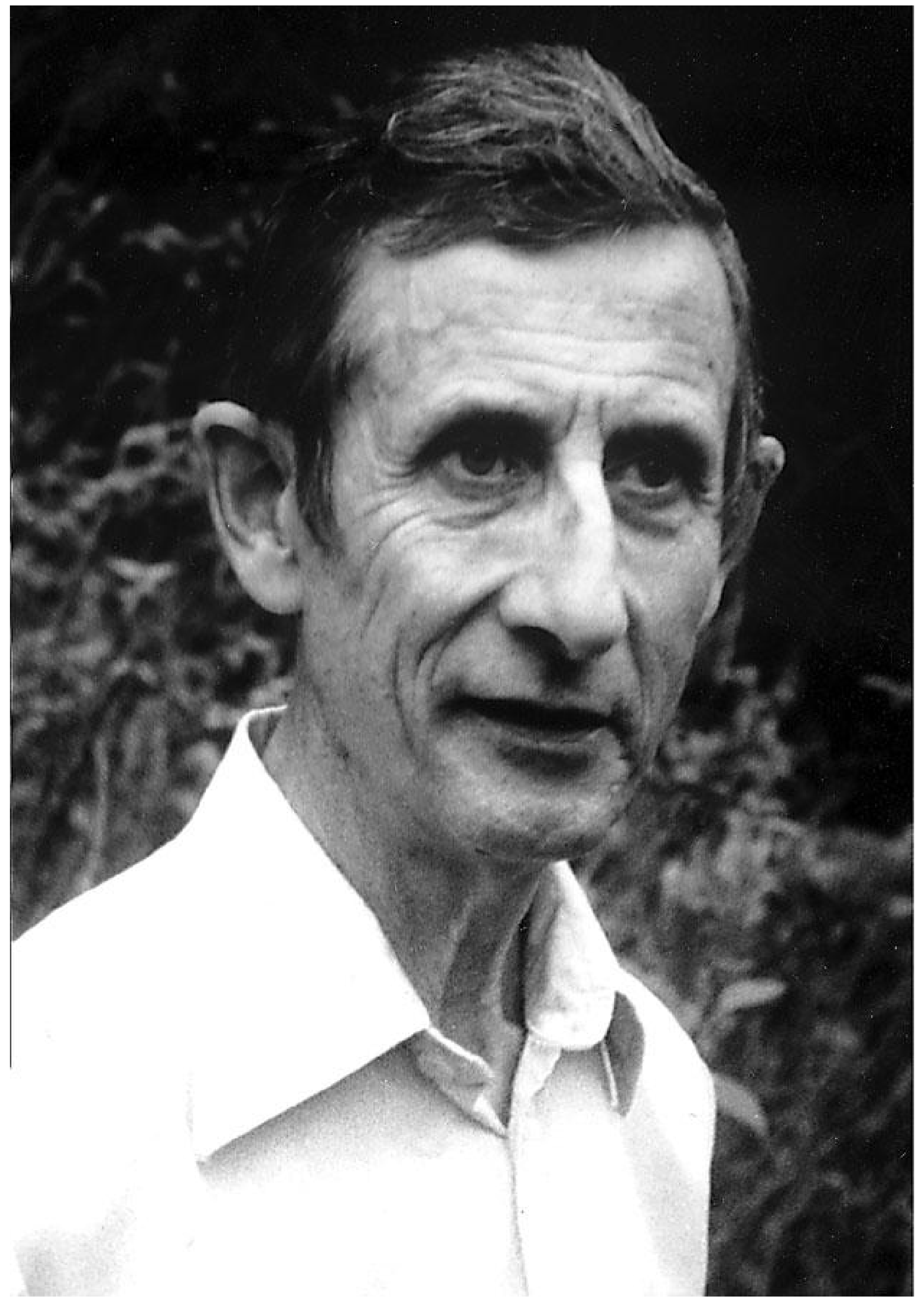


EOM and the Dyson equation

The equation of motion is determined by simply differentiating the contour-ordered Green's function with respect to time.
One term is complicated. Rather than evaluate it directly, we define it to be the convolution of the self-energy with the Green's function.
This becomes Dyson's equation.

$$i\frac{\partial}{\partial t}G^c_{k\sigma}(t,t') = \theta_c(t,t')\frac{1}{Z}\,Tr\,e^{-\beta H(-\infty)}T_c\frac{\partial}{\partial t}c_{k\sigma}(t)c^\dagger_{k\sigma}(t') + \delta_c(t,t')$$

$$i\frac{\partial}{\partial t}G^c_{k\sigma}(t,t') = \theta_c(t,t')\frac{i}{Z}\,Tr\,e^{-\beta H(-\infty)}T_c[H_H(t),c_{k\sigma}(t)]c^\dagger_{k\sigma}(t') + \delta_c(t,t')$$



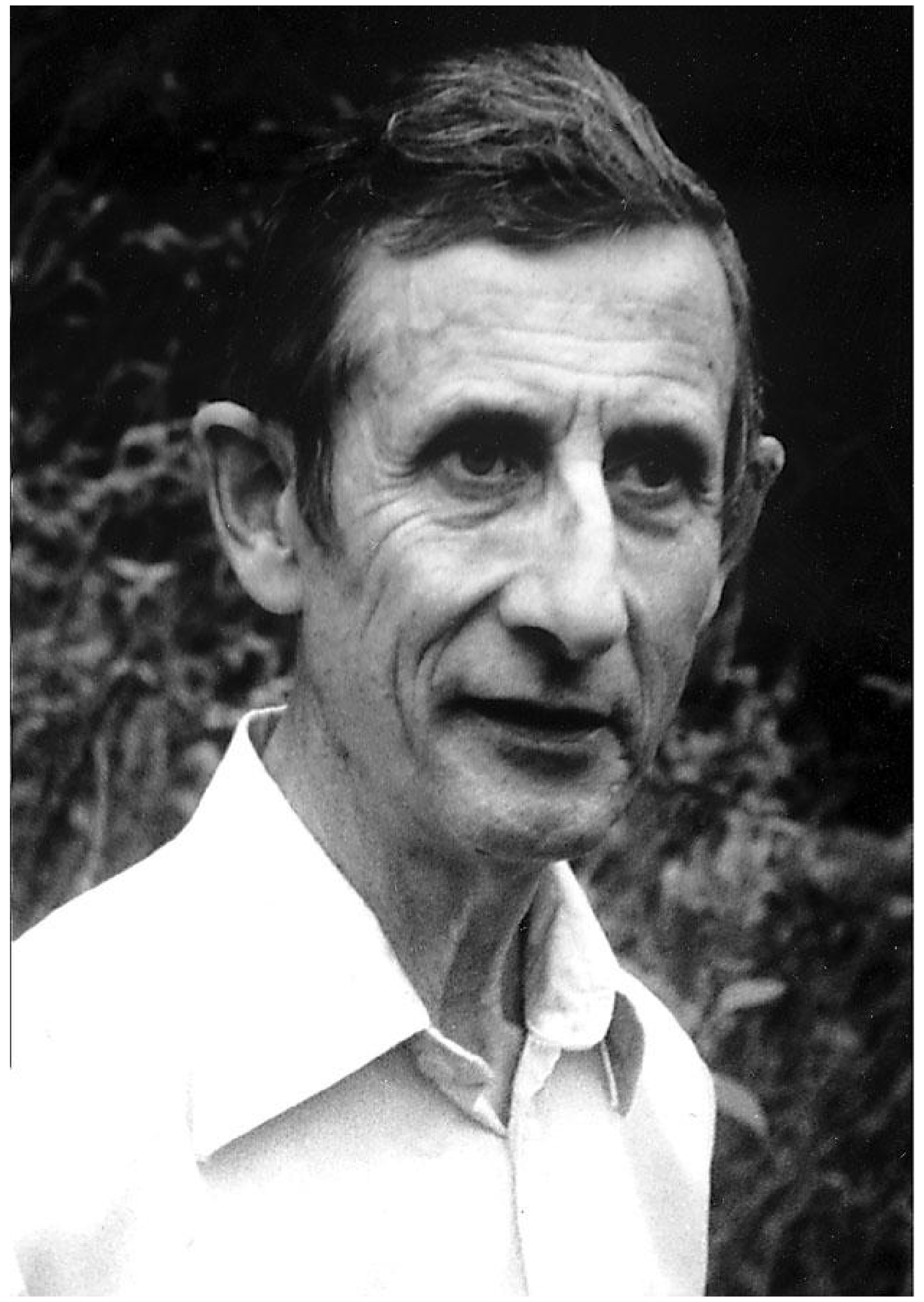
EOM and the Dyson equation

$$H_H(t) = \sum_{k\sigma} [\epsilon(\boldsymbol{k} - \boldsymbol{A}(t)) - \mu] c_{k\sigma}^{\dagger}(t) c_{k\sigma}(t) + V_H(t)$$

$$[H_H(t), c_{k\sigma}(t)] = -[\epsilon(\boldsymbol{k} - \boldsymbol{A}(t)) - \mu] c_{k\sigma}(t) + [V_H(t), c_{k\sigma}(t)]$$

$$[i\frac{\partial}{\partial t} + \mu - \epsilon(\boldsymbol{k} - \boldsymbol{A}(t))] G_{k\sigma}^{c}(t,t') = \int_c d\bar{t}\, \Sigma^c(t,\bar{t}) G_{k\sigma}^{c}(\bar{t},t') + \delta_c(t,t')$$

This is the definition of the contour-ordered self-energy



EOM and the Dyson equation

$$H_H(t) = \sum_{k\sigma} [\epsilon(\boldsymbol{k} - \boldsymbol{A}(t)) - \mu] c^{\dagger}_{k\sigma}(t) c_{k\sigma}(t) + V_H(t)$$

$$[H_H(t), c_{k\sigma}(t)] = -[\epsilon(\boldsymbol{k} - \boldsymbol{A}(t)) - \mu] c_{k\sigma}(t) + [V_H(t), c_{k\sigma}(t)]$$

$$[i\frac{\partial}{\partial t} + \mu - \epsilon(\boldsymbol{k} - \boldsymbol{A}(t))] G^c_{k\sigma}(t, t') = \int_c d\bar{t} \Sigma^c(t, \bar{t}) G^c_{k\sigma}(\bar{t}, t') + \delta_c(t, t')$$

$$\int_c d\bar{t} \left\{ \left[i\frac{\partial}{\partial t} + \mu - \epsilon(\boldsymbol{k} - \boldsymbol{A}(t)) \right] \delta_c(t, \bar{t}) - \Sigma^c(t, \bar{t}) \right\} G^c_{k\sigma}(\bar{t}, t') = \delta_c(t, t')$$

Solving the Dyson equation

$$\int_c d\bar{t}\left\{\left[i\frac{\partial}{\partial t}+\mu-\epsilon\left(\boldsymbol{k}-\boldsymbol{A}(t)\right)\right]\delta_c(t,\bar{t})-\Sigma^c(t,\bar{t})\right\}G^c_{k\sigma}(\bar{t},t')=\delta_c(t,t')$$

This is of the form $(G^c)^{-1}(G^c)=\mathbb{I}$ with a boundary conditon

$$G^c_{k\sigma}(t_{min},t_{min}-i\beta)=-\frac{i}{Z}\,Tr\;e^{-\beta H(t_{min})}c_{k\sigma}(t_{min})c^\dagger_{k\sigma}(t_{min}-i\beta)$$

$$G^c_{k\sigma}(t_{min},t_{min}-i\beta)=-\frac{i}{Z}\,Tr\;e^{-\beta H}c_{k\sigma}e^{-\beta H}c^\dagger_{k\sigma}e^{\beta H}$$

$$G^c_{k\sigma}(t_{min},t_{min}-i\beta)=-\frac{i}{Z}\,Tr\;e^{-\beta H}c^\dagger_{k\sigma}c_{k\sigma}=-G^c_{k\sigma}(t_{min},t_{min})$$

Self energy and memory effects

Often it is said that in nonequilibrium on has "memory effects"

But the EOM is a first order linear equation, so how can this be?

It happens when we employ a self-energy, because it enters via a convolution, which couples different times together.

If we could solve the problem just with the GFs, the memory effects would be gone, but we do not know how to do this.

Discretizing continuous matrix equations

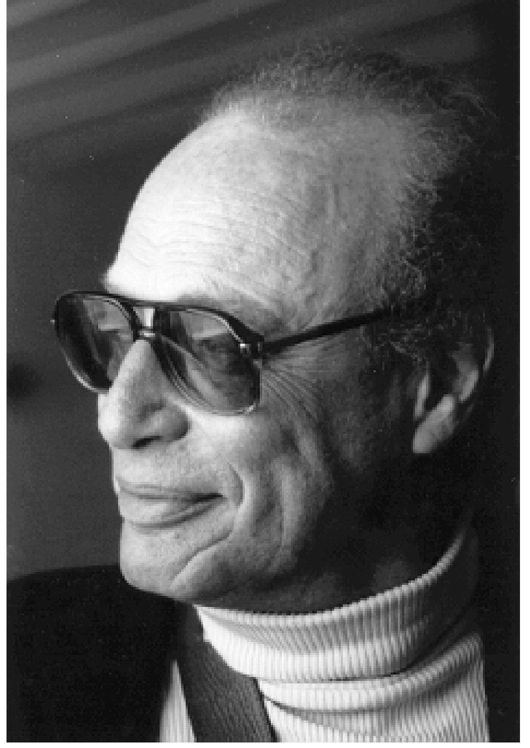
$$\left(i\frac{\partial}{\partial t}+\mu\right)\delta_c(t,t') = \frac{1}{W_j}M_{jk}\frac{1}{W_k}$$

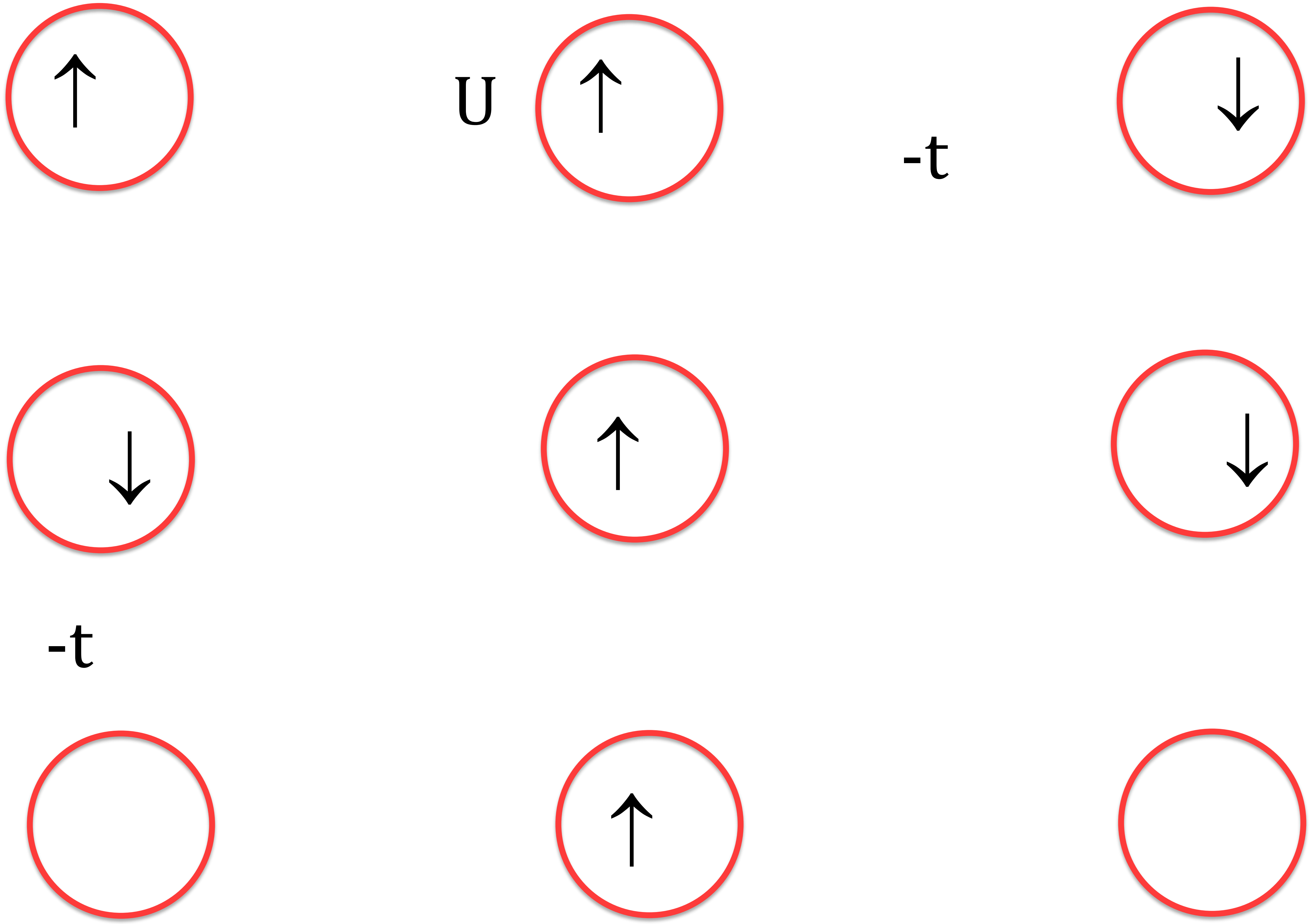
$$M_{jk}=\begin{pmatrix}
1 & 0 & 0 & & & & & \cdots & & & 1+i\Delta t\mu \\
-1-i\Delta t\mu & 1 & 0 & & & & & \cdots & & & 0 \\
0 & -1-i\Delta t\mu & 1 & 0 & & & & & & & \\
 & & & \ddots & & & & & & & \\
 & & & 0 & -1+i\Delta t\mu & 1 & 0 & & & & \\
 & & & & 0 & -1+i\Delta t\mu & 1 & & & & \\
 & & & & & & & \ddots & & & \\
 & & & & & & & & -1-\Delta\tau\mu & 1 & \\
 & & & & & & & & & \ddots & \\
 & & & & & & & & & -1-\Delta\tau\mu & 1
\end{pmatrix}$$

$$\det(M) = 1+(-1)^{2N_t+N_\tau-1}(1+i\Delta t\mu)(-1-i\Delta t\mu)^{N_t-1}(-1+i\Delta t\mu)^{N_t}(-1-\Delta\tau\mu)^{N_\tau}$$
$$= 1+(1+\Delta\tau\mu)^{N_\tau}+O\left(\Delta t^2\right) = 1+e^{\beta\mu}$$

Modeling electrons

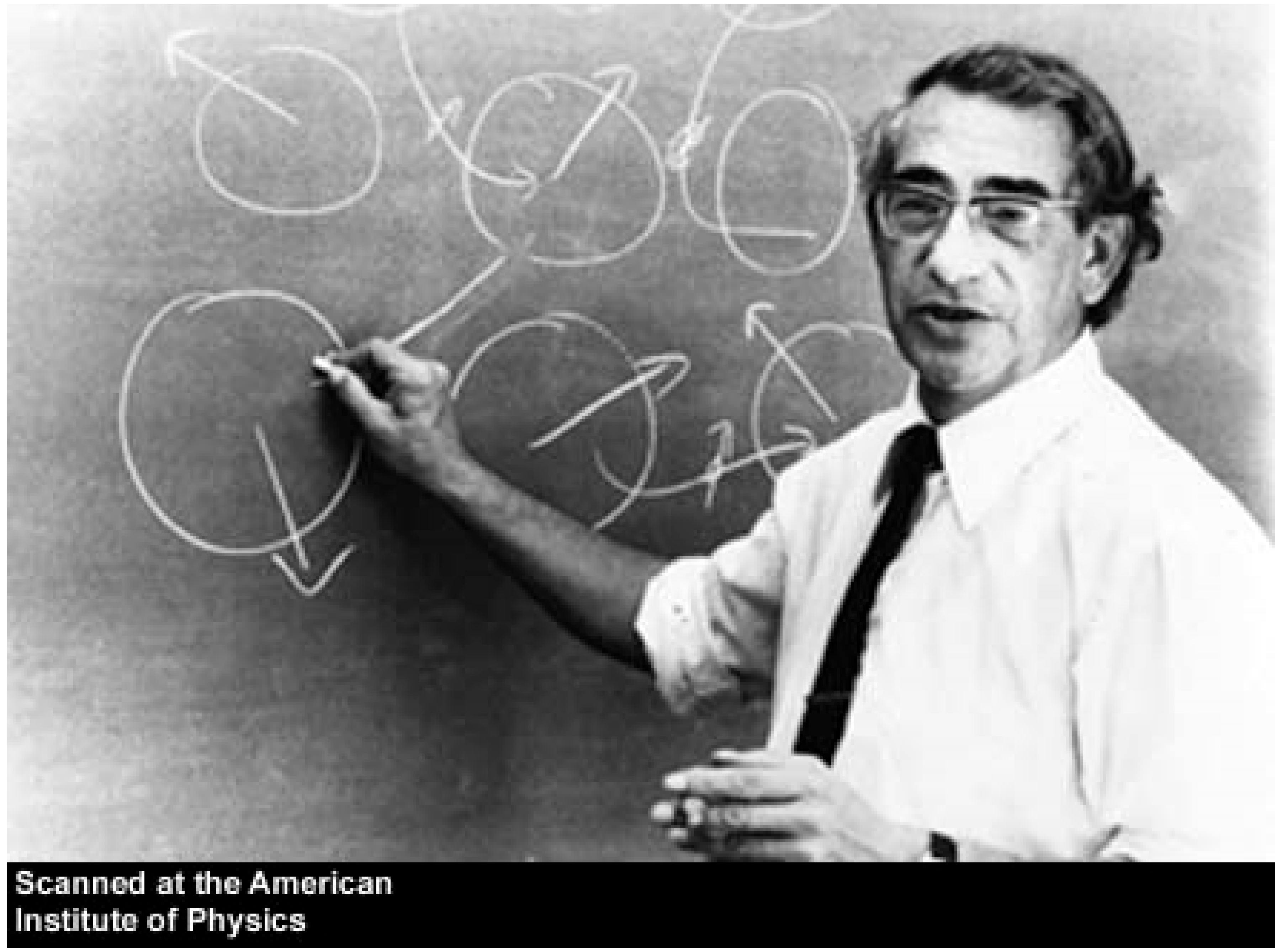
Modeling correlations with the Falicov-Kimball model

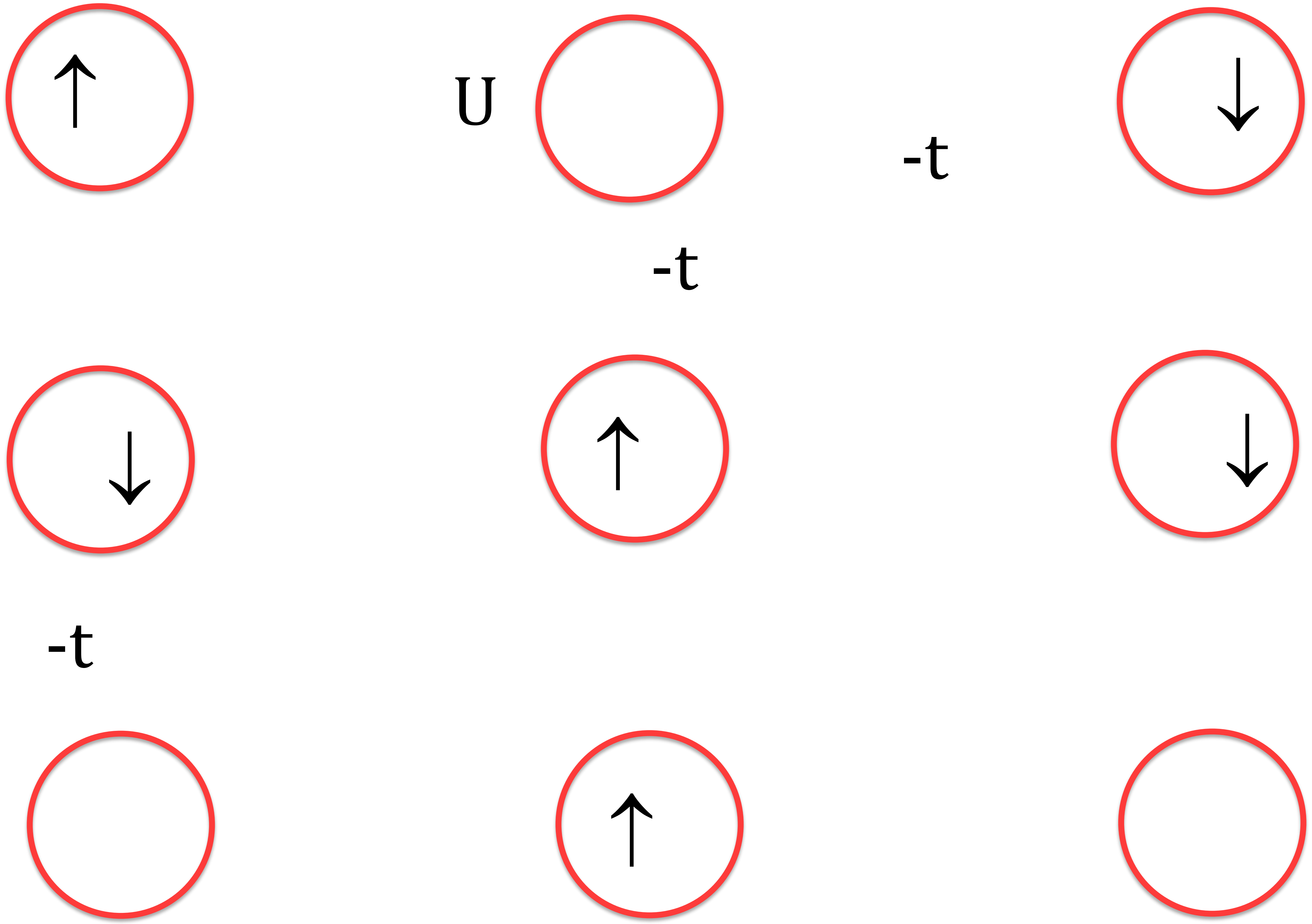




Down-spin electrons hop with strength $-t$ between lattice sites. They feel an interaction of U when two electrons are on the same site.

Hubbard model

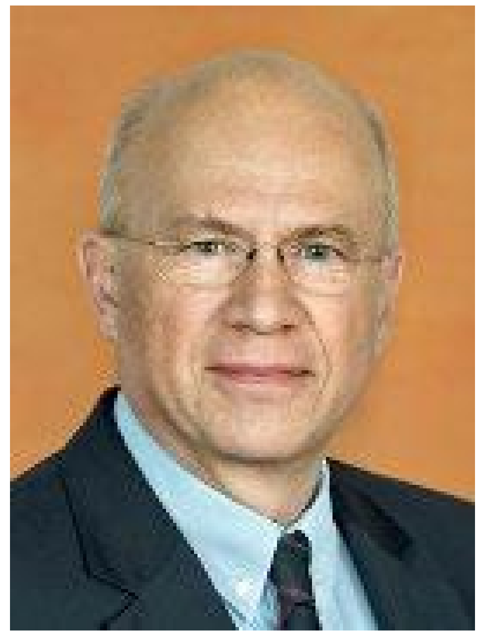


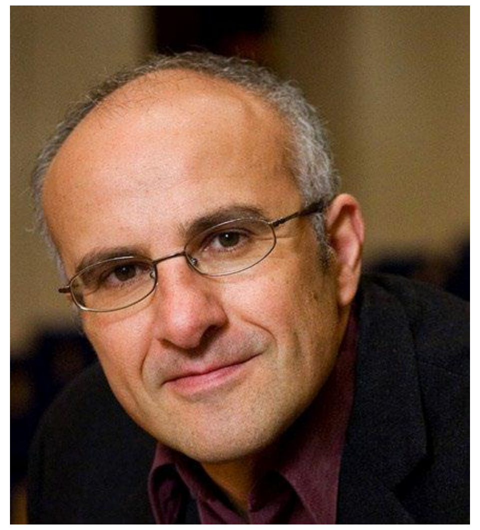
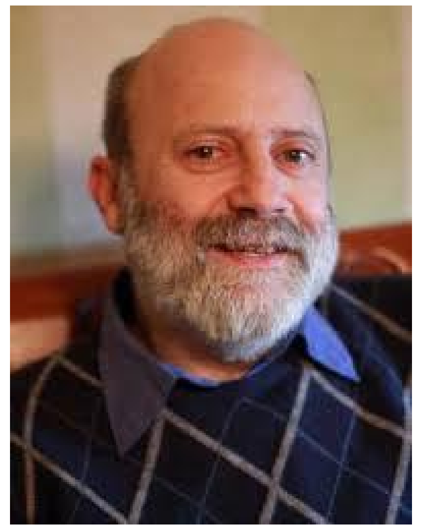


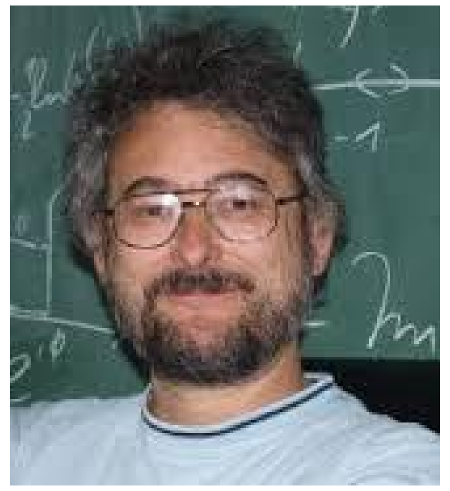
Electrons hop with strength $-t$ between lattice sites. Feel an interaction of U when two electrons are on the same site.

Dynamical mean-field theory

Dynamical mean-field theory



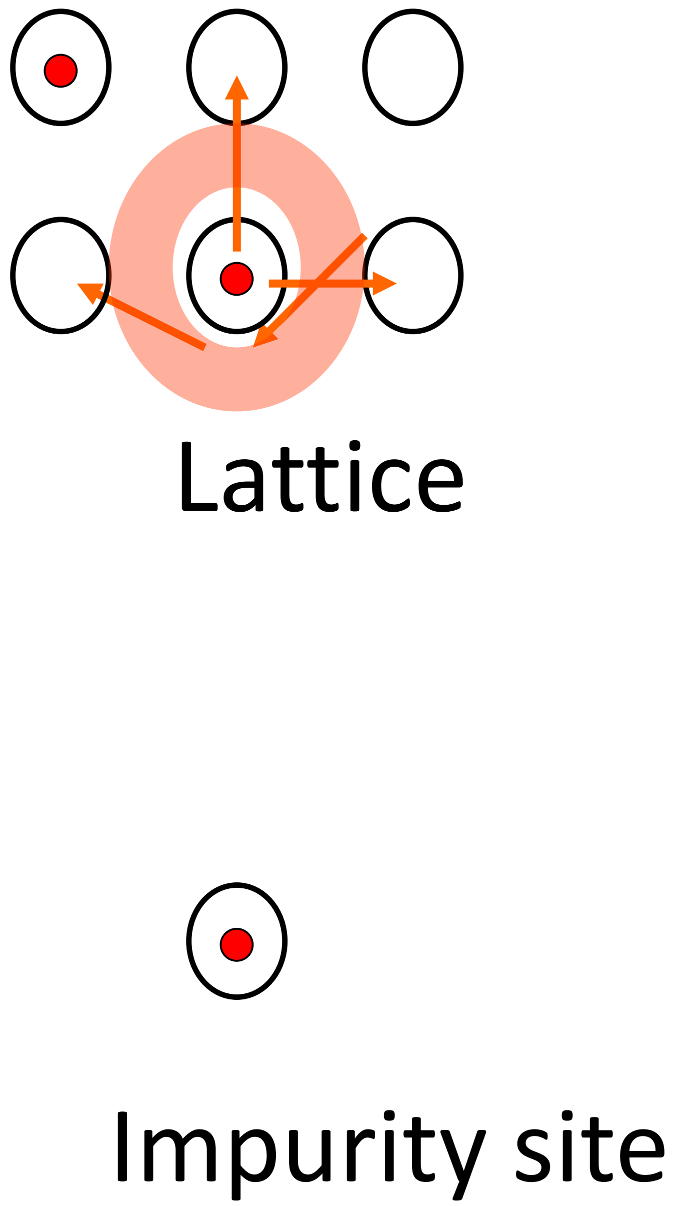





Dynamical mean-field theory introduced in the late 1980s.

Self-consistent solution of an impurity problem solves the lattice problem in large dimensions

Extension to nonequilibrium in 2006 follows by working in the time representation.



Self-consistency loop

Dyson equation

$\Sigma = G_0^{-1} - G_{loc}^{-1}$

Hilbert transform

$G_{loc} = \Sigma_k [G_k^{non-1}(E) - \Sigma]^{-1}$

Dyson equation

$G_0 = (G_{loc}^{-1} + \Sigma)^{-1}$

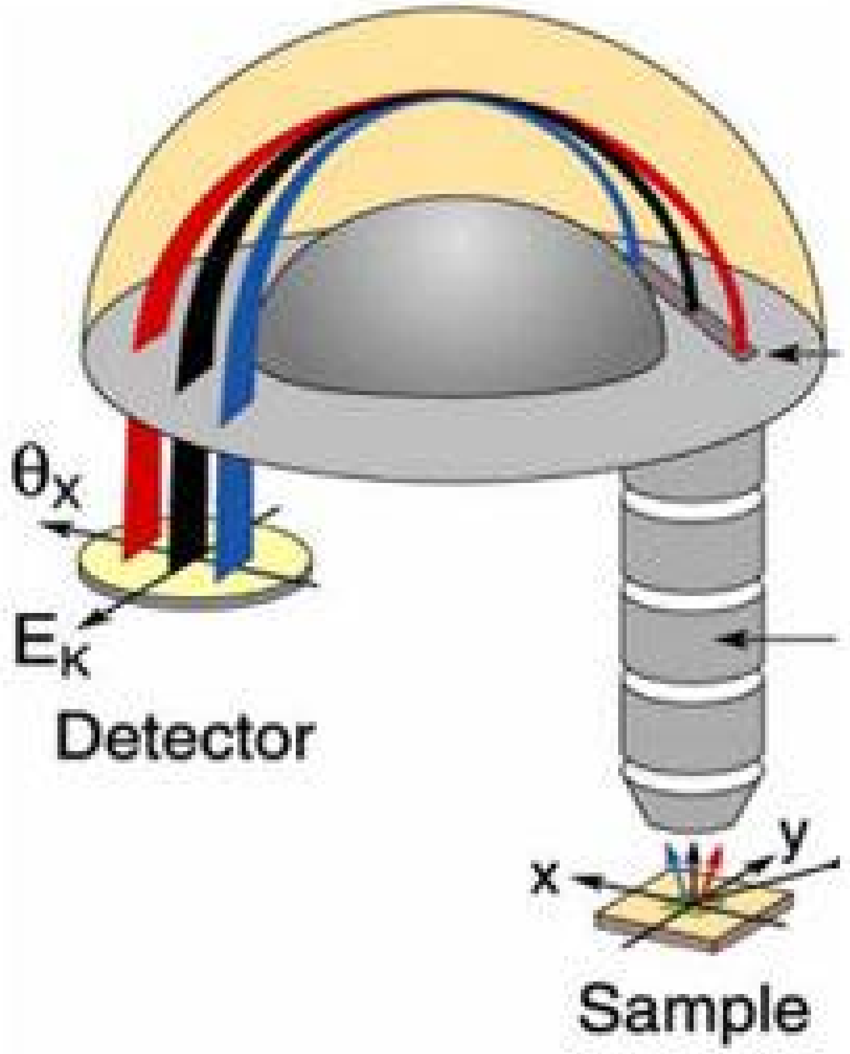
Solve impurity problem

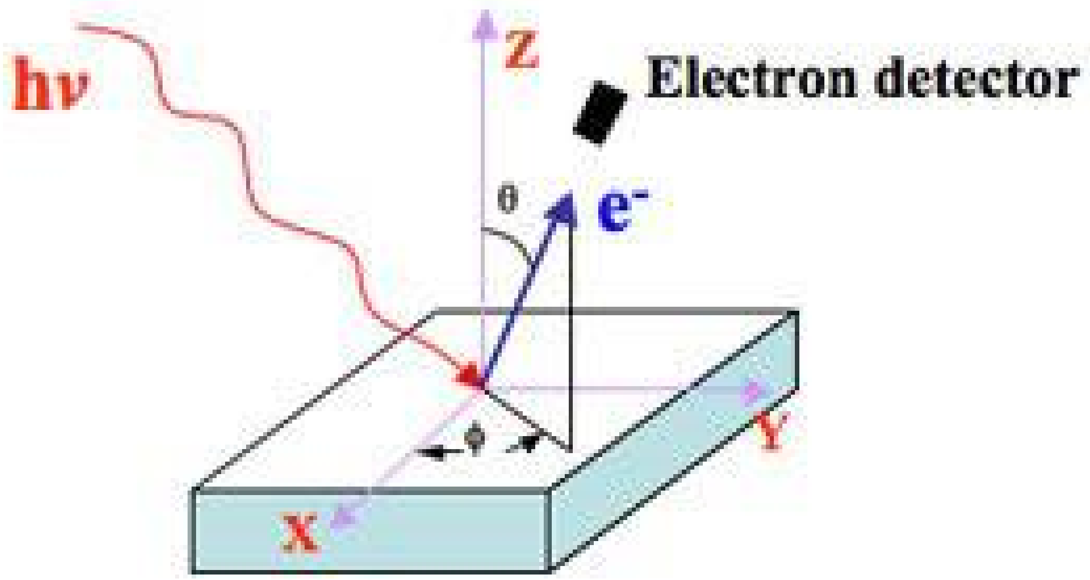
G_{loc}=Functional(G_0)
{example: FK model:
$G_{loc} = (1-w_1) G_0(\mu) + w_1 G_0(\mu - U)$}

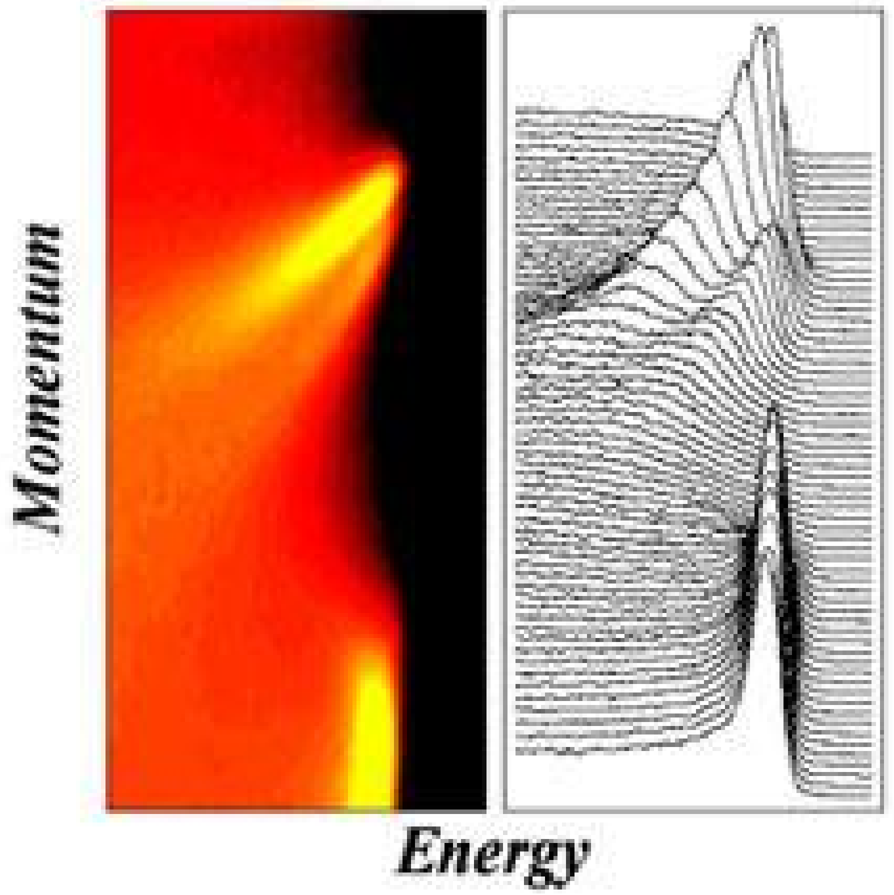
All objects (G and Σ) are matrices with each time argument lying on the contour.

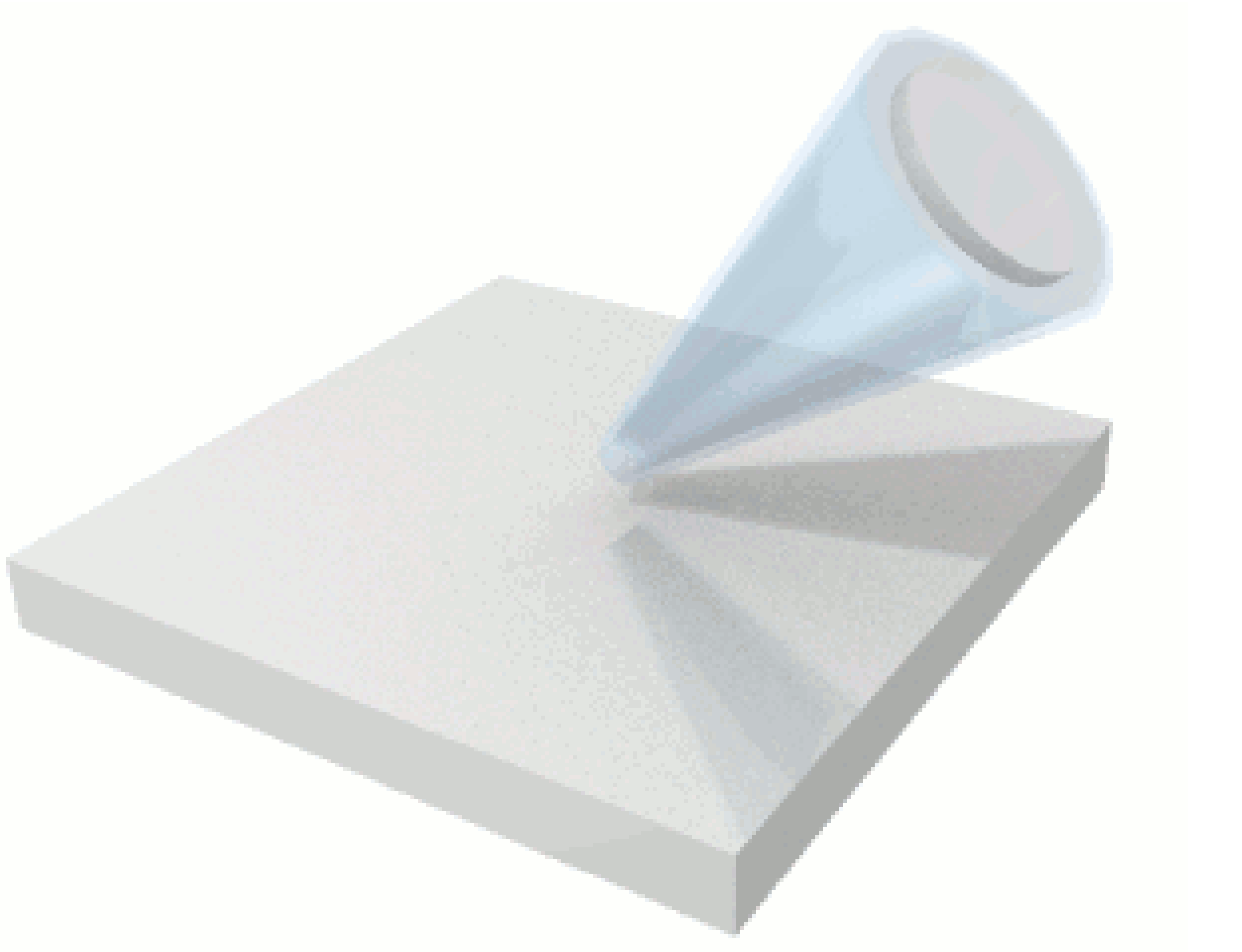
Experimental methodology

Experimental observables: photoemission





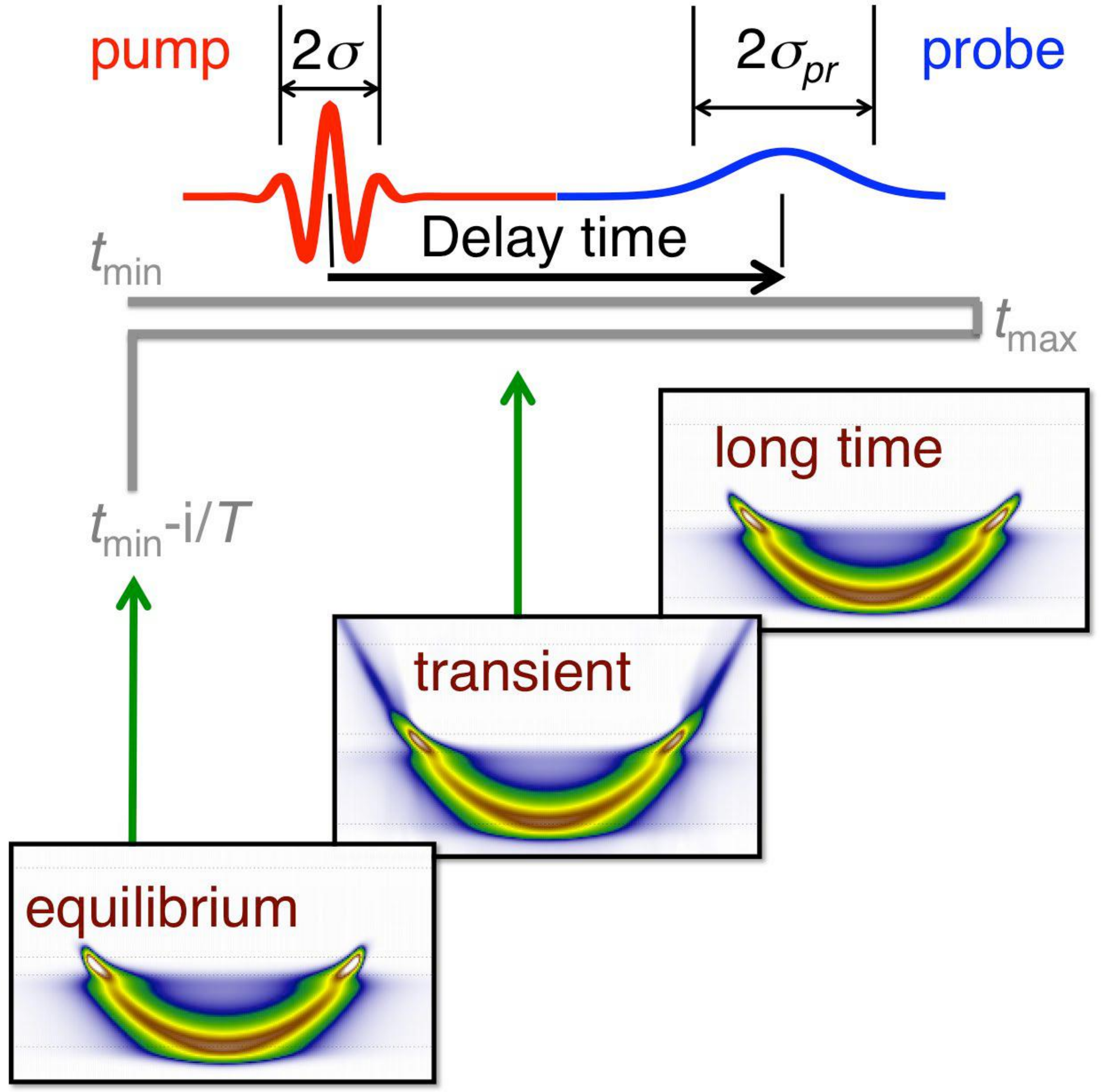


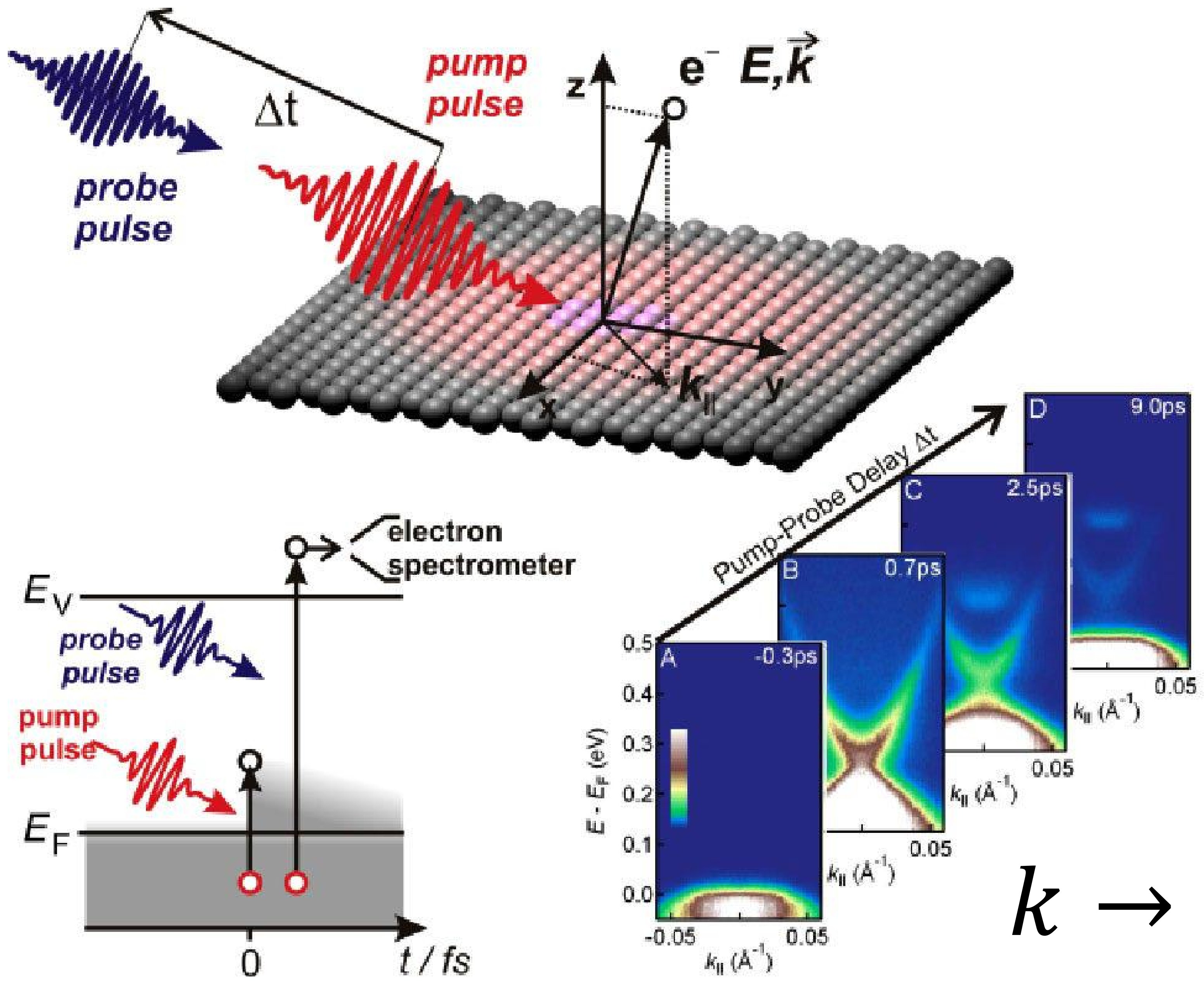


From the photoelectric effect to high-precision experiments on correlated electrons

Focus on the observables.
Forget your frequency-space biases.

Theoretical description of TR-ARPES



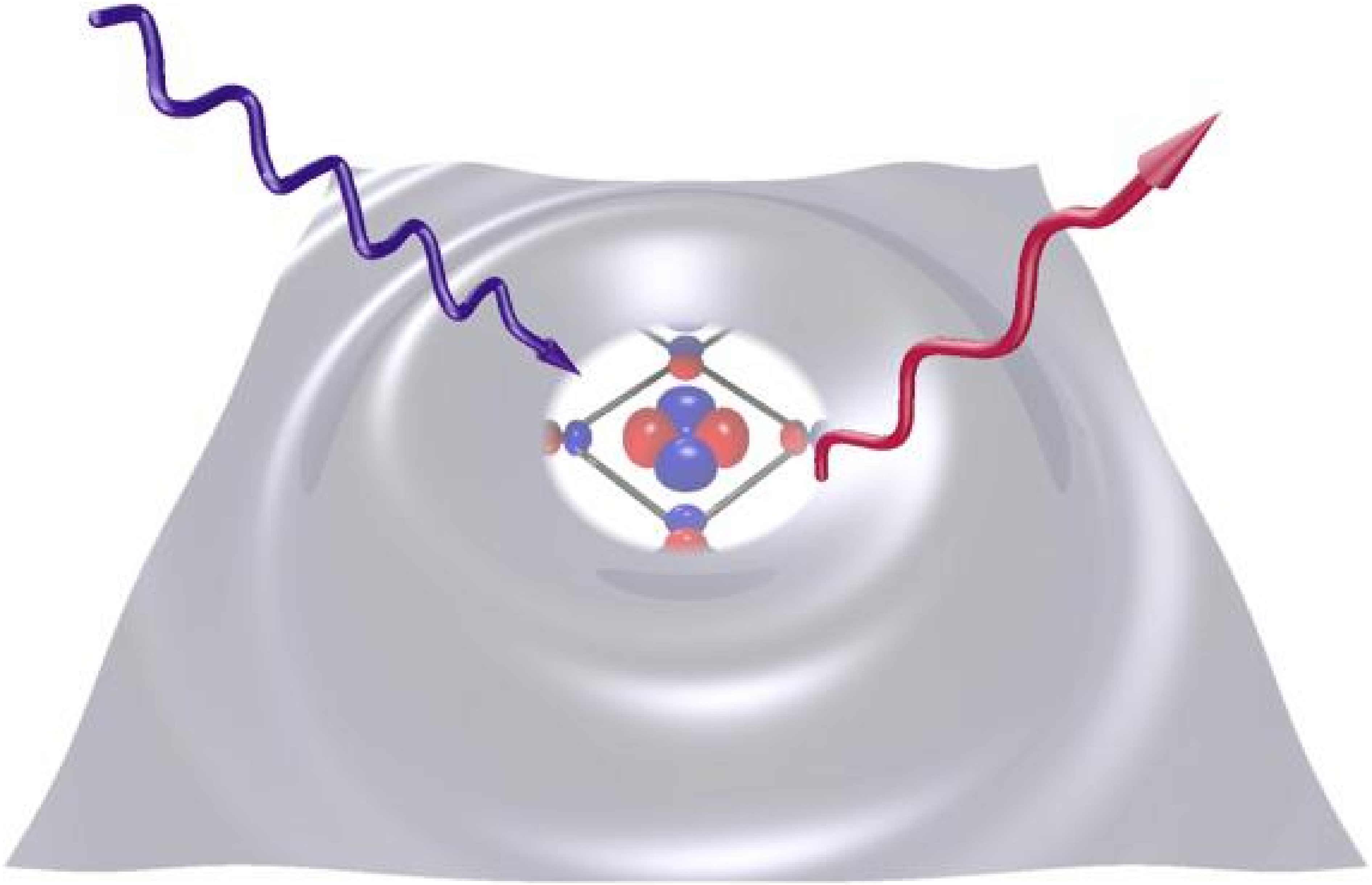


For angle-resolved calculations, we need to work with GAUGE-INVARIANT Green's functions

$$k \to k - \frac{1}{t-t'}\int_{-\frac{t-t'}{2}}^{\frac{t-t'}{2}} d\bar{t} A\left(\frac{t+t'}{2}+\bar{t}\right)$$

$$A_{\mathbf{k}}(\omega, t_0) = \mathrm{Im}\ \frac{1}{2\pi\sigma^2}\int dt dt' G^<_{\mathbf{k}}(t,t') e^{-(t-t_0)^2/2\sigma^2} e^{-(t'-t_0)^2/2\sigma^2} e^{i\omega(t-t')}$$

What is electronic Raman scattering



Inelastic scattering of light–one out of 10^{11} photons loses or gains energy when scattering. If the energy is lost or gained comes from electronic excitations, it is called electronic Raman scattering

Raman cross section vs. response function

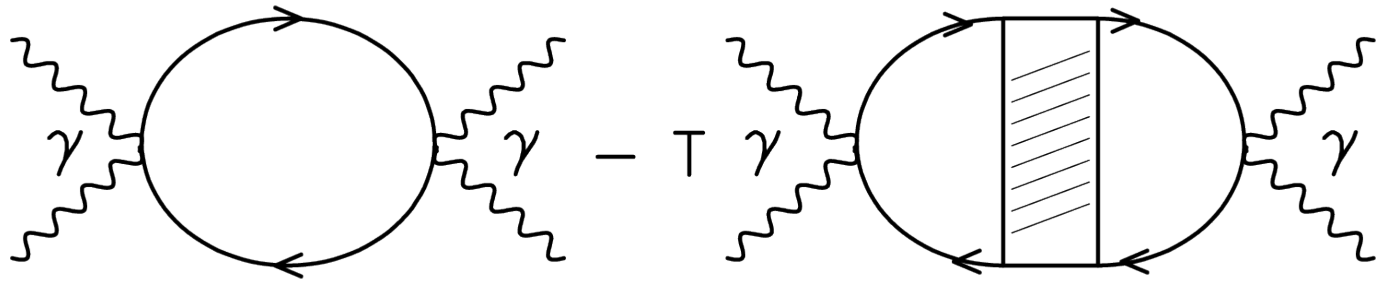
The nonresonant Raman cross-section $R_N(\Omega)$ is what one measures in an experiment; it comes from the greater correlation function.

The ratio of the signal when energy comes from the electrons (anti-Stokes) to the case when energy is transferred to electron (Stokes) is given by the temperature:

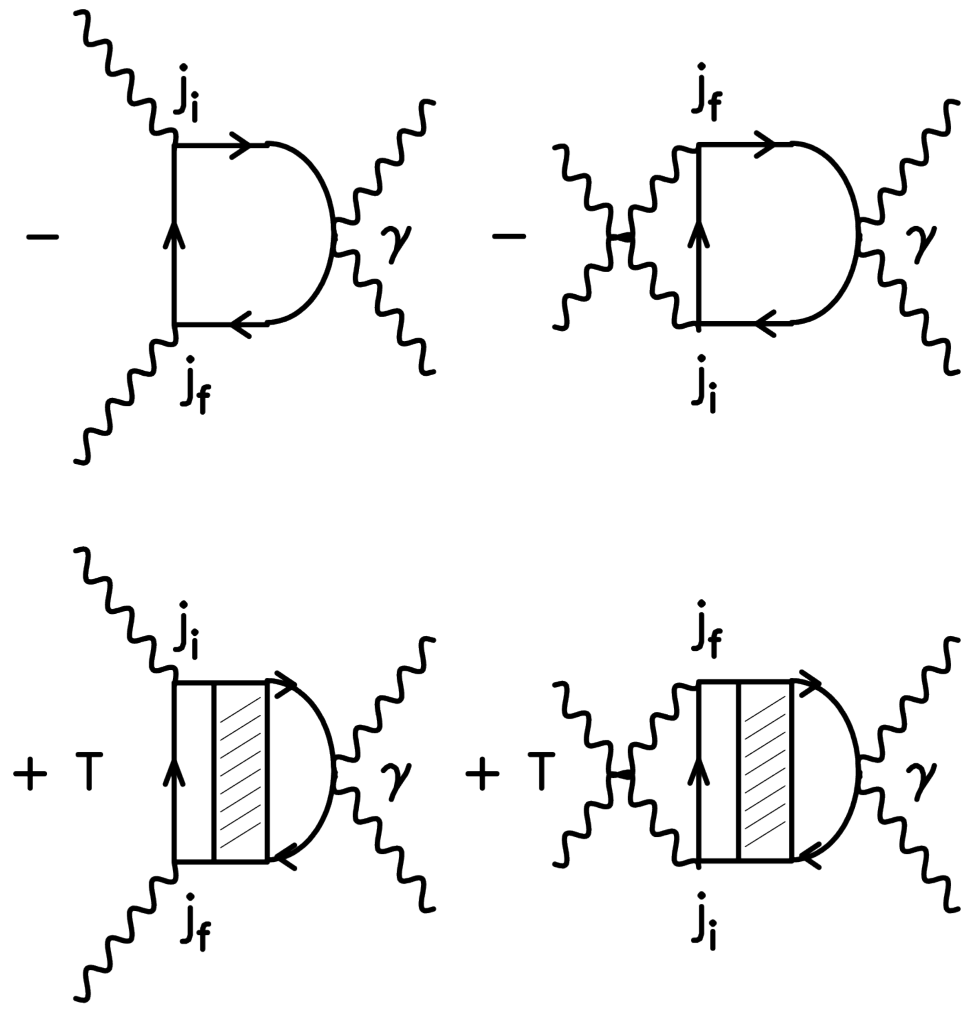
$$\frac{Stokes}{anti-Stokes} = \frac{R_N(\Omega)}{R_N(-\Omega)} = \exp(\beta\Omega)$$

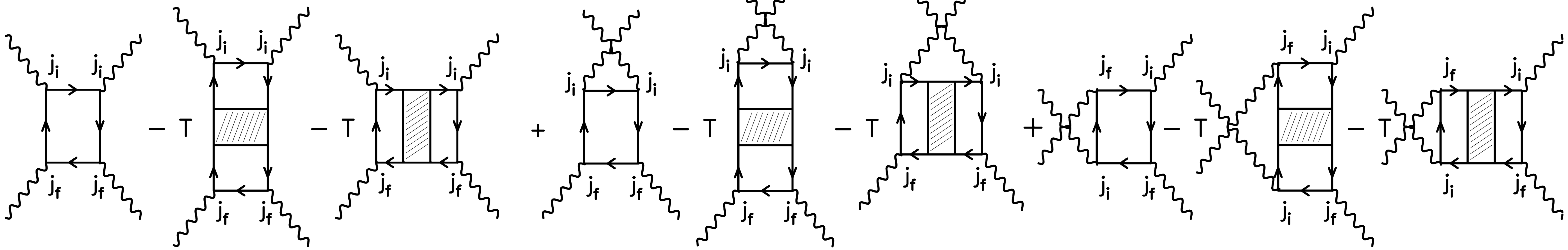
The Raman response function $\chi_N(\Omega)$ comes from the retarded Green's function. It is given by $R_N(\Omega) = [1 + \mathrm{n_B}(\Omega)]\chi_N(\Omega)$. χ_N is antisymmetric and real.

Diagrammatic representation

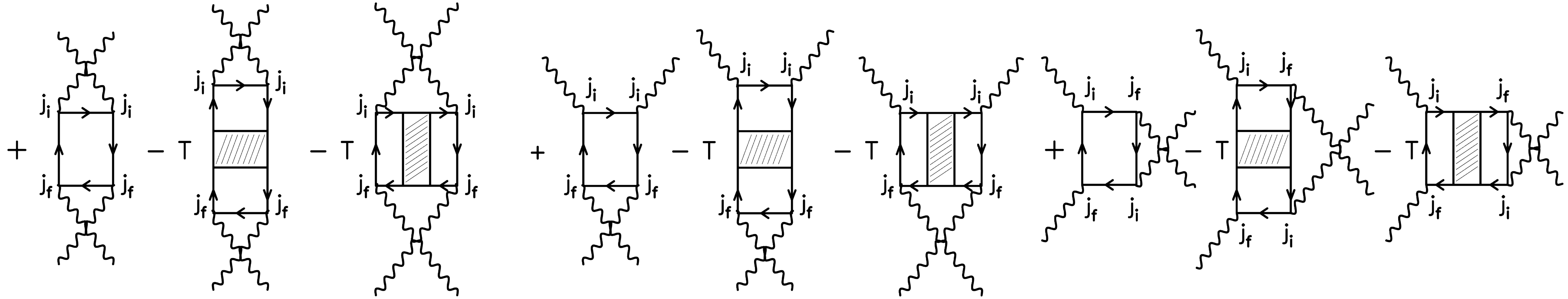


Nonresonant response





Mixed response



Resonant response

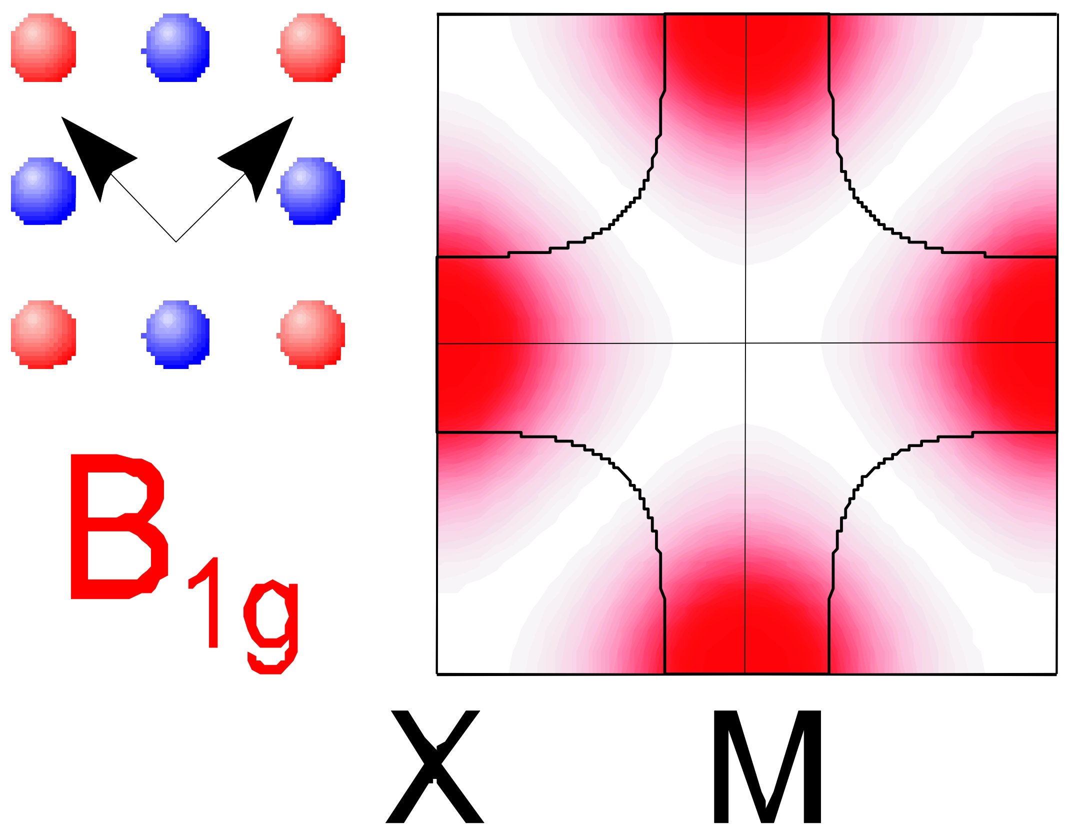
Polarization dependence

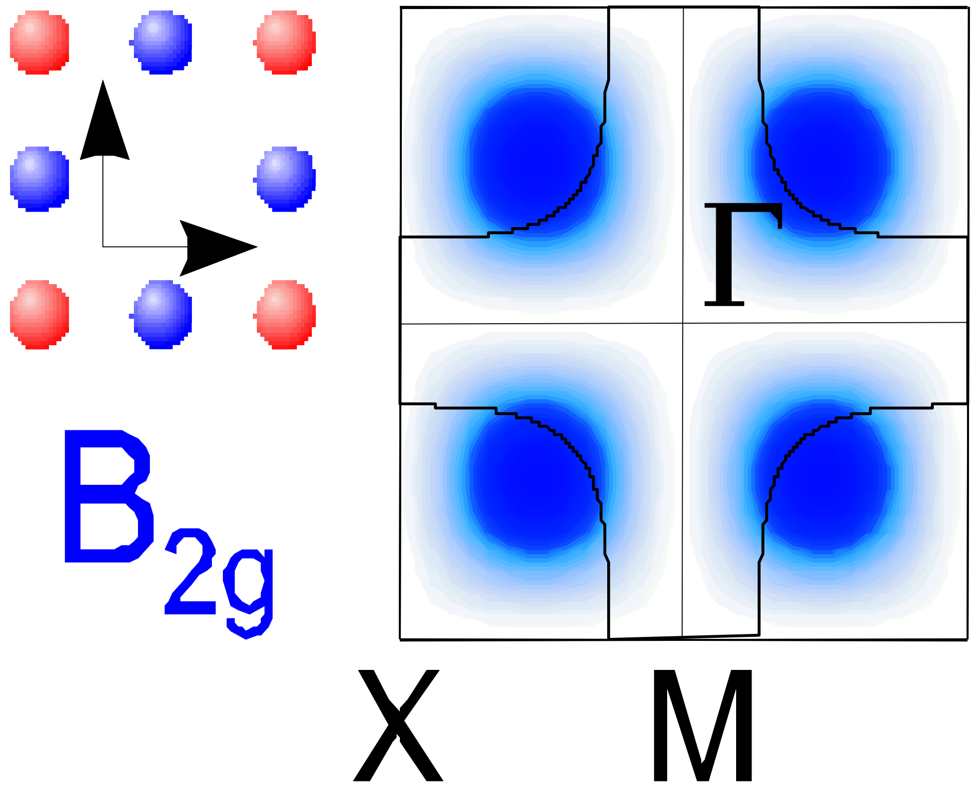
$$\gamma_R = (\hat{e}_i \cdot \nabla)(\hat{e}_f \cdot \nabla)\epsilon(k)c_k^\dagger c_k$$ Raman stress-tensor operator

A_{1g} symmetry–same symmetry as the lattice–polarizers in the same direction.

B_{2g} symmetry–d-wave symmetry–crossed polarizers in the axial direction.

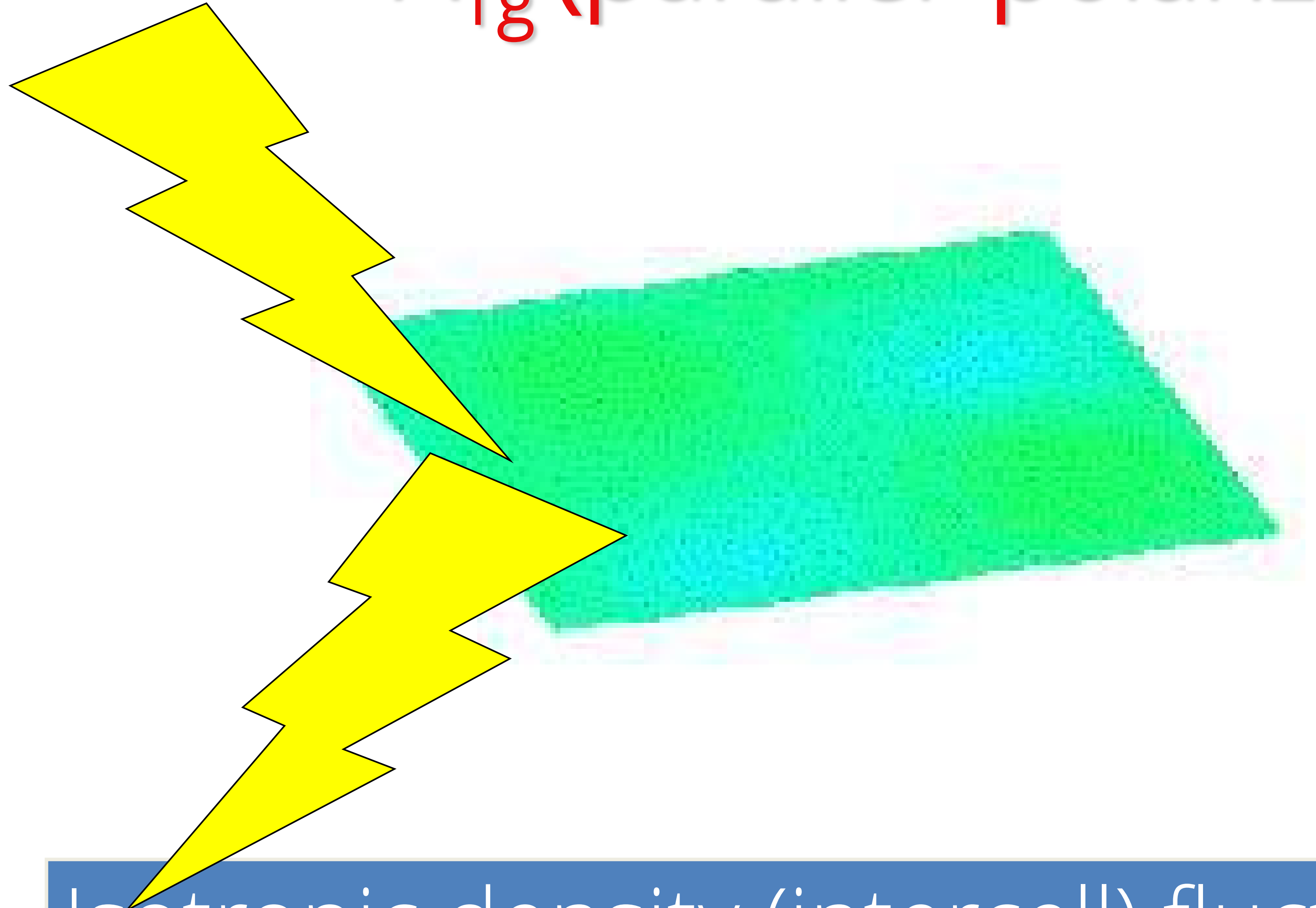
B_{1g} symmetry–d-wave symmetry–crossed polarizers in the diagonal direction.





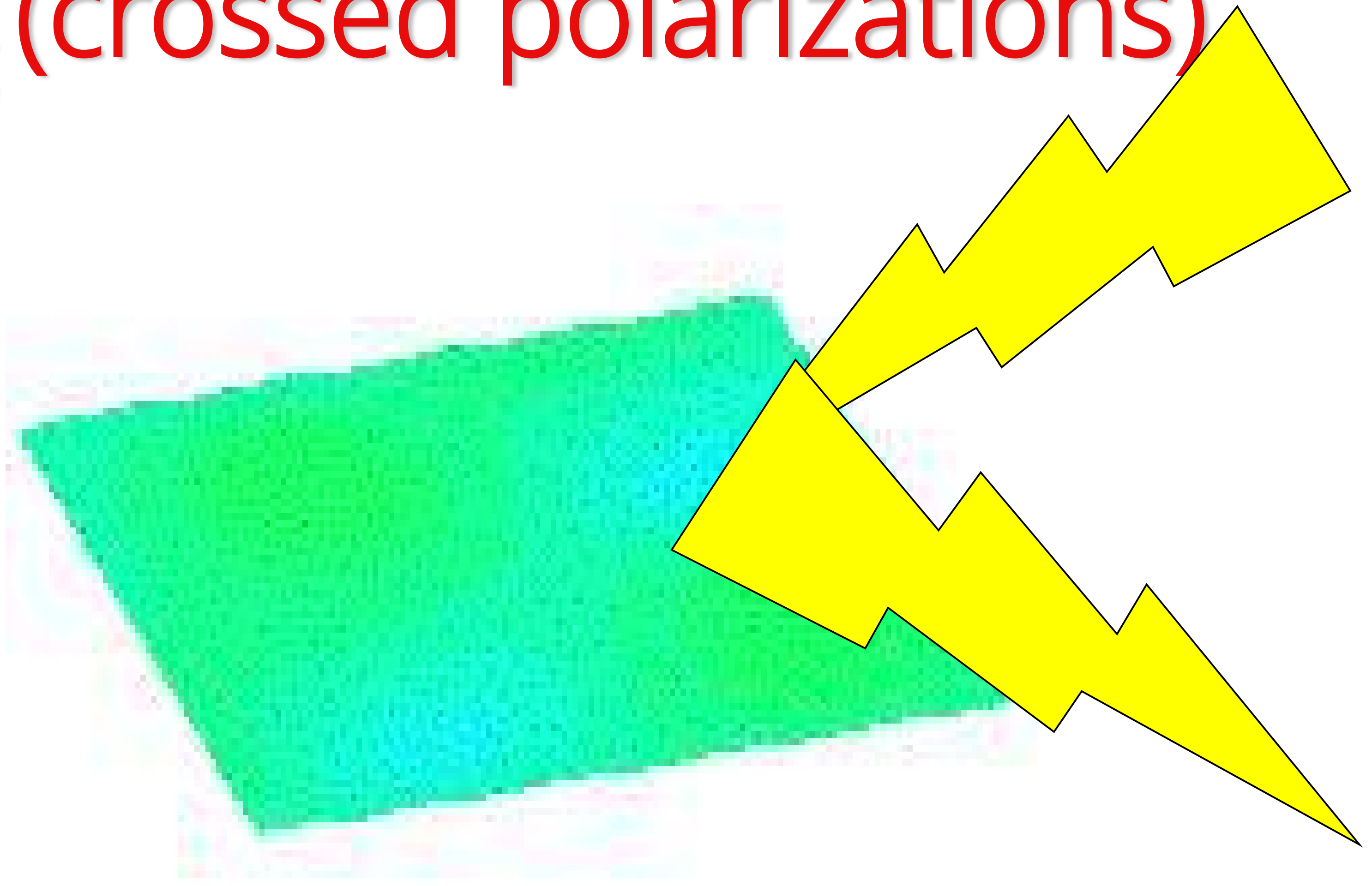
Polarization dependence determines the type of charge excitation

A_{1g} (parallel polarizations)



Isotropic density (intercell) fluctuations – couple to long-range Coulomb interactions ~ Im ($1/\varepsilon$)

B_{2g} (crossed polarizations)



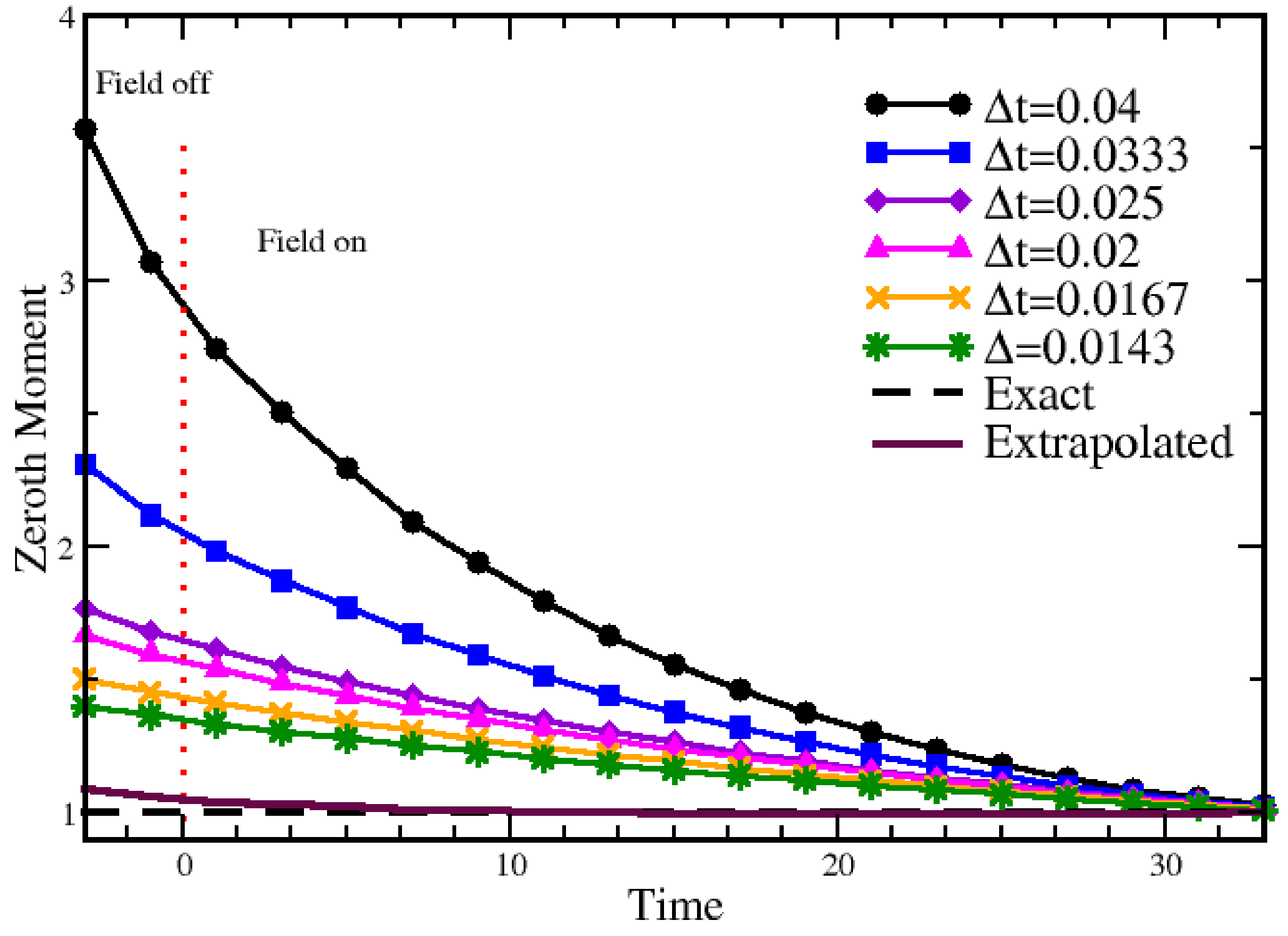
Anisotropic density (intracell) fluctuations – couple to short-range Coulomb interactions.

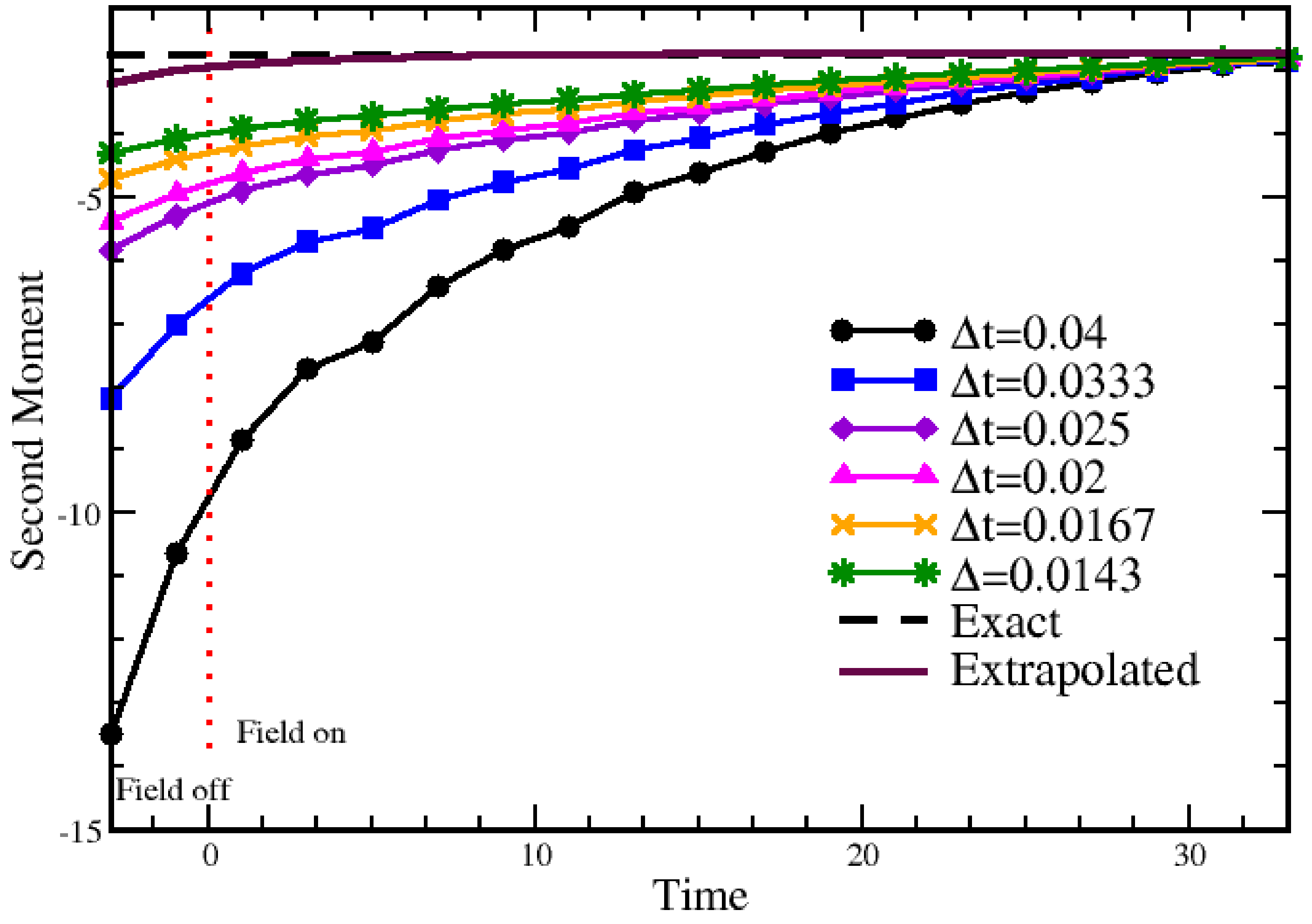
X-ray absorption spectroscopy

- Shine x-rays in, detect x-rays that come out
- Absorption of x-rays can be described by an interband optical conductivity
- Has an “edge singularity” in metals
- Singularity disappears in insulators
- Peaks of spectra are strongly T-dependent at high T

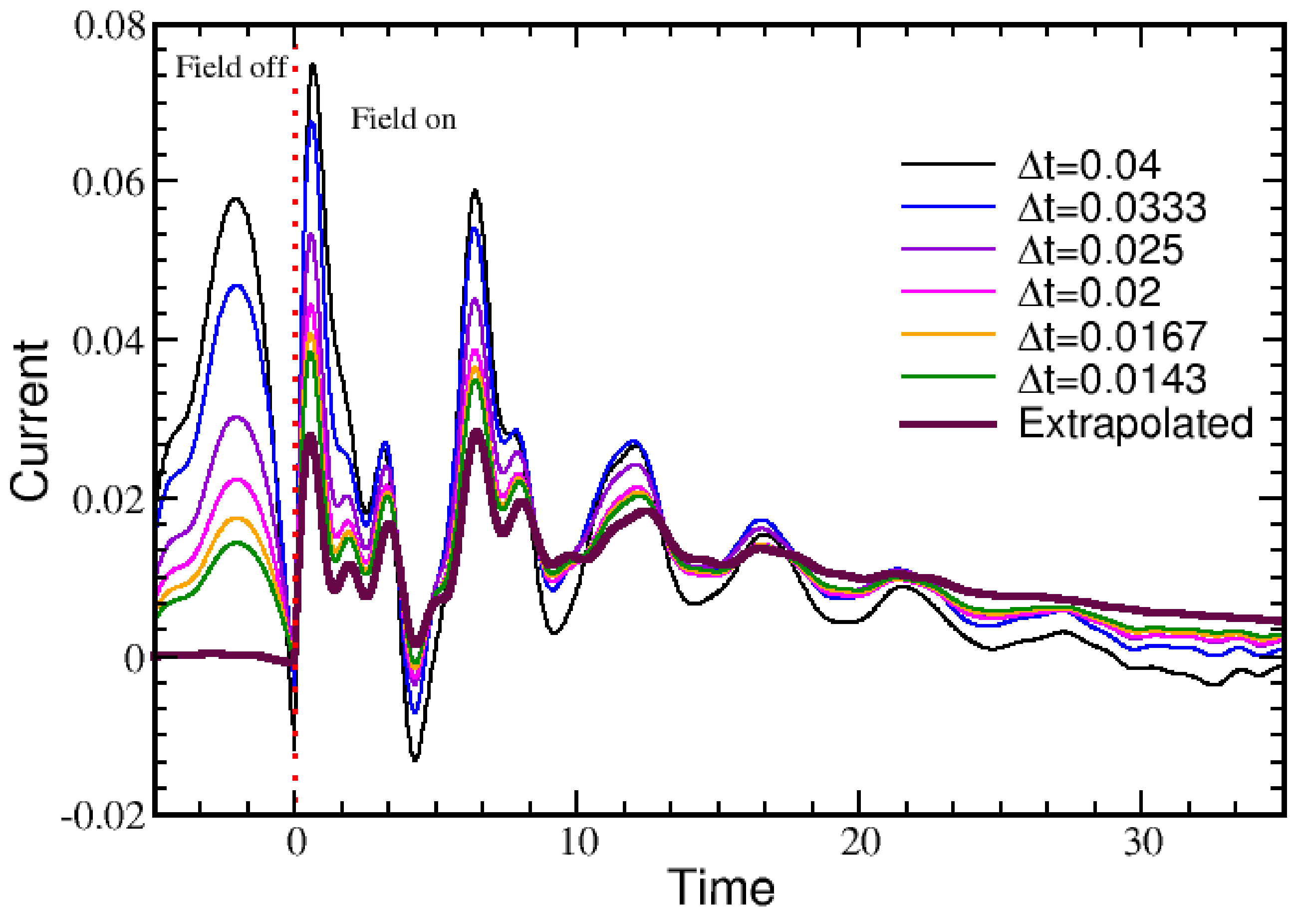
Numerics

Numerical issues





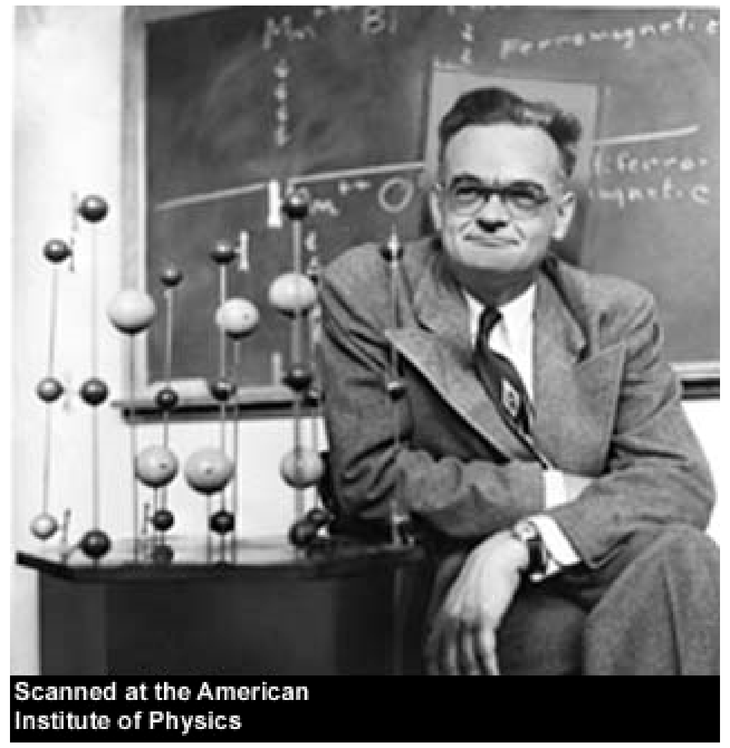
Need to scale results to continuum limit to satisfy sum rules



Scaling also needed to satisfy proper causality.

DC fields and Bloch oscillations

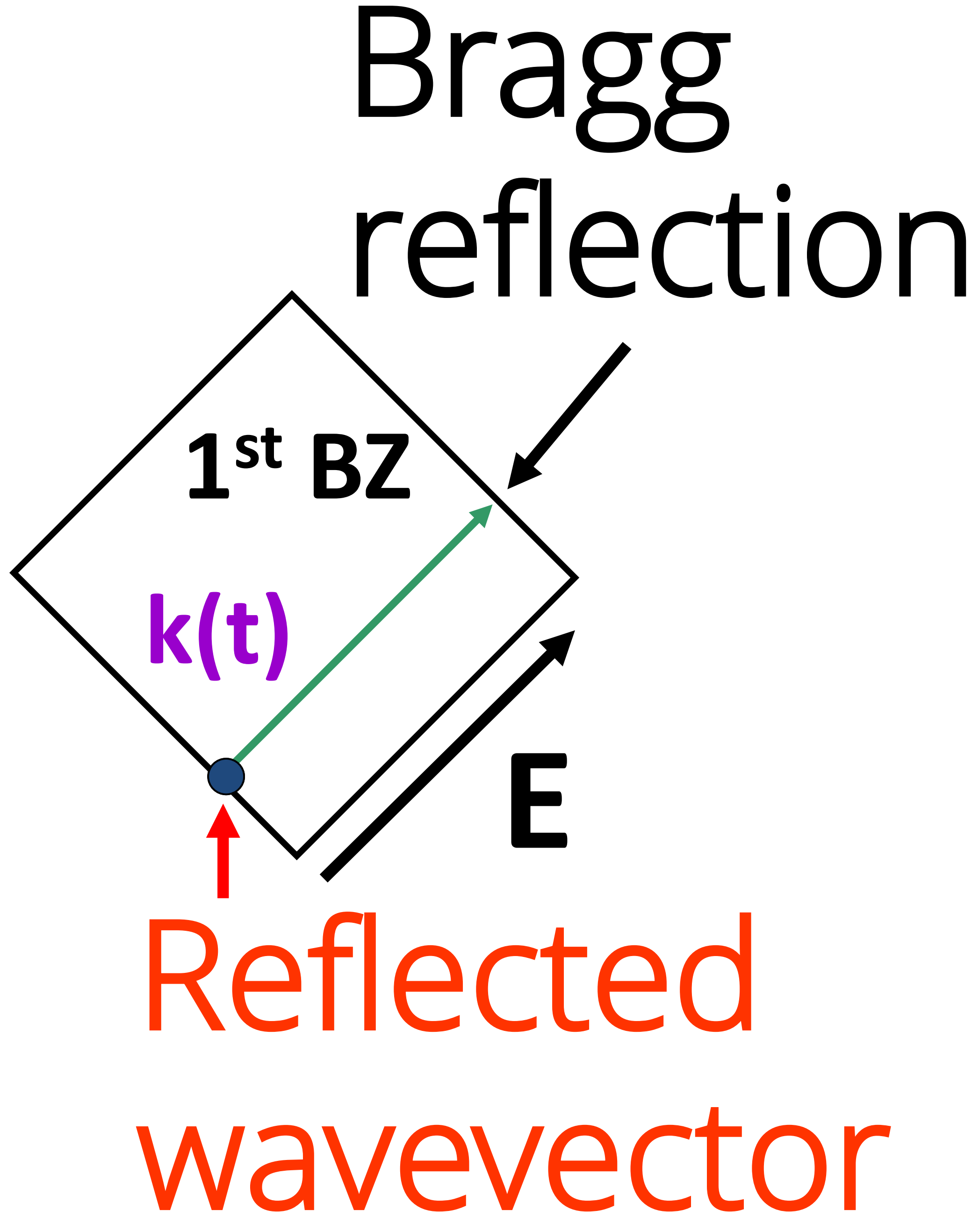
DC fields and Bloch oscillations



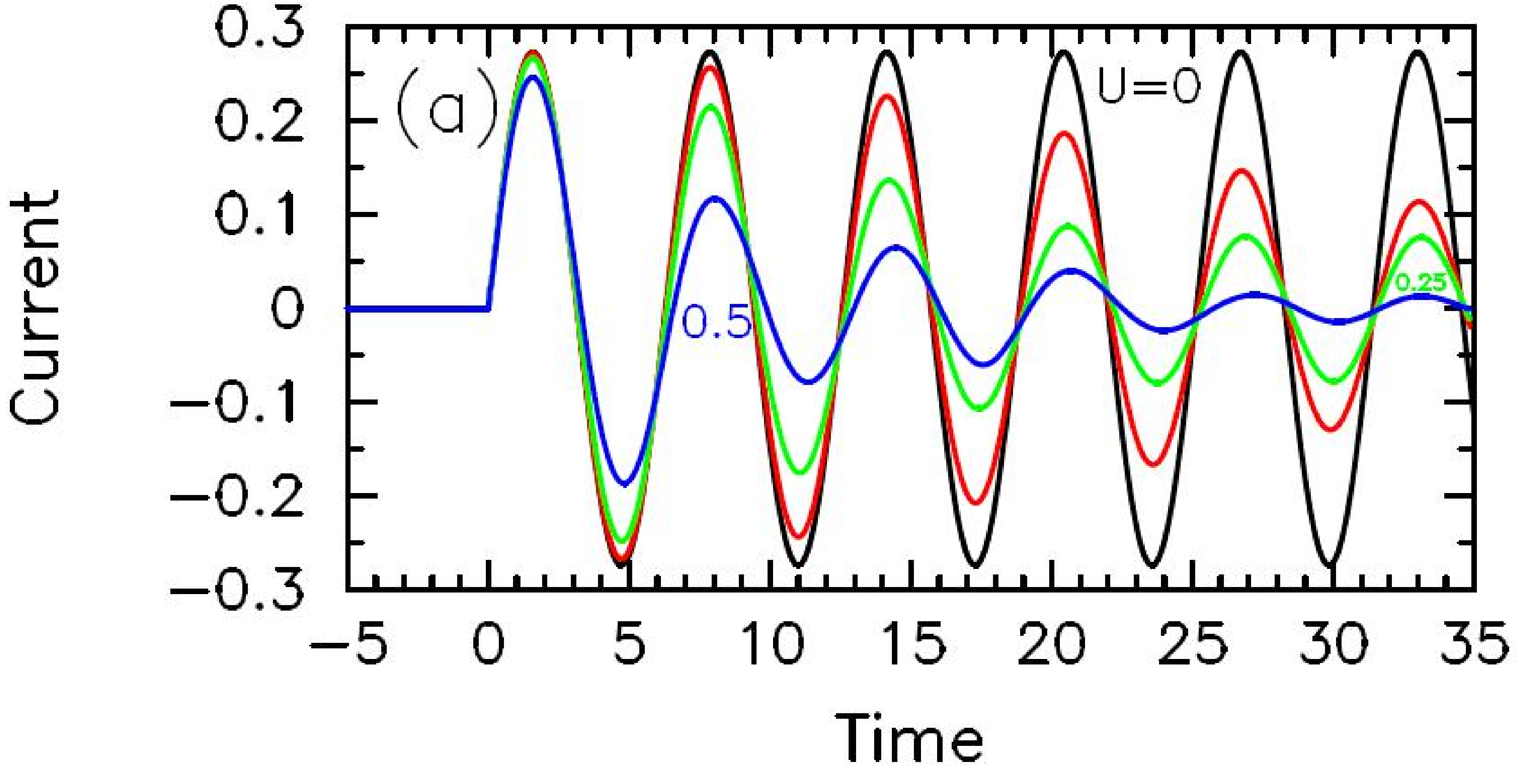
Electrons are uniformly accelerated in a dc field: $k(t) = \frac{eEt}{\hbar}$

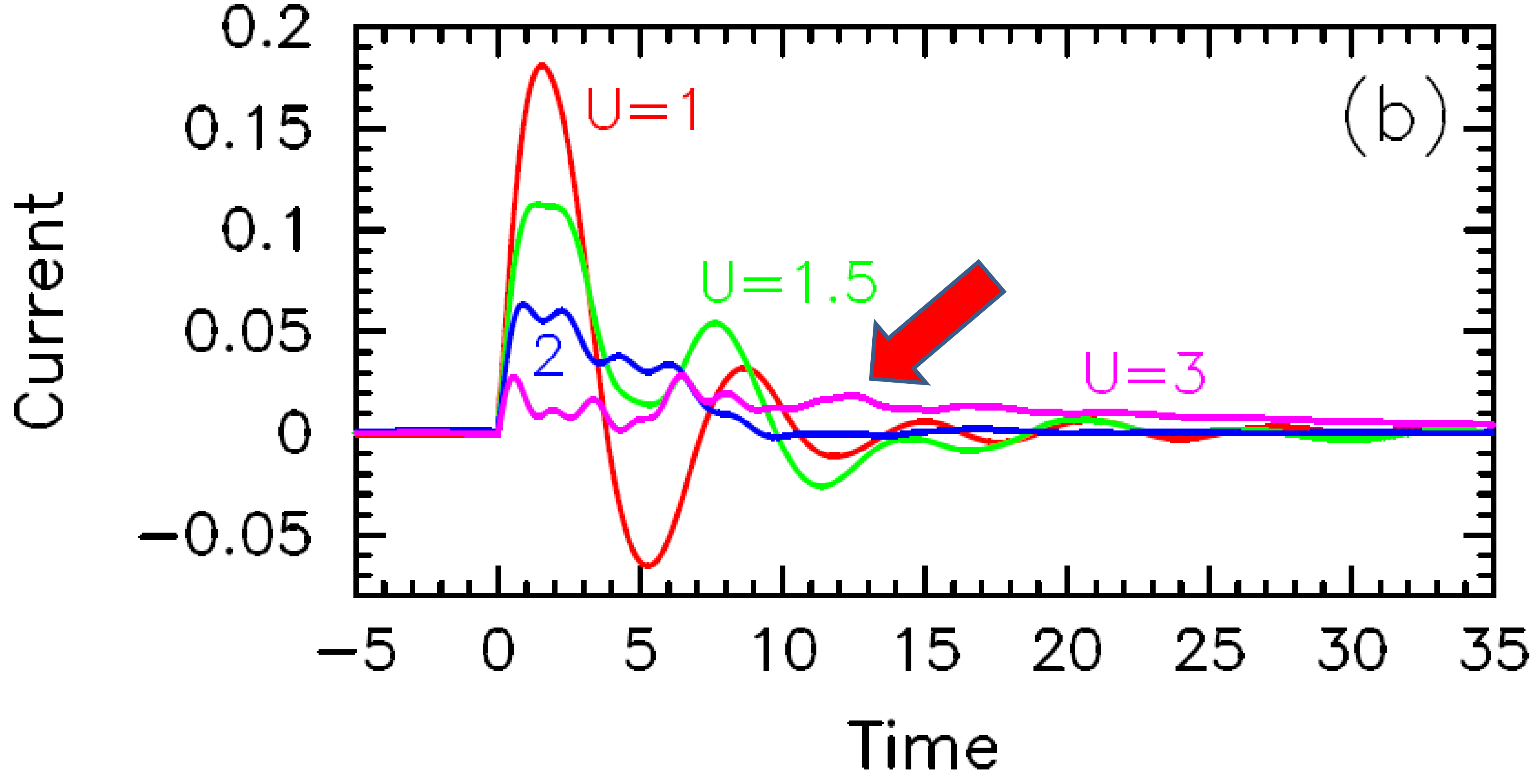
But, when the wavevector arrives at the Brillouin zone boundary, it is Bragg reflected.

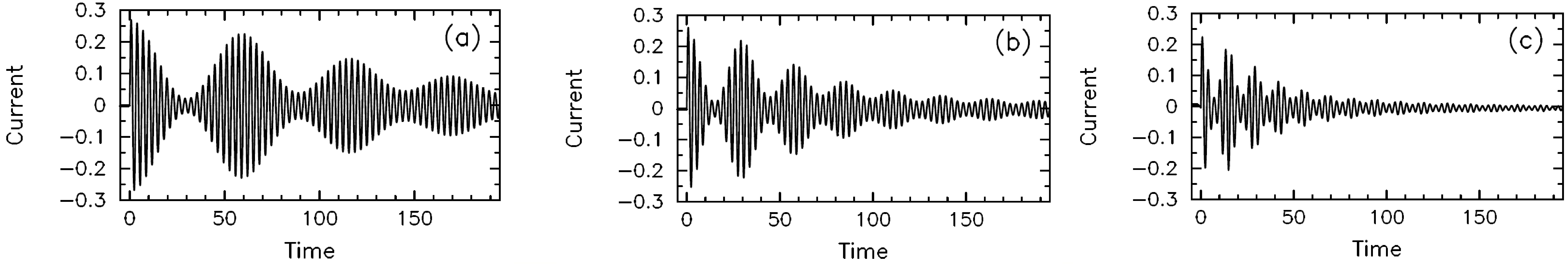
So a dc field induces an ac current with a period inversely proportional to E. This is called a Bloch-Zener oscillation.



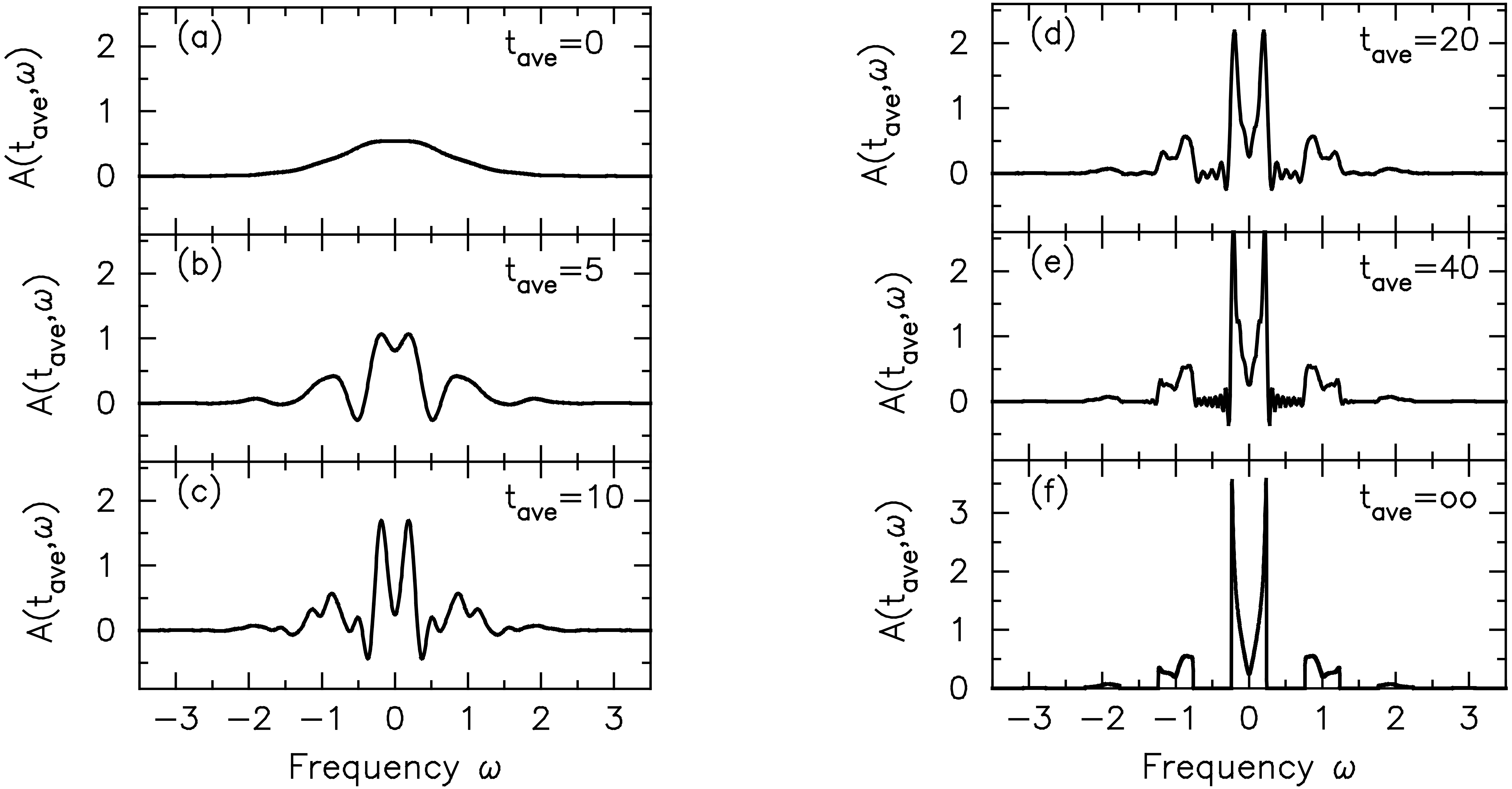
Bloch oscillations in metals and insulators



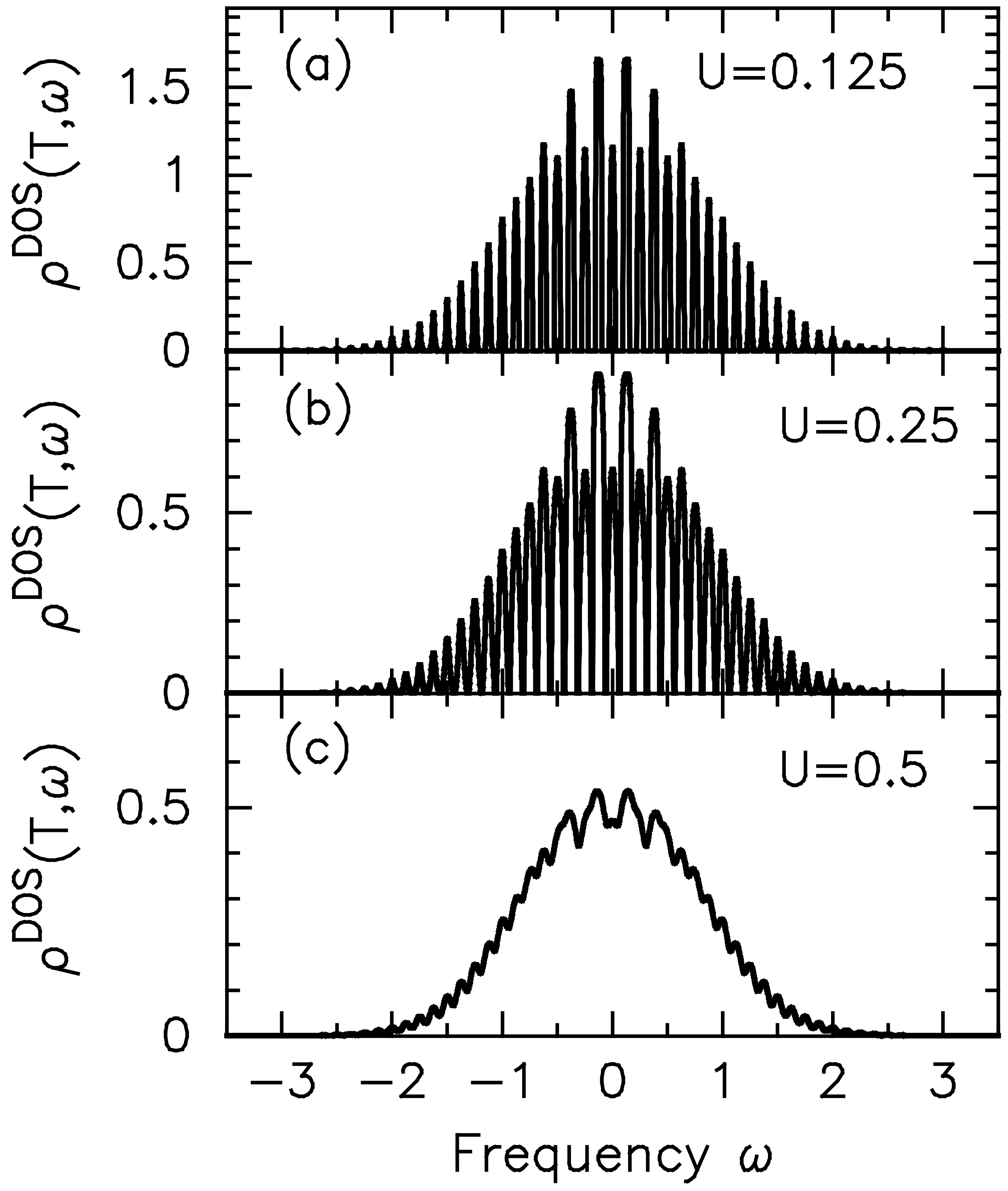


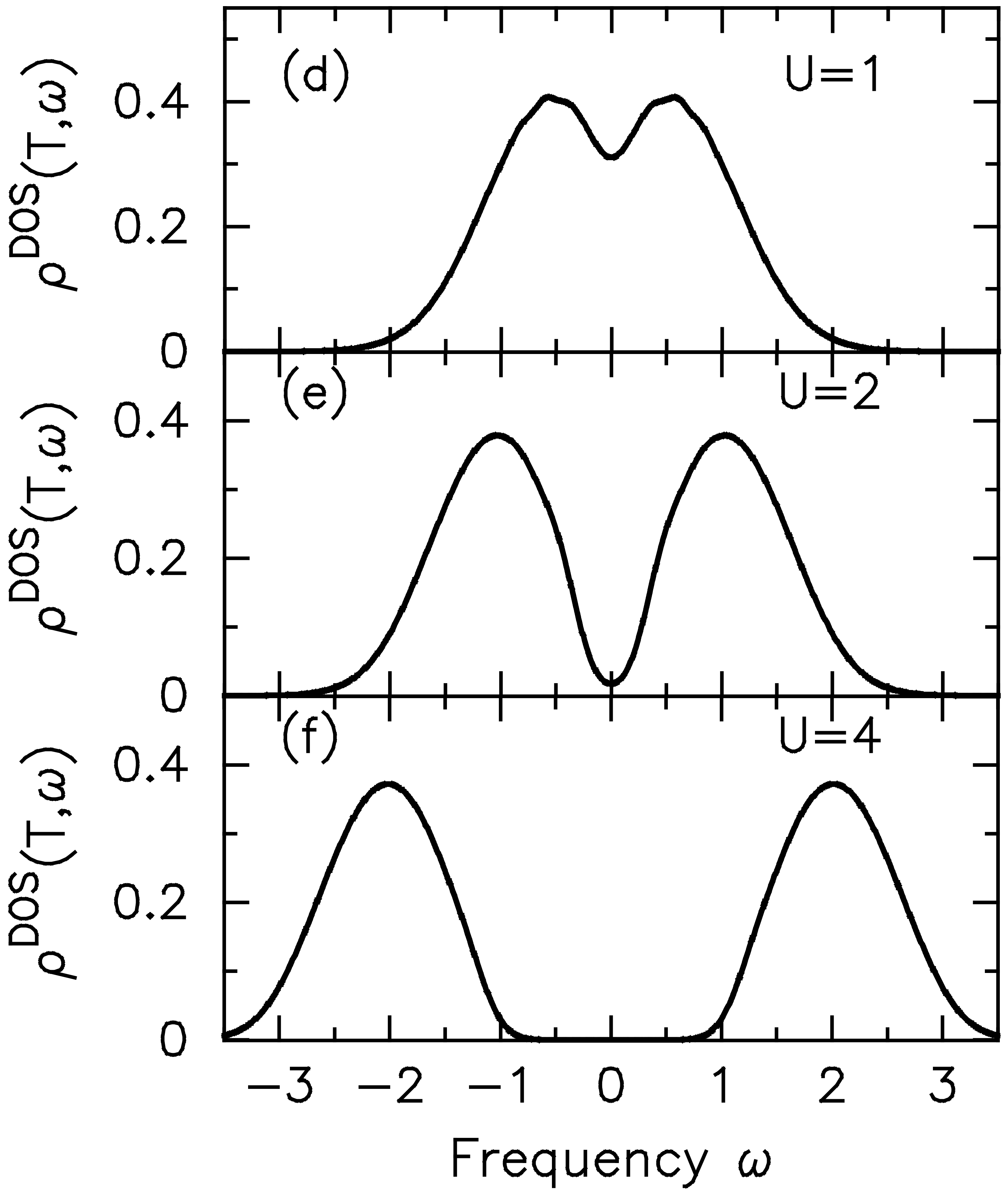


Transient local DOS metal (U=0.5,E=1)

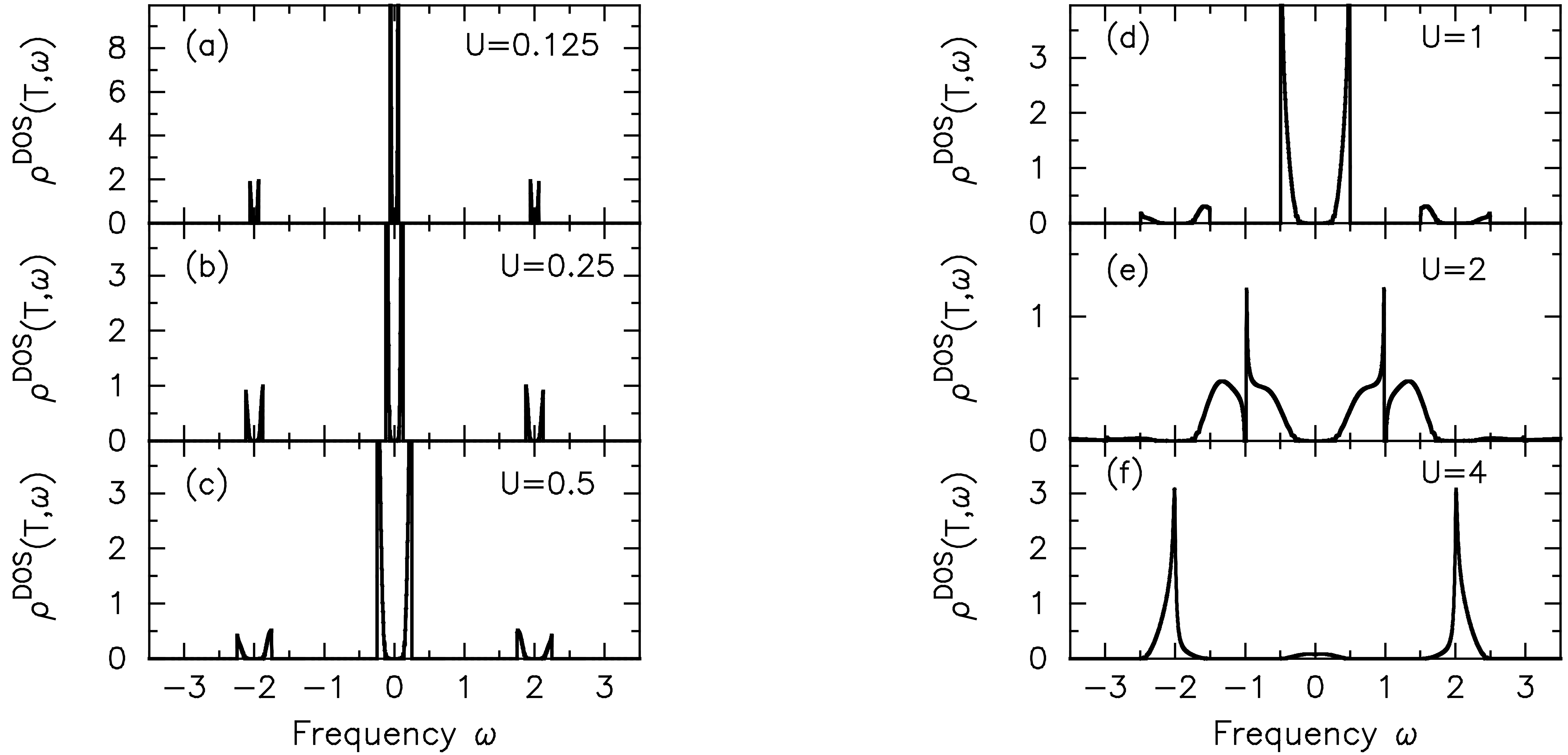


Long-time DOS weak field (E=0.125)

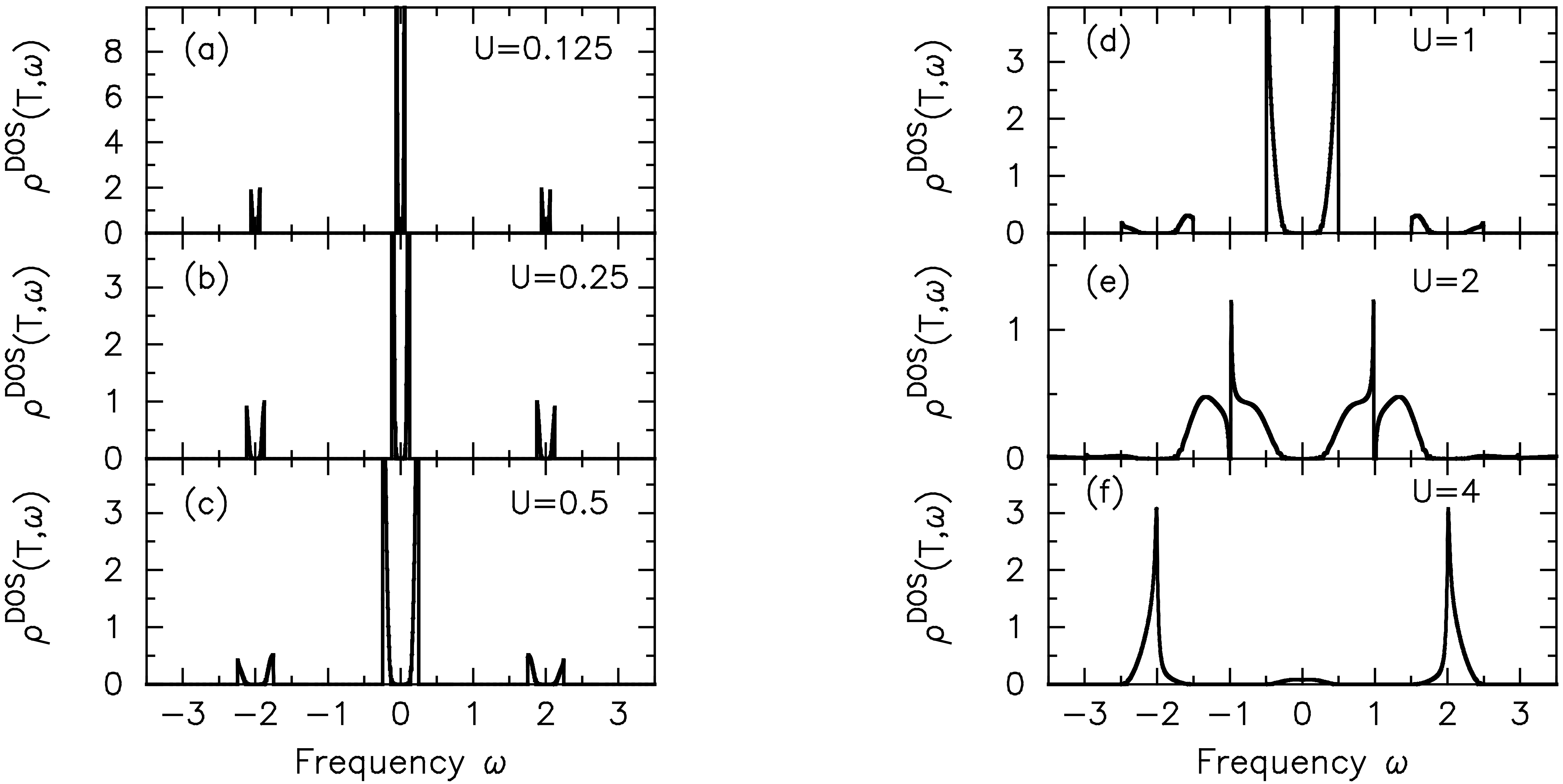




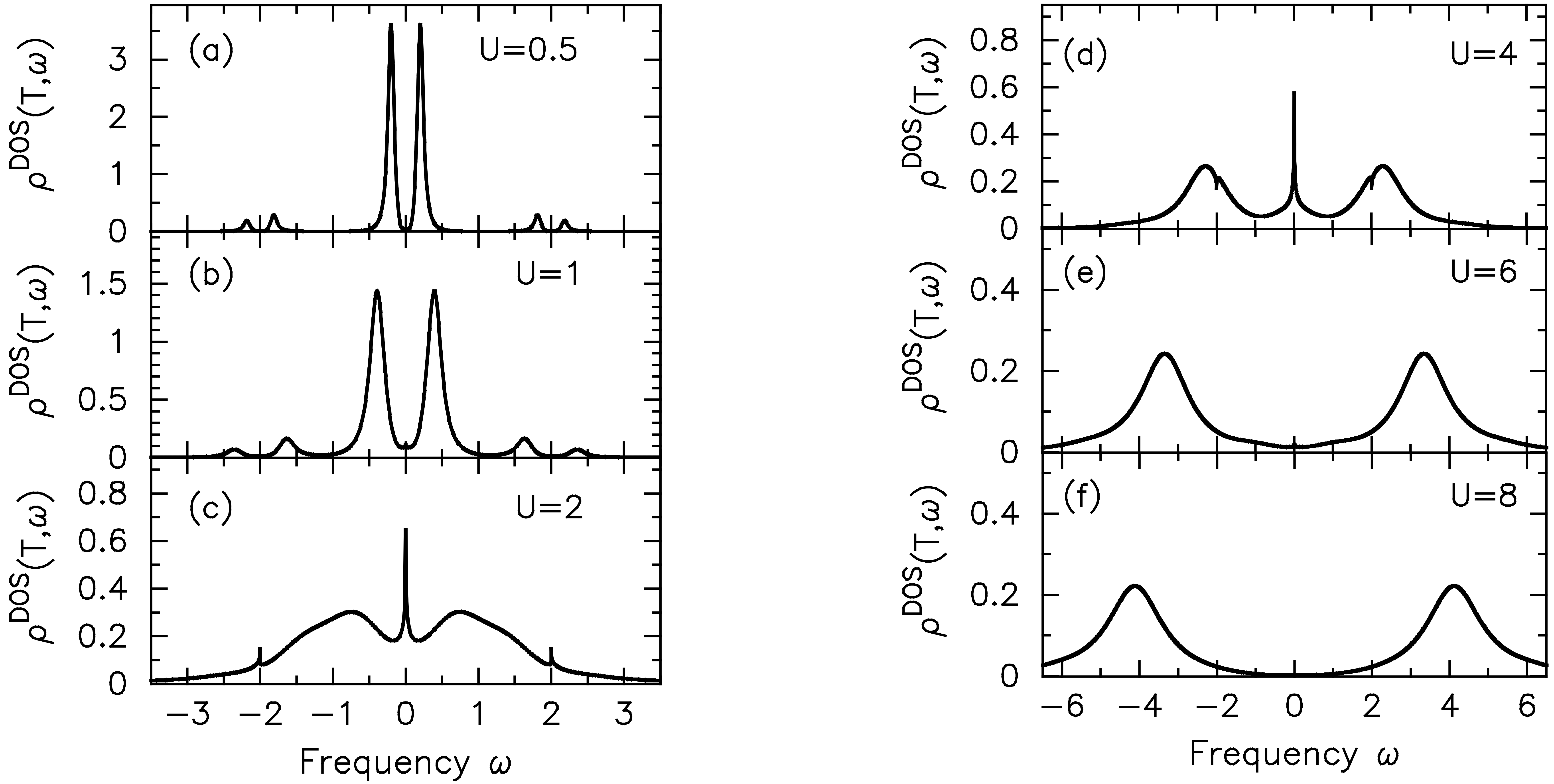
Long-time DOS moderate field (E=0.5)



Long-time DOS strong field (E=2)

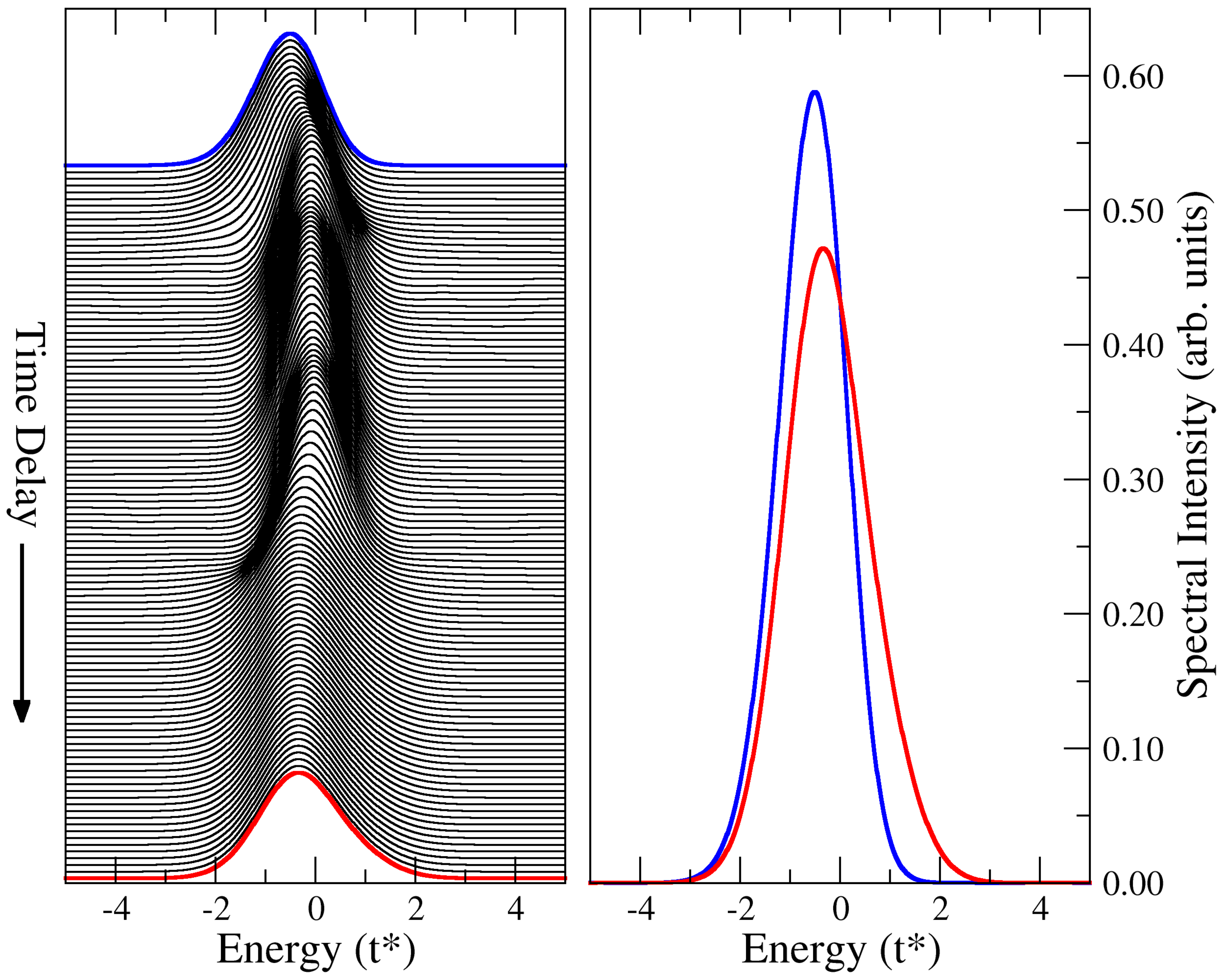


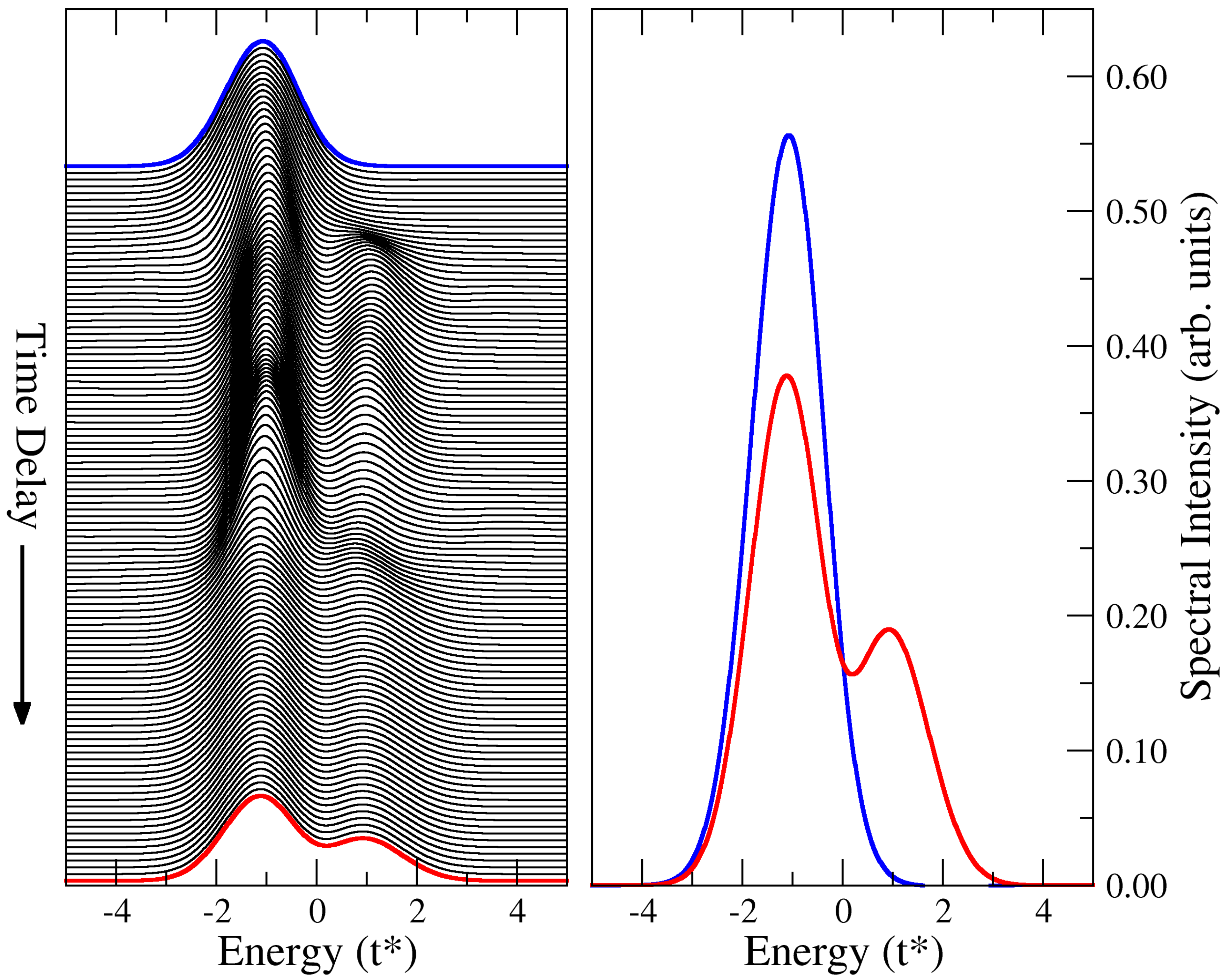
Long-time DOS Hubbard (E=2, approx)



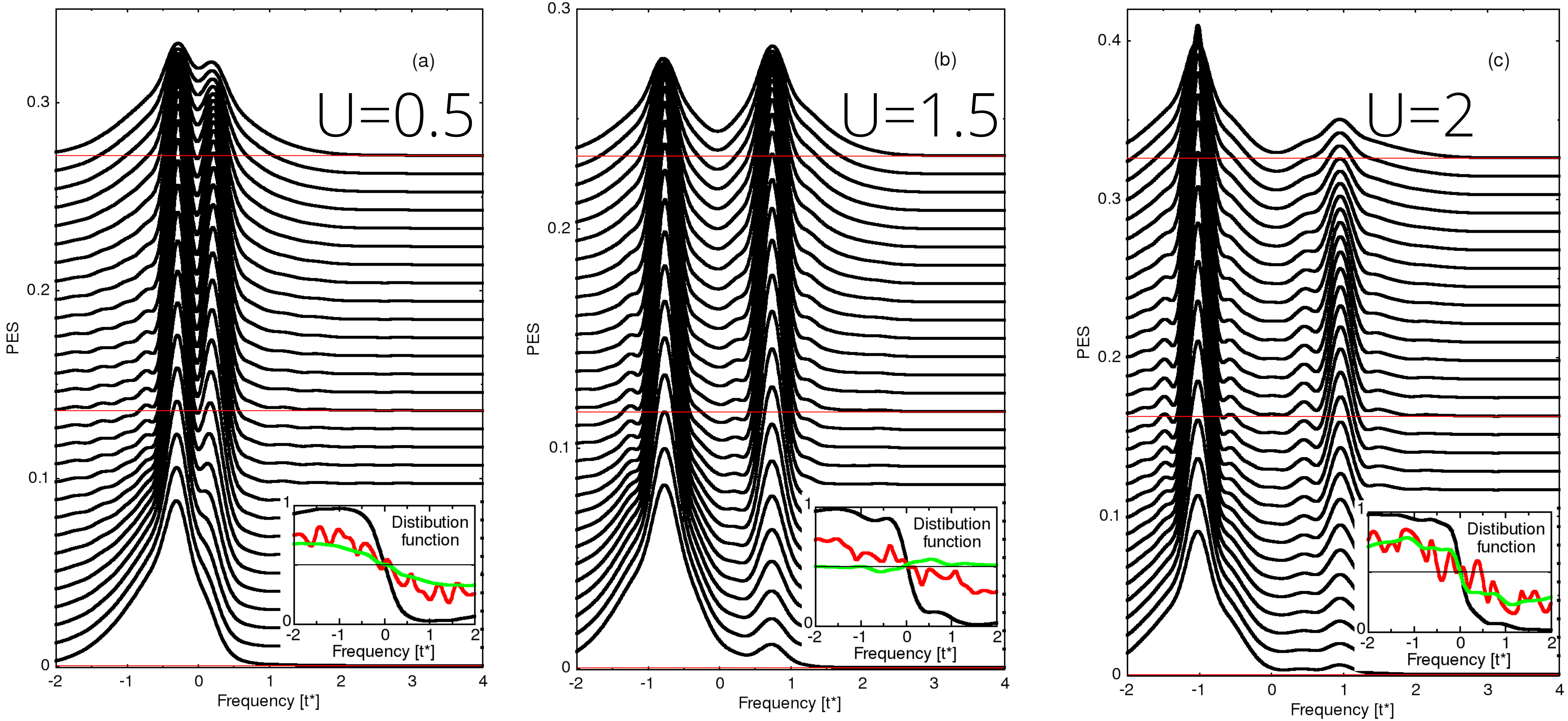
Time-resolved Photoemission

Time-resolved PES (normal state)

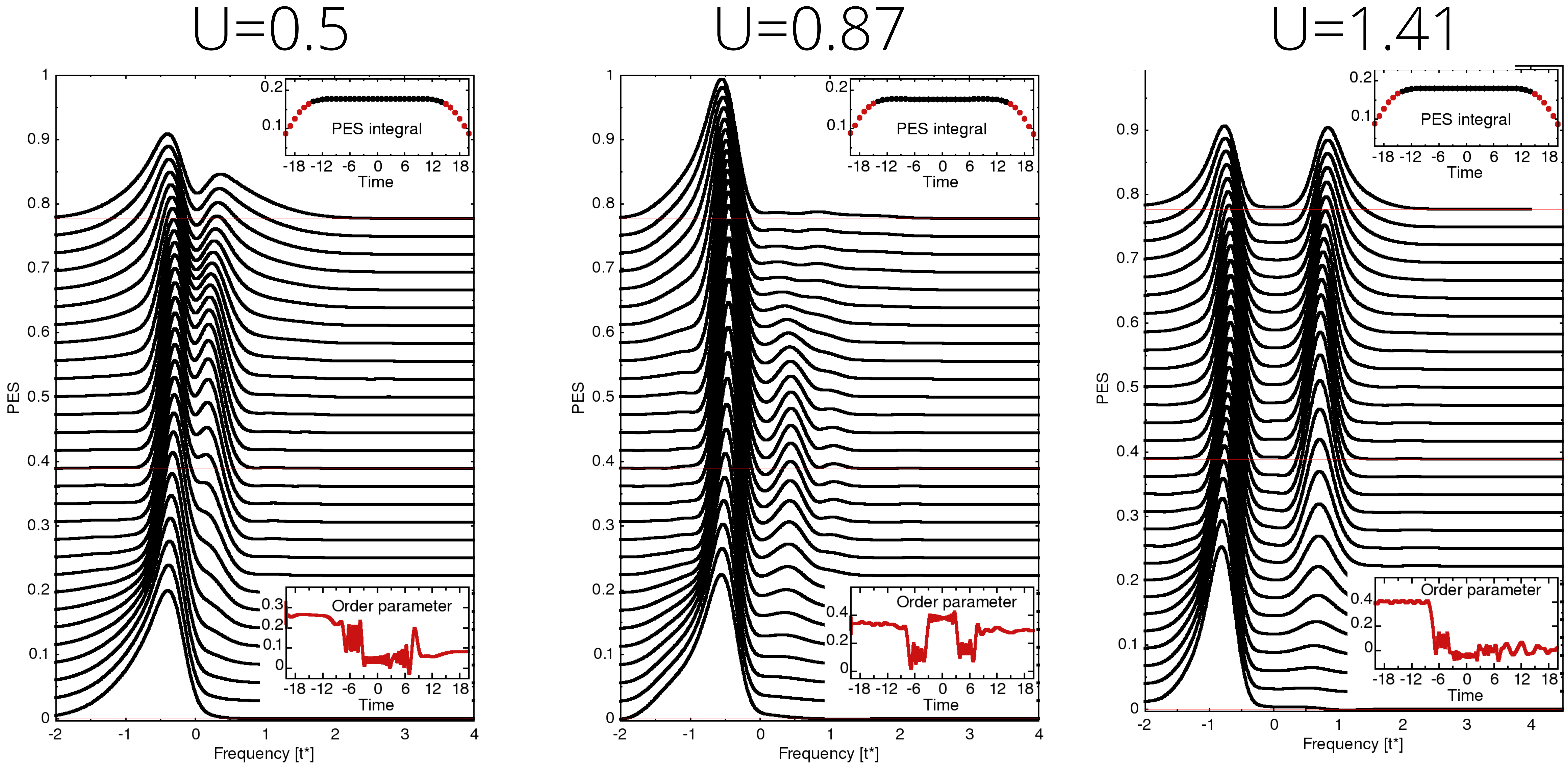




Time-resolved PES (normal state)

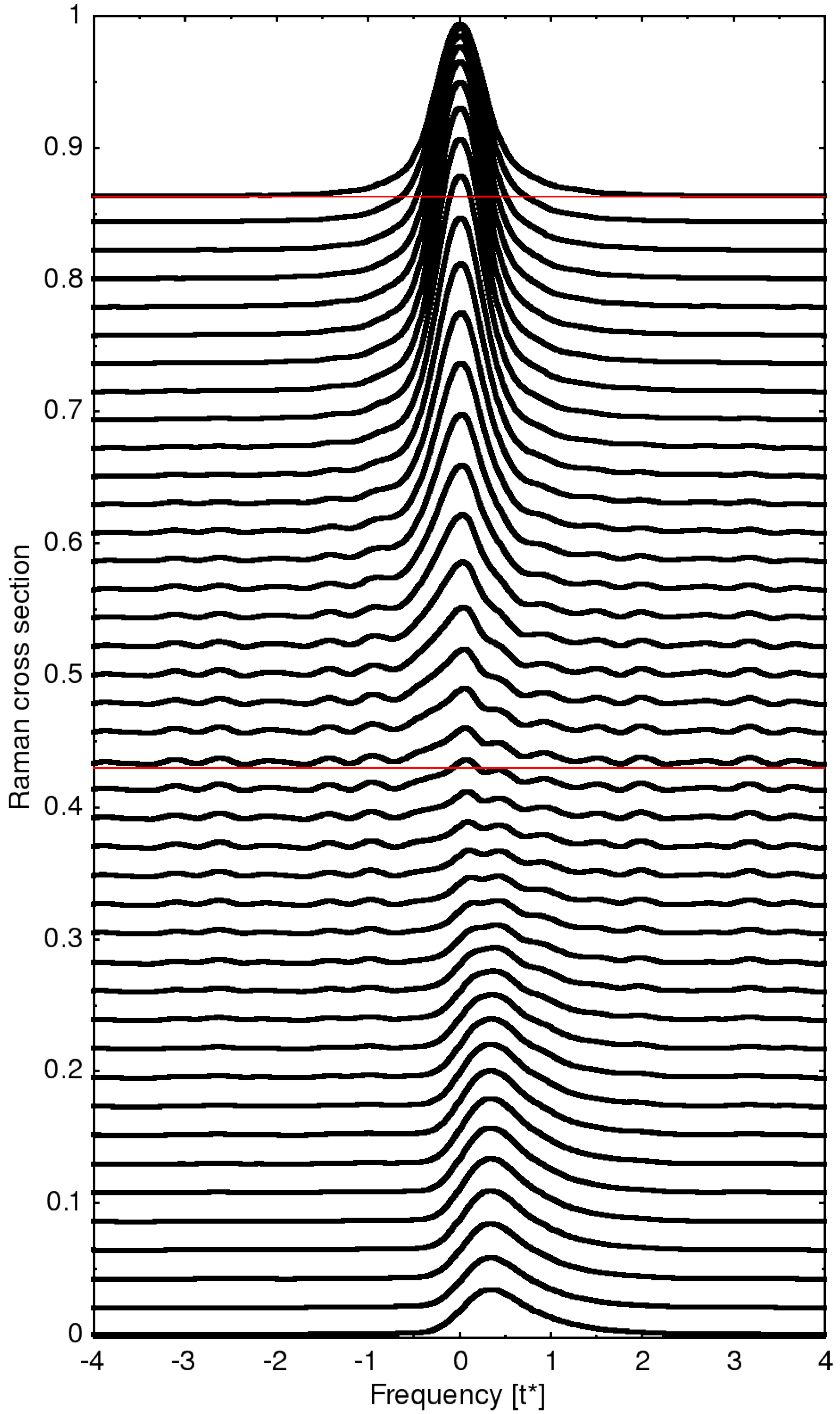


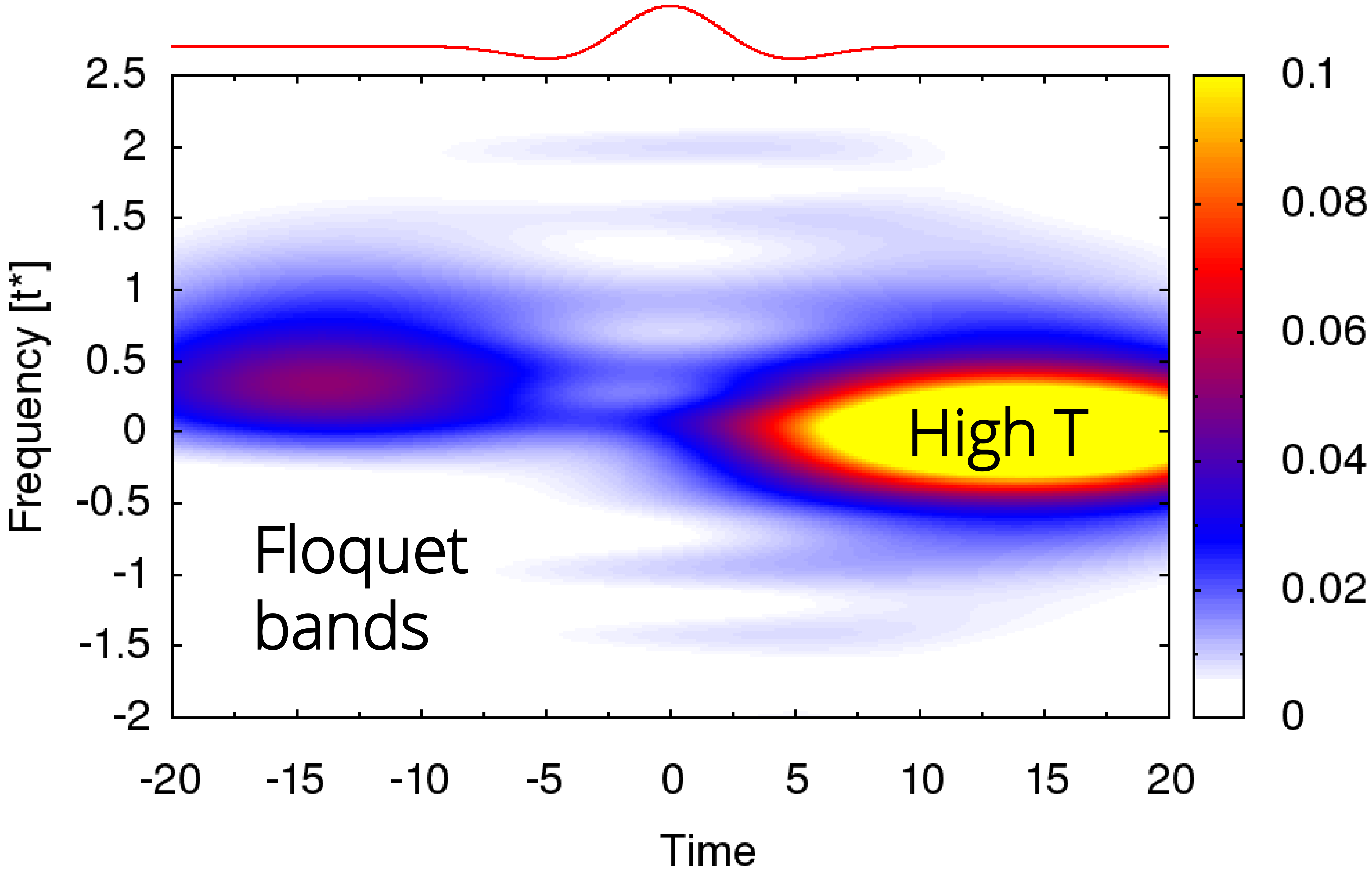
Time-resolved PES (CDW state)



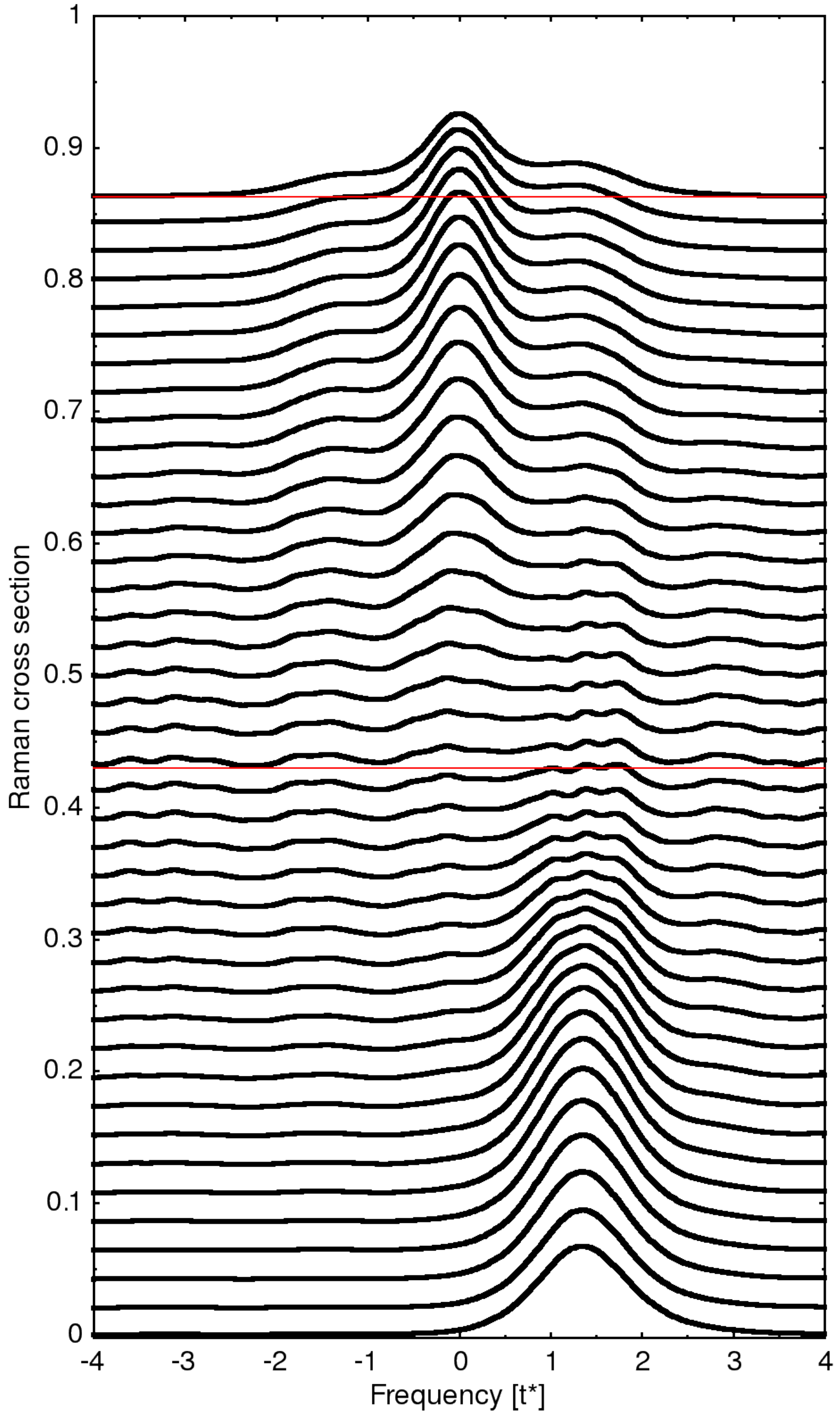
Time-resolved electronic Raman scattering

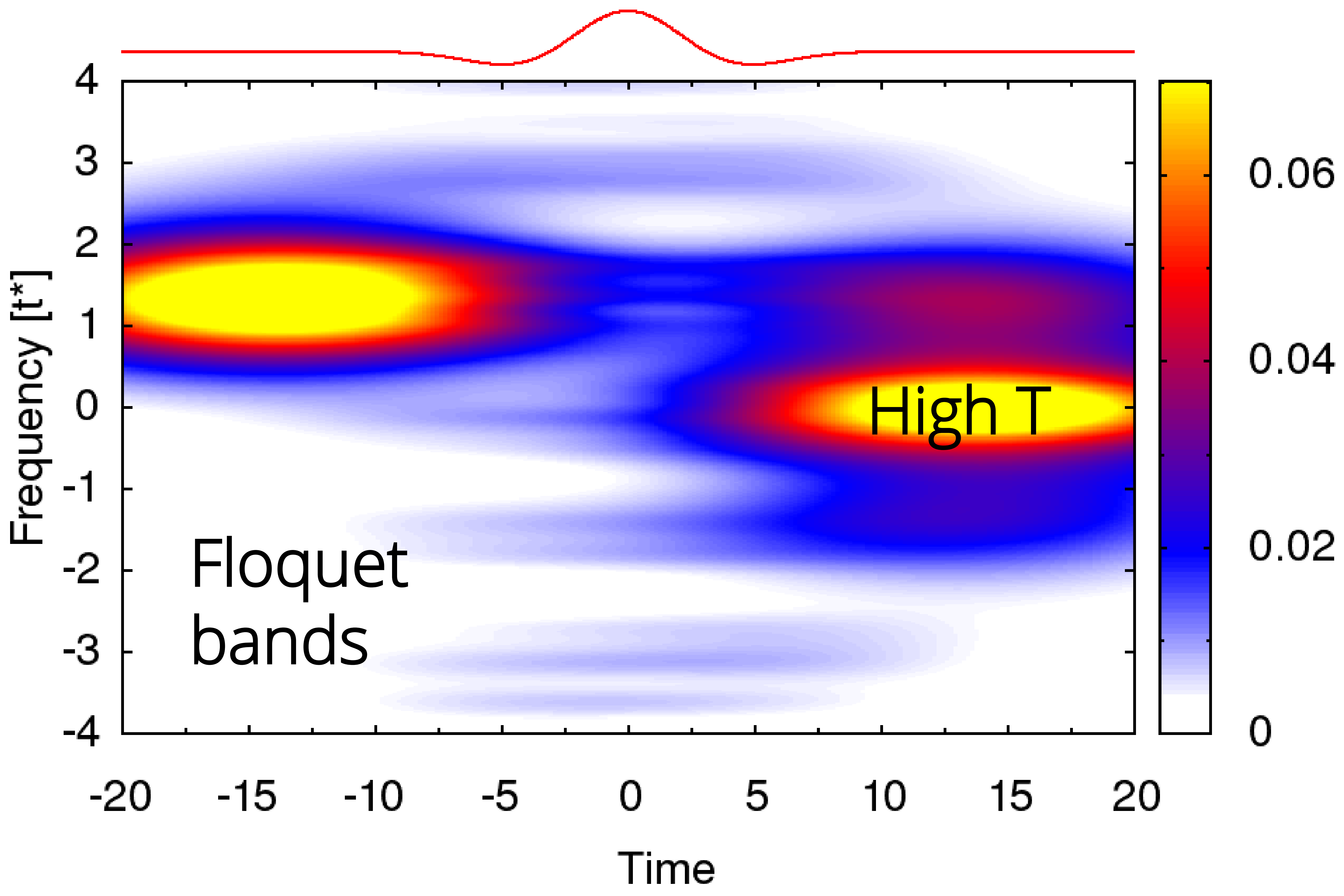
Strongly pumped metal



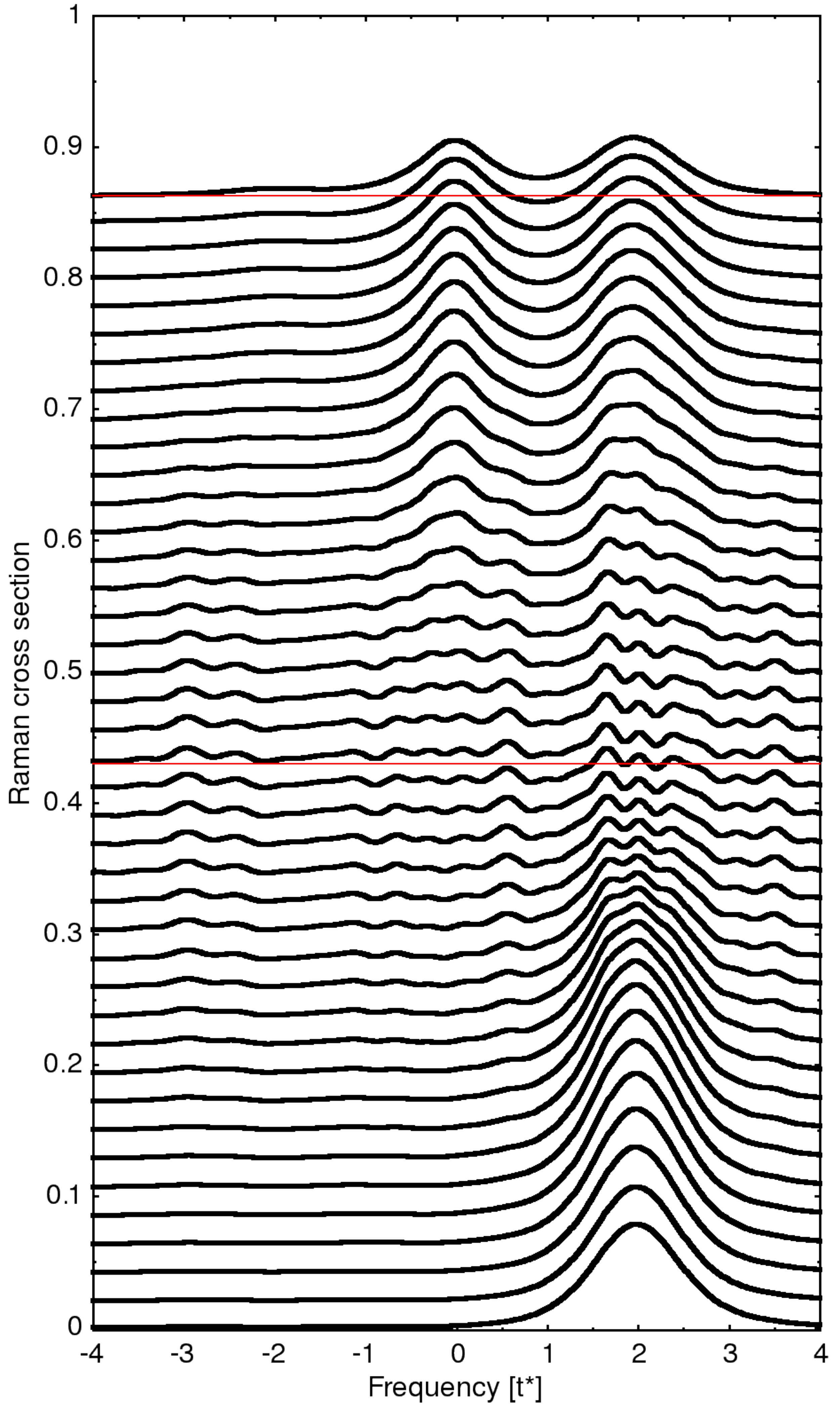


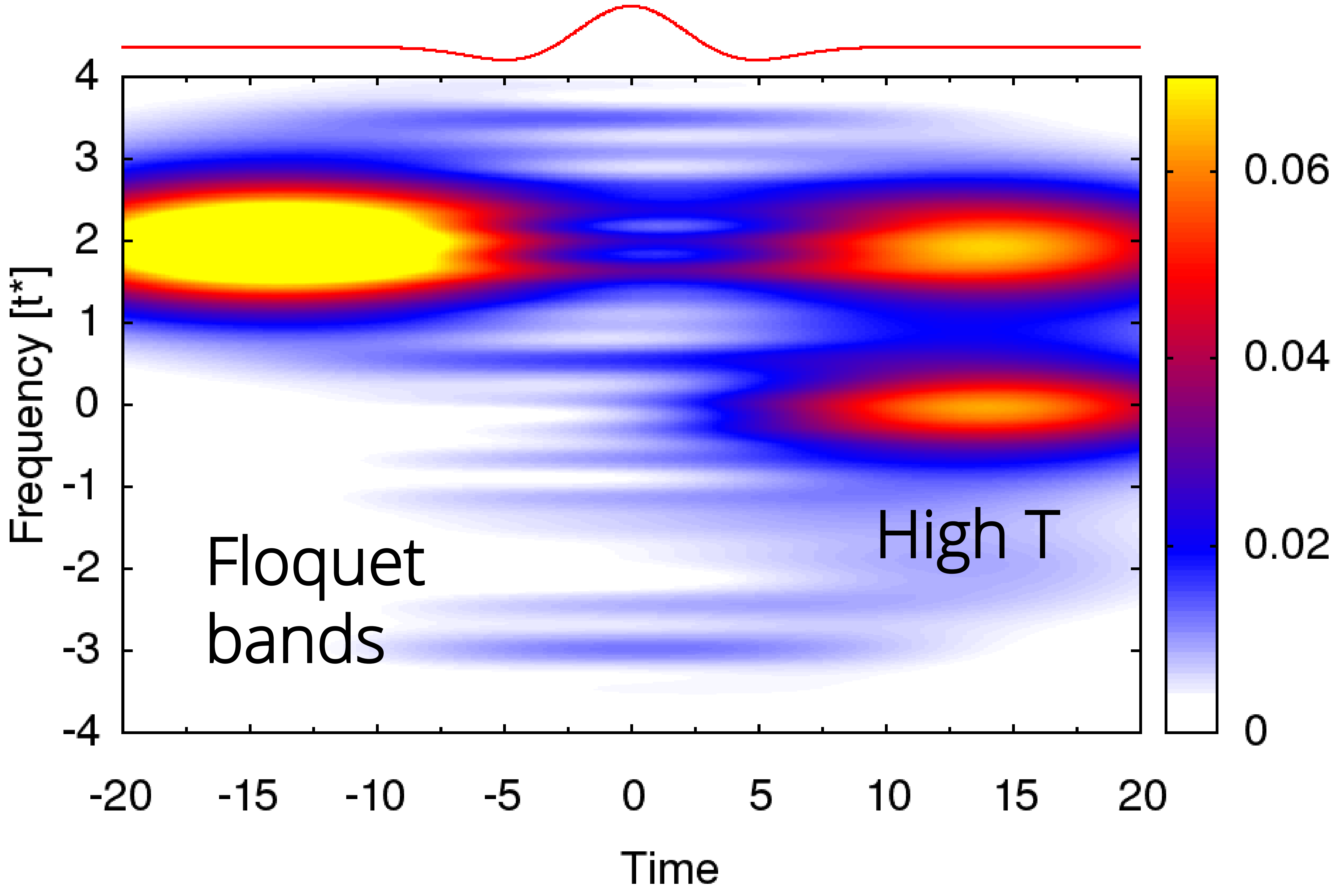
Strongly pumped at MIT





Strongly pumped insulator



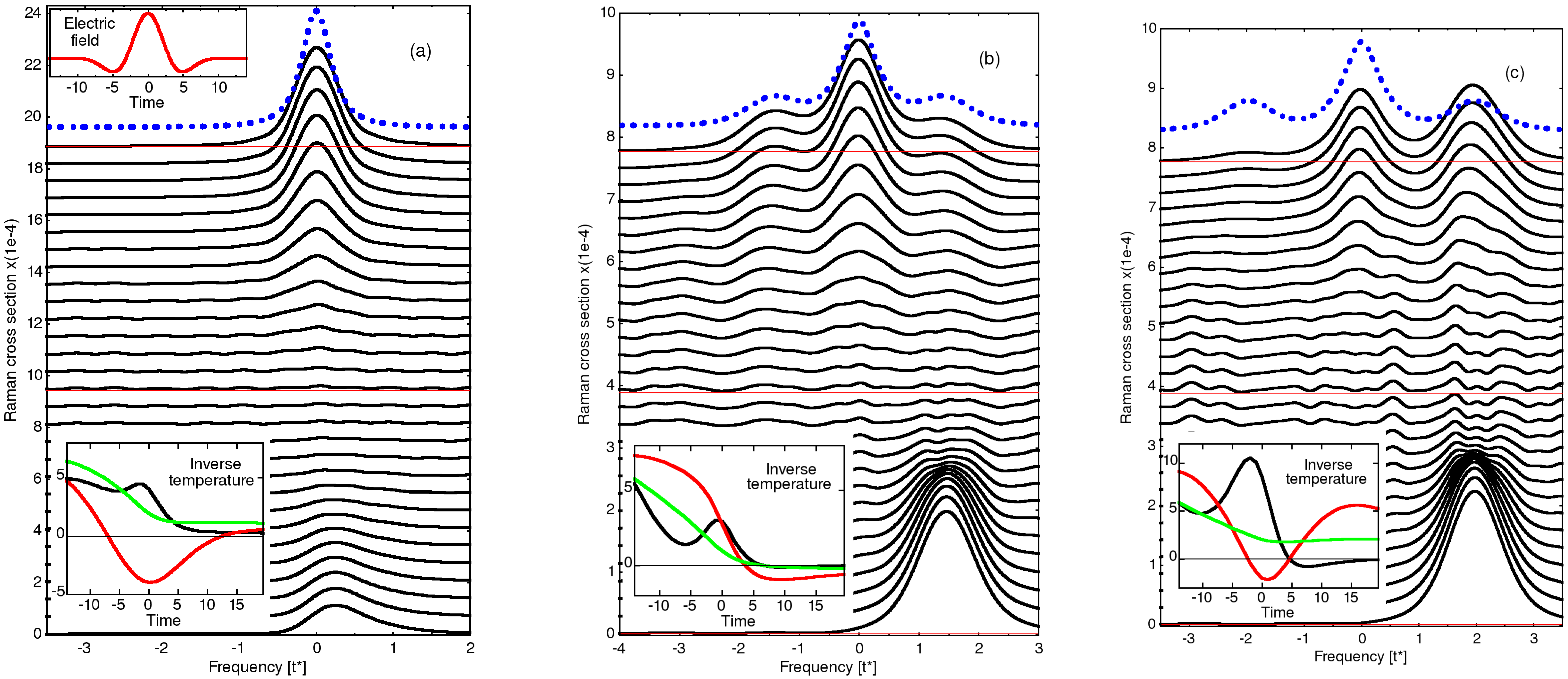


Ultrafast Thermometry

Extract fermionic T from PES and collective bosonic T from electronic Raman scattering

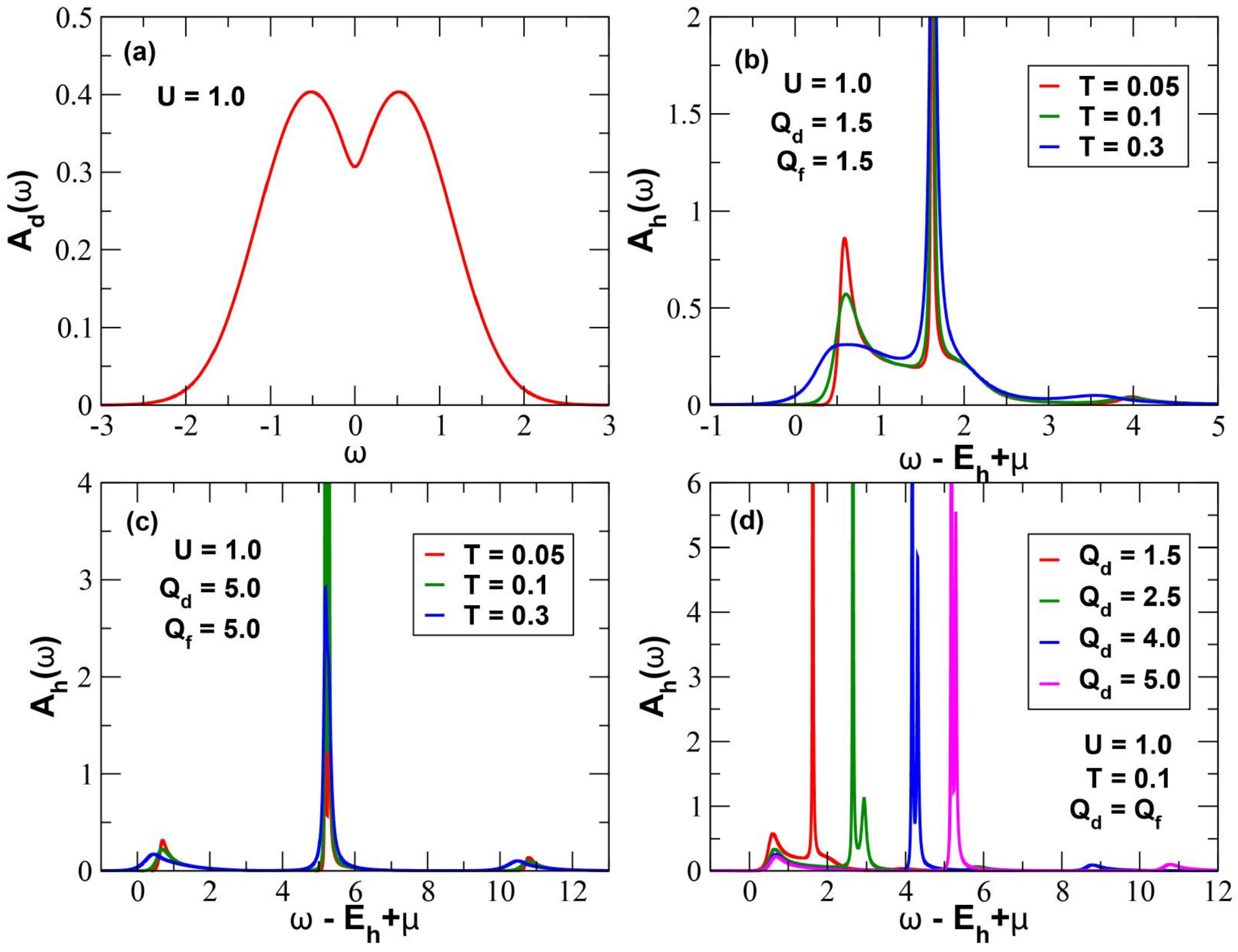
Thermalization occurs when they are the same!

Comparison of T_{PES} to T_{Raman}



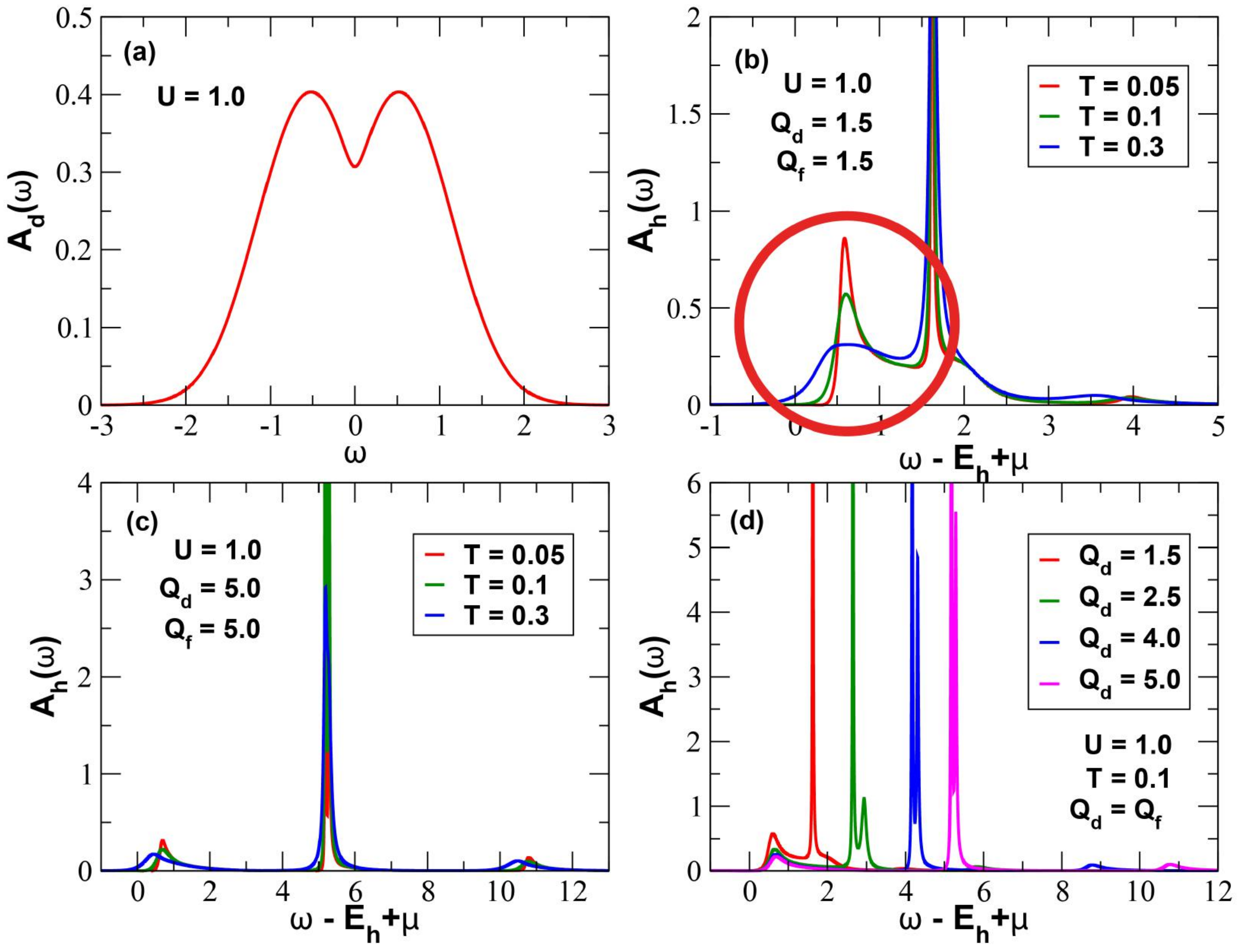
Application of similar ideas to XFELS

XPS and XAS have satellites with strong T dependence



XPS in correlated systems have satellite features split off from the main peak.
These satellites have strong T dependence at high T.
One should be able to measure these satellites in pump/probe experiments to determine $T_{eff}(t)$.
Similar behavior occurs for XAS.

XPS and XAS have satellites with strong T dependence



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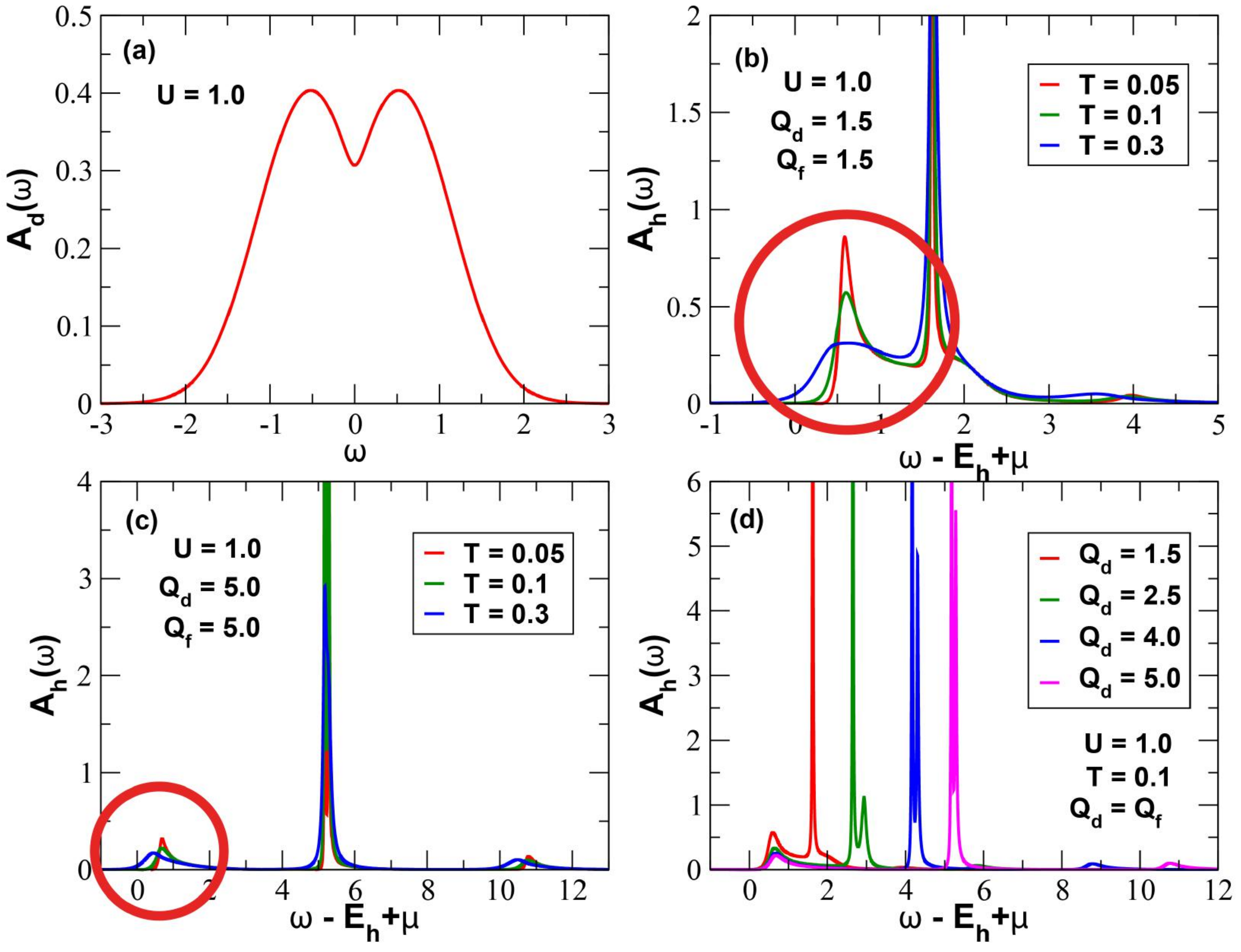

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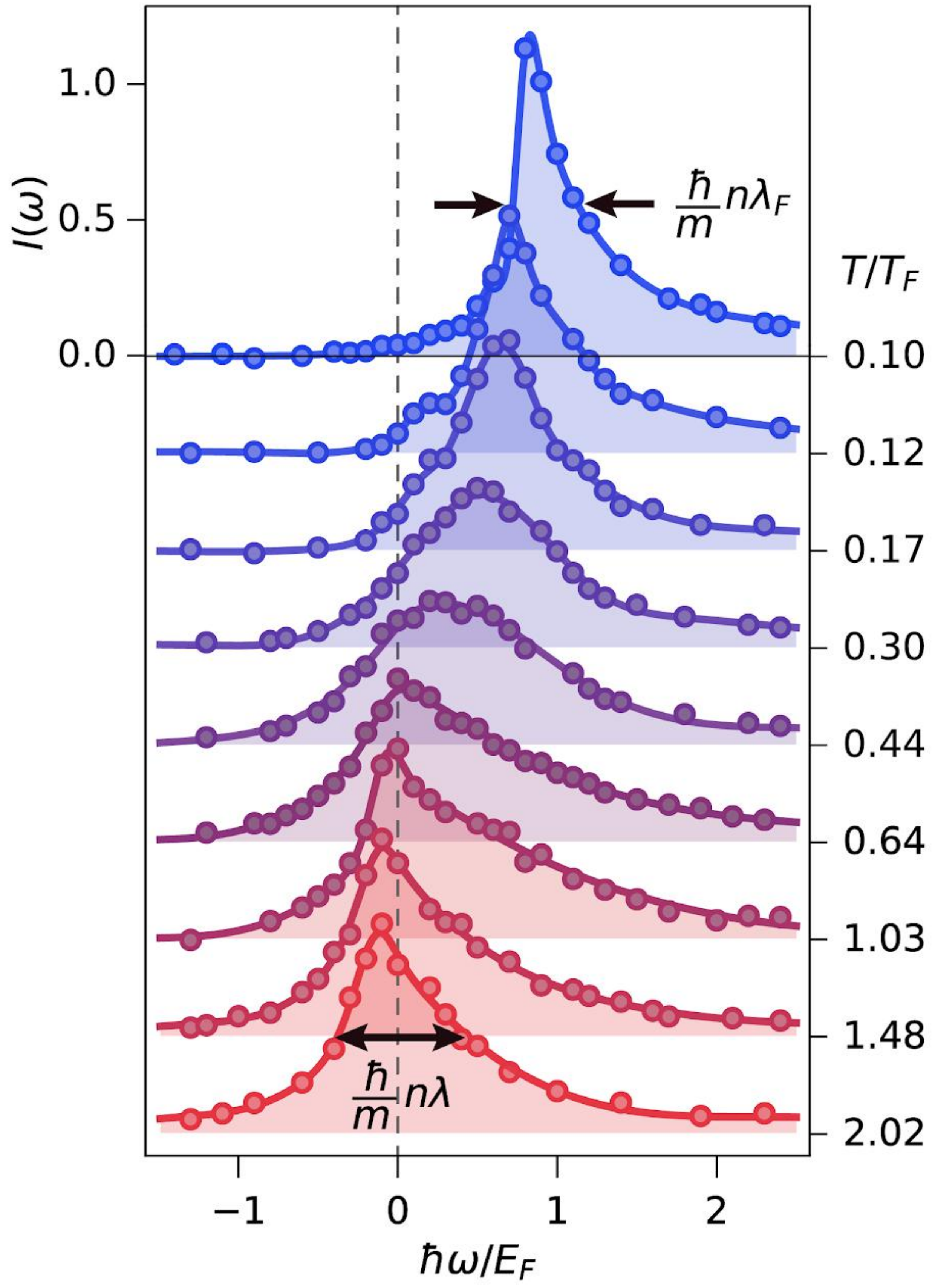
Spectral response of the Unitary Fermi gas:

Similar ideas have been used with cold atoms in Martin Zweirlein's group



Strongly temperature dependent

→ A local thermometer!

Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, J. Struck, M. Zwierlein, arXiv:1902.08548 (2019)