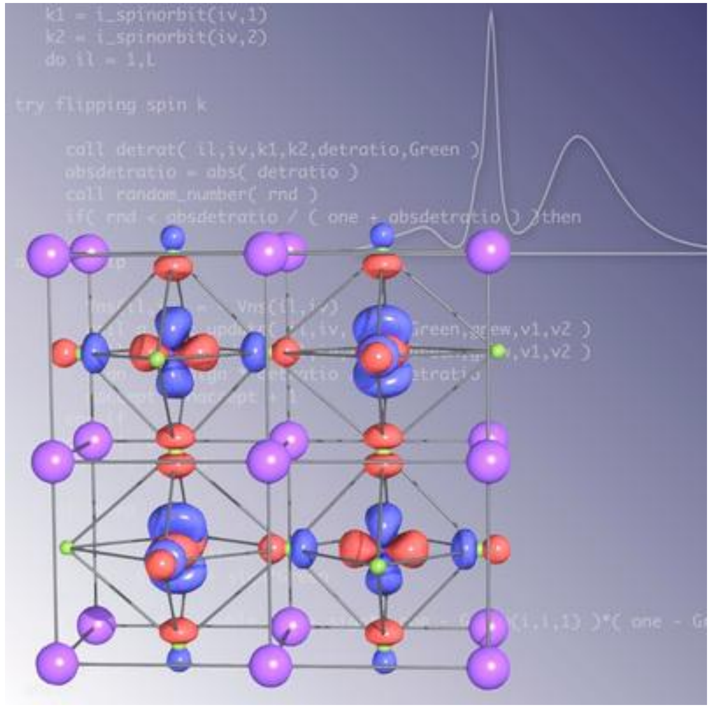
Aspects of Topological Unconventional Superconductors



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Topology, Entanglement and Strong Correlations

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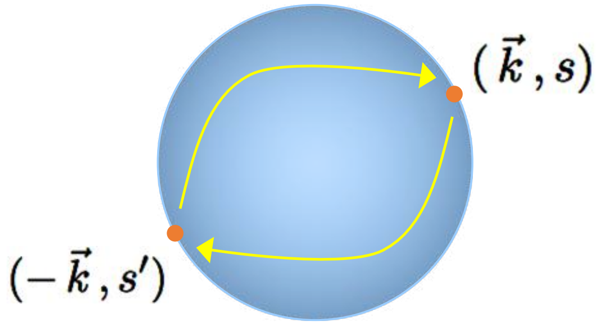
JÜLICH Forschungszentrum

Outline

- introduction of unconventional superconductivity and chiral phase
- topological invariant: Chern number for 2D superconductors
- edge states – bulk-edge correspondence
- spontaneous surface supercurrents
- quantized thermal Hall effect
- extension to 3D chiral superconductors
- other topological superconductors in 2D – helical edge states
- classification

Unconventional Superconductors

superconductivity
Fermi surface instability



$$\vec{k} \to -\vec{k} \qquad s \leftrightarrow s'$$

Pauli principle

totally antisymmetric

$$F_{ss'}(\vec{k}) = -F_{s's}(-\vec{k})$$

$$\hat{F}(\vec{k}) = -\hat{F}(-\vec{k})^T$$

BCS - coherent state of Cooper pairs

$$|\Phi_{BCS}\rangle = \bigotimes_{\vec{k},s,s'} \left\{ u_{\vec{k},ss'} + v_{\vec{k},ss'} |\vec{k}, s; -\vec{k}, s'\rangle \right\}$$

pair wavefunction $\quad F_{ss'}(\vec{k}) = \langle \Phi_{BCS} | \hat{c}_{-\vec{k}s'} \hat{c}_{\vec{k}s} | \Phi_{BCS} \rangle$

$$\hat{F}(\vec{k}) = \begin{cases} f_0(\vec{k})\hat{\sigma}^0 i\hat{\sigma}^y \\ \vec{f}(\vec{k}) \cdot \hat{\vec{\sigma}} i\hat{\sigma}^y \end{cases}$$

even-parity spin-singlet
$f_0(\vec{k}) = f_0(-\vec{k})$
$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

odd-parity spin-triplet
$\vec{f}(\vec{k}) = -\vec{f}(-\vec{k})$
$|\uparrow\uparrow\rangle, \; |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, \; |\downarrow\downarrow\rangle$

Unconventional Superconductors

real space onsite
pairing amplitude

$$I_0 = \sum_{\vec{k}\in \mathrm{BZ}} f_0(\vec{k})$$

$$\vec{I} = \sum_{\vec{k}\in \mathrm{BZ}} \vec{f}(\vec{k})$$

superconducting phase is

conventional $I_0 \neq 0$ ”*s*-wave”

unconventional $\begin{cases} I_0 = 0 \\ \vec{I} = 0 \end{cases}$ ”*p*-,*d*-wave” higher angular momentum

”real space pair wave function”

$$\hat{\Phi}(\vec{r} - \vec{r}') = \frac{1}{\Omega} \sum_{\vec{k}\in \mathrm{BZ}} \hat{F}(\vec{k}) e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}$$

only the pairing state with relative
angular momentum $\ell = 0$ s-wave
has finite onsite amplitude

$$\hat{\Phi}(\vec{r} - \vec{r}') \to |\vec{r} - \vec{r}'|^{\ell}$$

Unconventional Superconductors

real space onsite
pairing amplitude

$$I_0 = \sum_{\vec{k}\in \mathrm{BZ}} f_0(\vec{k})$$

$$\vec{I} = \sum_{\vec{k}\in \mathrm{BZ}} \vec{f}(\vec{k})$$

superconducting phase is

conventional $I_0 \neq 0$ *"s*-wave"

unconventional $\begin{cases} I_0 = 0 \\ \vec{I} = 0 \end{cases}$ "*p*-,*d*-wave" higher angular momentum

symmetry operations

crystal point group $g \in \mathcal{P}$

$$g \circ \hat{F}(\vec{k}) \quad \Rightarrow \quad \begin{cases} g \circ f_0(\vec{k}) = f_0(R_g \vec{k}) \\ g \circ \vec{f}(\vec{k}) = R_g \vec{f}(R_g \vec{k}) \end{cases}$$

time reversal

$$\mathcal{K} \circ f_\mu(\vec{k}) = f_\mu(\vec{k})^*$$

U(1) gauge

$$\Phi \circ f_\mu(\vec{k}) = f_\mu(\vec{k}) e^{i\phi}$$

Unconventional Superconductors

example: tetragonal crystal

classification by
irreducible representations

Γ^+	$f_0(\vec{k})$	Γ^-	$\vec{f}(\vec{k})$
A_{1g}	conventional 1	A_{1u}	$\hat{x}k_x + \hat{y}k_y$
A_{2g}	$k_xk_y(k_x^2 - k)y^2)$	A_{2u}	$\hat{x}k_y - \hat{y}k_x$
B_{1g}	$k_x^2 - k_y^2$	B_{1u}	$\hat{x}k_x - \hat{y}k_y$
B_{2g}	k_xk_y	B_{2u}	$\hat{x}k_y + \hat{y}k_x$
E_g	$\{k_xk_z, k_yk_z\}$	E_u	$\{\hat{z}k_x, \hat{z}k_y\}$

two-fold degeneracy

spontaneous symmetry breaking

Landau paradigm

$$\mathcal{G} = \mathcal{P} \times \mathcal{K} \times U(1)$$

orbital & spin

two-dimensional representation E_u

$$\vec{f}(\vec{k}) = \eta_x \hat{z}k_x + \eta_y \hat{z}k_y$$

two-component
order parameter

several symmetries can be
broken spontaneously in
the superconducting phase

Unconventional Superconductors

example: tetragonal crystal

$$\vec{f}(\vec{k}) = \eta_x \hat{z} k_x + \eta_y \hat{z} k_y$$

Ginzburg-Landau Theory

$$\vec{\eta} = (\eta_x , \eta_y)$$

two-component order parameter

$$F[\vec{\eta}] = a(T)|\vec{\eta}|^2 + b_1|\vec{\eta}|^4 + \frac{b_2}{2}\left\{\eta_x^{*2}\eta_y^2 + \eta_x^2\eta_y^{*2}\right\} + b_3|\eta_x|^2|\eta_y|^2$$

possible stable phases

$$\vec{\eta} = \eta_0(1, \pm i)$$

$$f_z(\vec{k}) = \eta_0(k_x \pm i k_y)$$

broken time reversal symmetry

chiral

$$\vec{\eta} = \eta_0(1, \pm 1)$$

$$f_z(\vec{k}) = \eta_0(k_x \pm k_y)$$

$$\vec{\eta} = \eta_0(1, 0), \eta_0(0, 1)$$

$$f_z(\vec{k}) = \eta_0 k_x ,\ \eta_0 k_y$$

broken crystal rotation symmetry

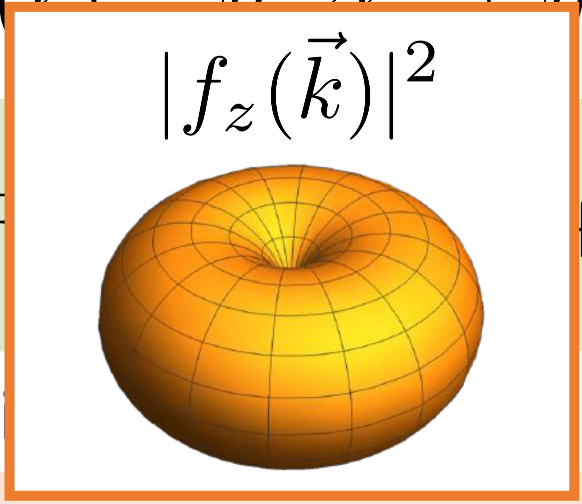
nematic

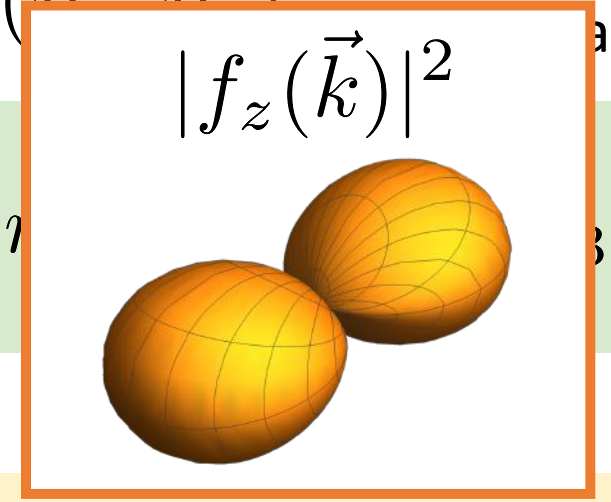
Unconventional Superconductors

example: tetragonal crystal

$$\vec{f}(\vec{k}) \ldots \eta_y \hat{z} k_y \qquad \vec{\eta} = (\ldots)$$

two-component … arameter





$$F \ldots + b_1|\vec{\eta}|^4 + \frac{b_2}{2}\{\eta_x^{*2} \ldots \ldots |\eta_x|^2|\eta_y|^2$$

possi…

$\vec{\eta} = \eta_0(1, \pm i)$

$f_z(\vec{k}) = \eta_0(k_x \pm i k_y)$

broken time reversal symmetry

chiral

$\vec{\eta} = \eta_0(1, \pm 1)$ | $\vec{\eta} = \eta_0(1,0), \eta_0(0,1)$

$f_z(\vec{k}) = \eta_0(k_x \pm k_y)$ | $f_z(\vec{k}) = \eta_0 k_x,\ \eta_0 k_y$

broken crystal rotation symmetry

nematic

Unconventional Superconductors

standard BCS-type of theory

$$\mathcal{H} = \sum_{\vec{k},s} \xi_{\vec{k}} \hat{c}^\dagger_{\vec{k}s} \hat{c}_{\vec{k}s} + \frac{1}{2} \sum_{\vec{k},\vec{k}'} \sum_{s_1,s_2,s_3,s_4} V^{s_1 s_2 s_3 s_4}_{\vec{k},\vec{k}'} \hat{c}^\dagger_{\vec{k}s_1} \hat{c}^\dagger_{-\vec{k}s_2} \hat{c}_{-\vec{k}'s_3} \hat{c}_{\vec{k}'s_4}$$

mean field gap function

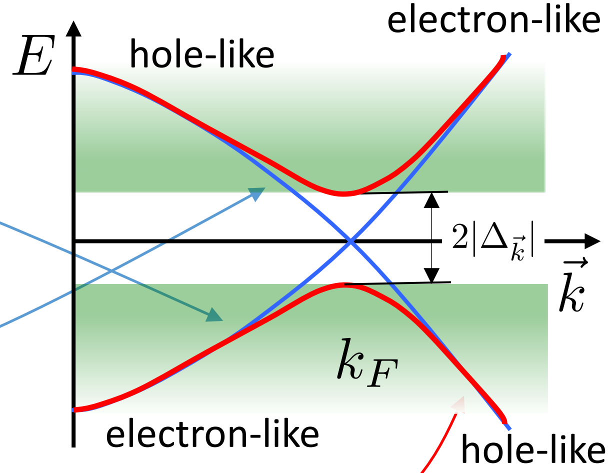
$$\Delta_{\vec{k}ss'} = - \sum_{\vec{k}'} \sum_{\tilde{s},\tilde{s}'} V^{ss'\tilde{s}\tilde{s}'}_{\vec{k},\vec{k}'} F_{\tilde{s}'\tilde{s}}(\vec{k})$$

Nambu space formulation

$$\mathcal{H}_{\text{mf}} = \frac{1}{2} \sum_{\vec{k}} \hat{C}^\dagger_{\vec{k}} H_{\vec{k}} \hat{C}_{\vec{k}}$$

$$H_{\vec{k}} = \begin{pmatrix} \xi_{\vec{k}} \hat{\sigma}^0 & \hat{\Delta}_{\vec{k}} \\ \hat{\Delta}^\dagger_{\vec{k}} & -\xi_{-\vec{k}} \hat{\sigma}^0 \end{pmatrix}$$

$$\hat{C}_{\vec{k}} = (\underbrace{\hat{c}_{\vec{k}\uparrow}, \hat{c}_{\vec{k}\downarrow}}_{\text{electron}}, \underbrace{\hat{c}^\dagger_{-\vec{k}\uparrow}, \hat{c}^\dagger_{-\vec{k}\downarrow}}_{\text{hole}})$$



quasiparticle energies

$$E_{\vec{k}} = \pm \sqrt{\xi^2_{\vec{k}} + |\Delta_{\vec{k}}|^2}$$

$$|\Delta_{\vec{k}}|^2 = \frac{1}{2} \text{Tr} \hat{\Delta}^\dagger_{\vec{k}} \hat{\Delta}_{\vec{k}}$$

Unconventional Superconductors

gap function

$$\hat{\Delta}(\vec{k}) = -\hat{\Delta}(-\vec{k})^T$$

$$\hat{\Delta} = \begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{pmatrix}$$

$$\hat{\Delta}(\vec{k}) = (d^0(\vec{k})\hat{\sigma}_0 + \vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}})i\hat{\sigma}_y$$

$$= \begin{pmatrix} -d_x + id_y & d_0 + d_z \\ -d_0 + d_z & d_x + id_y \end{pmatrix}$$

$d_0(\vec{k}) = d_0(-\vec{k})$	$\vec{d}(\vec{k}) = -\vec{d}(-\vec{k})$
even parity pairing	odd parity pairing
spin singlet	spin triplet

excitation gap

$$|\Delta_{\vec{k}}|^2 = |d_0(\vec{k})|^2$$

$$|\Delta_{\vec{k}}|^2 = |\vec{d}(\vec{k})|^2$$

Cooper pair spin orientation

$$" \vec{d} \cdot \vec{S} = 0 "$$

Chiral superconductor in 2D systems

gap function

$$\hat{\Delta}(\vec{k}) = -\hat{\Delta}(-\vec{k})^T$$

$$\hat{\Delta}(\vec{k}) = (d^0(\vec{k})\hat{\sigma}_0 + \vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}})i\hat{\sigma}_y$$

$$d_0(\vec{k}) = d_0(-\vec{k})$$

even parity pairing

spin singlet

$$\vec{d}(\vec{k}) = -\vec{d}(-\vec{k})$$

odd parity pairing

spin triplet

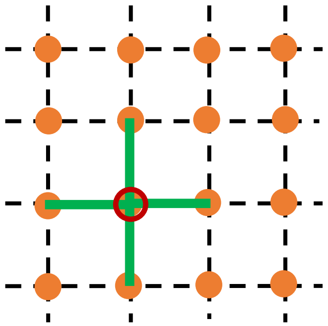
chiral p-wave state

tight-binding model

$$\xi_{\vec{k}} = -2t(\cos k_x + \cos k_y) - \mu$$

nearest-neighbor pairing
odd-parity spin-triplet

square lattice



$$\vec{d}^{\pm}(\vec{k}) = \Delta_0 \hat{\vec{z}}(\sin k_x \pm i \sin k_y)$$

- broken time reversal symmetry
- degeneracy 2
- nodeless gap in general
- topological

Chiral superconductor in 2D systems

chiral p-wave state – topological superconductor

$$\vec{d}^{\pm}(\vec{k}) = \Delta_0 \hat{\vec{z}}(\sin k_x \pm i \sin k_y)$$

spin configuration $(\uparrow\downarrow) + (\downarrow\uparrow)$

decoupled Nambu space Hamiltonians

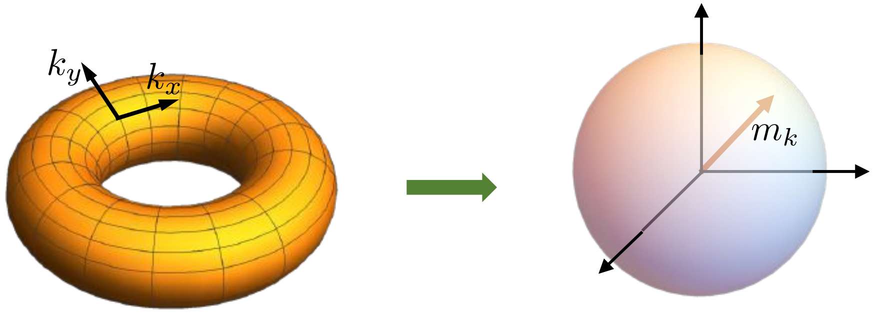
$(\hat{c}_{\vec{k}\uparrow}, \hat{c}^{\dagger}_{-\vec{k}\downarrow})$ and $(\hat{c}_{\vec{k}\downarrow}, \hat{c}^{\dagger}_{-\vec{k}\uparrow})$

$$h(\vec{k}) = \begin{pmatrix} \xi_{\vec{k}} & d_z(\vec{k}) \\ d_z^*(\vec{k}) & -\xi_{\vec{k}} \end{pmatrix} = \vec{h}_{\vec{k}} \cdot \hat{\vec{\tau}}$$

$$\vec{h}_{\vec{k}} = [Im(d_z(\vec{k}), Re(d_z(\vec{k}), \xi_{\vec{k}}]$$

"spinless"

mapping of BZ-torus to sphere



$|\vec{h}_{\vec{k}}| > 0$ $\qquad$ $\vec{m}_{\vec{k}} = \vec{h}_{\vec{k}}/|\vec{h}_{\vec{k}}|$

Chern number – topological invariant

$$N_C = \pi \int_{BZ} \frac{d^2k}{(2\pi)^2} \vec{m}_{\vec{k}} \cdot [\partial_{k_x} \vec{m}_{\vec{k}} \times \partial_{k_y} \vec{m}_{\vec{k}}]$$

integer number of coverings of torus on sphere

Chiral superconductor in 2D systems

Chern number – simpler method

$$N_C = \pi \int_{\mathrm{BZ}} \frac{d^2k}{(2\pi)^2} \vec{m}_{\vec{k}} \cdot [\partial_{k_x} \vec{m}_{\vec{k}} \times \partial_{k_y} \vec{m}_{\vec{k}}]$$

$$\vec{h}_{\vec{k}} = [Im(d_z(\vec{k}), Re(d_z(\vec{k}), \xi_{\vec{k}}] \qquad \vec{m}_{\vec{k}} = \vec{h}_{\vec{k}} / |\vec{h}_{\vec{k}}|$$

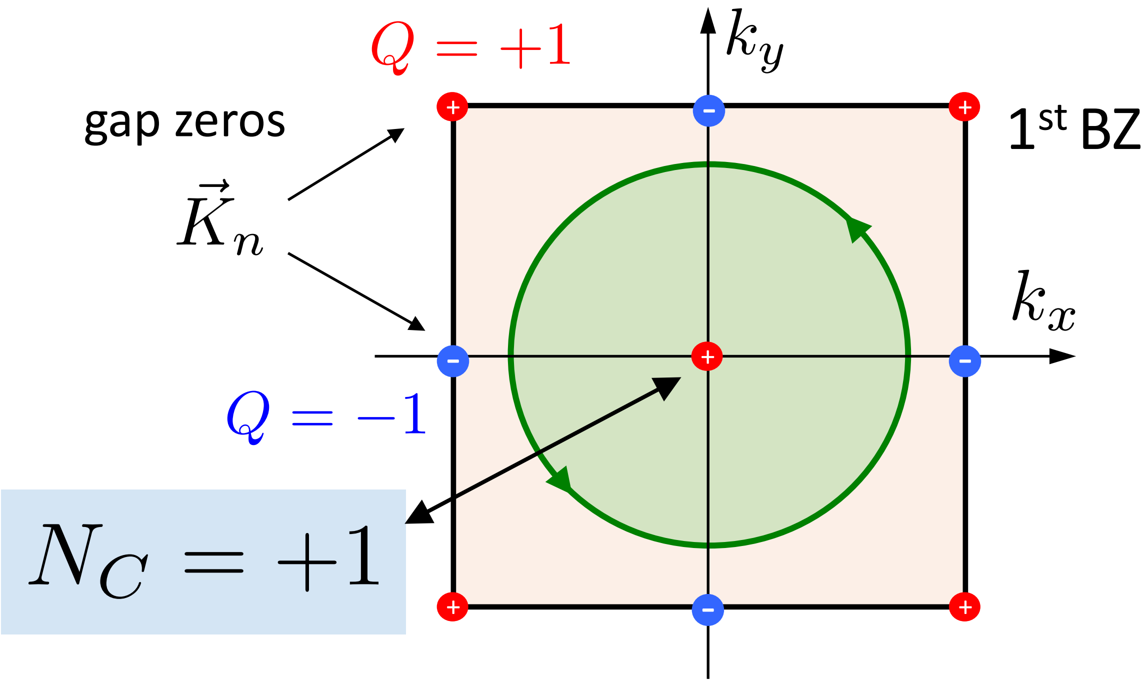
properties of gap function

$$d_z(\vec{k}) = \Delta_0(\sin k_x + i \sin k_y)$$

gap zeros in the Brillouin Zone with winding numbers

$$d_z(\vec{K}_n) = 0$$

$$d_z(\vec{K}_n + \vec{q}) = \Delta_0 q e^{iQ_n \theta_q}$$



Chern number as winding number

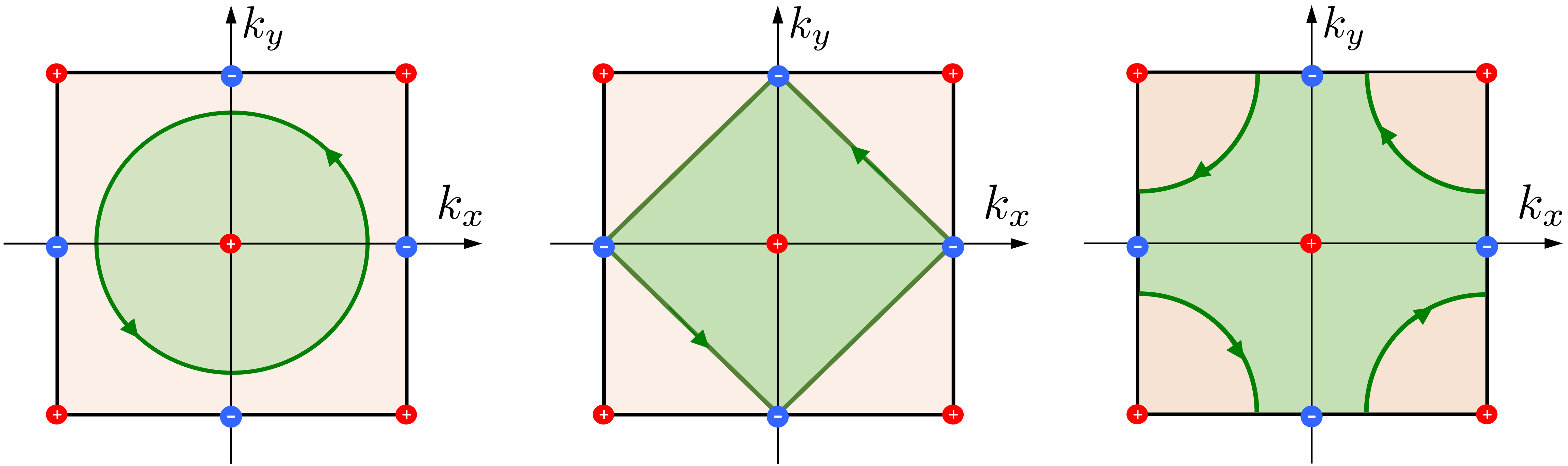
$$N_{\mathrm{C}} = \frac{1}{2\pi} \oint_{\mathrm{FS}} d\vec{k} \cdot \vec{\nabla}_{\vec{k}} \arg[d_z(\vec{k})]$$

counts windings of enclosed gap zeros

Chiral superconductor in 2D systems

Chern number

$$N_{\mathrm{C}} = \frac{1}{2\pi} \oint_{\mathrm{FS}} d\vec{k} \cdot \vec{\nabla}_{\vec{k}} \arg[d_z(\vec{k})]$$



$N_C = +1$

gap closing on FS

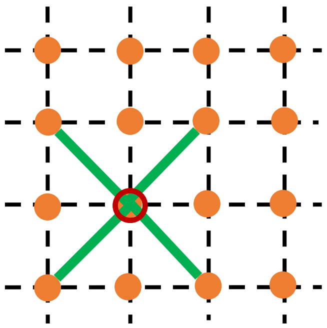
topological transition

$N_C = -1$

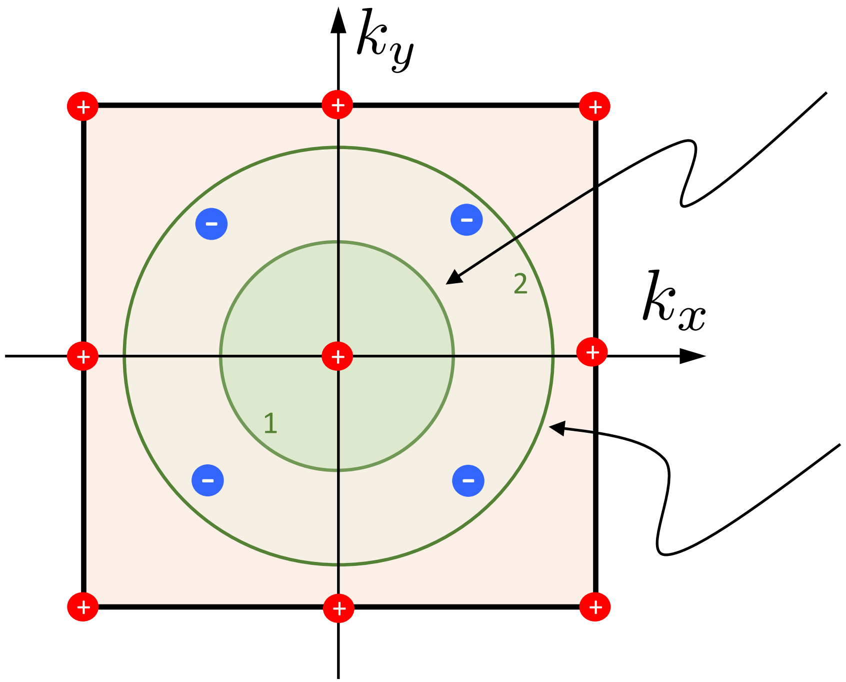
Chiral superconductor in 2D systems

example: chiral p-wave from
next-nearest neighbor pairing

square lattice



$$d_z(\vec{k}) = \Delta_0(\cos k_y \sin k_x + i \cos k_x \sin k_y)$$



FS 1: $N_C = +1$

topological transition
with gap closing

FS 2: $N_C = +1 - 4 = -3$

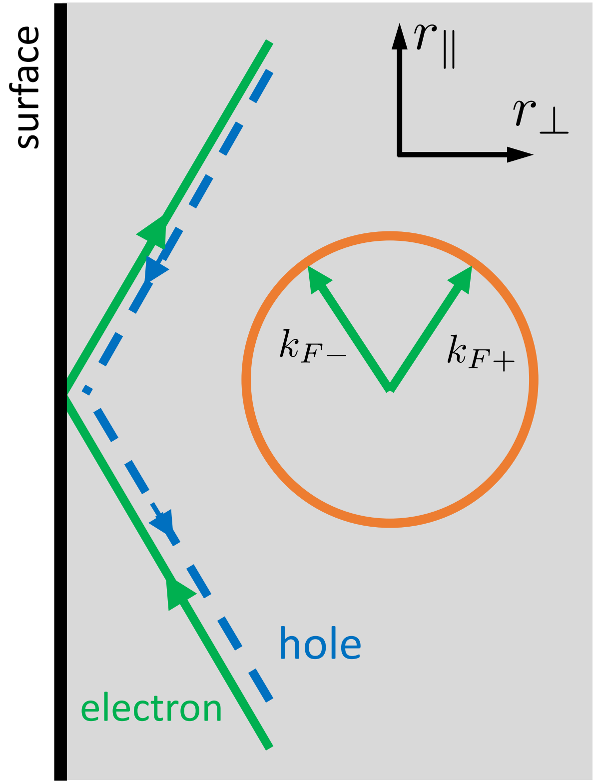
same
order parameter
symmetry

but

new zeros and
new windings

Chiral superconductor in 2D systems

edge states



Bogolyubov-de Gennes: Andreev approximation

$$\begin{pmatrix} \hbar\vec{v}_F \cdot (i\vec{\nabla} - \vec{k}_F) & d_z(\vec{k}_F) \\ d_z(\vec{k}_F)^* & -\hbar\vec{v}_F \cdot (i\vec{\nabla} - \vec{k}_F) \end{pmatrix} \begin{pmatrix} u_{\vec{k}_F}(\vec{r}) \\ v_{\vec{k}_F}(\vec{r}) \end{pmatrix} = E \begin{pmatrix} u_{\vec{k}_F}(\vec{r}) \\ v_{\vec{k}_F}(\vec{r}) \end{pmatrix}$$

electron

hole

Ansatz for bound state

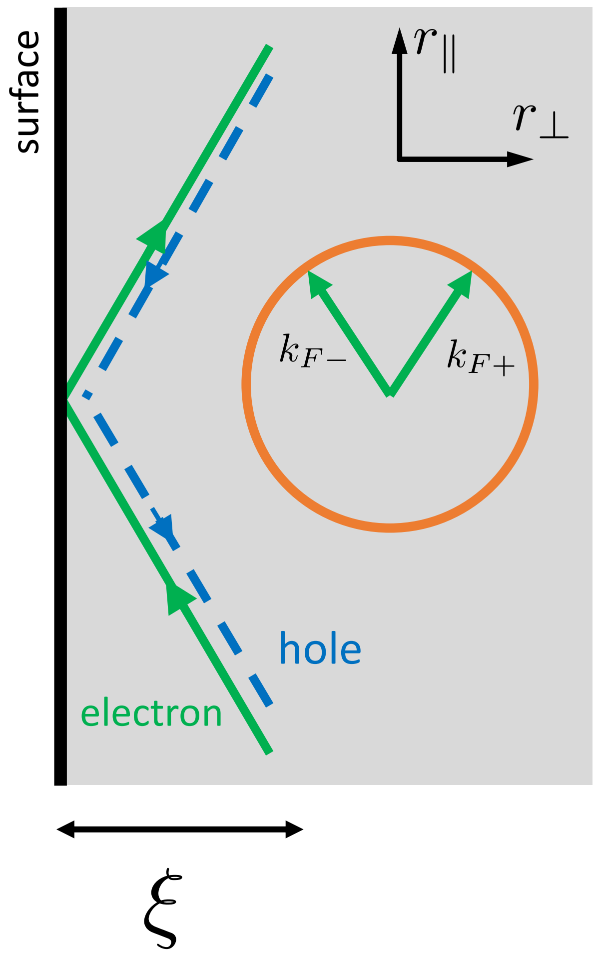
$$\begin{pmatrix} u_{\vec{k}_F}(\vec{r}) \\ v_{\vec{k}_F}(\vec{r}) \end{pmatrix} = b_1 \begin{pmatrix} A^+_{\vec{k}_{F+}} \\ r_{\vec{k}_{F+}} A^-_{\vec{k}_{F+}} \end{pmatrix} e^{+iq_x x + i\vec{k}_{F+}\cdot\vec{r}} + b_2 \begin{pmatrix} r^*_{\vec{k}_{F-}} A^-_{\vec{k}_{F-}} \\ A^+_{\vec{k}_{F-}} \end{pmatrix} e^{-iq_x x + i\vec{k}_{F-}\cdot\vec{r}}$$

$$r_{\vec{k}_{F+}} r^*_{\vec{k}_{F-}} = \frac{E + \sqrt{E^2 - |d_z(\vec{k}_F)|^2}}{E - \sqrt{E^2 - |d_z(\vec{k}_F)|^2}}$$

$$r_{\vec{k}_F} = \frac{d_z(\vec{k}_F)^*}{|d_z(\vec{k}_F)|}$$

Chiral superconductor in 2D systems

edge states



$$r_{\vec{k}_{F+}} r^*_{\vec{k}_{F-}} = \frac{E + \sqrt{E^2 - |d_z(\vec{k}_F)|^2}}{E - \sqrt{E^2 - |d_z(\vec{k}_F)|^2}}$$

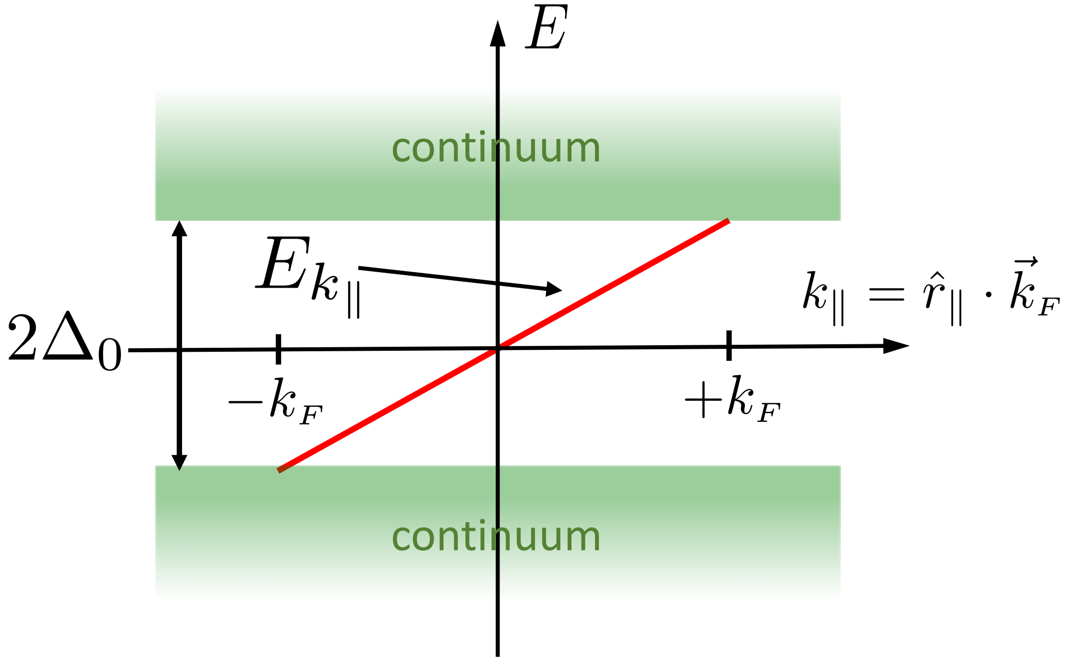
$$r_{\vec{k}_F} = \frac{d_z(\vec{k}_F)^*}{|d_z(\vec{k}_F)|}$$

input
phase winding

bulk-edge
correspondence

$$N_C = +1$$

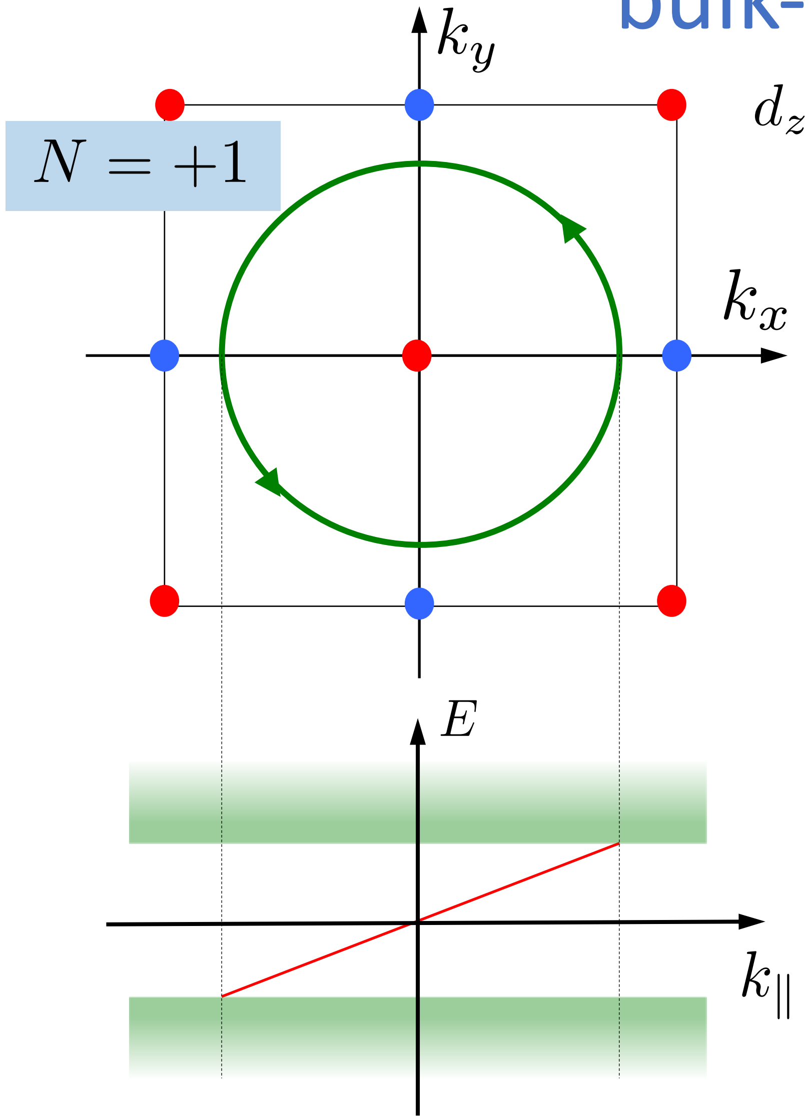
Andreev bound states localized at edge
penetration depth ξ coherence length

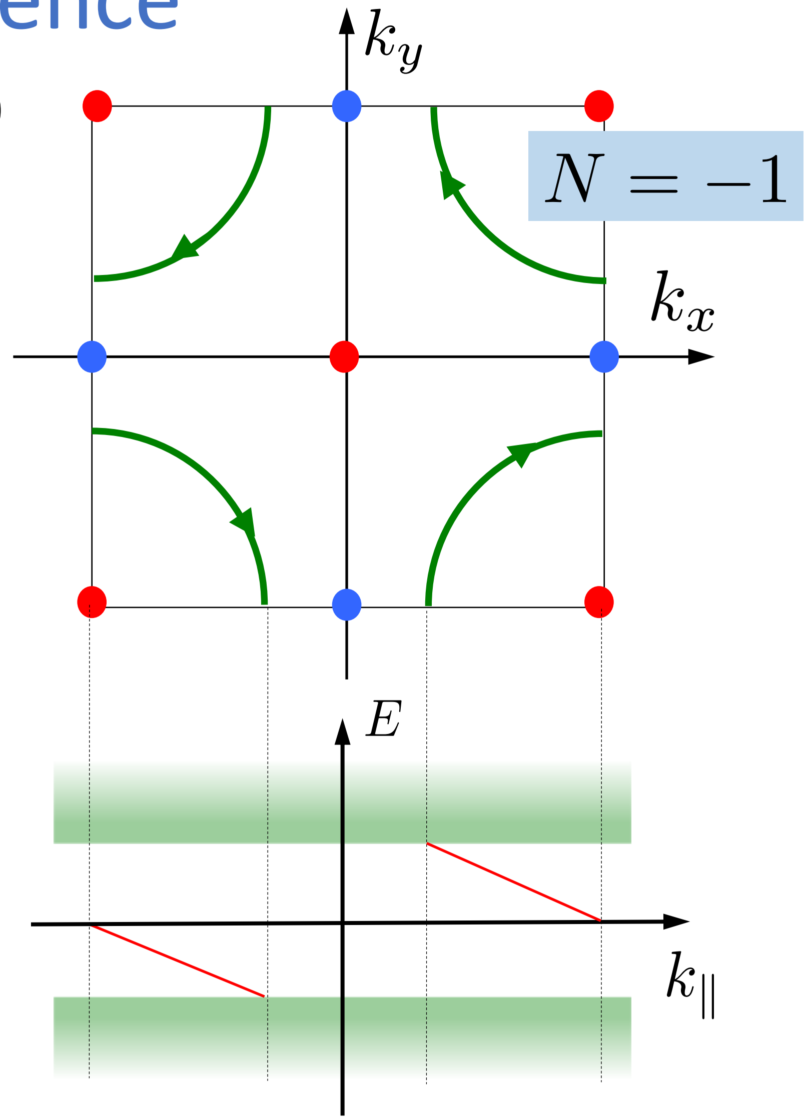


Chiral superconductor in 2D systems

bulk-edge correspondence

$$d_z(\vec{k}) = \Delta_0(\sin k_x + i \sin k_y)$$

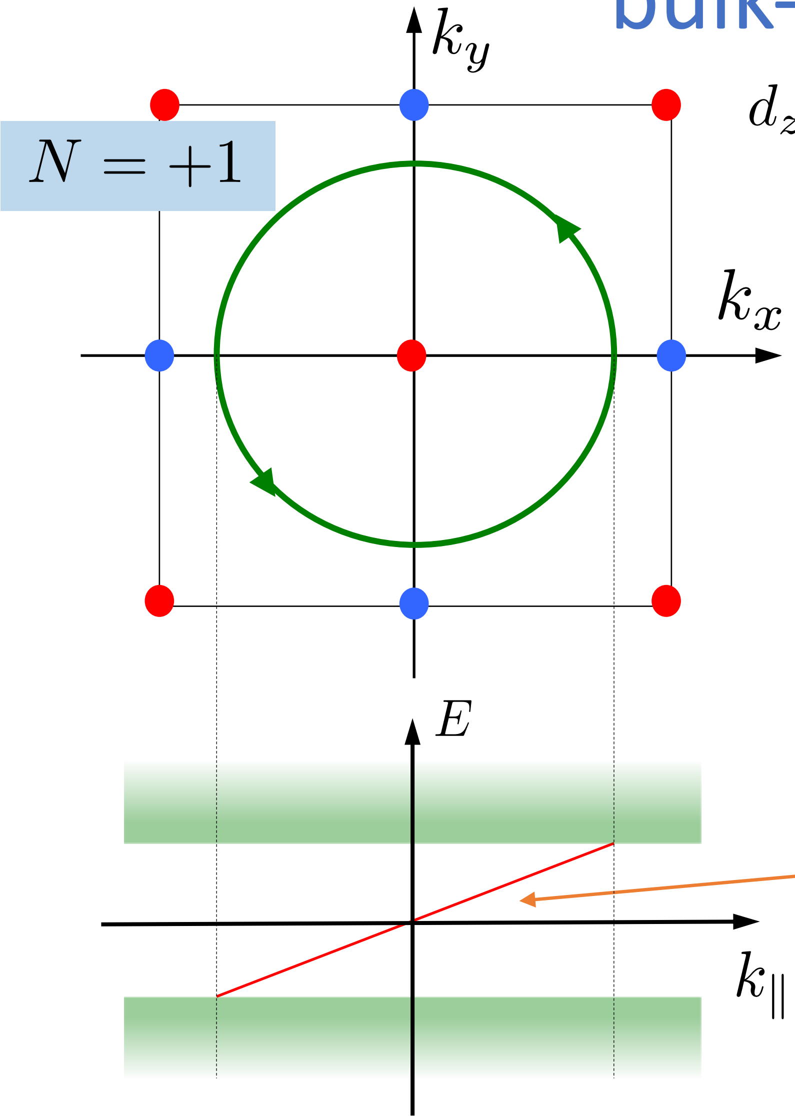




Chiral superconductor in 2D systems

bulk-edge correspondence

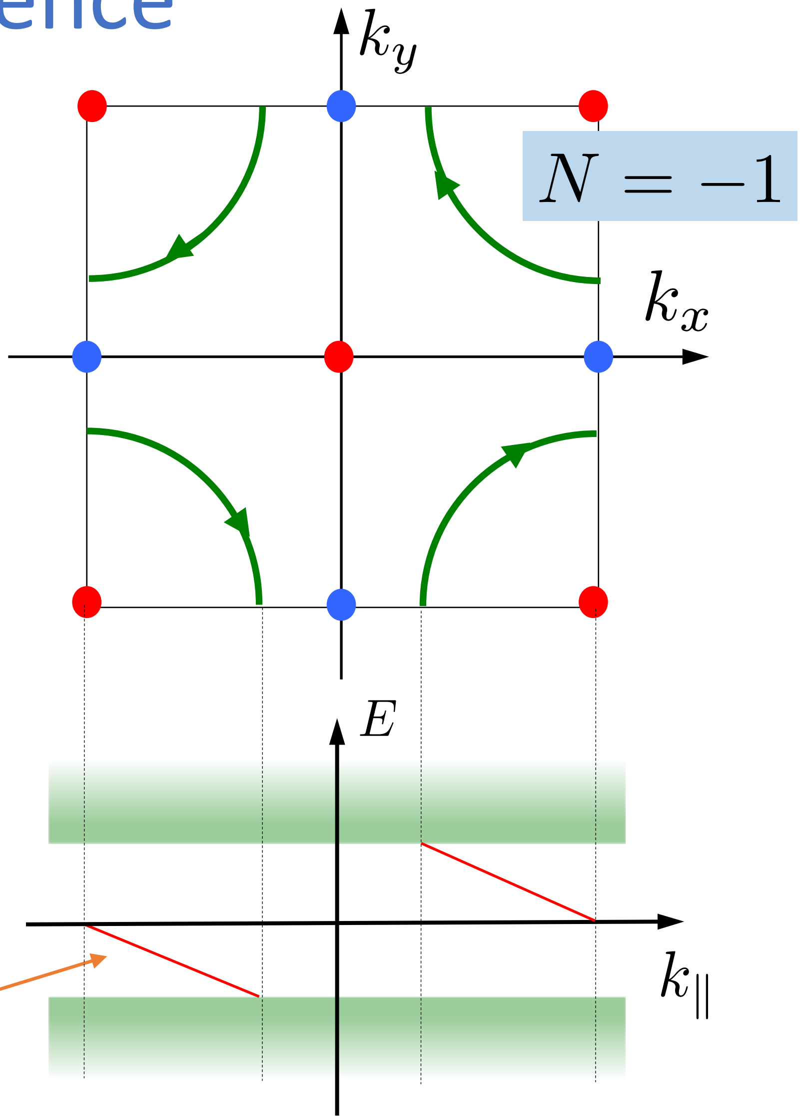
$$d_z(\vec{k}) = \Delta_0(\sin k_x + i \sin k_y)$$



quasiparticle velocity

$$v_{\mathrm{QP}} = \frac{1}{\hbar}\frac{dE_{k_\parallel}}{dk_\parallel}$$

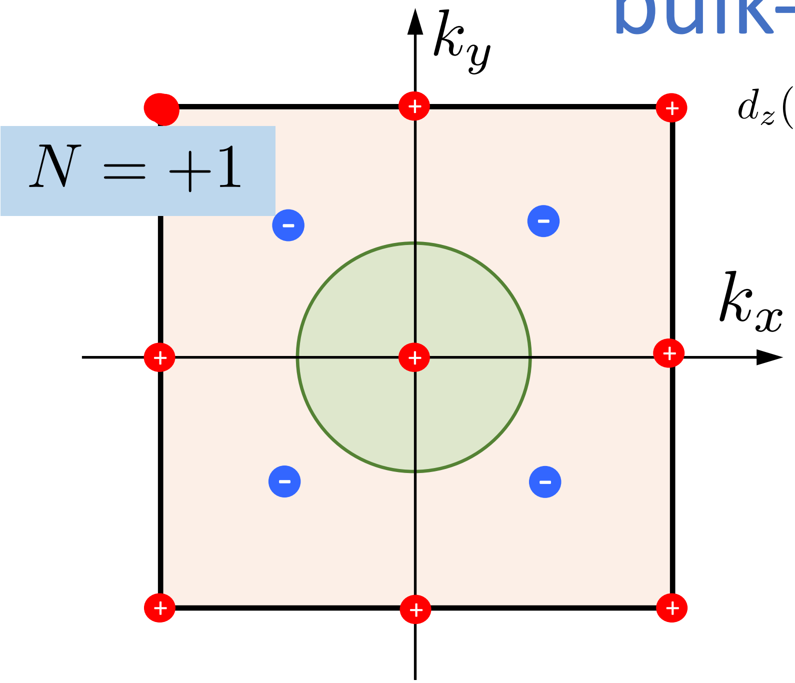
sign tied to chirality

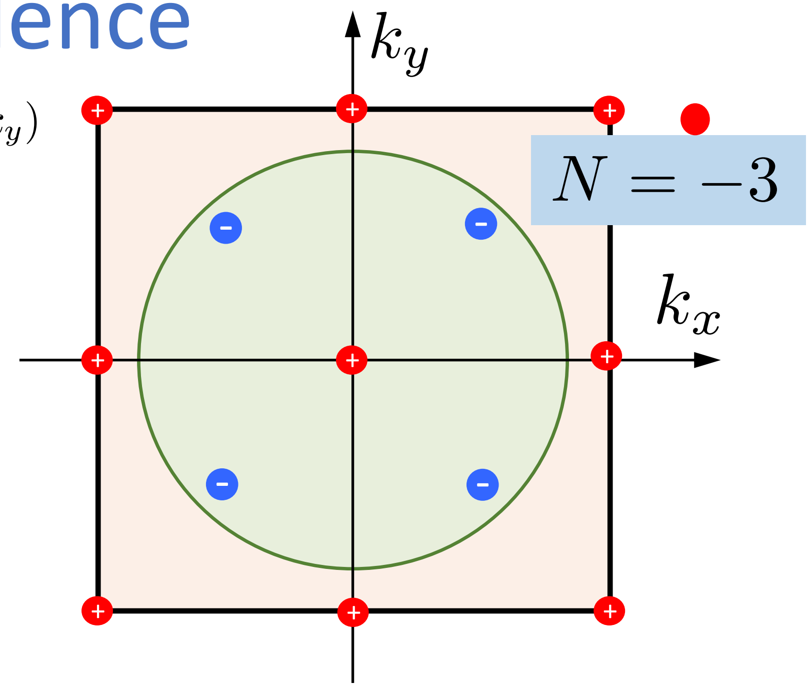


Chiral superconductor in 2D systems

bulk-edge correspondence

$$d_z(\vec{k}) = \Delta_0(\cos k_y \sin k_x + i \cos k_x \sin k_y)$$

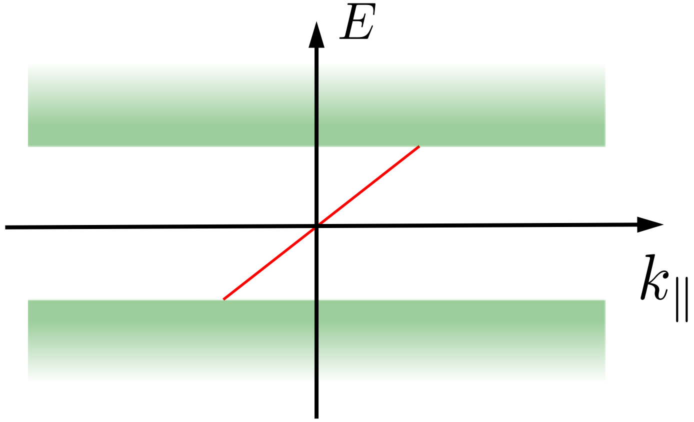


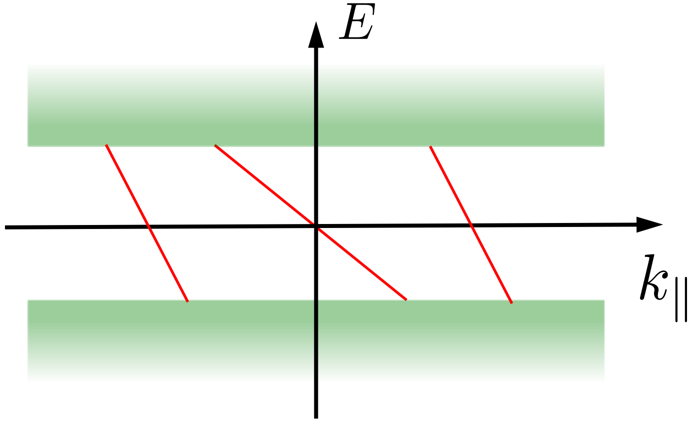


quasiparticle velocity

$$v_{\mathrm{QP}} = \frac{1}{\hbar}\frac{dE_{k_\parallel}}{dk_\parallel}$$

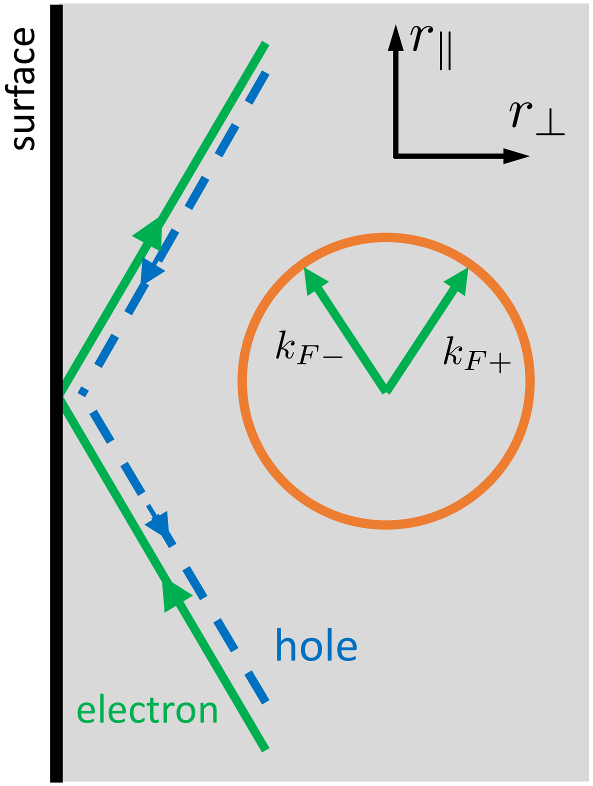
sign and number of zero crossing tied to chirality

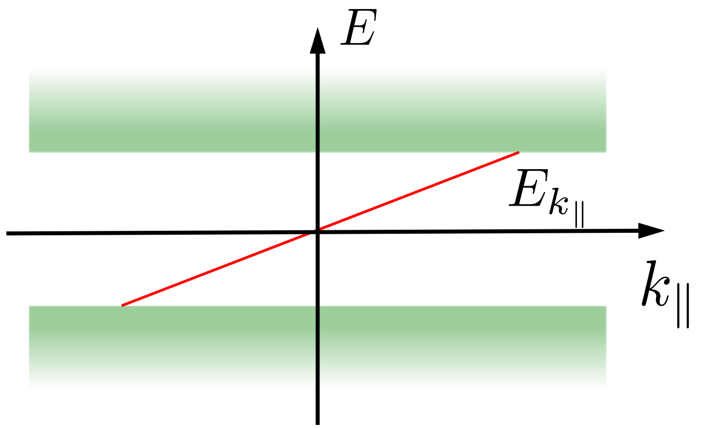




Chiral superconductor in 2D systems

surface currents



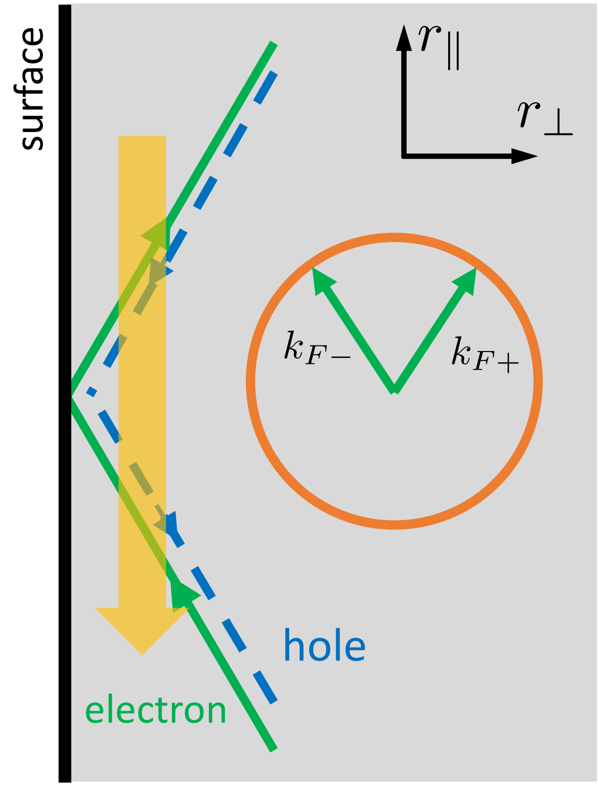


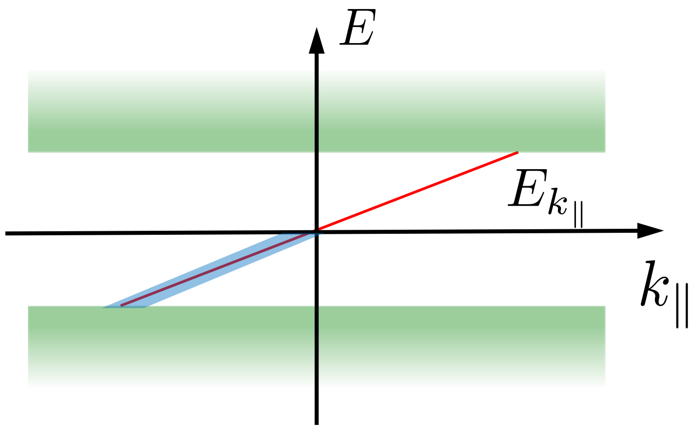
analog to ν=1
Quantum Hall edge state

universal surface current ?

Chiral superconductor in 2D systems

surface currents





focus on edge states

but also continuum contributes to current

net electric current parallel to surface

electron   hole

$$J_\parallel(x) = -\frac{2e}{L} \sum_{k_\parallel = \vec{k}_F \cdot \hat{r}_\parallel} v_{F\parallel} \left[|u_{\vec{k}_F}(x)|^2 f(E_{k_\parallel}) - |v_{\vec{k}_F}(x)|^2 f(-E_{k_\parallel}) \right]$$

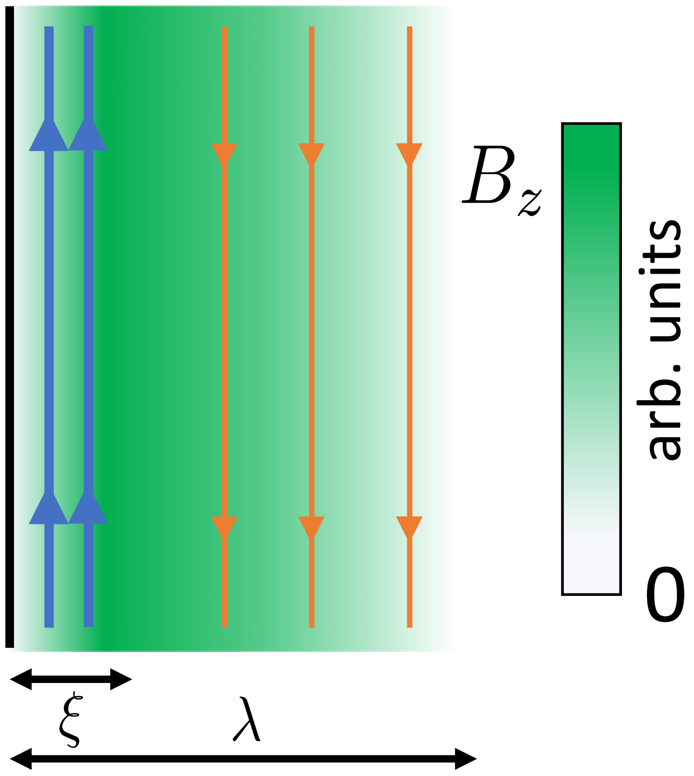
$$= -\frac{2e}{L} \sum_{k_\parallel} v_{F\parallel} \left[|u_{\vec{k}_F}(x)|^2 + |v_{\vec{k}_F}(x)|^2 \right] f(E_{k_\parallel})$$

material specific Fermi velocity

surface current is non-universal

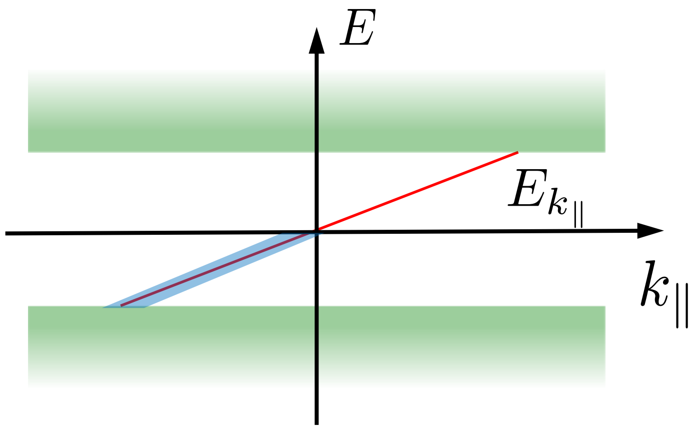
Chiral superconductor in 2D systems

surface currents



Meissner screening

magnitude depends on material/sample parameters



focus on edge states

but also continuum contributes to current

net electric current parallel to surface

electron — hole

$$J_\parallel(x) = -\frac{2e}{L} \sum_{k_\parallel = \vec{k}_F \cdot \hat{r}_\parallel} v_{F\parallel} \left[|u_{\vec{k}_F}(x)|^2 f(E_{k_\parallel}) - |v_{\vec{k}_F}(x)|^2 f(-E_{k_\parallel}) \right]$$

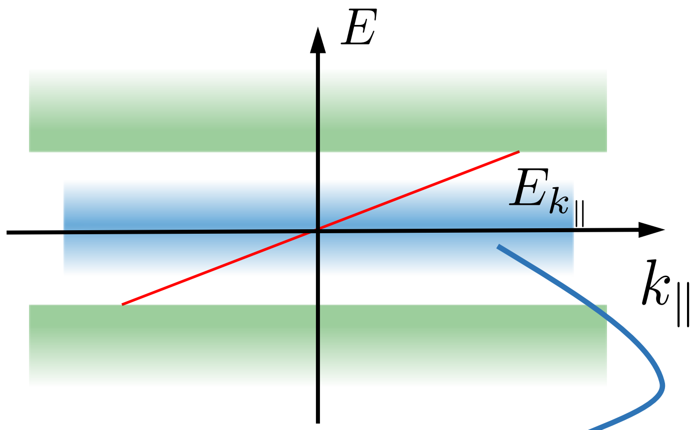
$$= -\frac{2e}{L} \sum_{k_\parallel} v_{F\parallel} \left[|u_{\vec{k}_F}(x)|^2 + |v_{\vec{k}_F}(x)|^2 \right] f(E_{k_\parallel})$$

material specific Fermi velocity

surface current is non-universal

Chiral superconductor in 2D systems

surface heat currents



heat current density

$$J_\parallel^{(Q)}(x) = \frac{1}{L}\sum_{k_\parallel} E_{k_\parallel} v_{k_\parallel} \left[|u_{\vec{k}_F}(x)|^2 + |v_{\vec{k}_F}(x)|^2\right] f(E_{k_\parallel})$$

$$\frac{1}{\hbar}\frac{dE_{k_\parallel}}{dk_\parallel}$$ quasiparticle velocity

total heat current along surface

$$I^{(Q)}(T) = \int \frac{dk_\parallel}{2\pi} E_{k_\parallel} v_{k_\parallel} f(E_{k_\parallel})$$

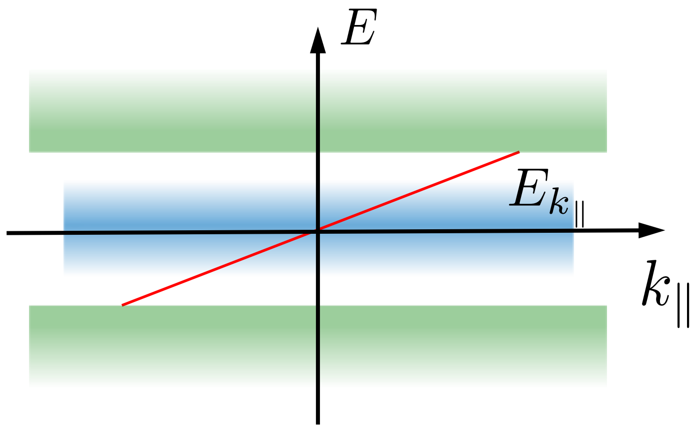
limit $T \ll T_c$

Sommerfeld expansion

$$I^{(Q)}(T) = \int \frac{dk_\parallel}{2\pi} v_{k_\parallel} \left\{ E_{k_\parallel}\Theta(-E_{k_\parallel}) - k_B T \frac{\beta^2 E_{k_\parallel}^2}{4\cosh^2(\beta E_{k_\parallel}/2)} + \cdots \right\}$$

Chiral superconductor in 2D systems

surface heat currents



heat current density

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$$\frac{1}{\hbar}\frac{dE_{k_\parallel}}{dk_\parallel}$$

quasiparticle velocity

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limit $T \ll T_c$

Sommerfeld expansion

$$I^{(Q)}(T) = I_0^{(Q)} - \frac{\pi}{6}\frac{(k_B T)^2}{\hbar} + O(T^4)$$

contribution by edge state zero-energy quasiparticles

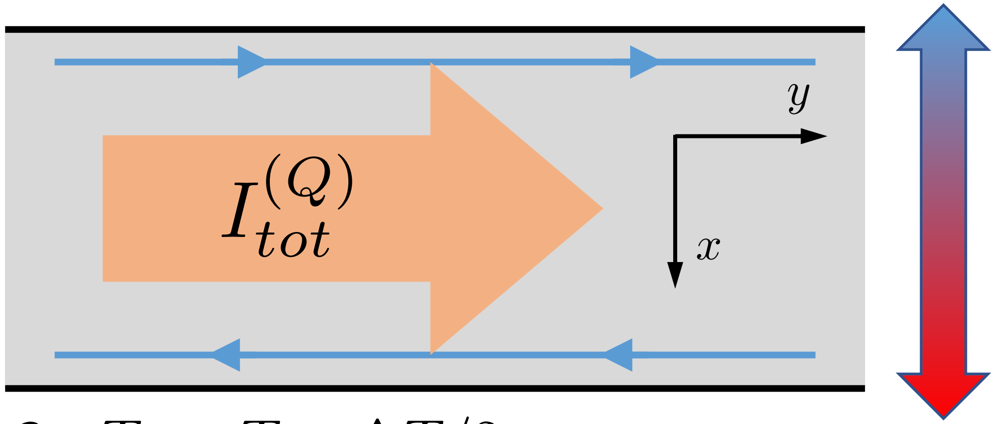
Chiral superconductor in 2D systems

thermal Hall bar

Rigghi-Leduc effect

$$I_1^{(Q)} = I_0^{(Q)} - \frac{\pi}{6}\frac{k_B^2 T_1^2}{\hbar}$$

1 $T_1 = T - \Delta T/2$



2 $T_2 = T + \Delta T/2$

$$I_2^{(Q)} = -I_0^{(Q)} + \frac{\pi}{6}\frac{k_B^2 T_2^2}{\hbar}$$

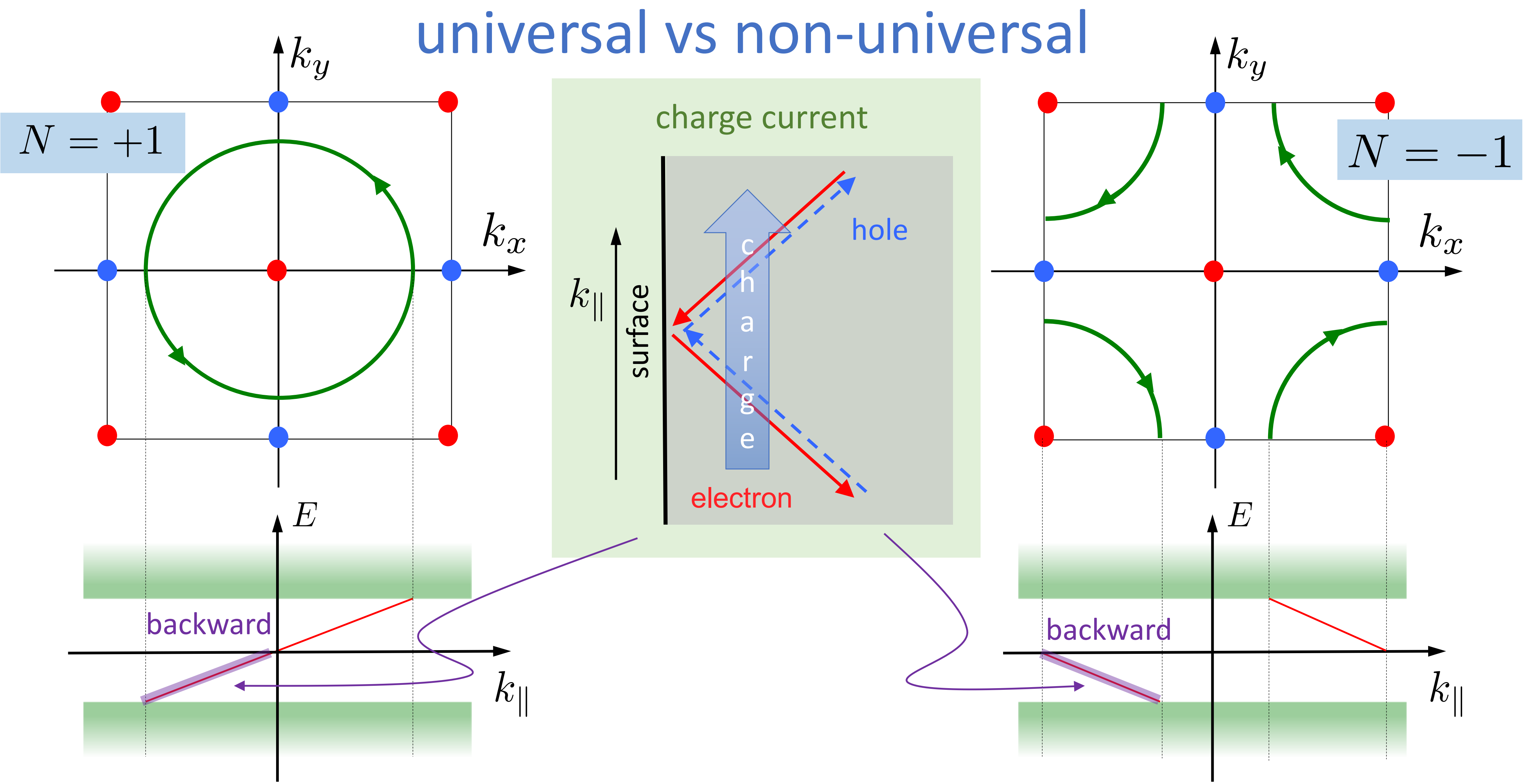
$$I_{tot}^{(Q)} = I_1^{(Q)} + I_2^{(Q)} = \frac{\pi}{6}\frac{k_B^2}{\hbar}(T_2^2 - T_1^2)$$

$$= -\frac{\pi}{6}\frac{k_B^2 T}{\hbar}\Delta T = \kappa_{yx}\Delta T$$

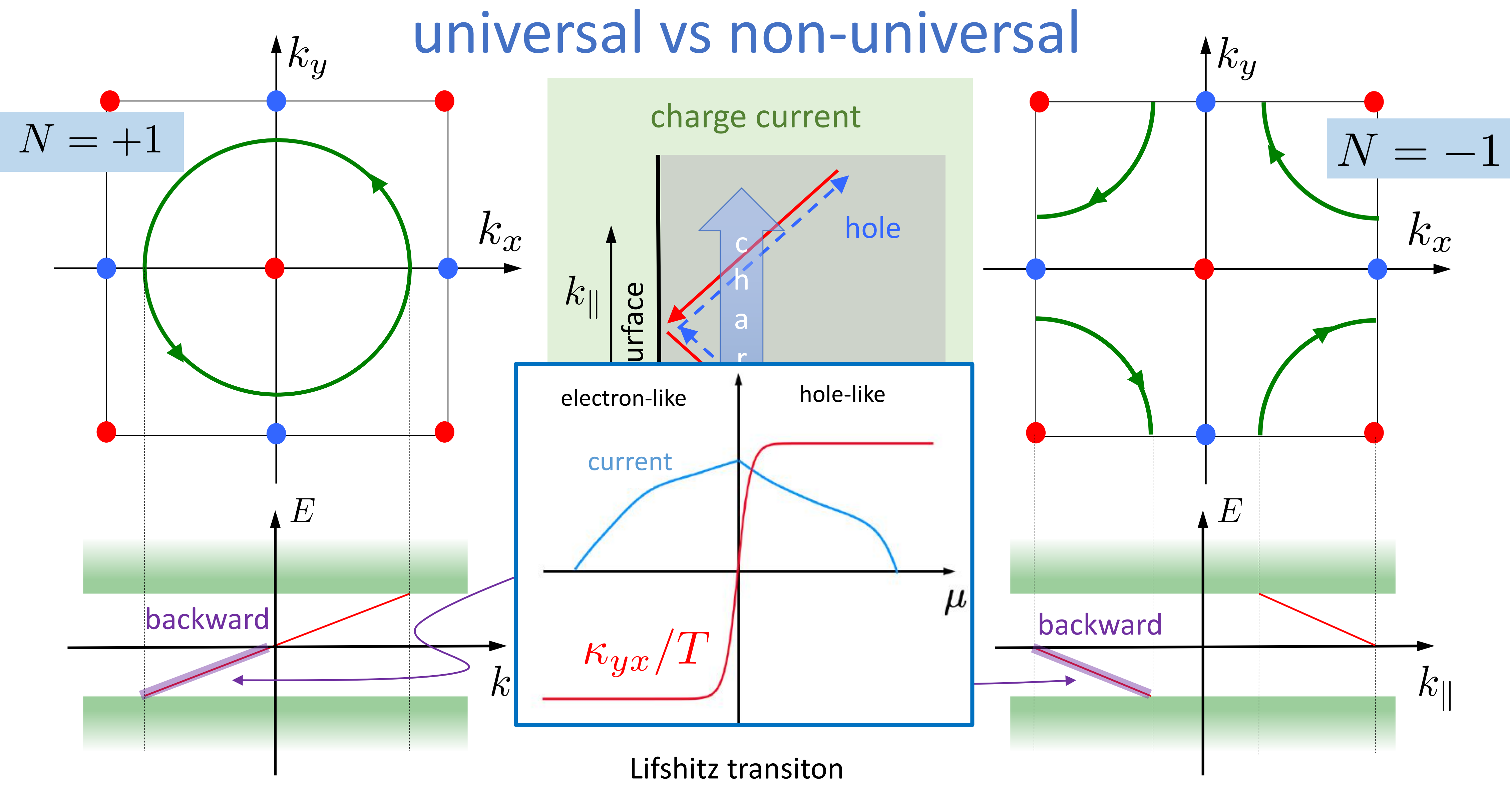
universal thermal Hall effect

$$\kappa_{xy} = -\kappa_{yx} = \frac{\pi}{6}\frac{k_B^2 T}{\hbar} N_C$$

Chiral superconductor in 2D systems



Chiral superconductor in 2D systems



Chiral superconductor in 3D systems

chiral p-wave

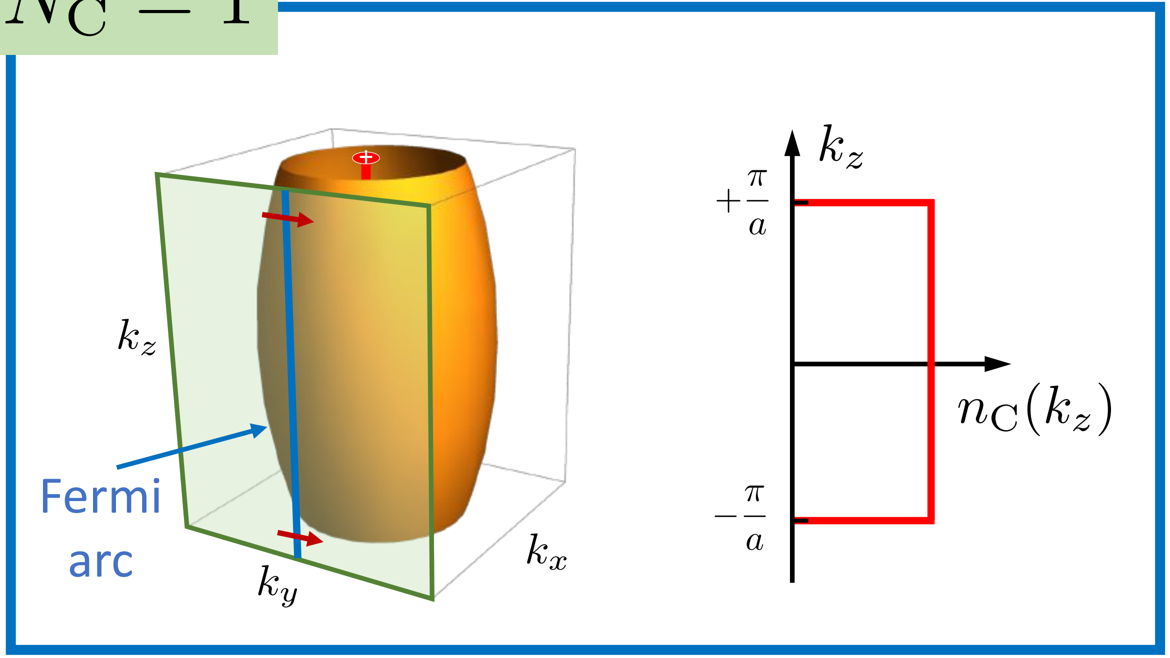
$$d_z(\vec{k}) = \Delta_0(\sin k_x + i \sin k_y)$$

sliced Chern numbers (integer valued)

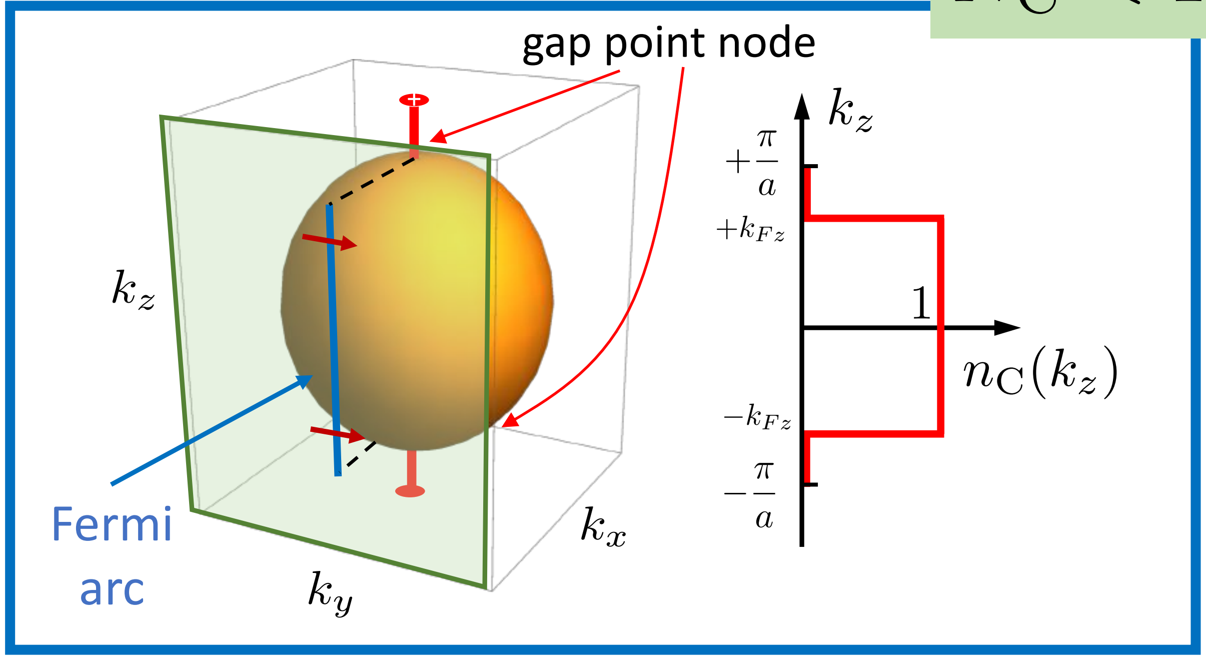
$$n_{\mathrm{C}}(k_z) = \frac{1}{2\pi} \oint_{\mathrm{FS}(k_z)} d\vec{k} \cdot \vec{\nabla}_{\vec{k}} \arg[d_z(\vec{k})]$$

$$N_{\mathrm{C}} = \frac{a}{2\pi} \int_{-\pi/a}^{+\pi/a} \frac{dk_z}{2\pi} n_{\mathrm{C}}(k_z)$$

$N_{\mathrm{C}} = 1$



$N_{\mathrm{C}} < 1$



Chiral superconductor in 3D systems

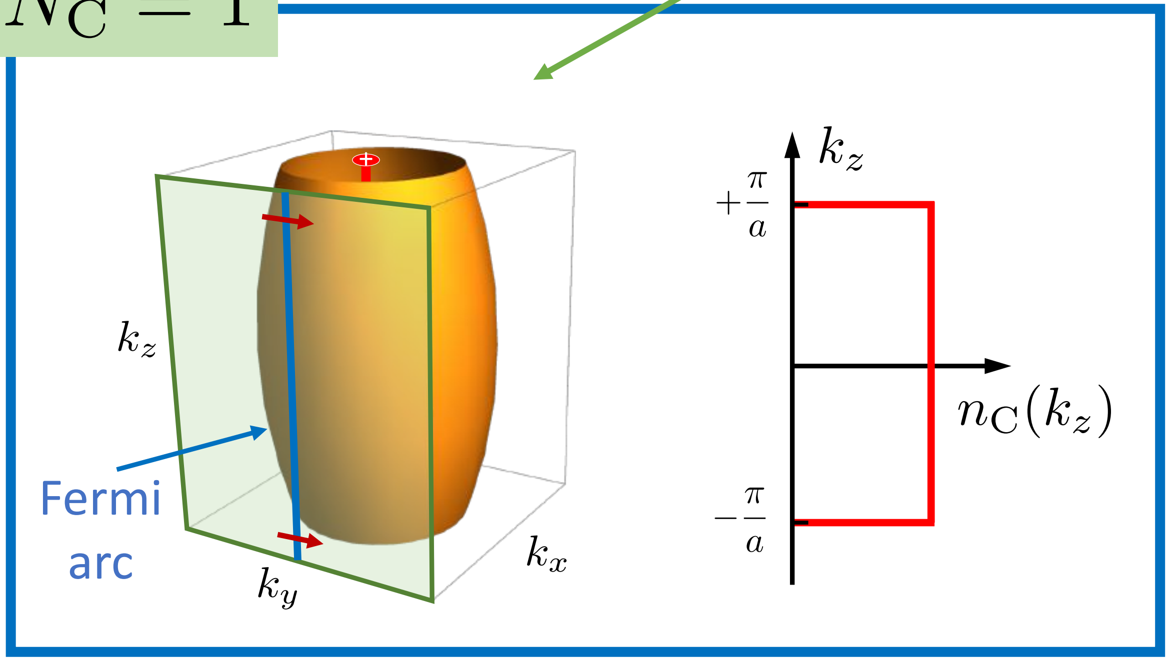
- non-universal surface currents
- thermal Hall effect

$$\kappa_{xy} = \frac{\pi}{6}\frac{k_B^2 T}{\hbar} N_{\mathrm{C}}$$

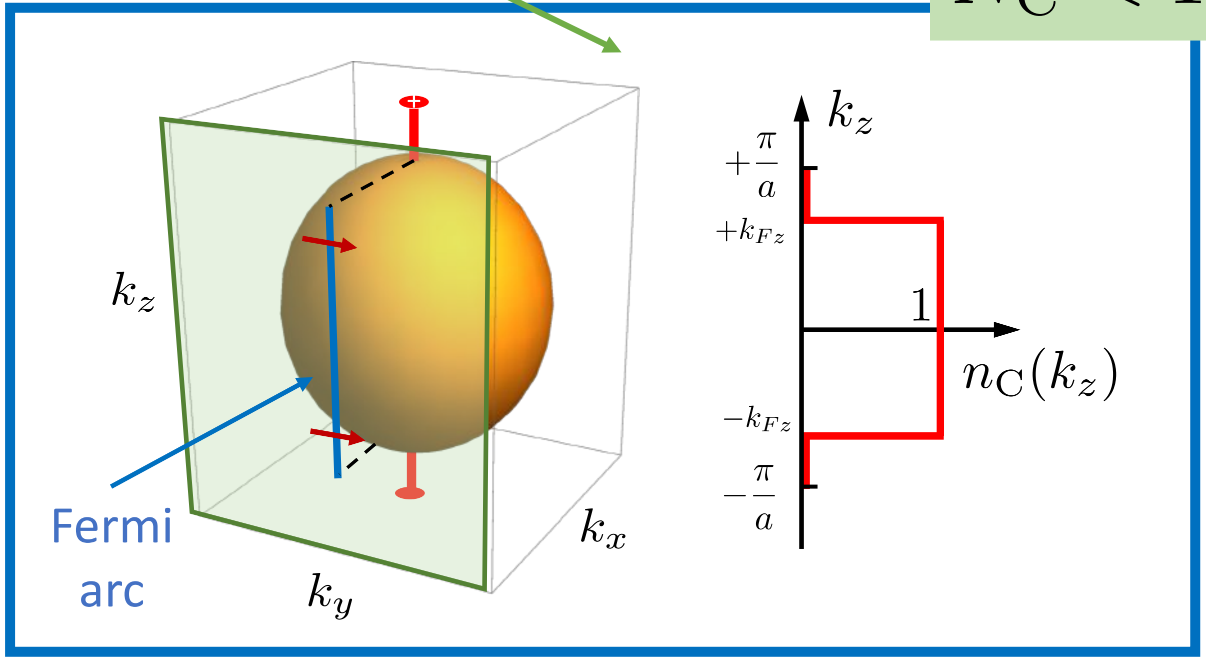
universal

non-universal
measures length
of Fermi arc

$N_{\mathrm{C}} = 1$

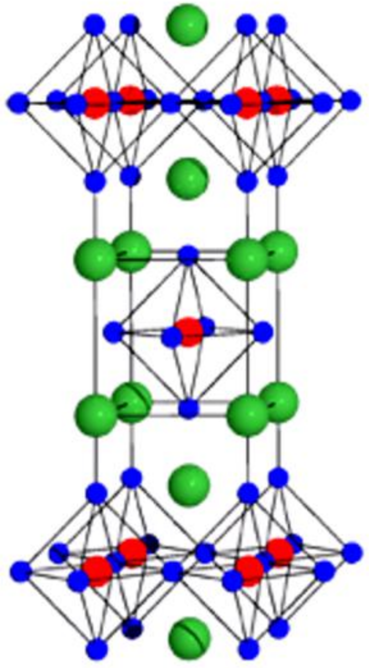


$N_{\mathrm{C}} < 1$



possible chiral superconductors

Sr_2RuO_4



tetragonal

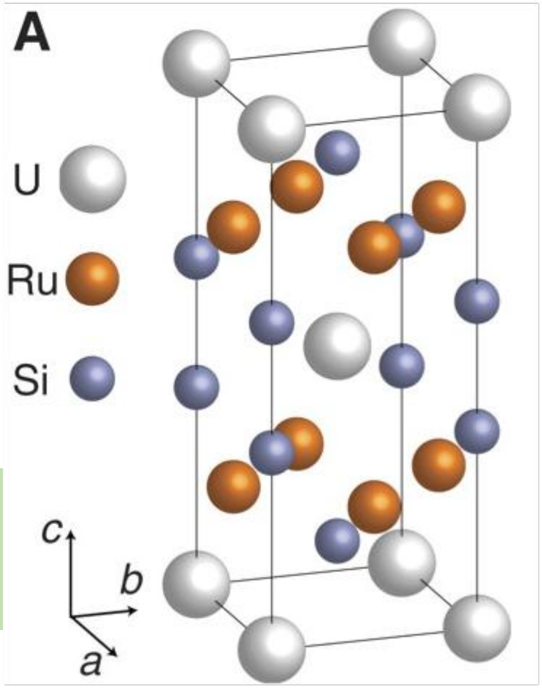
crystal structure

odd-parity

$$d_z(\vec{k}) = \Delta_0(k_x \pm i k_y)$$

chiral *p*-wave

URu_2Si_2



even-parity

$$d_0(\vec{k}) = \Delta_0(k_x \pm i k_y) k_z$$

nodal gap chiral *d*-wave

μSR
polar Kerr effect

polar Kerr effect

possible chiral superconductors

Sr_2RuO_4

under debate now!

new NMR-Knight shift data cast doubt on odd-parity pairing

μSR
polar Kerr effect

tetragonal

crystal structure

odd-parity

$$\Delta_0(k_x \pm i k_y)$$

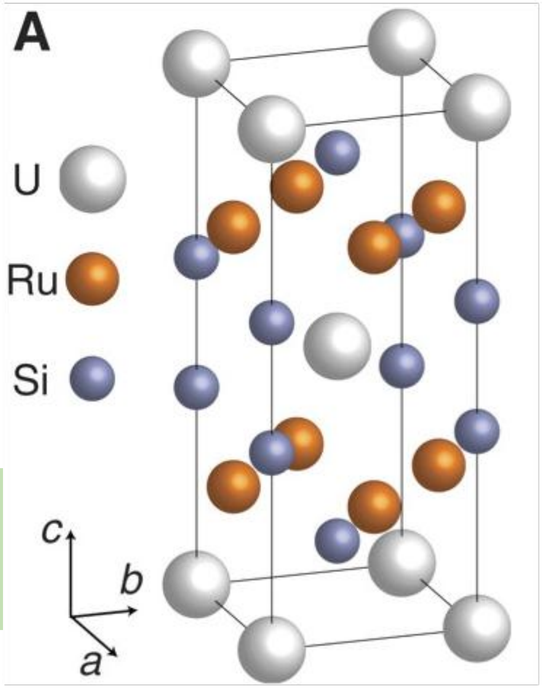
wave

even-parity

$$d_0(\vec{k}) = \Delta_0(k_x \pm i k_y) k_z$$

nodal gap chiral *d*-wave

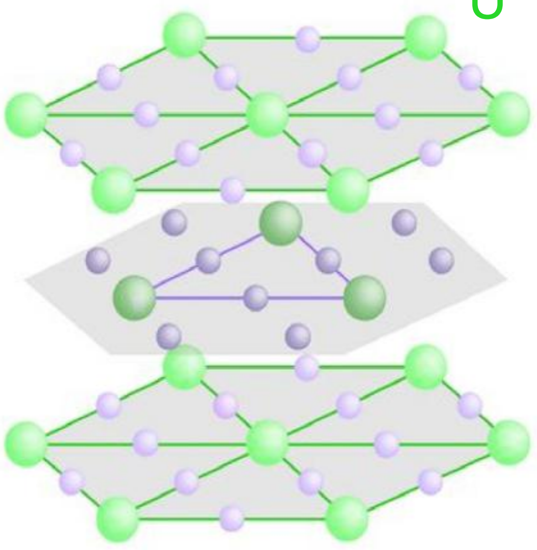
URu_2Si_2



polar Kerr effect

possible chiral superconductors

UPt_3



μSR
polar Kerr effect

hexagonal
crystal structure

odd-parity

$$d_z(\vec{k}) = \Delta_0(k_x \pm ik_y)^2 k_z$$

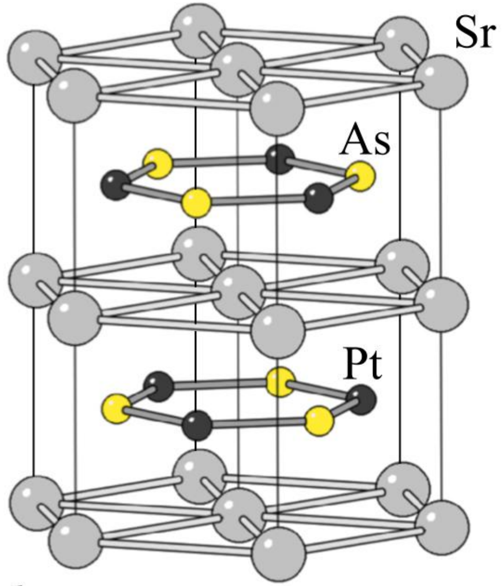
chiral *f*-wave   nodal gap

even-parity

$$d_0(\vec{k}) = \Delta_0(k_x \pm ik_y)^2$$

chiral *d*-wave

SrPtAs



μSR

other topological superconductors in 2D

odd-parity superconductors with time reversal symmetry

example on square lattice (nearest-neighbor pairing)

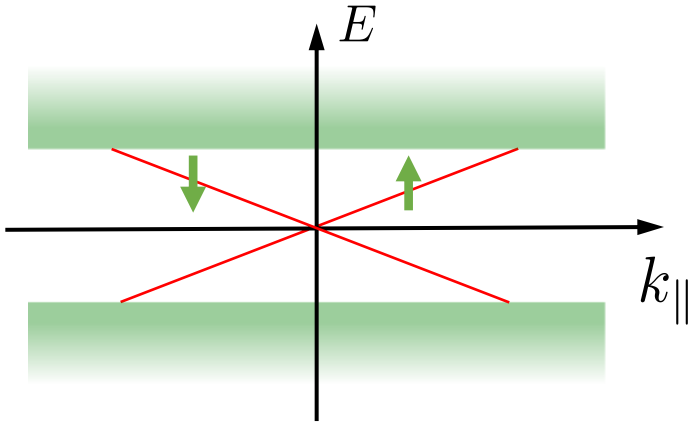
$$\vec{d}(\vec{k}) = \Delta_0\{\hat{\vec{x}} \sin k_x + \hat{\vec{y}} \sin k_y\}$$

$$N_{C\uparrow} = -1$$

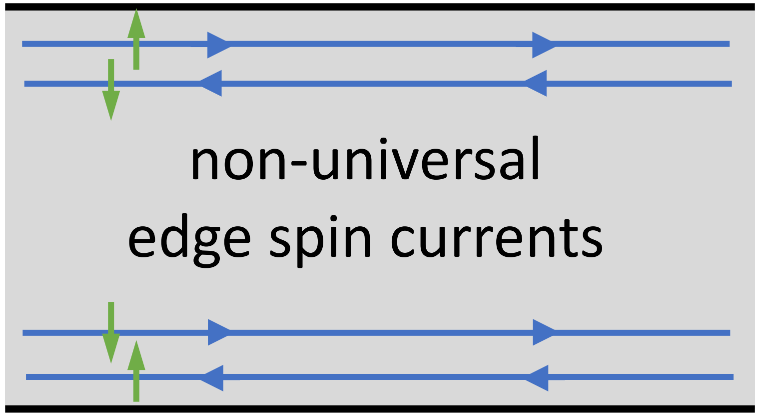
$$\hat{\Delta}_{\vec{k}} = \begin{pmatrix} -\Delta_0(\sin k_x - i \sin k_y) & 0 \\ 0 & \Delta_0(\sin k_x + i \sin k_y) \end{pmatrix}$$

$$N_{C\downarrow} = +1$$

spin dependent chirality



helical edge states



other topological superconductors in 3D

odd-parity superconductors with time reversal symmetry

example on cubic lattice (nearest-neighbor pairing)

$$\vec{d}(\vec{k}) = \Delta_0 \{ \hat{\vec{x}} \sin k_x + \hat{\vec{y}} \sin k_y + \hat{\vec{z}} \sin k_z \}$$

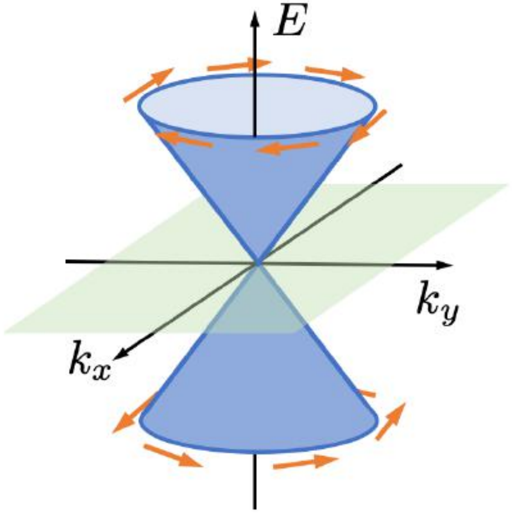
$$\hat{\Delta}_{\vec{k}} = \Delta_0 \begin{pmatrix} -\sin k_x + i \sin k_y & \sin k_z \\ \sin k_z & \sin k_x + i \sin k_y \end{pmatrix}$$

analog to BW-phase - B-phase of superfluid ^{3}He

completely gapped spectrum in general

$$|\Delta_{\vec{k}}|^2 = |\vec{d}(\vec{k})|^2 = |\Delta_0|(\sin^2 k_x + \sin^2 k_y + \sin^2 k_z)$$

edge states with spin texture



other topological superconductors in 3D

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example on cubic lattice (nearest-neighbor pairing)

$$\vec{d}(\vec{k}) = \Delta_0 \{ \hat{\vec{x}} \sin k_x + \hat{\vec{y}} \sin k_y + \hat{\vec{z}} \sin k_z \}$$

$$\hat{\Delta}_{\vec{k}} = \Delta_0 \begin{pmatrix} -\sin k_x + i \sin k_y & \sin k_z \ldots \\ \sin k_z & \sin k_x + i \, \mathrm{s}\ldots \end{pmatrix}$$

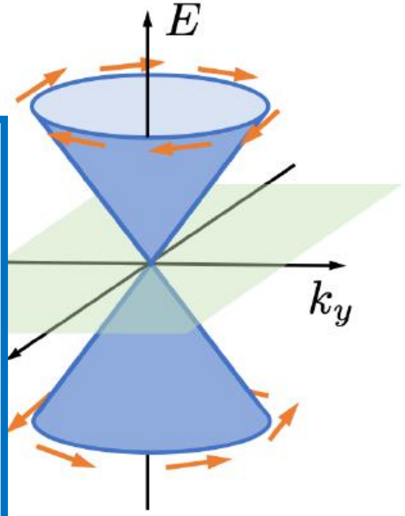
analog to BW-phase - B-phase of superflu…

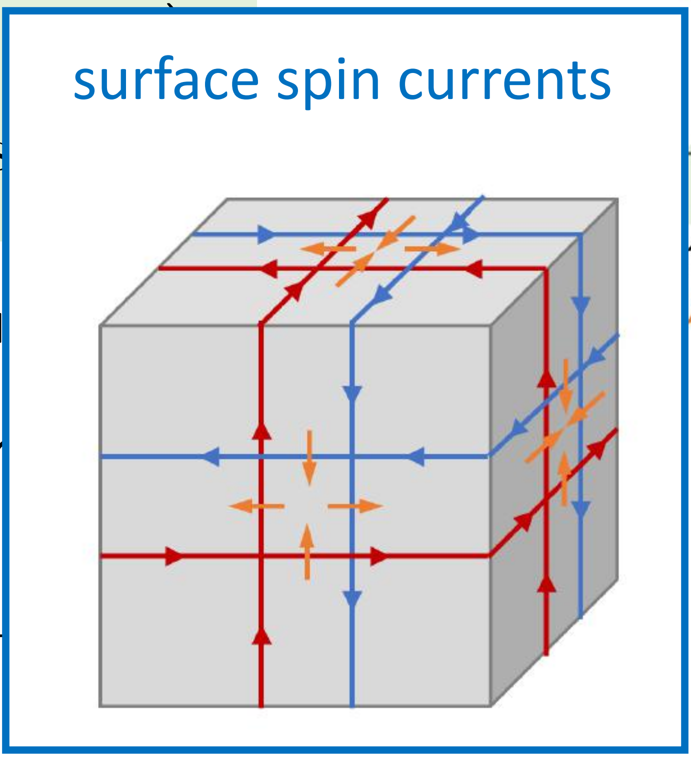
completely gapped spectrum in gener…

$$|\Delta_{\vec{k}}|^2 = |\vec{d}(\vec{k})|^2 = |\Delta_0|(\sin^2 k_x + \sin^2 k_y + \ldots$$

edge states

with spin texture





topological superconducting phases

classes of Bogolyubov-de Gennes Hamiltonians with non-trivial edge states

AZ class	$SU(2)$	TRS	parity	examples	edge states
D	×	×	odd	spinless chiral p-wave	chiral
DIII	×	○	odd	BW p-wave	helical
A	△	×	odd	spinfull chiral p-wave	chiral
AIII	△	○	odd	nematic	zero-energy
C	○	×	even	chiral d-wave	chiral
CI	○	○	even	nematic	zero-energy

○ present
× absent
△ restricted

Conclusion

topologically non-trivial phases in unconventional superconductors

chiral superconductors

time reversal symmetry broken

- non-universal spontaneous surface currents
- quantized thermal Hall effect

Bogolyubov quasiparticles
conserve energy, but not charge

helical superconductors

time reversal symmetry broken

- non-universal spontaneous surface spin current
- quantized thermal spin Hall effect

Conclusion

topologically non-trivial phases in unconventional superconductors

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Superfluid ^3He