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A bird-eye view of Fermi liquids

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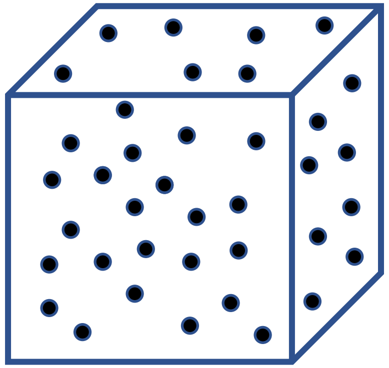
1. The Fermi liquid

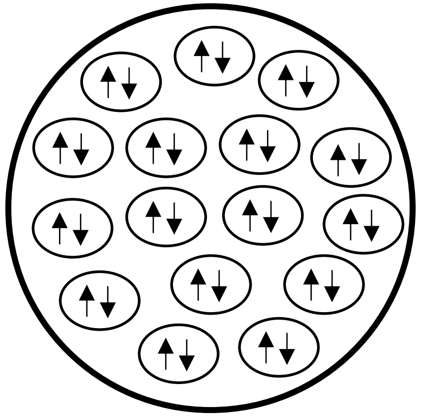
When many interacting fermions condense in a state close to the minimum energy allowed by the Pauli exclusion principle one obtains a *Fermi liquid*. Physical realizations of the Fermi liquids concept range from interacting electrons in metals and semiconductors to liquid ^{3}He, to gases of cold Fermionic atoms, to nuclear matter, electrons in white dwarves, and neutron stars.



$$n = N/V$$

density

Real space: N particles in a box of volume V



$$k_F = (3\pi^2 n)^{1/3}$$

Fermi wave vector

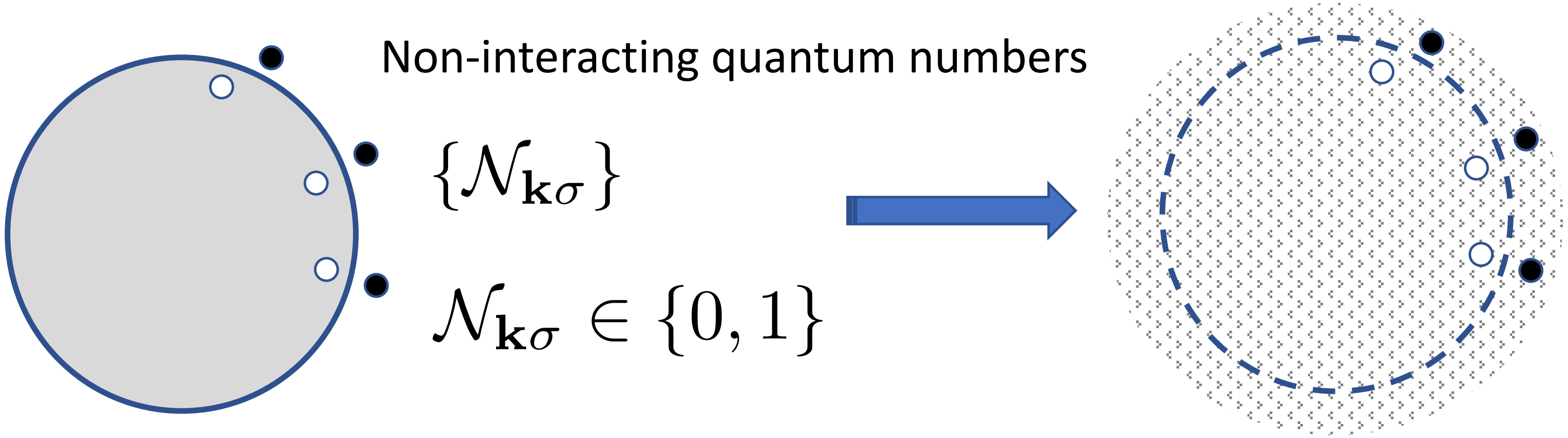
Momentum space: N particles in a sphere of radius k_F

The behavior of Fermi liquids confronts us with a puzzle: in spite of strong mutual interactions the particles appear to behave as if they were non-interacting.

It was not until the late 1950s that this puzzling state of affairs was clarified from a theoretical point of view by L. D. Landau.

2. Principle of adiabatic continuity

The low-lying states of the interacting liquid are in one-to-one correspondence with those of the ideal Fermi gas.

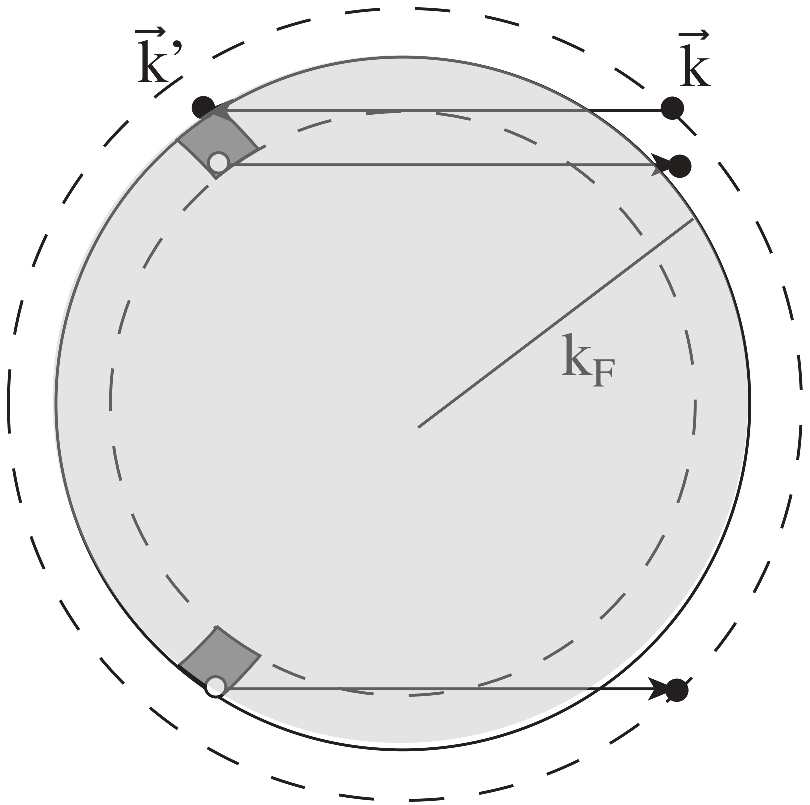


$$n_{\mathbf{k}\sigma} \neq \mathcal{N}_{\mathbf{k}\sigma}$$

$n_{\mathbf{k}\sigma}$ discontinuous at k_F

Note: the adiabatic continuation principle does not say anything about the microscopic structure of the interacting states. It only says that these states can be labelled in terms of “quasiparticles”, which are defined relative to the Fermi sphere.

3. Adiabatic continuity and quasiparticle lifetime



At low temperature, the Pauli principle greatly reduces the phase space available to the mutual scattering of particles under constraint of momentum and energy conservation. In 3D this leads to

$$\frac{1}{\tau_k} \sim \frac{v_F (k - k_F)^2}{k_F} \qquad (T = 0)$$

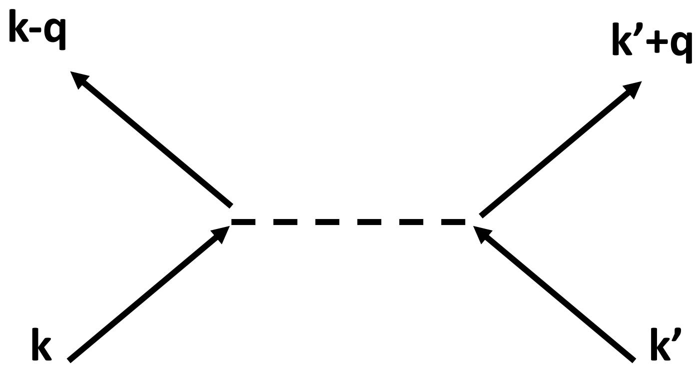
$$\frac{1}{\tau_k} \sim \frac{(k_B T)^2}{\hbar E_F} \qquad (k = k_F)$$

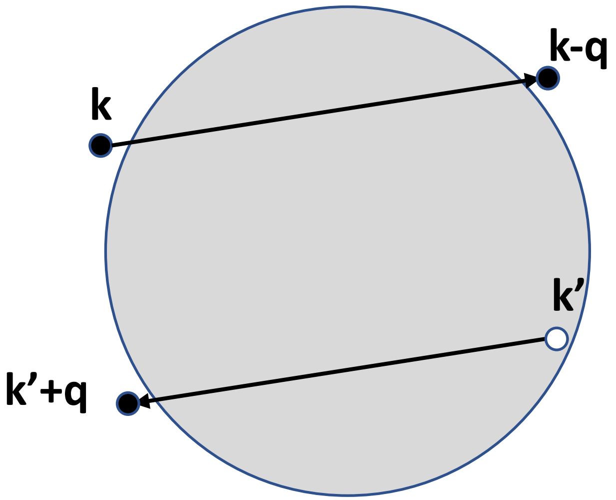
The adiabatic "switching on" of interactions is justified on a time scale that is long on the scale of excitation frequencies, yet short on the scale of the (diverging) lifetime.

4. "Golden rule" calculation of the quasiparticle lifetime

(GV Chapter 8.4)

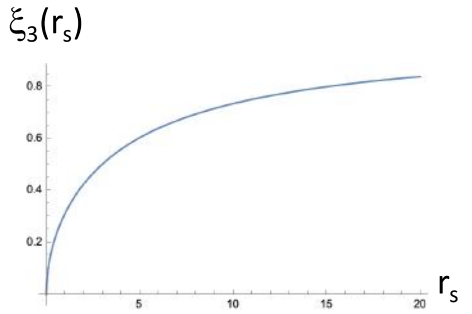
$$\frac{1}{\tau^{(e)}_{\vec{k}\sigma}} = \frac{2\pi}{\hbar} \sum_{\vec{q}\vec{k}'\sigma'} \left| \frac{W(\vec{q})}{L^d} \right|^2 n_{\vec{k}'\sigma'}(1-n_{\vec{k}'+\vec{q}\sigma'})(1-n_{\vec{k}-\vec{q}\sigma})\delta(\varepsilon_{\vec{k}-\vec{q}\sigma}+\varepsilon_{\vec{k}'+\vec{q}\sigma'}-\varepsilon_{\vec{k}\sigma}-\varepsilon_{\vec{k}'\sigma'})$$





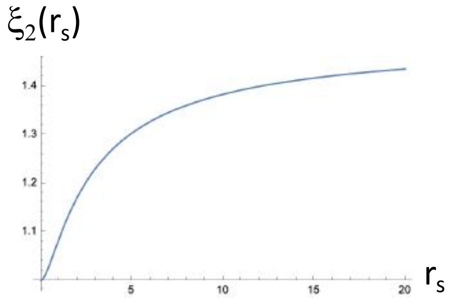
3D electron gas with Thomas-Fermi screened interaction:

$$\begin{cases} \dfrac{1}{\tau^{(e)}_{\vec{k}\sigma}} \simeq \dfrac{\pi}{8\hbar\epsilon_F} \dfrac{(\varepsilon_{\vec{k}\sigma}-\epsilon_F)^2+(\pi k_B T)^2}{1+e^{-\beta(\varepsilon_{\vec{k}\sigma}-\epsilon_F)}}\xi_3(r_s) \\ \xi_3(r_s) = \sqrt{\dfrac{\alpha_3 r_s}{4\pi}}\tan^{-1}\sqrt{\dfrac{\pi}{\alpha_3 r_s}} + \dfrac{1}{2\left(1+\frac{\pi}{\alpha_3 r_s}\right)} \end{cases}$$

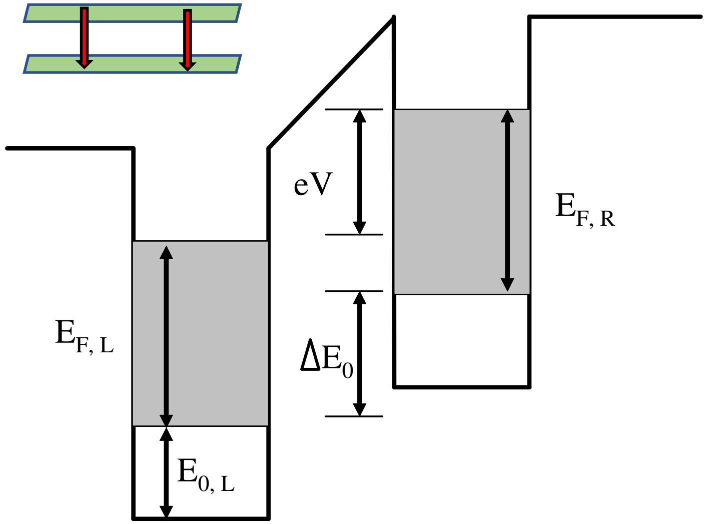


2D electron gas with Thomas-Fermi screened interaction:

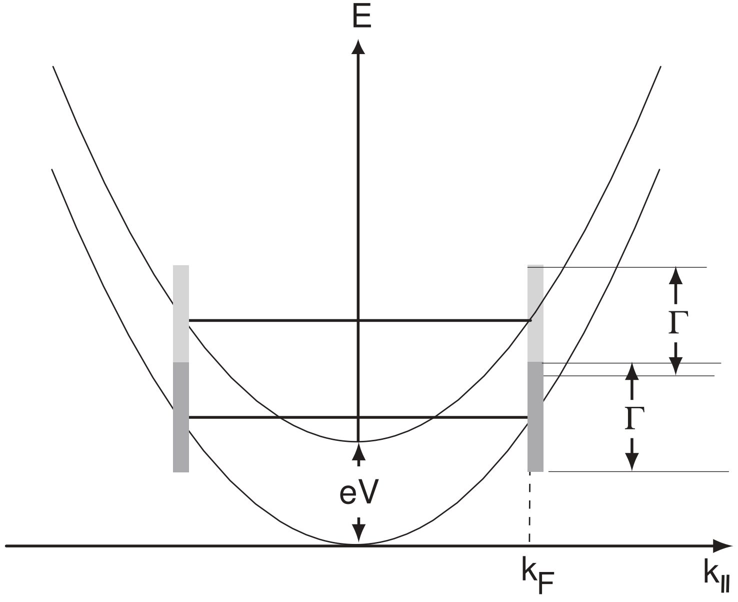
$$\begin{cases} \dfrac{1}{\tau^{(e)}_{\vec{k}\sigma}} \simeq \xi_2(r_s)\dfrac{(\varepsilon_{\vec{k}\sigma}-\epsilon_F)^2}{4\pi\hbar\epsilon_F}\ln\dfrac{4\epsilon_F}{|\varepsilon_{\vec{k}\sigma}-\epsilon_F|}, \quad k_B T \ll |\varepsilon_{\vec{k}\sigma}-\epsilon_F|. \\ \dfrac{1}{\tau^{(e)}_{\vec{k}\sigma}} \simeq \xi_2(r_s)\dfrac{(\pi k_B T)^2}{8\pi\hbar\epsilon_F}\ln\dfrac{4\epsilon_F}{k_B T}, \quad |\varepsilon_{\vec{k}\sigma}-\epsilon_F| \ll k_B T \\ \xi_2(r_s) = 1+\dfrac{1}{2}\left(\dfrac{r_s}{r_s+\sqrt{2}}\right)^2 \end{cases}$$

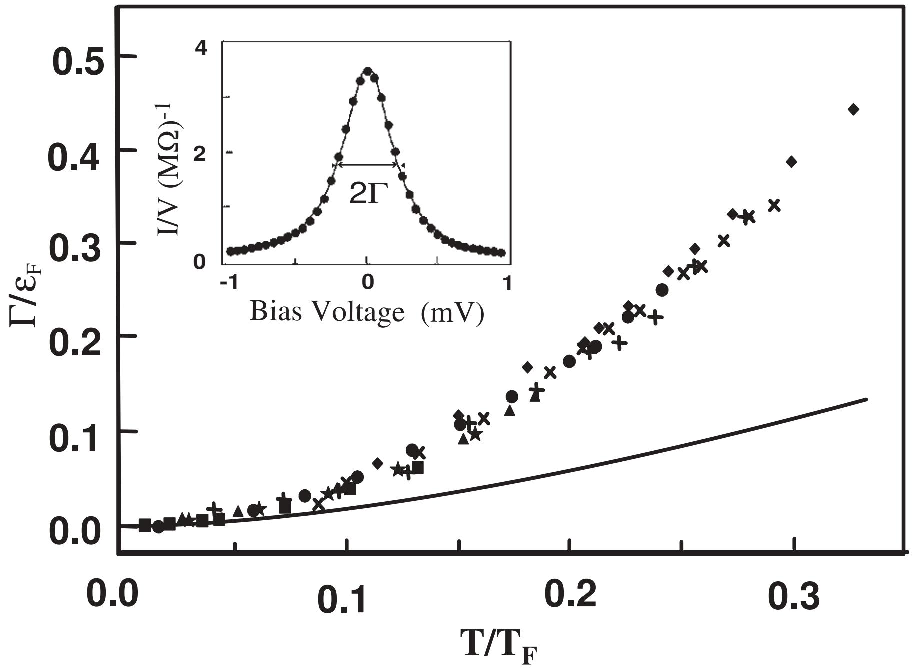


5. Measuring the quasiparticle lifetime



Momentum-conserving tunneling between two identical quantum wells separated by a potential difference eV . Because of energy conservation, the tunneling probability decreases rapidly when eV exceeds the spectral width Γ of the single-particle states in each band. Adapted from Sheena Murphy (2004)





6. The Landau energy functional

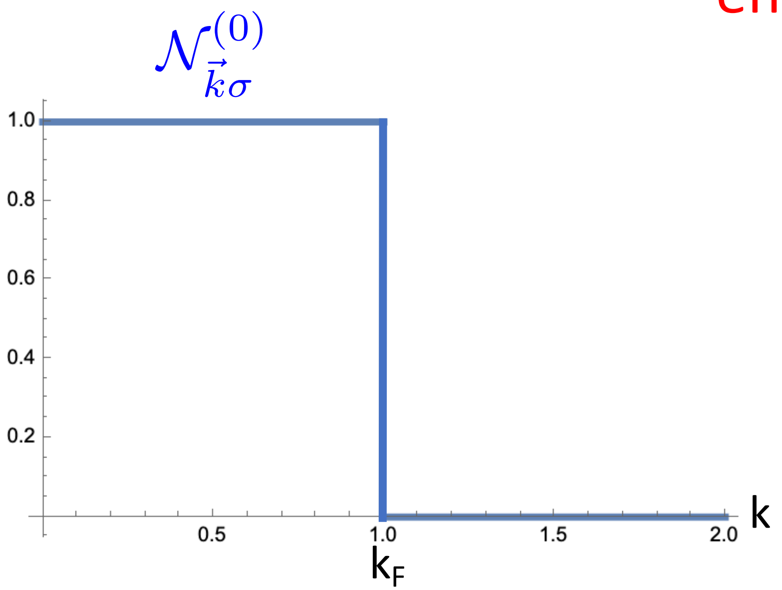
Expand the energy to second order in the deviation of the quasiparticle distribution from the ground state distribution $\mathcal{N}^{(0)}_{\vec{k}\sigma}$

$$E[\{\mathcal{N}_{\vec{k}\sigma}\}] = E_0 + \sum_{\vec{k}\sigma} \mathcal{E}_{\vec{k}\sigma} \delta\mathcal{N}_{\vec{k}\sigma} + \frac{1}{2} \sum_{\vec{k}\sigma,\vec{k}'\sigma'} f_{\vec{k}\sigma,\vec{k}'\sigma'} \delta\mathcal{N}_{\vec{k}\sigma} \delta\mathcal{N}_{\vec{k}'\sigma'}$$

$$\delta\mathcal{N}_{\vec{k}\sigma} = \mathcal{N}_{\vec{k}\sigma} - \mathcal{N}^{(0)}_{\vec{k}\sigma}$$

Quasiparticle energy

Interaction function



Quasiparticle effective mass

$$\mathcal{E}_{k_F\sigma} = \mu$$

$$\mathcal{E}_{\vec{k}\sigma} \simeq \mu + \hbar v_F^*(k - k_F)$$

$$v_F^\star = \frac{1}{\hbar} \left| \frac{\partial \mathcal{E}_{\vec{k}\sigma}}{\partial \vec{k}} \right|_{k=k_F} = \frac{\hbar k_F}{m^\star}$$

7. Going to finite temperature

We obtain the equilibrium distribution of quasiparticles at finite temperature by maximizing the entropy

$$S[\{\mathcal{N}_{\vec{k}\sigma}\}] = -k_B \sum_{\vec{k}\sigma} [\mathcal{N}_{\vec{k}\sigma} \ln \mathcal{N}_{\vec{k}\sigma} + (1 - \mathcal{N}_{\vec{k}\sigma}) \ln (1 - \mathcal{N}_{\vec{k}\sigma})]$$

subject to the constraints of constant energy and number

$$\sum_{\vec{k}\sigma} \mathcal{N}_{\vec{k}\sigma} = N \qquad \sum_{\vec{k}\sigma} \mathcal{N}_{\vec{k}\sigma} \mathcal{E}_{\vec{k}\sigma} = E$$

This gives the usual Fermi-Dirac distribution function

$$\mathcal{N}^{eq}_{\vec{k}\sigma} = \frac{1}{e^{(\mathcal{E}_{\vec{k}\sigma} - \mu)/k_B T} + 1}$$

Notice that, to the order of our expansion $\mathcal{N}^{eq}_{\vec{k}\sigma}$ is not affected by the interaction function.

8. Calculation of macroscopic properties

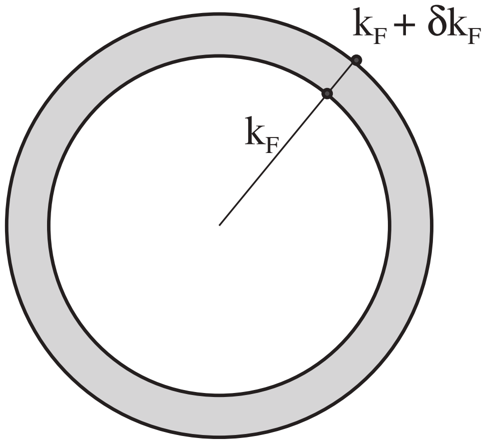
(i) Heat capacity depends only on the effective mass. No contribution from interaction function.

$$c_v(T) = \frac{\pi^2}{3} N^*(0) L^d k_B^2 T$$

Landau parameters

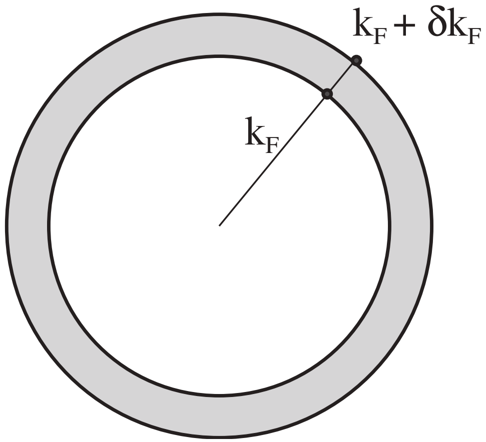
$$F_\ell^{s,a} = \frac{L^d N^*(0)}{2} \int \frac{d\Omega_d}{\Omega_d} [f_{\uparrow\uparrow}(\cos\theta) \pm f_{\uparrow\downarrow}(\cos\theta)] \begin{cases} P_\ell(\cos\theta), & 3D \\ \cos\ell\theta, & 2D \end{cases}$$

(ii) Compressibility



$$\frac{K}{K_0} = \frac{m^\star/m}{1+F_0^s}$$

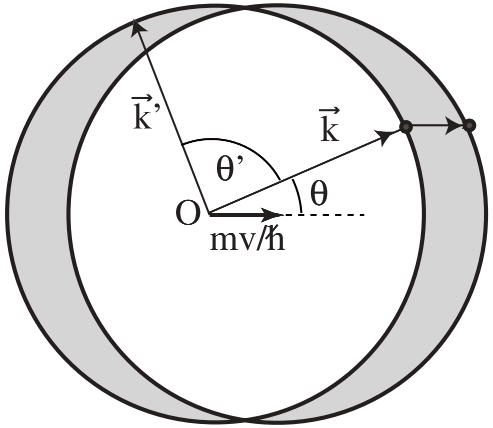
(iii) Spin susceptibility



$$\frac{\chi_S}{\chi_{S0}} = \frac{m^\star/m}{1+F_0^a}$$

(iv) Effective mass

(Galilean invariant system)



$$\frac{m^*}{m} = 1 + F_1^s$$

9. Connection with microscopic theory

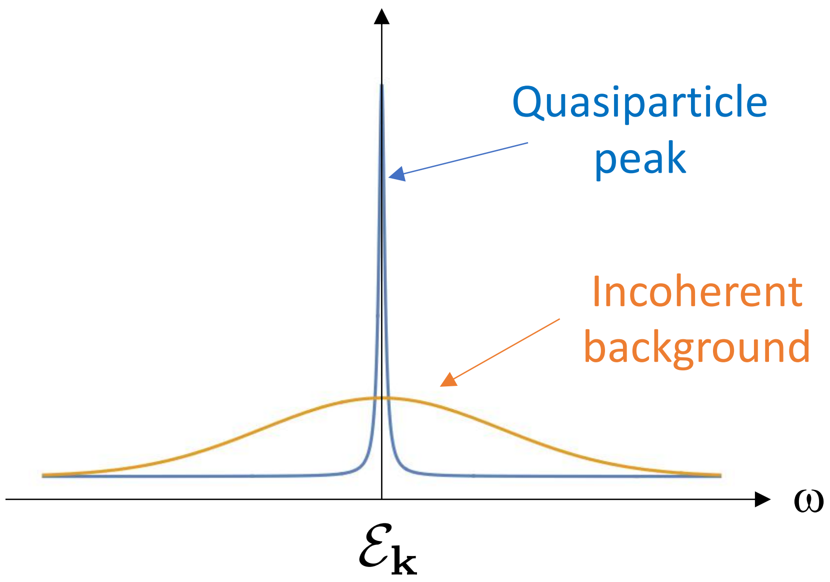
(i) Existence of quasiparticles

The 1-particle Green's function $G(k,\omega) = -i\int_{-\infty}^{\infty} dt \langle T\hat{a}_k(t)\hat{a}_k^{\dagger}(0)\rangle e^{i\omega t}$

has a pole at the quasiparticle energy $\mathcal{E}_{\mathbf{k}}$

$$G(k,\omega) = \frac{1}{\omega - \epsilon_k - \Sigma(k,\omega)} = G^{(reg)}(\vec{k},\omega) + \frac{Z_{\vec{k}}}{\omega - \frac{\mathcal{E}_{\vec{k}}}{\hbar} + \frac{i}{2\tau_{\vec{k}}}}$$

$$A(k,\omega) = -\frac{1}{\pi}\Im m G^{\mathrm{ret}}(k,\omega)$$



The renormalization constant Z_k - a positive number between 0 and 1 – measures the strength of the quasiparticle peak. Physically, Z_{k_F} is the probability that the liquid remains in the ground state after injecting a particle at the Fermi surface.

10. The central role of the self-energy

(ii) Quasiparticle energy, renormalization constant, effective mass, and quasiparticle lifetime can all be calculated from the "self-energy", Σ(k,ω), here shown in the so-called GW approximation

Effective mass

$$\frac{m}{m^*} = Z_{k_F}\left[1 + \frac{m}{\hbar^2 k_F}\frac{\partial}{\partial k}\Re e\Sigma^{ret}(k,\epsilon_F)\Big|_{k=k_F}\right]$$

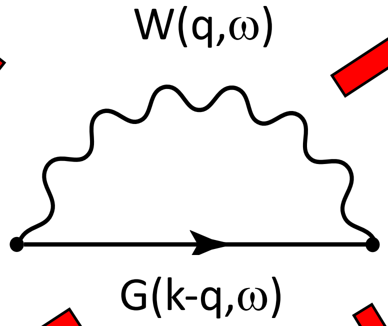
Renormalization constant

$$Z_{\vec{k}} \equiv \left(1 - \frac{1}{\hbar}\frac{\partial}{\partial\omega}\Re e\Sigma^{ret}(k,\omega)\Big|_{\hbar\omega=\mathcal{E}_{\vec{k}}}\right)^{-1}$$

Quasiparticle energy

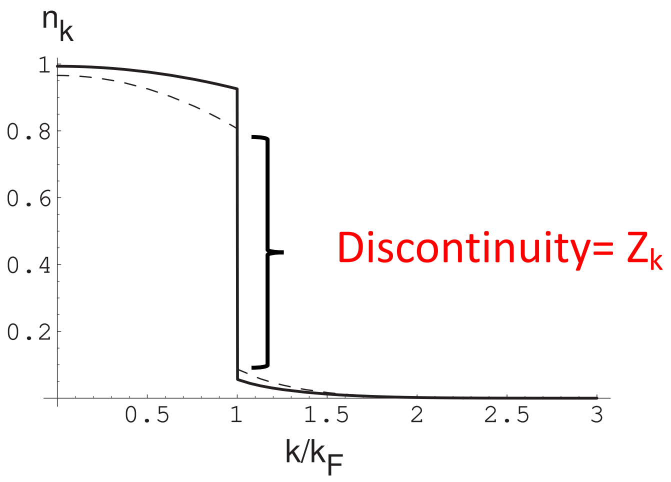
$$\mathcal{E}_{\vec{k}} = \varepsilon_{\vec{k}} + \Re e\Sigma_\sigma^{ret}(\vec{k},\mathcal{E}_{\vec{k}})$$

$\Sigma(k,\omega)$ =



Quasiparticle lifetime

$$\frac{\hbar}{2\tau_{\vec{k}}} \equiv Z_{\vec{k}}|\Im m\Sigma^{ret}(k,\mathcal{E}_{\vec{k}\sigma})|$$



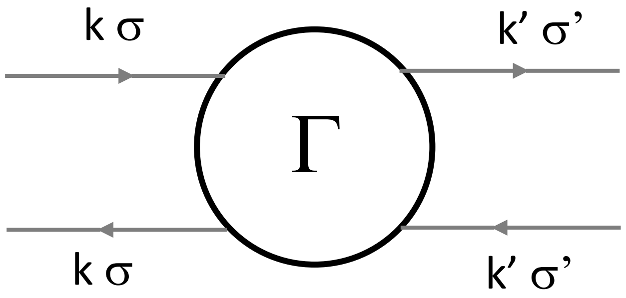
Momentum occupation numbers

11. What about the Landau interaction function?

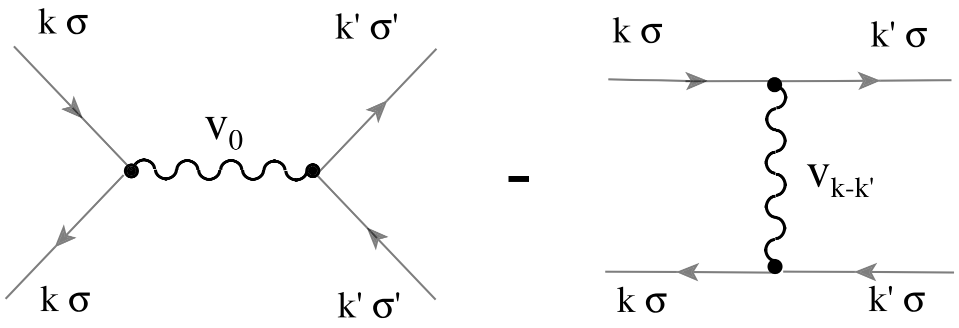
$$f_{\vec{k},\vec{k}'} = Z_{\vec{k}} Z_{\vec{k}'} \frac{\delta \Sigma(\vec{k}, \mathcal{E}_{\vec{k}})}{\delta \mathcal{N}_{\vec{k}'}} = Z_{\vec{k}} Z_{\vec{k}'} \lim_{\omega \to 0} \lim_{q \to 0} \Gamma_{\vec{k}\mathcal{E}_{\vec{k}};\vec{k}'\mathcal{E}_{\vec{k}'}}(q, \omega)$$

Notice the order
of the limits

Diagrammatically



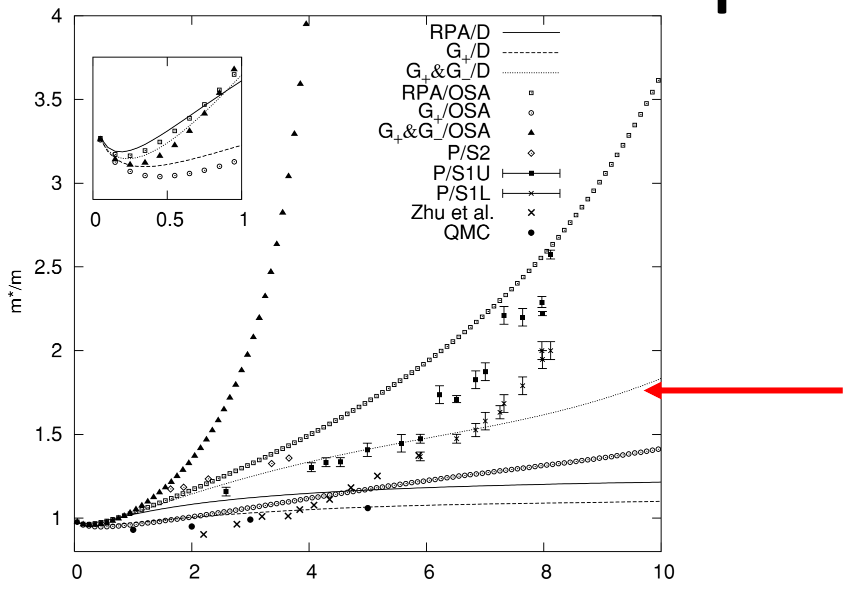
Lowest order approximation



12a. Some numerical results for the electron liquid

r_s	F_0^s	F_0^a	F_1^s	F_1^a	F_2^s	F_2^a
1	-0.21	-0.17	-0.04	-0.0645	-0.0215	-0.0181
2	-0.37	-0.25	-0.03	-0.0825	-0.0168	-0.0126
3	-0.55	-0.32	-0.02	-0.0915	-0.0107	-0.0073
4	-0.74	-0.37	0.0	-0.0956	-0.0047	-0.0022
5	-0.95	-0.40	0.03	-0.0965	+0.0009	+0.0023

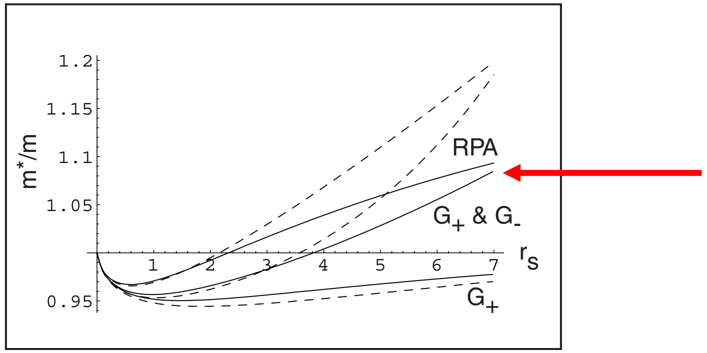
Landau Fermi liquid parameters of the three-dimensional electron liquid.



Effective mass enhancements for the two-dimensional electron liquid.

r_s	K_0/K	χ_P/χ_S	F_0^s	F_0^a	F_1^s
1	0.533	0.691	−0.45	−0.29	0.03
2	0.018	0.525	−0.98	−0.40	0.15
3	−0.538	0.421	−1.68	−0.47	0.26
5	−1.737	0.296	−3.48	−0.58	0.43
8	−3.657	0.196	−7.03	−0.68	0.65

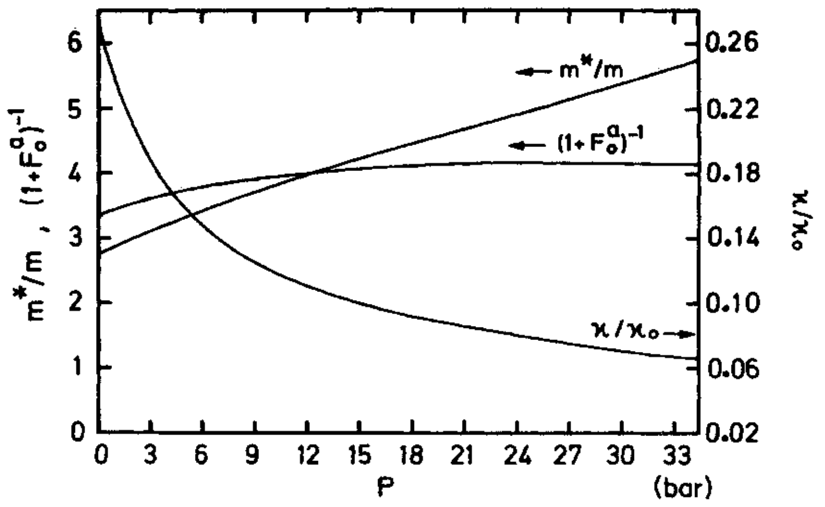
Landau Fermi liquid parameters of the two-dimensional electron liquid.



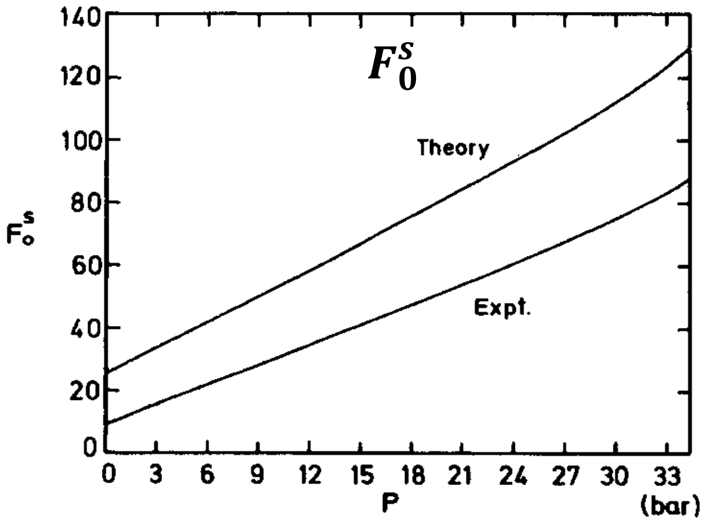
Effective mass enhancements for the three-dimensional electron liquid.

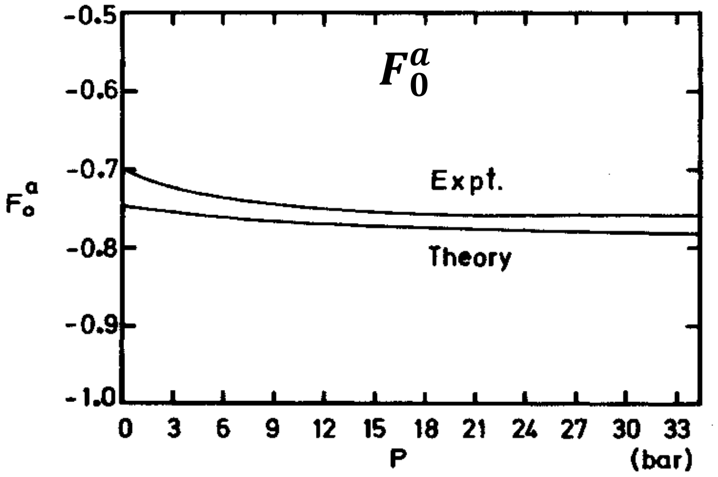
12b. Some results for ^{3}He

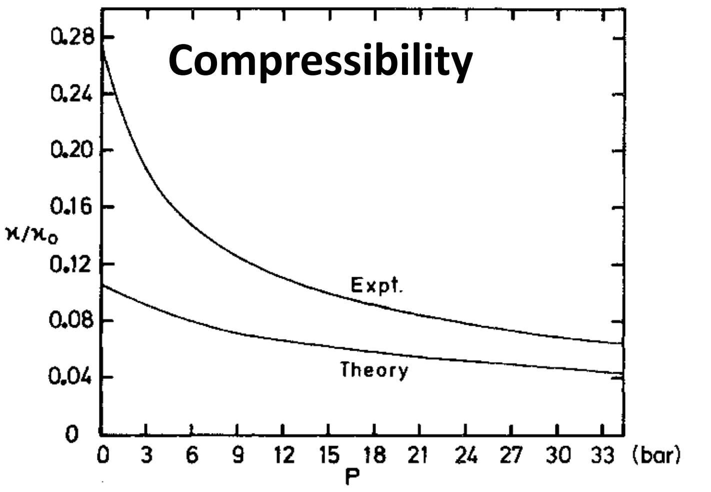
D. Vollhardt, Rev. Mod. Phys. **56**, 99 (1984)

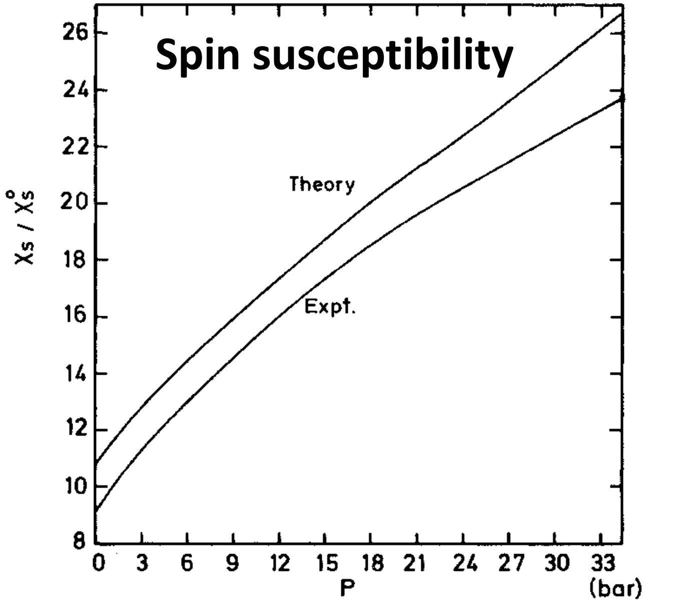


Experimental values (Greywall, 1983) of effective mass, normalized compressibility and spin susceptibility of ^{3}He









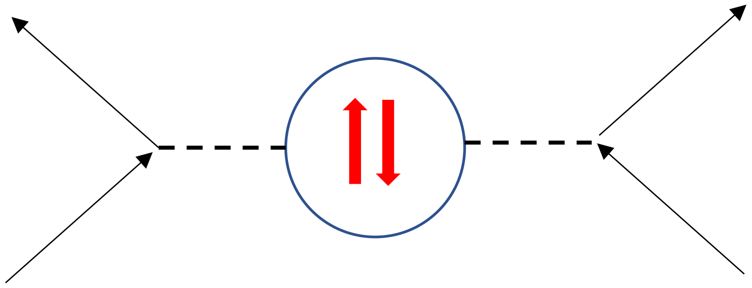
12c. Fermi liquid description of the Kondo effect at low temperature

P. Nozieres, J. Low Temp. Phys. **17**, 31 (1974)

Kondo s-d model

$$H = \sum_{\mathbf{k},\sigma} \varepsilon_k c^*_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\substack{\mathbf{k}\sigma \\ \mathbf{k}'\sigma'}} (J/N)\mathbf{S}\cdot\mathbf{s}_{\sigma\sigma'} c^*_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma'}$$

The electron gas for $T \ll T_K$ is a Fermi liquid, with Landau parameters that can be calculated perturbatively (electrons interact with each other via the polarization they induce in the Kondo singlet)



13. The kinetic equation

This is the technical centerpiece of the Landau theory of Fermi liquids. Based on a classical Hamiltonian for quasiparticles

$$H_{cl}(\vec{r}, \hbar\vec{k}, \sigma) = \mathcal{E}_{\vec{k}\sigma} \overbrace{- e\phi_\sigma(\vec{r}, t) + \sum_{\vec{k}'\sigma'} f_{\vec{k}\sigma,\vec{k}'\sigma'} \delta\mathcal{N}_{\vec{k}'\sigma'}(\vec{r}, t)}^{\text{mean-field potential}}$$

the following Boltzmann equation is deduced for the phase space distribution function

$$\frac{\partial \mathcal{N}_{\vec{k}\sigma}(\vec{r}, t)}{\partial t} + \frac{1}{\hbar}\frac{\partial H_{cl}}{\partial \vec{k}} \cdot \frac{\partial \mathcal{N}_{\vec{k}\sigma}(\vec{r}, t)}{\partial \vec{r}} - \frac{1}{\hbar}\frac{\partial H_{cl}}{\partial \vec{r}} \cdot \frac{\partial \mathcal{N}_{\vec{k}\sigma}(\vec{r}, t)}{\partial \vec{k}} = \left(\frac{\partial \mathcal{N}_{\vec{k}\sigma}(\vec{r}, t)}{\partial t}\right)_{coll}$$

The linearized version of this equation (for small deviations from equilibrium) is

$$\frac{\partial \delta\mathcal{N}_{\vec{k}\sigma}(\vec{r}, t)}{\partial t} + \vec{v}_{\vec{k}\sigma} \cdot \frac{\partial \delta\mathcal{N}_{\vec{k}\sigma}(\vec{r}, t)}{\partial \vec{r}} + \vec{v}_{\vec{k}\sigma} \cdot \vec{\mathcal{F}}_{\vec{k}\sigma}(\vec{r}, t)\delta(\mathcal{E}_{\vec{k}\sigma} - \mu) = \left(\frac{\partial \delta\mathcal{N}_{\vec{k}\sigma}(\vec{r}, t)}{\partial t}\right)_{coll}$$

$$\vec{\mathcal{F}}_{\vec{k}\sigma}(\vec{r}, t) = -\vec{\nabla}_{\vec{r}}\left[-e\phi_\sigma(\vec{r}, t) + \sum_{\vec{k}'\sigma'} f_{\vec{k}\sigma,\vec{k}'\sigma'} \delta\mathcal{N}_{\vec{k}'\sigma'}(\vec{r}, t)\right] \qquad \left(\frac{\partial \mathcal{N}_{\vec{k}\sigma}}{\partial t}\right)_{coll} = -\frac{\mathcal{N}_{\vec{k}\sigma}}{\tau^{(e)}_{\vec{k}\sigma}} + \frac{1 - \mathcal{N}_{\vec{k}\sigma}}{\tau^{(h)}_{\vec{k}\sigma}}$$

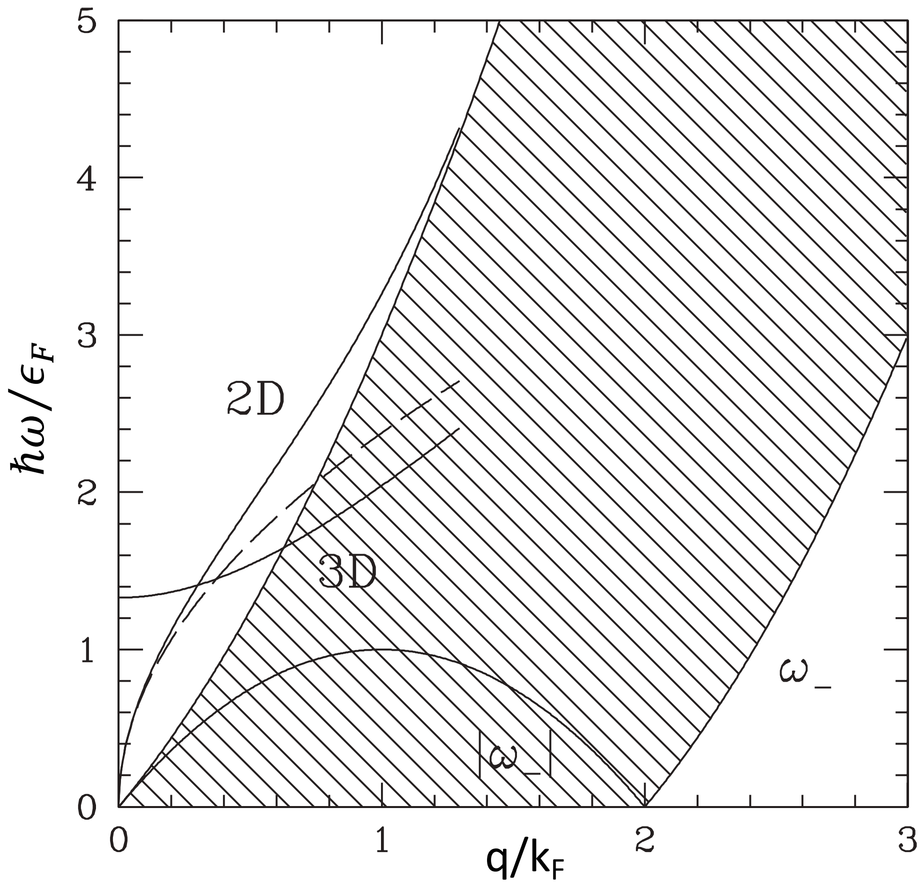
14a. Notable applications of the kinetic equation I

Collective modes/Plasmons (Collisionless)

$$\left(\omega - \vec{q}\cdot\vec{v}_{\vec{k}\sigma}\right)\delta\mathcal{N}_{\vec{k}\sigma}(\vec{q},\omega) + \vec{q}\cdot\vec{v}_{\vec{k}\sigma}\delta(\mathcal{E}_{\vec{k}\sigma}-\mu)\sum\left[v_q + f_{\vec{k}\sigma,\vec{k}'\sigma'}\right]\delta\mathcal{N}_{\vec{k}'\sigma'}(\vec{q},\omega) = 0$$

$$v_q = \frac{4\pi e^2}{q^2} \quad (3D) \quad v_q = \frac{2\pi e^2}{q} \quad (2D)$$

A nontrivial solution (not identically zero) of this equation exists only if ω equals the plasmon frequency $\omega_p^2(q)=4\pi ne^2/m$ in 3D or $\omega_p^2(q)=2\pi ne^2 q/m$ in 2D. On a microscopic level this plasmons are quite different from hydrodynamic sound.



14b. Notable applications of the kinetic equation II

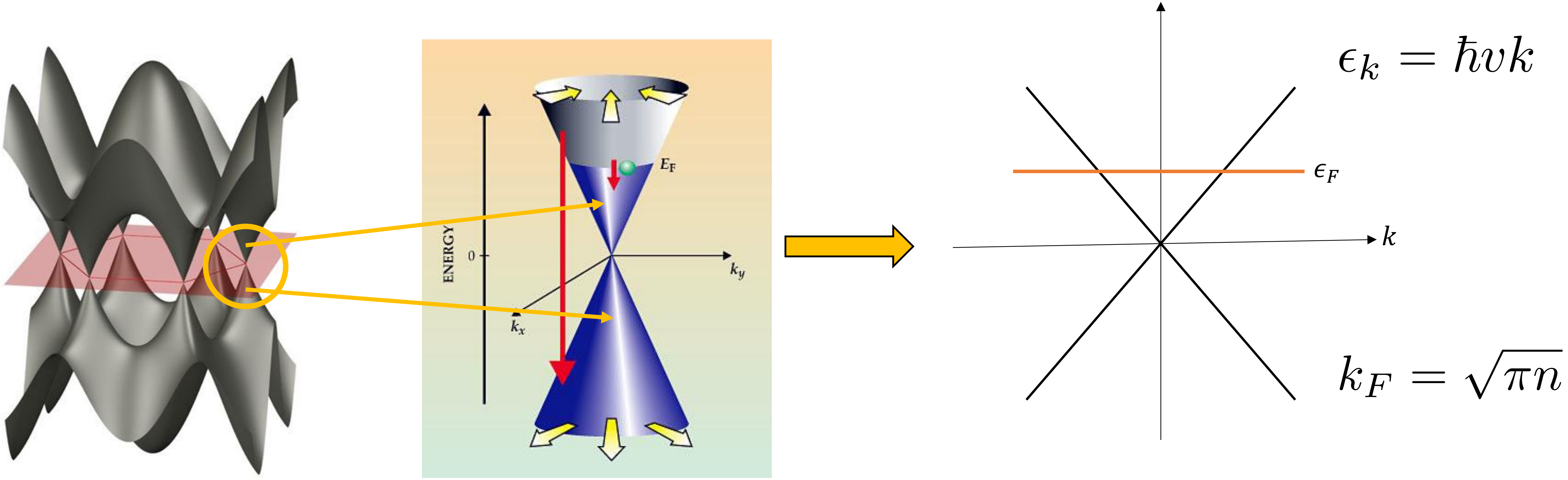
Transport coefficients

(A.A. Abrikosov and I. M. Khalatnikov, 1959)

$$D_s \sim v_F^2 \tau_s \,, \quad \eta \sim S\tau_\eta \,, \quad \zeta \sim B\tau_\zeta \,, \quad \kappa = n c_v v_F^2 \tau_q$$

The transport scattering times are similar to quasiparticle lifetimes, but there are differences in detail. For example , in the 2D electron gas, the scattering times associated with spin diffusion and thermal conductivity diverge as $1/(T^2 \ln T)$ (same as the quasiparticle lifetime), but the scattering time associated with the viscosity diverges as $1/T^2$.

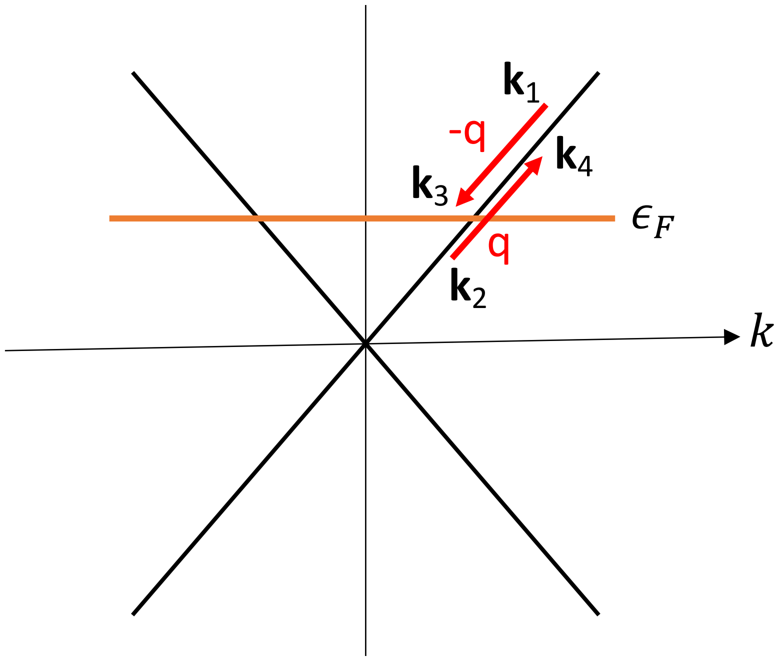
15. Fermi liquid theory of massless Dirac fermions



The linear energy-momentum relation $\epsilon_k = \hbar v k$ is characteristic of massless relativistic particles. However, its form is indistinguishable from that of an ordinary Fermi liquid in the vicinity of the Fermi energy. The role of (non-interacting) effective mass is played by the "cyclotron mass", $m_c = \hbar k_F / \mathrm{v}$, which is density-dependent.

$$\epsilon_k = \epsilon_F + \frac{\hbar k_F}{m_c}(k - k_F)$$

16. The quasiparticle lifetime – Collinear singularity

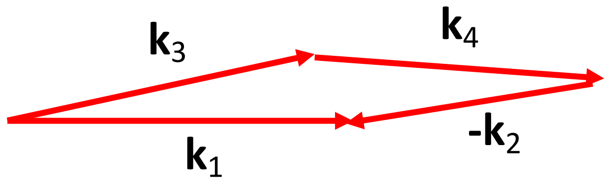


The dominant contribution to the quasiparticle decay rate comes from nearly collinear (forward) scattering processes. The density of states for such processes diverges as $\frac{1}{\sqrt{q^2-\omega^2/v^2}}$ because momentum conservation

$$\mathbf{k_1} = \mathbf{k_3} + \mathbf{k_4} - \mathbf{k_2}$$

coincides with energy conservation

$$k_1 = k_3 + k_4 - k_2$$



This is known as the *collinear singularity*.

However, the dielectric screening also diverges when $\omega \to vq$, so the decay rate remains finite. At the same time, non-collinear back-scattering processes with $2k_F$ are strongly suppressed.

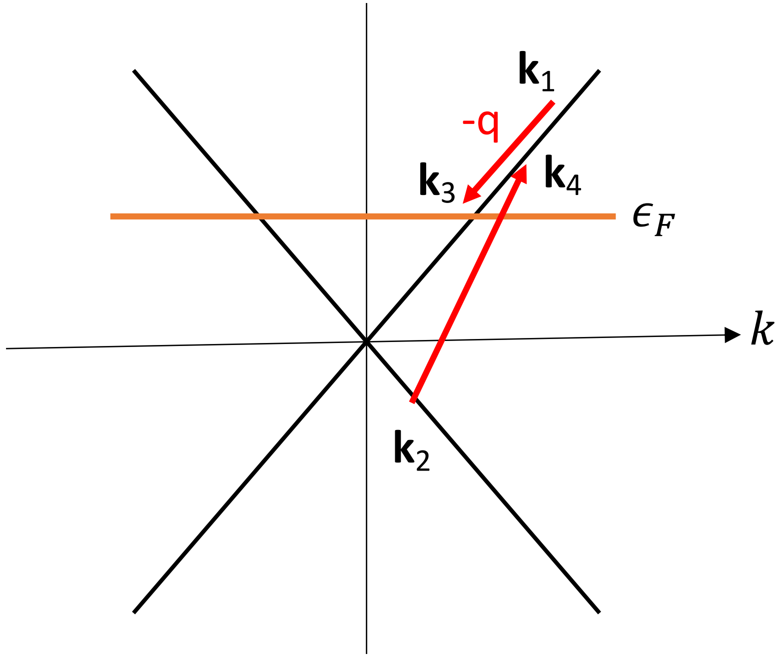
17. The quasiparticle lifetime I: Intra-band transitions

The results are very similar to those obtained for the two-dimensional electron gas, except that large momentum scattering does not contribute to the dominant (logarithmic) contribution to the decay rate.

$$\begin{cases} \dfrac{1}{\tau^{(\mathrm{e})}_{\boldsymbol{k},+}} \simeq \dfrac{\varepsilon_{\mathrm{F}}}{\hbar} \dfrac{1}{\pi N(0)} \left(\dfrac{\xi_{\boldsymbol{k},+}}{\varepsilon_{\mathrm{F}}}\right)^2 \ln\left(\dfrac{\Lambda}{\xi_{\boldsymbol{k},+}}\right) & \text{T=0} \\ \\ \dfrac{1}{\tau^{(\mathrm{e})}_{\boldsymbol{k},+}} \simeq \dfrac{\varepsilon_{\mathrm{F}}}{\hbar} \dfrac{\pi}{2N(0)} \left(\dfrac{k_{\mathrm{B}}T}{\varepsilon_{\mathrm{F}}}\right)^2 \ln\left(\dfrac{\Lambda}{k_{\mathrm{B}}T}\right) & \xi_{\mathbf{k}}\text{=0} \end{cases}$$

Notice that these formulas (valid in the low temperature/energy limit) do not depend on the strength of the interaction coupling constant $\alpha = \dfrac{e^2}{\varepsilon\hbar v}$. Here Λ is an ultraviolet energy cutoff which marks the limit of validity of the linear dispersion model.

18. What about interband transitions?

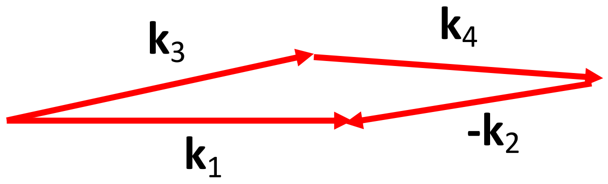


In this case momentum and energy conservation are in conflict with each other:

$$\mathbf{k_1} = \mathbf{k_3} + \mathbf{k_4} - \mathbf{k_2}$$

makes

$$k_1 = k_3 + k_4 + k_2$$



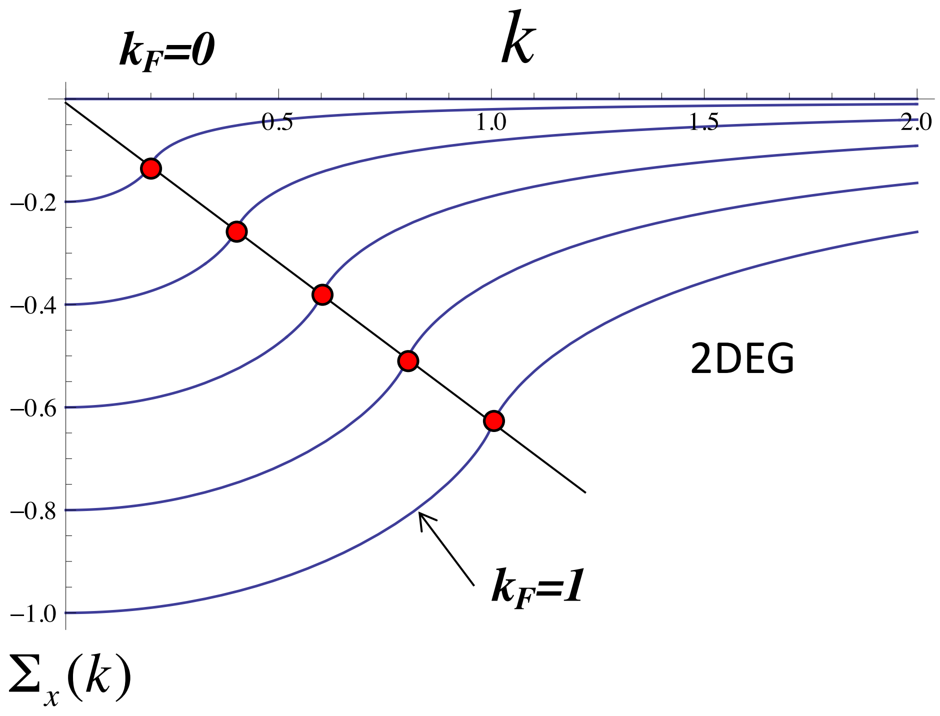
impossible!

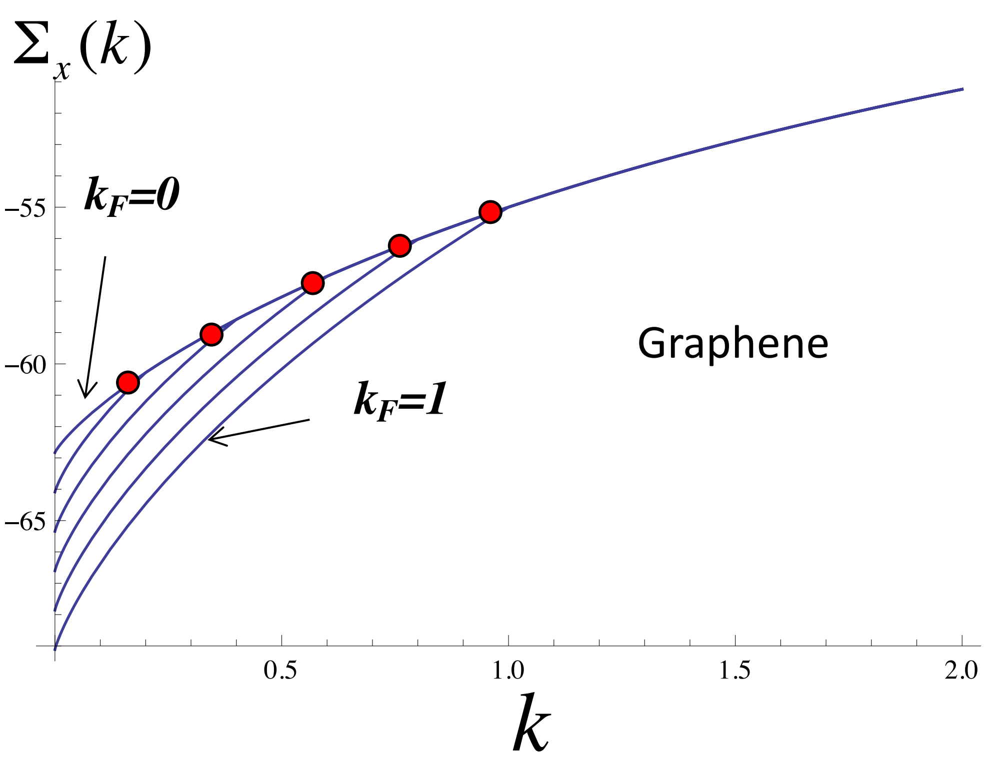
Thus, interband transitions do not contribute to the decay rate in a Fermi golden rule approximation (they can contribute, however, at higher order in the interactions)

19. Anomalous behaviors I

The Fermi liquid of massless Dirac fermions exhibits some remarkable anomalies.

The effect of interactions on the compressibility and spin susceptibility is the opposite of what one finds in the two-dimensional electron liquid: these responses are reduced by interactions rather than enhanced. This is due to the interaction of electrons at the Fermi level with "spectator electrons" in the occupied bands and can be understood by examining the behavior of the exchange self-energy.

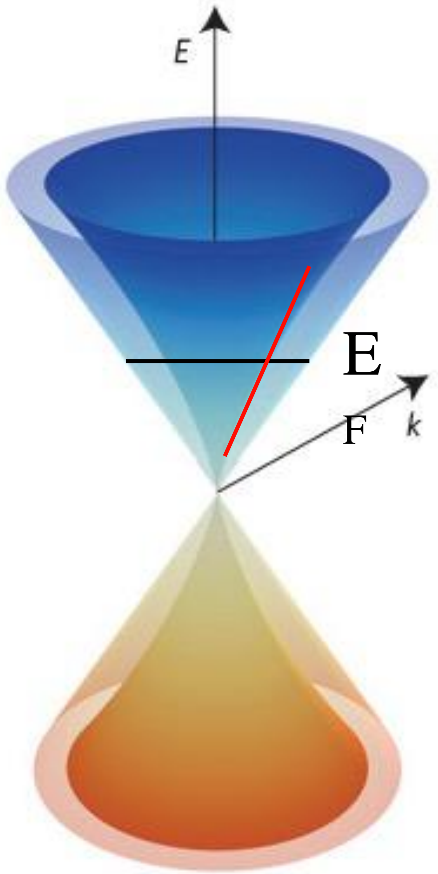


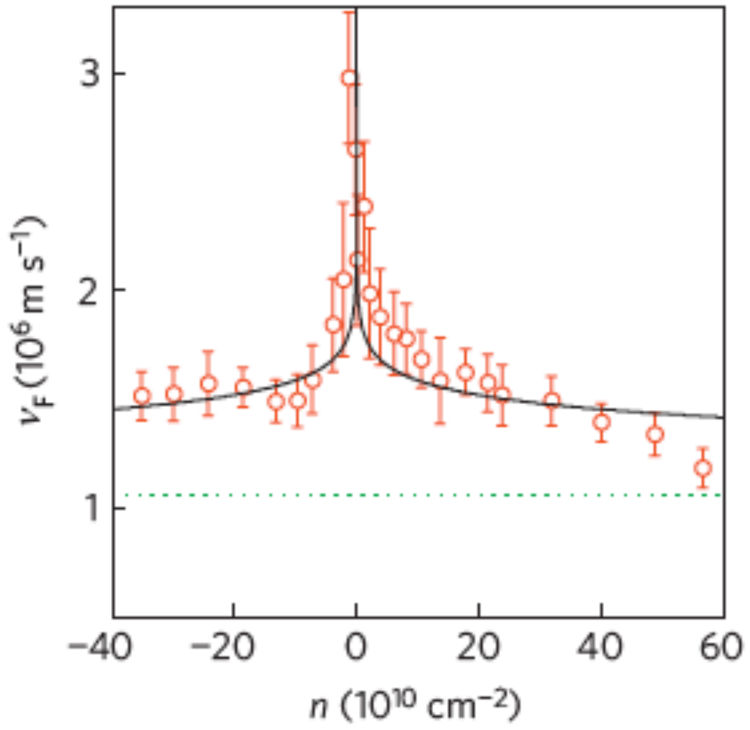


20. Anomalous behaviors II

The Fermi velocity diverges logarithmically as k_F tends to 0.

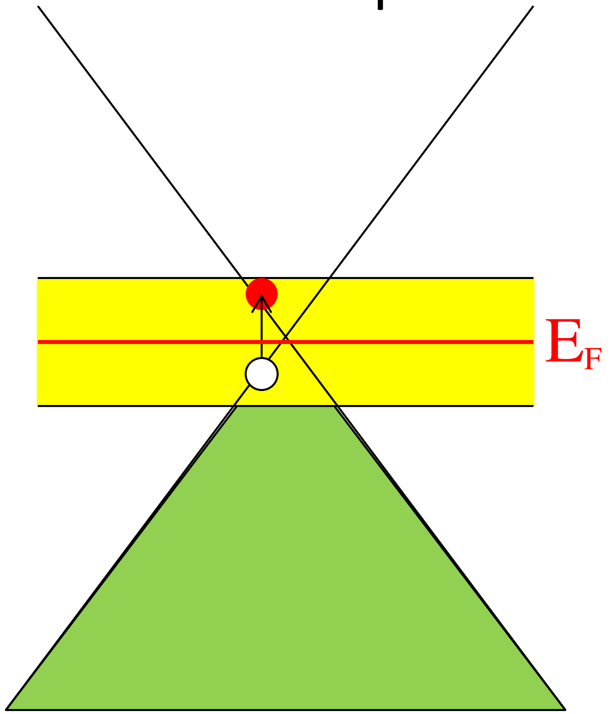
$$v_F = \left.\frac{d\varepsilon_k}{dk}\right|_{k=k_F} = v\left(1 + \frac{\alpha}{4}\ln\frac{\Lambda}{k_F}\right) \qquad \alpha = \frac{e^2}{\varepsilon\hbar v}$$





21. The breakdown of Fermi liquid theory at $\varepsilon_F, k_F=0$

Dirac Liquid



Low-energy excitations are electrons and holes (possibly bound in pairs)

Their decay rate of electrons and holes, turns out to be [Trushin, PRB 94,205306, (2016)]

$$\frac{1}{\tau} \simeq \frac{\alpha^2 k_B T}{\hbar \ln \alpha}$$

This is known as "Planckian" regime, and in this regime the liquid is not a Fermi liquid anymore (because the energy uncertainty is comparable to the excitation energy).

While the viscosity of the Fermi liquid is very high, the viscosity of the Dirac liquid is very low, closed to a conjectured lower bound [Müller et al., PRL 103, 025301 (2009)] :

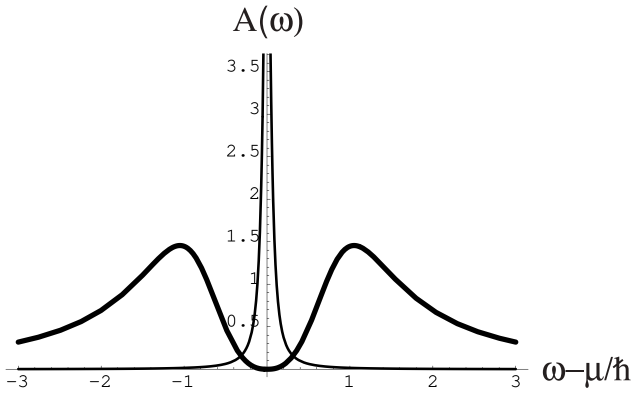
$$\eta \simeq 0.449 \frac{(k_B T)^2}{\hbar v^2 \alpha^2}$$

22. Non-Fermi liquid behavior

Deviations from standard Fermi liquid behavior are observed in a variety of situations.

(i) Disordered electronic systems : Scattering rates of quasiparticles scale with non-standard exponents, such as $T^{3/2}$ in 3D and T ln T in 2D. In the latter case the electron liquid is found to be a marginal Fermi liquid.

(ii) One-dimensional liquids. These systems are known as Luttinger liquids because the fermionic quasiparticles do not exist (no pole in the Green's function). The low-energy excitations are bosons.



Schematic behavior of the spectral function $A(k_F,\omega)$ (at T=0) for a Luttinger liquid in the weak coupling regime (thin line) and in the strong coupling regime (thick line). Notice the absence of the quasiparticle δ-function peak at $\omega = \mu$.

(iii) Two-dimensional quantum liquids, in particular the quantum Hall liquid in the two-dimensional electron gas at high magnetic field.