Correlations and Phase Transitions

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Lecture Notes of the Autumn School on

Correlated Electrons 2024

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This is NOT the Meissner effect



Heit 44. 3. 11. 1933]



Ein neuer Effekt bei Eintritt der Supraleitfähigkeit.

Bringt zylindrischen Supraleiter, z. B. Blei oder Zinn, eines Sprungpunktes in ein senkrecht tetes homogenes Magnetfeld, so gehen zu seiner A die Kraftli der sehr geringen Suszeptibilität de schwach paramagnetisch, Blei diama-Supraleite gnetisch) fast ungehindert durch sie hindurch. Nach den bisherigen Anschauungen war zu erwarten, daß die Kraftlinienverteilung unverändert bleibt, wenn man die Temperatur, ohne an dem äußeren Magnetfeld etwas zu ändern, bis unter den Sprungpunkt erniedrigt. Unsere Versuche an Zinn und Blei haben im Gegensatz hierzu folgendes ergeben 1. Beim Unterschreiten des Sprungpunktes andert sich die Kraftlinienverteilung in der äußeren Umgebung der Supraleiter und wird nahezu so, wie es bei der Permeabilität des Suprao, also der diamagnetischen Suszeptibilität leiters zu erwarten wäre.



The Meissner effect seems to violate the laws of classical physics

- (1) Law of inertia
- (2) Faraday's law
- (3) Law of momentum conservation
- (4) Laws of thermodynamics



Correspondence Principle

How do superconductors manage to do this???

Easy, it's quantum mechanics at play



Meissner effect puzzle

How does the current overcome the Faraday field E_F (counter-emf) that wants to stop it?

 $\varepsilon = \oint \vec{E}_F \cdot \vec{d}\,\ell = -\frac{\partial}{\partial t} \int \vec{B} \cdot \vec{d}S$





How is the momentum of the supercurrent compensated?









For a cylinder of R=1cm, h=5cm, H~500 G, λ_L =500A: I~ 2,000 Amps P_I~ 1 mg moving at 1 mm/s K_I~ 2.7 erg

A simple answer (1911 to 1933):

As the system becomes normal, resistivity becomes non-zero. I decays by collision processes with phonons and/or impurities, its kinetic energy (K_I) is dissipated as Joule heat Q, its momentum (P_I) is transmitted in the collisions to the body as a whole, body starts to rotate.

Simple answer is WRONG.

Discovery of the Meissner effect (1933) suggested Joule heat is zero

Joule heat was measured to high accuracy, it is zero

Keesom (1934-1938)

closed. We call $r/T + \sigma$ the increase in entropy which occurs in passing the magnetic threshold curve at (H, T), σ being due to some possible irreversible process going on at that event.



Considering the numbers of the last column we conclude that a definite indication of irreversibility is not present. Comparing those numbers with the values of r (the differences do not reach 2%) seems to entitle us to admit that the process mentioned may be considered as reversible as far as thermodynamical consequences go, viz. that

Keesom (1934)

Till now we imagined that the surplus work served to deliver the Joule-heat developed by the persistent currents the metal getting resistance while passing to the non-supraconductive condition. As, however, the conception of J o u l e-heat can rather difficultly be reconciled with reversibility we think now that there must be going on another process that absorbs energy. So we admit that in passing the threshold value curve, what happens first is the penetrating of the magnetic field into the supraconductive material. By this process which is to be considered as reversible, the persistent currents are annihilated by induction. At this process an amount of energy is absorbed equal to twice the energy of the magnetic field that comes into existence. It is not clear for what purpose the surplus of absorbed energy serves. It is to be supposed that some reversible process is involved which probably is connected with the transition to the nonsupraconductive state. In this picture of what happens it is essential that the persistent currents have been annihilated before the material gets resistance, so that no J o u l e-heat is developed.

how does the supercurrent get 'annihilated' without Joule heat?











An Introduction to Magnetohydrodynamics



P.A. DAVIDSON

Figure 2.8 An example of Alfvén's theorem. Flow through a magnetic field causes the field lines to bow out. P. A. Davidson. Introduction to Magnetohydrodynamics

Outflows & Jets: Theory & Observations MHD theory

3) MHD equations, flux freezing



field, motions transverse to the field carry the field with them.



Integrate induction equation $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$, with Gauss' theorem $\int_{V} \nabla \cdot \mathbf{A} dV = \int_{S} \mathbf{A} \cdot \mathbf{dS}$, (S is a closed surface enclosing volume V) and with Stokes' theorem $\int_{C} \nabla \times \mathbf{A} \cdot \mathbf{dS} = \int_{C} \mathbf{A} \cdot \mathbf{dI}$, (**C** is a closed curve around the open surface **S**; $d\mathbf{S} = \hat{\mathbf{n}} dS$ with the outward unit normal $\hat{\mathbf{n}}$) (i) Since for all time $\nabla \cdot \mathbf{B} = 0 \implies 0 = \int_{V} \nabla \cdot \mathbf{B} dV = \int_{C} \mathbf{B} \cdot \mathbf{dS}, \quad \forall t, \text{ (closed surface S)}$ (ii) Time behaviour of the magnetic flux Φ through closed curve **C**, around an open surface **S**₁:

$$\Phi = \int_{S_1} \mathbf{B}(\mathbf{r}, t) \cdot \mathbf{dS}.$$

Now Φ changes in time since **B** = **B**(t) and since curve **C** changes in response to plasma motions. www.mpia-hd.mpg.de/homes/fendt/Lehre/Lecture_OUT/lect_jets4.pdf











The 'fluid' flowing out has to carry zero charge and zero mass

The 'fluid' flowing out is electrons + holes

Holes flowing out = mass flowing **N**

1988 / F	Theory of hole superconductivity
0 r	References: http://physics.ucsd.edu/~jorge/hole.html
A	Collaborators: <u>Frank Marsiglio;</u> S. Tang, X. Q. Hong, H.Q. Lin
	* Key to the cuprates is O ⁼ Negatively charged anions
Ē	* Key to Fe-As compounds is As ³⁻
s u	* Key to MgB ₂ is B- <u>Hole</u> conduction necessary
р е	* Superconductivity is driven by lowering of kinetic energy
r c	Models:
0	* Dynamic Hubbard models * Correlated hopping model
d	Correlated hopping model
u c	<u>Experimental support:</u> * I _c versus hole concentration * Tunneling asymmetry (theory 1989, eyp. 1995-2012)
t o	* Optical sum rule violation (theory 1992, exp. 1999-2012)
r 2024	* Explains dynamics of the Meissner effect- BCS can't



Fig. 5.1 The hole is denoted by the dotted circle. It moves from the second to the third place in the row, i.e. to the right. Equivalently, an electron moved to the left, from third to second place.

SUPERCONDUCTIVITY BEGINS WITH H

both properly understood, and misunderstood Superconductivity basics rethought





Fig. 5.1 The hole is denoted by the dotted circle. It moves from the second to the third place in the row, i.e. to the right. Equivalently, an electron moved to the left, from third to second place.

ENERGY **Electrons and Holes** O empty electron states or holes in Semiconductors filled electron states WITH APPLICATIONS TO TRANSISTOR ELECTRONICS By 5 Conduction band WILLIAM SHOCKLEY electrons Member of the Technical Staff Ec ----BELL TELEPHONE LABORATORIES, INC. Forbidden band holes E_v-Valence band k-space n

Zum Paulischen Ausschließungsprinzip

W. Heisenberg

DOI: 10.1002/andp.19314020710 Copyright © 1931 WILEY-VCH Verlag GmbH &

The conductivity in metals with a small number of holes can then in every respect be described as the conductivity in metals with a small number of positive conduction electrons. From this follows directly the anomalous Hall effect for such metals.

Elektronentheorie der Metalle.

Von R. PEIERLS, Zürich. (1932)

A band that has only a few electrons behaves, in every respect, like a band where there is only room left for a few electrons, with the difference, that the empty places have to be assigned an opposite – i.e. positive – charge. Since the conductivity is independent of the sign of the charge, the difference will not be apparent in the conductivity.



1931

Annalen der Physik Volume 402, Issue 7, 888–904, 1931







Electron-hole symmetry in solids

Holes

(Ashcroft and Mermin book)⁻

One of the most impressive achievements of the semiclassical model is its explanation for phenomena that free electron theory can account for only if the carriers have a positive charge. The most notable of these is the anomalous sign of the Hall coefficient in some metals (see page 58). There are three important points to grasp in understanding how the electrons in a band can contribute to currents in a manner suggestive of positively charged carriers:

1. Since electrons in a volume element $d\mathbf{k}$ about \mathbf{k} contribute $-e\mathbf{v}(\mathbf{k})d\mathbf{k}/4\pi^3$ to the current density, the contribution of all the electrons in a given band to the current density will be

$$\mathbf{j} = (-e) \int_{\text{occupied}} \frac{d\mathbf{k}}{4\pi^3} \mathbf{v}(\mathbf{k}), \qquad (12.19)$$

where the integral is over all occupied levels in the band.²³ By exploiting the fact that a completely filled band carries no current,

$$0 = \int_{\text{zone}} \frac{d\mathbf{k}}{4\pi^3} \mathbf{v}(\mathbf{k}) = \int_{\text{occupied}} \frac{d\mathbf{k}}{4\pi^3} \mathbf{v}(\mathbf{k}) + \int_{\text{unoccupied}} \frac{d\mathbf{k}}{4\pi^3} \mathbf{v}(\mathbf{k}), \qquad (12.20)$$

we can equally well write (12.19) in the form:

$$\mathbf{j} = (+e) \int_{\text{unoccupied}} \frac{d\mathbf{k}}{4\pi^3} \mathbf{v}(\mathbf{k}). \qquad (12.21)$$

Thus the current produced by occupying with electrons a specified set of levels is precisely the same as the current that would be produced if (a) the specified levels were unoccupied and (b) all other levels in the band were occupied but with particles of charge +e (opposite to the electronic charge).

Thus, even though the only charge carriers are electrons, we may, whenever it is convenient, consider the current to be carried entirely by fictitious particles of positive charge that fill all those levels in the band that are unoccupied by electrons.²⁴ The fictitious particles are called *holes*.



holes

R_H>0

E£₽

R_H=Hall coefficient

The simple physics that explains the Meissner effect: 1) Electrons flow <u>radially outward</u> (generate Meissner current) $\vec{F}_B = \frac{e}{c} \vec{v} \times \vec{B}$

Lorentz force



- The simple physics that explains the Meissner effect:
- 1) Electrons flow <u>radially outward</u> (generate Meissner current)

$$\vec{F}_B = \frac{e}{c}\vec{v} \times \vec{B}$$

2) Backflow of normal electrons Lor transfers momentum to the body



Lorentz force



Explanation



Normal hole outflow Lorentz force canceled by Faraday force holes move <u>radially</u> out transfer azimuthal momentum to ions

without collisions!



What is the difference between electrons and holes? Hall effect



 $F_{Amp} = \bigoplus_{E_y}$ no net transverse force on electrons no other force on ions

 $F_{Amp} \neq \bigoplus \underbrace{F_{E}}$ no net transverse force on FE holes there is another force on ions

What is the difference between electrons and holes?Hall effect


Explanation



Normal antibonding electron backflow move radially in transfer azimuthal momentum to ions without collisions! angular momentum is conserved body rotation EF ion on-latt superfluid \dot{F} hole electron **I**_{Meissner}

What is the difference between electrons and holes?





==> electrons near the top of the band exert a large force ON the lattice ==> transfer momentum to the lattice





Negative Hall coefficient=electron carriers Positive Hall coefficient=hole carriers

н	, : superconductors;															He	
Li	Be				: nor	non-superconductors							с	N	0	F	Ne
Na	Mg	E : magnetic										AI	Si	Р	s	CI	Ar
ĸ	Са	Sc	• _{Ti}	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Тс	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Те	I	Xe
Cs	Ва	Lu	Hf	Та	w	Re	Os	Ir	Pt	Au	Hg	TI	Рь	Bi	Po	At	Rn
Fr	Ra	Ac	Th	Ра	U	Np	Pu	Am	Cm	Bk	Cf	E	Fm	Md	No	Lw	
Lanthanides (rare earths) La Ce					Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Но	Er	Tm	Yb	
	Transition Transition (only unde	element : element : r pressur	supercon supercon e)	ductors			Supera (only L Extrap	conducting inder pres	g sure)			[Rar	e earths a gnetic trar	nd transu	ranic eler ments	ments

JANUARY 9, 1932]

NATURE

Hall Effect and Superconductivity.

As can be seen from these data, the superconductors show usually a relatively small value of R and especially of $R\sigma$. I. KIKOIN.

I. KIKOIN. BORIS LASAREW.

Magnetic Department, Physical-Technical Institute, Sosnovka 2, Leningrad (21).

ON A POSSIBLE CRITERION FOR SUPERCONDUCTIVITY 1962

I. M. Chapnik

Novosibirsk State University (Presented by Academician I. K. Kikoin, June 1, 1961) Translated from Doklady Akademii Nauk SSSR, Vol. 141, No. 1, pp. 70-73, November, 1961

• Original article submitted February 1, 1961

Thus, we have as criteria for the appearance of superconductivity: the presence of a sufficient concentration of holes $N_D > 10^{19} \text{ cm}^{-3}$, a restriction on the min-

1932

ON THE EMPIRICAL CORRELATION BETWEEN THE SUPERCONDUCTING $T_{\rm c}$ AND THE HALL COEFFICIENT

(1979)

I.M. CHAPNIK Cavendish Laboratory, Cambridge CB3 OHE, England

favorable condition for superconductivity would apparently be a positive value of the Hall coefficient.

R. Feynman, Rev.Mod.Phys.29, 205 (1957)

There may be small regions in momentum space, for instance, where the electrons behave as positively charged particles, that is, places where the conductivity is by holes and other regions where they behave normally. There is some indication that this is the case because it has been noticed that the Hall effect is very small when the material has a tendency to be superconductive. The Hall effect is very small when the positive and negative carriers cancel. Thus some people think that this, in conjunction with the lattice vibrations, may have something to do with superconductivity. Of course, that makes the problem more complicated, because it would mean that if Frohlich and Bardeen could solve their model exactly, they still would not find superconductivity, since it would still involve only negative carriers.

(1948)

Theory of Superconductivity

On the other hand, we have found an empirical rule that indicates a correlation between superconductivity and lattice structure; namely, that those metals are superconductive for which the Fermi surface, supposed to be a sphere, lies in very close proximity to one set of the corners formed by the boundary planes of a Brillouin zone.

This table encourages the view that a correct theory of superconductivity should be based on the ionic forces. It is known that these ionic forces have predominant influence on the behaviour of the electrons, if the Fermi surface is close to the boundary planes of the Brillouin zone.

June 19, 1948

KAI CHIA CHENG

MAX BORN



Why hole carriers give rise to superconductivity and electron carriers do not



Orbital expansion upon double-occupancy



Orbital expansion upon double-occupancy







Generalized Hubbard model

$$H = -\sum_{ij\sigma} t_{ij} [c_{i\sigma}^{+} c_{j\sigma} + hc.] + \sum (ij |1/r|kl) c_{i\sigma}^{+} c_{j\sigma}^{+} c_{l\sigma} c_{k\sigma}$$

$$H = -\sum_{ij\sigma} t_{ij} [c_{i\sigma}^{+} c_{j\sigma} + hc.] + U \sum_{i} n_{i\uparrow} n_{i\downarrow} + V \sum_{\langle ij \rangle} n_{i} n_{j} \left(+\Delta t \sum_{\langle ij \rangle} (n_{i,-\sigma} + n_{j,-\sigma}) (c_{i\sigma}^{+} c_{j\sigma} + h.c.) \right)$$

$$\Delta t = (ii |1/r|ij) = \int |\varphi_{i}(r)|^{2} \frac{e^{2}}{|r-r'|} \varphi_{i}(r') \varphi_{j}(r') \quad \text{correlated hopping}$$

$$n_{i,-\sigma} c_{i\sigma}^{+} c_{j\sigma} \text{ is } \begin{cases} < 0 & + \frac{1}{|r-r'|} \varphi_{i}(r') \varphi_{j}(r') \\ > 0 & + \frac{1}{|r-r'|} \varphi_{i}(r') \varphi_{j}(r') \end{cases} \quad \text{correlated hopping}$$

* Δt breaks electron hole symmetry

* Δt lowers the kinetic energy of hole pairs



Hamiltonian is, in hole representation

1

$$H = \sum_{k\sigma} (\varepsilon_{k} - \mu) c_{k\sigma}^{\dagger} c_{k\sigma} + \frac{1}{N} \sum_{kk'q} (V(q) + \alpha(\varepsilon_{k} + \varepsilon_{k+q} + \varepsilon_{k'} + \varepsilon_{k'-q})) c_{k+q\uparrow}^{\dagger} c_{k'\downarrow}^{\dagger} c_{k\downarrow} c_{k\uparrow} c_{k\uparrow} c_{k\uparrow} (59a)$$

$$V(q) = \sum_{j} e^{iqR_{j}} V_{0j} \qquad V_{00} = U, \text{ and } \alpha = \Delta t_{ij} / \bar{t}_{ij}.$$

$$V_{01} = V$$

$$H_{red} = \sum_{k\sigma} (\varepsilon_{k} - \mu) c_{k\sigma}^{\dagger} c_{k\sigma} + \sum_{kk'} V_{kk'} c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger} c_{-k'\downarrow} c_{k'\uparrow} \qquad D = 2z\bar{t},$$

$$K = 2z\Delta t$$

$$V_{kk'} = V(\varepsilon_{k}, \varepsilon_{k'}) = U + \frac{K}{D/2} (\varepsilon_{k} + \varepsilon_{k'}) + \frac{W}{D/2} \varepsilon_{k} \varepsilon_{k'} \qquad W = zV.$$

$$\Delta_{k} = -\frac{1}{N} \sum_{k'} V(\varepsilon_{k}, \varepsilon_{k'}) \Delta_{k'} \frac{\tanh(\beta E_{k'}/2)}{2E_{k'}} \qquad \Delta(\varepsilon_{k}) = \Delta_{m} \left(-\frac{\varepsilon_{k}}{D/2} + c\right)$$

$$I = K(I_{1} + cI_{0}) - W(I_{2} + cI_{1})$$

$$c = K(I_{2} + cI_{1}) - U(I_{1} + cI_{0})$$

$$I_{\ell} = \frac{1}{N} \sum_{k} \left(-\frac{\varepsilon_{k}}{D/2}\right)^{\ell} \frac{\tanh(\beta E_{k'}/2)}{2E_{k'}}$$

$$V_{kk} = U - \frac{2\Delta t}{t} (\varepsilon_k + \varepsilon_{k'})$$



Is repulsive/attractive for $\varepsilon_k < 0$ (electrons) / $\varepsilon_k > 0$ (holes)



 $1 = 2KI_1 - WI_2 - UI_0 + (K^2 - WU)(I_0I_2 - I_1^2)$



$$1 = 2KI_1 - WI_2 - UI_0 + (K^2 - WU)(I_0I_2 - I_1^2)$$

$$k > \sqrt{(1+u)(1+w)} - 1$$

$$k = K/D \qquad D = 2z\bar{t},$$

$$u = U/D \qquad K = 2z\Delta t$$

$$w = W/D \qquad W = zV.$$

$$T_{c} = (e^{\gamma}/\pi)\sqrt{n(2-n)} De^{-a/b},$$

$$a = 1 + 2k(1-n) - w(1-3n+\frac{3}{2}n^{2})$$

$$+ (k^{2} - wu)(1-n)^{2},$$

$$b = 2k(1-n) - w(1-n)^{2} - u$$

$$+ (k^{2} - wu)(1-n+\frac{1}{2}n^{2}),$$

$$E_{k} = \sqrt{(\varepsilon_{k}-\mu)^{2} + \Delta_{k}^{2}}.$$

$$E_{k} = \sqrt{a^{2}(\varepsilon_{k}-\mu-\nu)^{2} + \Delta_{0}^{2}}$$

$$a = \sqrt{1 + (\frac{\Delta_{m}}{D/2})^{2}}, \quad \Delta_{0} = \frac{\Delta(\mu)}{a} \quad \text{and} \quad \nu = \frac{1}{a} \frac{\Delta_{m}}{D/2} \Delta_{0}$$

Effective low energy Hamiltonian: Hubbard model with correlated hopping

$$H_{eff} \cong -\sum_{ij\sigma} [t_h + \Delta t(\tilde{n}_{i,-\sigma} + \tilde{n}_{j,-\sigma})] [\tilde{c}_{i\sigma}^+ \tilde{c}_{j\sigma} + hc.] + U \sum_i \tilde{n}_{i\uparrow} \tilde{n}_{i\downarrow}$$

Leads to, when a band is almost full:

* pairing and superconductivity
 * negative charge expulsion from interior to surface

driven by kinetic energy lowering

Negative charge expulsion in dynamic Hubbard model

FIG. 3. Hole site occupation per spin for a cylinder of radius R = 11 as a function of r/R, with r the distance to the center, for a cubic lattice of side length 1. There are 377 sites in a cross-sectional area ($\pi R^2 = 380.1$). The average occupation (both spins) is n = 0.126 hole per site and the temperature is $k_B T = 0.02$.

 $t(n_h) = t_h + n_h \Delta t$

PRB 87, 184506

FIG. 4. Diameters of the circles are proportional to the hole occupation at the site. Note that for finite Δt the hole occupation increases in the interior and is depleted near the surface. Parameters correspond to the cases shown in Fig. 3.

FIG. 5. Kinetic, potential, and total energy per site for $\Delta t = 0.25$ as a function of the number of iterations starting with a uniform hole distribution.

Negative charge expulsion in dynamic Hubbard model

Conventional Hubbard model electrons move out **Dynamic Hubbard model** $\Delta t=0.25$ $\Delta t = 0$ FIG. 4. Diameters of the circles are proportional to the hole Negative charge expulsion occupation at the site. Note that for finite Δt the hole occupation increases in the interior and is depleted near the surface. Parameters **Kinetic energy lowering** correspond to the cases shown in Fig. 3. band almost full band *→*many electrons, high kin.energy negative ions \rightarrow a lot of negative charge **>** system expels electrons

macroscopic charge inhomogeneity

An outward-pointing electric field exists in the interior of superconductors at zero temperature

PRB 87, 184506

(2013)

The Electromagnetic Equations of the Supraconductor (1935)

SUPRALEITUNG UND DIAMAGNETISMUS (1935)

von F. und H. LONDON

diese noch unbekannte Koppelung eine Verfestigung der Elektronenwellenfunktion in erster Näherung auch gegen elektrische Störungen zustande bringt ¹). Dann würde man zu einer Gleichung

$$\rho$$
=charge density $\rho = -\frac{e^2n}{mc^2} \varphi \phi$ =electric potential(12b)

An Experimental Examination of the Electrostatic Behaviour of Supraconductors (1936)

By H. LONDON, Clarendon Laboratory, Oxford It follows from these measurements that no electrostatic fields exist in a pure supraconductor, not even in a thin surface layer, at least to the approximation to which this is true for normal conductors. Accordingly

Derivation of conventional London equation:

New London-like equations for superconductors (JEH, PRB69, 214515(2004)

1)
$$J = -\frac{ne^2}{mc}A = -\frac{c}{4\pi\lambda_L^2}A \quad ; \quad \frac{1}{\lambda_L^2} = \frac{4\pi ne^2}{mc^2}$$

2)
$$\nabla \cdot A + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0 \quad ; \quad \text{(Lorenz gauge)}$$

$$\nabla \cdot J = -\frac{c}{c}\nabla \cdot A \quad \text{continuity equation: } \nabla \cdot I + \frac{\partial\rho}{\partial t} = 0$$

$$\nabla \cdot J = -\frac{c}{4\pi\lambda_L^2} \nabla \cdot A \quad \text{, continuity equation: } \nabla \cdot J + \frac{\partial \rho}{\partial t} = 0 \implies$$

$$\frac{\partial \rho}{\partial t} = -\frac{1}{4\pi\lambda_L^2}\frac{\partial \phi}{\partial t}$$

integrate in time, 1 integration constant
$$\rho_0$$
, ...

$$\implies \rho(r,t) - \rho_0 = -\frac{1}{4\pi\lambda_L^2} [\phi(r,t) - \phi_0(r)] \qquad \phi_0(r) = \int d^3r' \frac{\rho_0}{|r-r'|}$$

Electrostatics:

(JEH, PRB69, 214515(2004)

$$\nabla^{2}(\phi(r) - \phi_{0}(r)) = \frac{1}{\lambda_{L}^{2}}(\phi(r) - \phi_{0}(r)) \qquad \nabla^{2}(\rho(r) - \rho_{0}) = \frac{1}{\lambda_{L}^{2}}(\rho(r) - \rho_{0})$$

$$\nabla^{2}(\phi(r) - \rho_{0}) = \frac{1}{\lambda_{L}^{2}}(\rho(r) - \rho_{0})$$

$$\nabla^{2}(\vec{r} - \vec{r}_{0}) = \frac{1}{\lambda_{L}^{2}}(\vec{r} - \vec{r}_{0})$$

 $\nabla^2 \phi(r) = 0$ outside supercond.

+assume $\phi(\mathbf{r})$ and its normal derivative are continuous at surface

Solution for sphere of radius R:

$$\rho(\mathbf{r}) = \rho_0 \left(1 - \frac{R^3}{3\lambda_L^2} \frac{\sinh(r/\lambda_L)}{R/\lambda_L \cosh(R/\lambda_L) - \sinh(R/\lambda_L)}\right)$$
$$\vec{E}(\mathbf{r}) = \frac{4}{3}\pi \rho_0 \left[1 - \frac{R^3}{r^3} \frac{r/\lambda_L \cosh(r/\lambda_L) - \sinh(r/\lambda_L)}{R/\lambda_L \cosh(R/\lambda_L) - \sinh(R/\lambda_L)}\right]\vec{r}$$

No electric field outside sphere

Elliptical shape

test experimentally by measuring electric fields in the neighborhood of superconducting small particles

Electrostatics:

$$\nabla^{2}(\phi(r) - \phi_{0}(r)) = \frac{1}{\lambda_{L}^{2}}(\phi(r) - \phi_{0}(r)) \quad \nabla^{2}(\rho(r) - \rho_{0}) = \frac{1}{\lambda_{L}^{2}}(\rho(r) - \rho_{0})$$

$$\nabla^2 \phi(r) = -4\pi\rho(r) \quad \nabla^2 \phi_0(r) = -4\pi\rho_0$$

$$\nabla^2 (\vec{E} - \vec{E}_0) = \frac{1}{\lambda_L^2} (\vec{E} - \vec{E}_0)$$

electric screening length is $\lambda_{\rm L}$

Diamagnetic susceptibility

angular momentum of supercurrent:

Electron-doped cuprates

letters to nature

Nature 337, 345 - 347 (26 January 1989); doi:10.1038/337345a0

A superconducting copper oxide compound with electrons as the charge carriers

VOLUME 73, NUMBER 9

Anomalous Transport Properties in Superconducting Nd_{1.85}Ce_{0.15}CuO_{4±δ}

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We report a comprehensive study of the in-plane transport properties of $Nd_{2-x}Ce_xCuO_{4\pm\delta}$ epitaxial thin films and crystals by both increasing and decreasing δ with Ce content fixed at $x \approx 0.15$. We find a remarkable correlation between the appearance of superconductivity and (1) a positive magnetoresistance in the normal state, (2) a positive contribution to the otherwise negative Hall coefficient, and (3) an anomalously large Nernst effect. These results strongly suggest that both holes and electrons participate in the charge transport for the superconducting phase of $Nd_{2-x}Ce_xCuO_{4\pm\delta}$.

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Hole superconductivity in the electron-doped superconductor Pr_{2-x}Ce_xCuO₄

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We measure the resistivity and Hall angle of the electron-doped superconductor $Pr_{2-x}Ce_xCuO_4$ as a function of doping and temperature. The resistivity ρ_{xx} at temperatures 100 K < T < 300 K is mostly sensitive to the electrons. Its temperature behavior is doping independent over a wide doping range and even for nonsuperconducting samples. On the other hand, the transverse resistivity ρ_{xy} , or the Hall angle θ_H , where $\cot(\theta_H)$ $=\rho_{xx}/\rho_{xy}$, is sensitive to both holes and electrons. Its temperature dependence is strongly influenced by doping, and $\cot(\theta_H)$ can be used to identify optimum doping (the maximum T_c) even well above the critical temperature. These results lead to a conclusion that in electron doped cuprates holes are responsible for the superconductivity. 1 Feb 2019 · Vol 5, Issue 2 · DOI: 10.1126/sciadv.aap7349

CONDENSED MATTER PHYSICS

Hole pocket-driven superconductivity and its universal features in the electron-doped cuprates

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After three decades of intensive research attention, the emergence of superconductivity in cuprates remains an unsolved puzzle. One major challenge has been to arrive at a satisfactory understanding of the unusual metallic "normal state" from which the superconducting state emerges upon cooling. A second challenge has been to achieve a unified understanding of hole- and electron-doped compounds. Here, we report detailed magnetoresistance measurements for the archetypal electron-doped cuprate $Nd_{2-x}Ce_xCuO_{4+\delta}$ that, in combination with previous data, provide crucial links between the normal and superconducting states and between the electron- and hole-doped parts of the phase diagram. The characteristics of the normal state (magnetoresistance, quantum oscillations, and Hall coefficient) and those of the superconducting state (superfluid density and upper critical field) consistently indicate two-band (electron and hole) features and point to hole pocket–driven superconductivity in these nominally electron-doped materials. We show that the approximate Uemura scaling between the superconducting transition temperature and the superfluid density found for hole-doped cuprates also holds for the small hole component of the superfluid density in electron-doped cuprates.

Isotope effect: $T_c \sim M^{-\alpha}$





To the same natural effects we must, as far as possible, assign the same causes.

Summary

- * The Meissner effect is the most fundamental property of solution.
- * Expulsion of magnetic field requires radial flow of charge
- * Only hole carriers can transfer momentum to by Without dissipati
- * Hole carriers pair through kinetic energy lowering, electrons don't
- * Superconductors are like "giant atoms"
- * Superconductors have macroscopy zero-point motion



* Superfluid electrons where orbital angular momentum $L = \hbar/2$

* The electron-phonon interaction is irrelevant to superconductivity

* It is impossible for BCS theory to explain the Meissner effect! * Alo mains to be understood!

The Bohr superconductor

The Dirac superconductor

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PACS 74.20.-z – Theories and models of superconducting state

PACS 74.20.Mn - Nonconventional mechanisms

PACS 74.25.N- - Response to electromagnetic fields

Abstract – Superconductors have often been described as "giant atoms". The simplest description of atoms that heralded their quantum understanding was proposed by Bohr in 1913. The Bohr atom starts from some simple assumptions and deduces that the angular momentum of the electron in Bohr orbits is quantized in integer units of \hbar . This remarkable result, which does not appear to be implicit in the assumptions of the model, can be interpreted as a "theoretical proof" of the model's validity to describe physical reality at some level. Similarly we point out here that from some simple assumptions it can be deduced that electrons in superconductors reside in mesoscopic orbits with orbital angular momentum $\hbar/2$. This implies that both in superconductors and in ferromagnets the long-range order results from elementary units of identical angular momentum. Similarly to the case of the Bohr atom we propose that this remarkable result is compelling evidence that this physics, which is not part of conventional BCS theory, describes physical reality at some level and heralds a qualitatively new understanding of superconductors.