Unconventional superconducting pairing: Theoretical mechanisms and spectroscopic probes

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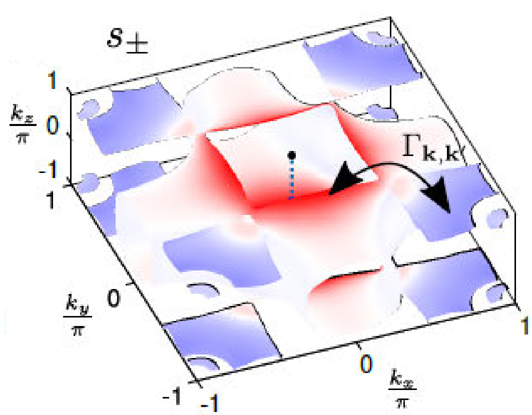
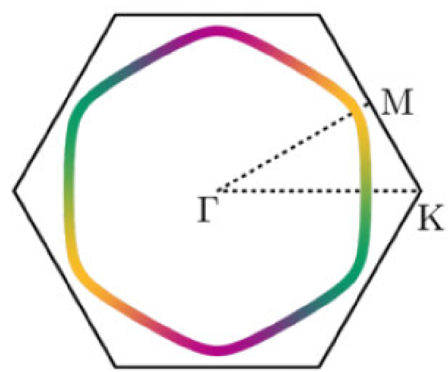
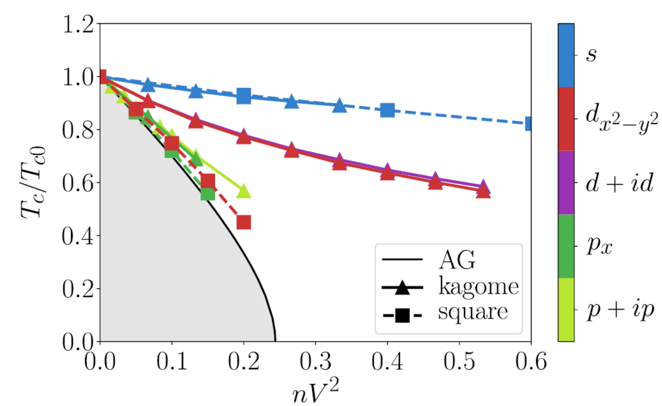






Outline

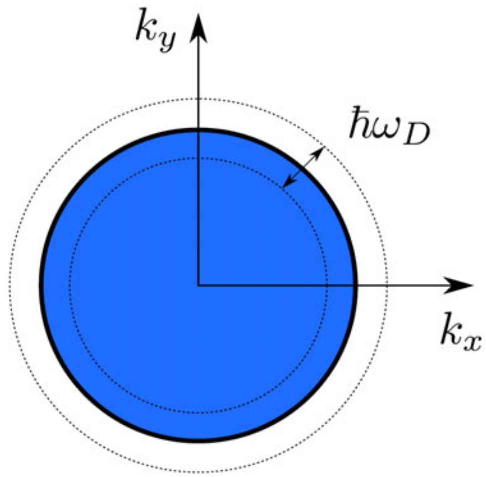
- Unconventional superconductivity
  - What is it?
  - Some material candidates
- Superconducting pairing: mean-field theory
- Spin fluctuation pairing mechanism
- (selected) spectroscopic probes
  - Quasiparticle tunneling
  - Neutron scattering: resonance
  - Spin relaxation rate: Hebel-Slichter peak

Introduction

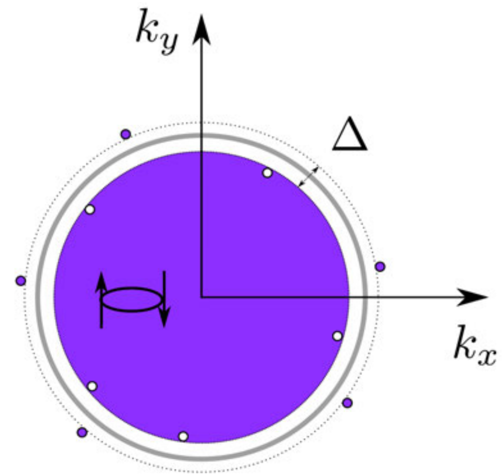
- Superconductivity (main phenomenology)
  - Zero resistance
  - Persistent currents in rings
  - Perfect diamagnetism
  - Energy gap opens in the SC state

Superconductivity

- Instability of Fermi sea due to interactions



T>T_c: normal metal, filled Fermi sea

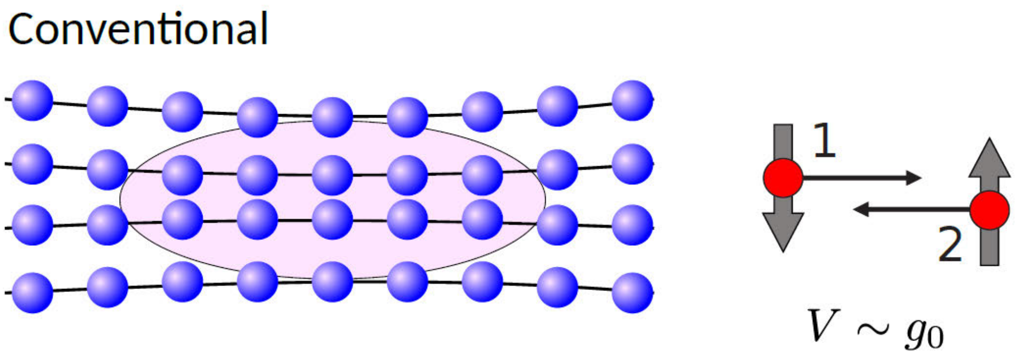


T<T_c gas of Cooper pairs: zero resistivity, Meissner effect, excitation gap

- Order parameter $\Delta_{\mathbf{k}} \sim \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$ generically complex-valued

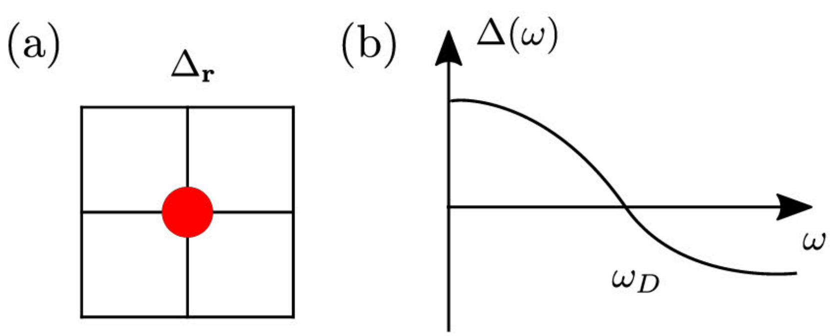
Conventional superconductivity

- Origin of pairing interaction: Phonons



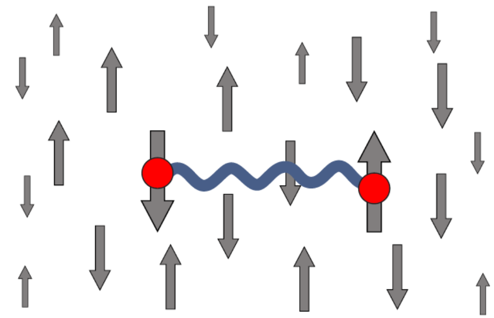
Retarted interaction: electrons avoid repulsive part of Coulomb interaction in time:
Effective attractive interaction in energy shell around E_F

- Order parameter (local)



Unconventional superconductivity

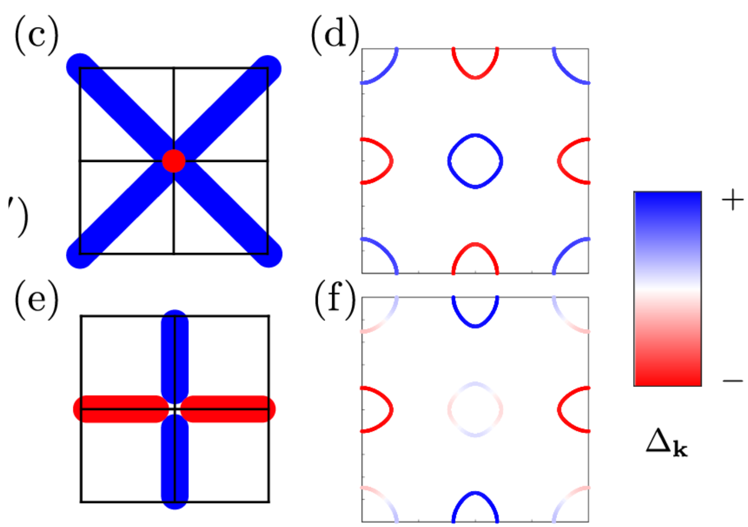
- Pairing interaction momentum dependent



$$V \sim V(\mathbf{k}-\mathbf{k}')$$

Electrons avoid repulsive part of Coulomb interaction in space rather than time! (Kohn-Luttinger, 1965)

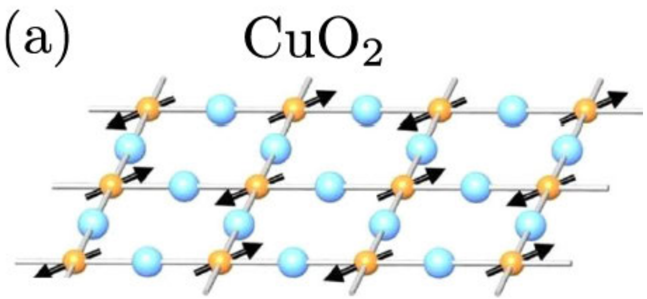
- Order parameter non-local (momentum-dependent)



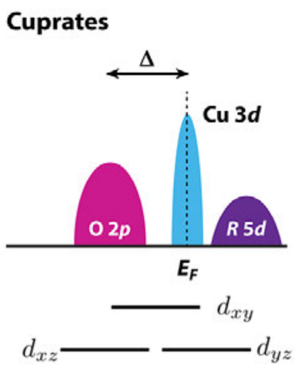
Possible materials (1)

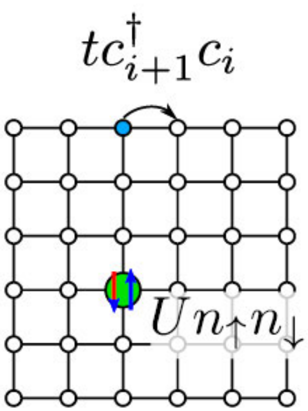
Michael R. Norman Physics **13**, 85 (2020)

- Cuprates

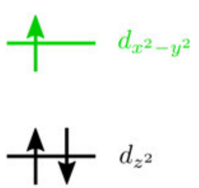


Electronic structure at low energy: single Cu-d orbital (hybridized with O-p orbitals)





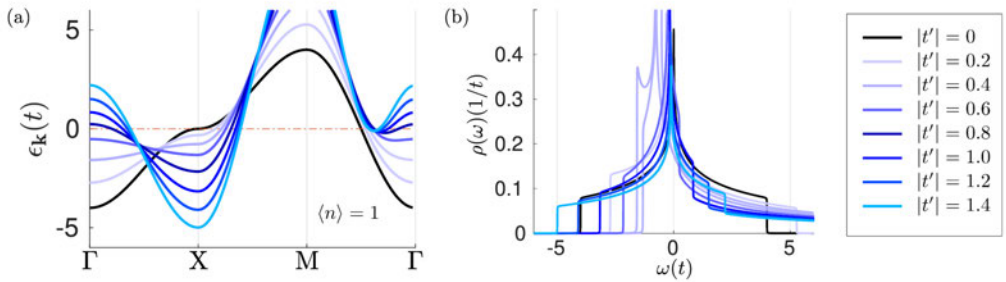
Sketch of model often discussed to represent phenomenology



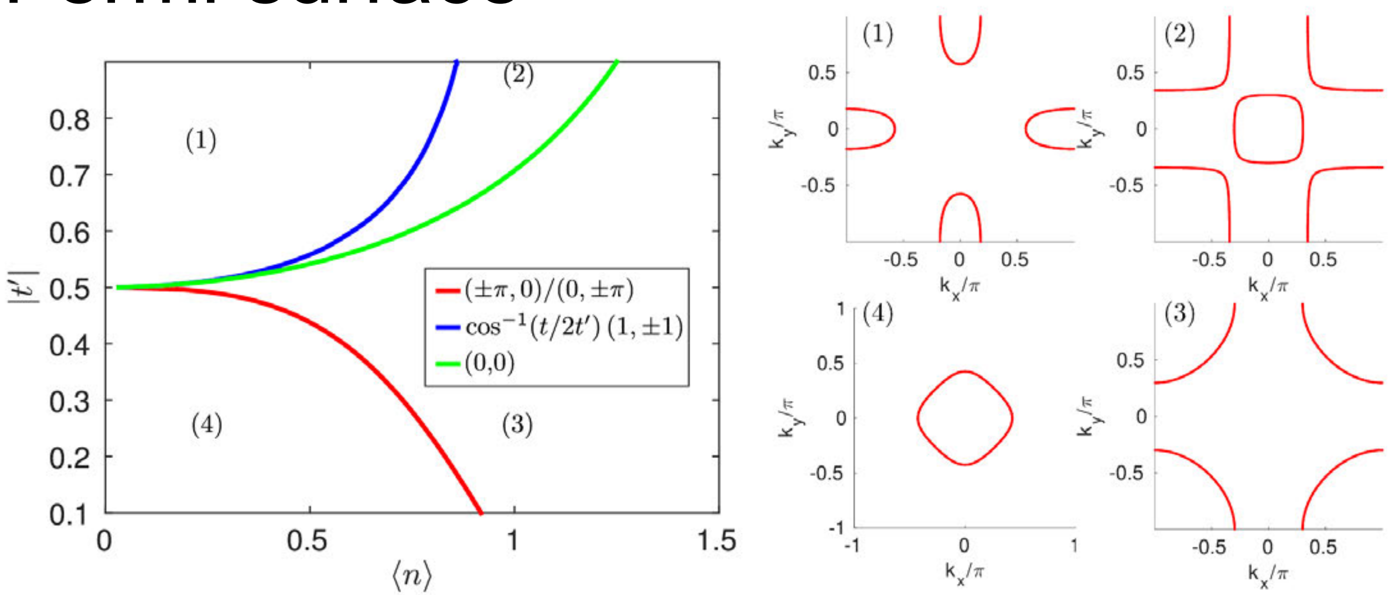
Single band model

- Band structure/density of states

$$\epsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y - \mu$$

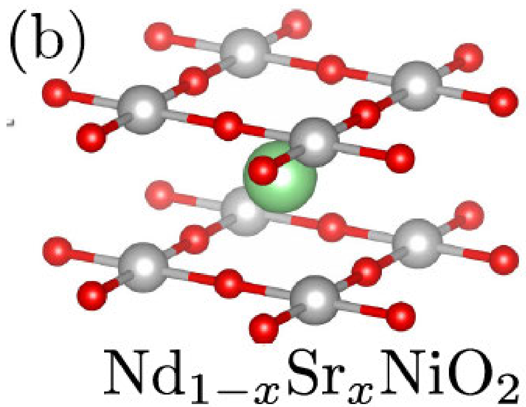


- Fermi surface

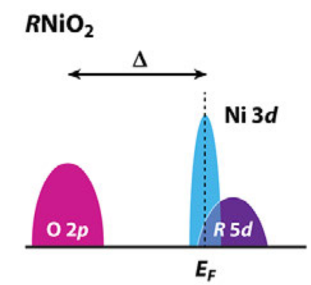


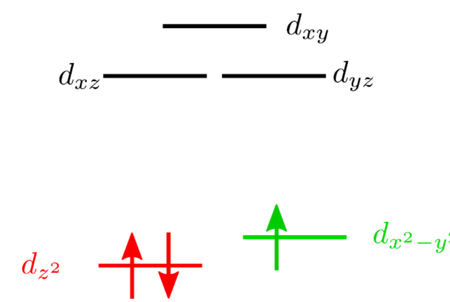
Possible materials (2)

- Nickelates

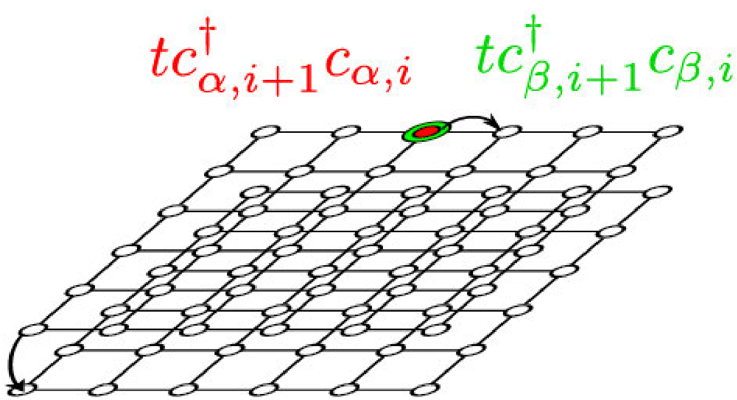


Electronic structure at low energy: two Ni d orbitals



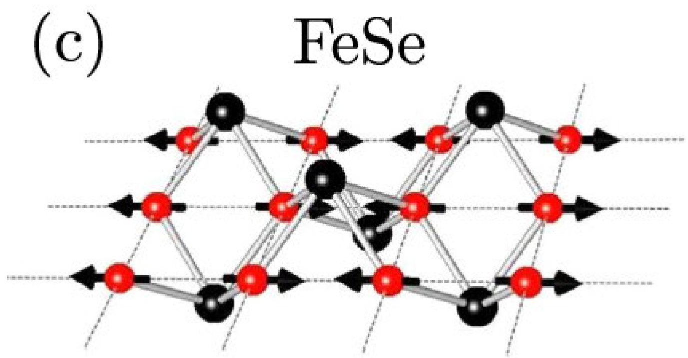


Multiband model?!

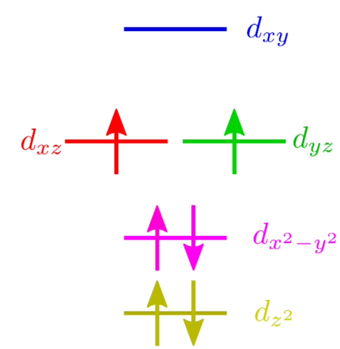


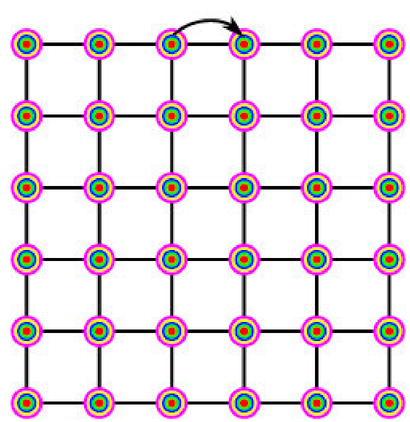
Possible materials (3)

- Fe-based superconductors



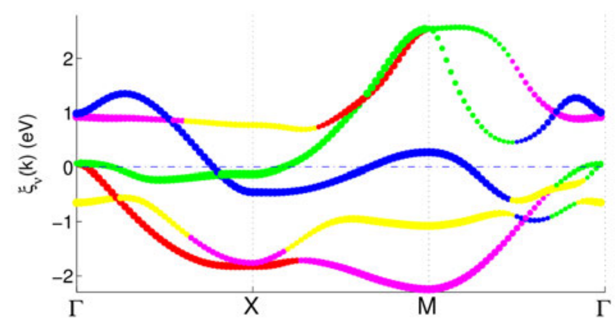
Electronic structure at low energy: all 5 Fe-d orbitals relevant





$$U n_\uparrow n_\downarrow + U' \ldots + \ldots$$

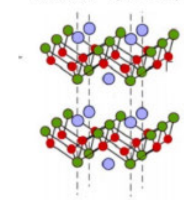
Multiband model with Hubbard-Kanamouri interaction term

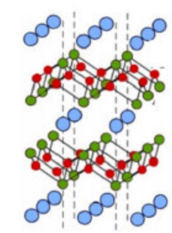


Chi, et al., PRB **94**, 134515 (2016) [LiFeAs]

1111, 111, 122, 1122...

LiFeAs

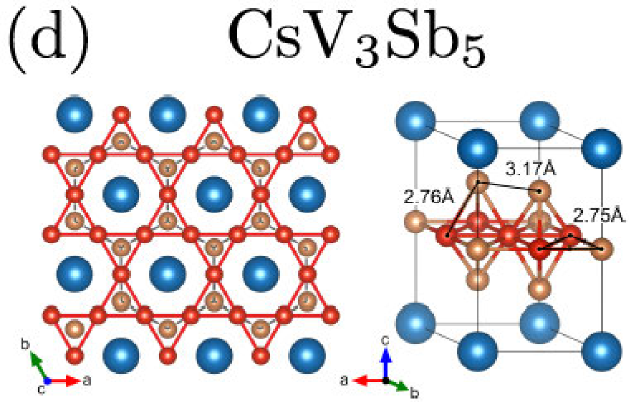




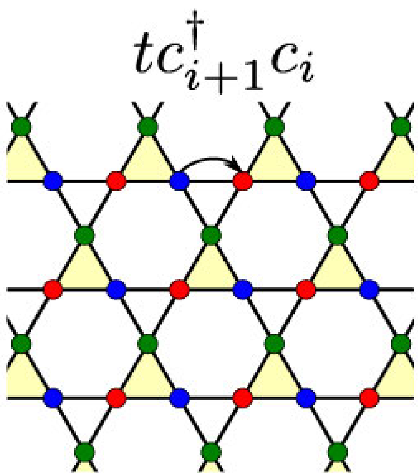
$K_xFe_2Se_2$

Possible materials (4)

- Kagome superconductors



Simplistic electronic structure: single orbital model: sublattice structure



3 band model
Interactions: Onsite and nearest neighbor Coulomb interactions

Mean field theory

- Generic Hamiltonian including interaction term

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}'\uparrow} c_{\mathbf{k}\downarrow} + \frac{1}{2N} \sum_{\mathbf{k},\mathbf{k}'} [V(\mathbf{k},\mathbf{k}') c^{\dagger}_{\mathbf{k}'\uparrow} c^{\dagger}_{-\mathbf{k}'\downarrow} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + \mathrm{H.c.}]$$

- Mean field theory

$$H = AB$$

$$H \to H_{MF} = \langle A \rangle B + A \langle B \rangle - \langle A \rangle \langle B \rangle$$

- Here: superconducting channel

$$A = c^{\dagger}_{\mathbf{k}'\uparrow} c^{\dagger}_{-\mathbf{k}'\downarrow} \qquad B = c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}$$

$$H_{MF} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} V(\mathbf{k},\mathbf{k}') \Big[\langle c^{\dagger}_{\mathbf{k}'\uparrow} c^{\dagger}_{-\mathbf{k}'\downarrow} \rangle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + c^{\dagger}_{\mathbf{k}'\uparrow} c^{\dagger}_{-\mathbf{k}'\downarrow} \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$$

$$- \langle c^{\dagger}_{\mathbf{k}'\uparrow} c^{\dagger}_{-\mathbf{k}'\downarrow} \rangle \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle + \mathrm{H.c.} \Big].$$

BCS theory

- Define $$\Delta_{\mathbf{k}}^{s/t} = -\frac{1}{N}\sum_{\mathbf{k}'} V^{s/t}(\mathbf{k},\mathbf{k}')\langle c^{\dagger}_{\mathbf{k}'\uparrow} c^{\dagger}_{-\mathbf{k}'\downarrow}\rangle$$ $$V^{s/t}(\mathbf{k},\mathbf{k}') = \frac{1}{2}[V(\mathbf{k},\mathbf{k}') \pm V(-\mathbf{k},\mathbf{k}')]$$

$$H_{MF} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \Delta_{\mathbf{k}}^{s/t*} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \mp \sum_{\mathbf{k}} \Delta_{\mathbf{k}}^{s/t} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} + \mathrm{const.}$$

- Bogoliubov transformation to quasiparticle operators

$$\begin{pmatrix} \gamma_{\mathbf{k}\uparrow} \\ \gamma^{\dagger}_{-\mathbf{k}\downarrow} \end{pmatrix} = \begin{pmatrix} u^*_{\mathbf{k}} & -v_{\mathbf{k}} \\ v^*_{\mathbf{k}} & u_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^{\dagger}_{-\mathbf{k}\downarrow} \end{pmatrix}$$

- Want: diagonal Hamiltonian

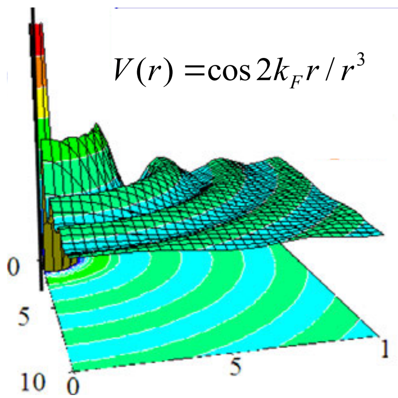
$$H_{BCS} = \sum_{\mathbf{k}\sigma} E_{\mathbf{k}} \gamma^{\dagger}_{\mathbf{k}\sigma} \gamma_{\mathbf{k}\sigma}$$ $$E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}^{s/t}|^2}.$$

- Self-consistency condition

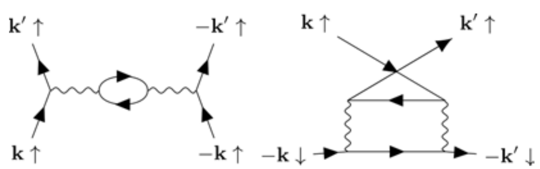
$$\Delta_{\mathbf{k}}^{s/t} = -\frac{1}{N}\sum_{\mathbf{k}'} V^{s/t}(\mathbf{k},\mathbf{k}') \frac{\Delta_{\mathbf{k}'}^{s/t}}{2E_{\mathbf{k}'}} \tanh\left(\frac{\beta E_{\mathbf{k}'}}{2}\right)$$

Unconventional superconductivity: Pairing from spin fluctuations

- Electrons avoid repulsive part of Coulomb interaction in space rather than time! (Kohn-Luttinger, 1965)



Friedel: Screened interaction

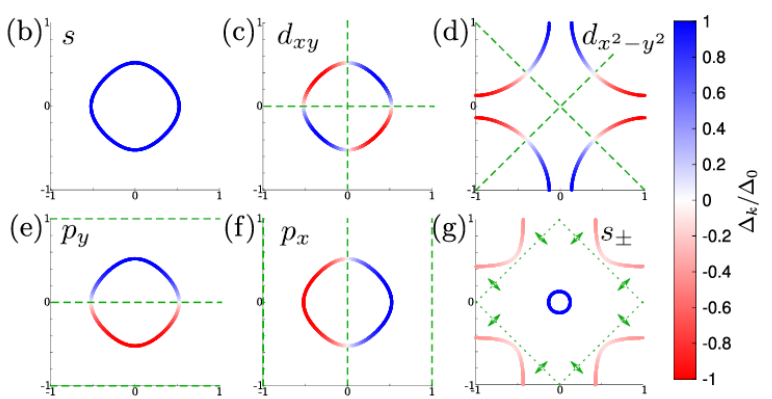


Momentum dependent interaction: $\Gamma(\mathbf{k}-\mathbf{k}') > 0$

- channels $$\Delta^{s/t}_{\mathbf{k}} = -\frac{1}{N}\sum_{\mathbf{k}'} V^{s/t}(\mathbf{k},\mathbf{k}')\frac{\Delta^{s/t}_{\mathbf{k}'}}{2E_{\mathbf{k}'}}\tanh\left(\frac{\beta E_{\mathbf{k}'}}{2}\right)$$

Note: order parameter (matrix) needs to be antisymmetric in all quantum numbers (Pauli principle)

- singlet $\Delta^s_{\mathbf{k}} = \Delta^s_{-\mathbf{k}}$
- triplet $\Delta^t_{\mathbf{k}} = -\Delta^t_{-\mathbf{k}}$.



Rømer, et al., PRB **92**, 104505 (2015)

Pairing glue from spin fluctuations

- Spin susceptibility

$$\chi_0^{+-}(\mathbf{q},\tau) = \frac{1}{N}\sum_{\mathbf{k},\mathbf{k}'}\langle T_\tau S^+(\mathbf{q},\tau)S^-(-\mathbf{q},0)\rangle$$
$$= \frac{1}{N}\sum_{\mathbf{k},\mathbf{k}'}\langle T_\tau c^\dagger_{\mathbf{k}+\mathbf{q},\uparrow}(\tau)c_{\mathbf{k},\downarrow}(\tau)c^\dagger_{\mathbf{k}'-\mathbf{q},\downarrow}(0)c_{\mathbf{k}',\uparrow}(0)\rangle$$

$$\vec{S}(\mathbf{r},\tau) = \tfrac{1}{2}c^\dagger_{\mathbf{r},\alpha}(\tau)\vec{\sigma}_{\alpha\beta}c_{\mathbf{r},\beta}(\tau)$$

- Evaluate in band basis

$$\chi_0^{+-}(\mathbf{q},i\nu_n) = -\frac{1}{\beta N}\sum_{\mathbf{k},i\omega_m} G_0^\uparrow(\mathbf{k}+\mathbf{q},i\omega_m+i\nu_n)G_0^\downarrow(\mathbf{k},i\omega_m)$$
$$= -\frac{1}{N}\sum_{\mathbf{k}}\frac{n_F(\epsilon_{\mathbf{k}+\mathbf{q}})-n_F(\epsilon_{\mathbf{k}})}{i\nu_n+\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}}$$

- Hubbard interaction

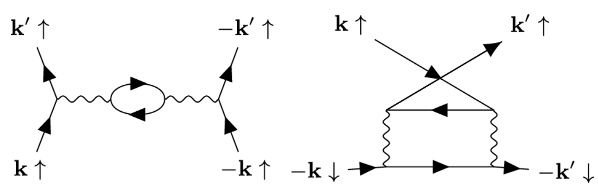
$$\chi_s(\mathbf{k}-\mathbf{k}') = \frac{\chi_0^{zz}(\mathbf{k}-\mathbf{k}')}{1-U\chi_0^{zz}(\mathbf{k}-\mathbf{k}')}$$

$$H_{\text{int}} = U\sum_{\mathbf{r}} c^\dagger_{\mathbf{r},\uparrow}c_{\mathbf{r},\uparrow}c^\dagger_{\mathbf{r},\downarrow}c_{\mathbf{r},\downarrow}$$

$$\chi_c(\mathbf{k}-\mathbf{k}') = \frac{\chi_0^{zz}(\mathbf{k}-\mathbf{k}')}{1+U\chi_0^{zz}(\mathbf{k}-\mathbf{k}')}$$

- Longitudinal and transverse contributions

$$V(\mathbf{k},\mathbf{k}') = U + V_{\text{lo}}^{\text{RPA}}(\mathbf{k}-\mathbf{k}') + V_{\text{tr}}^{\text{RPA}}(\mathbf{k}+\mathbf{k}')$$



$$V_{\text{lo}}^{\text{RPA}}(\mathbf{k}-\mathbf{k}') = \frac{U^3\chi_0^{zz}(\mathbf{k}-\mathbf{k}')^2}{1-U^2\chi_0^{zz}(\mathbf{k}-\mathbf{k}')^2} \qquad V_{\text{tr}}^{\text{RPA}}(\mathbf{k}+\mathbf{k}') = \frac{U^2\chi_0^{+-}(\mathbf{k}+\mathbf{k}')}{1-U\chi_0^{+-}(\mathbf{k}+\mathbf{k}')}$$

Pairing from spin fluctuations

- Back to example

$$H = \sum_{ij,\sigma} t_{ij} c^\dagger_{i,\sigma} c_{j,\sigma} + \sum_i U n_{i\uparrow} n_{i\downarrow}$$

$$\Gamma^s(\mathbf{q}) \sim \frac{3}{2}(U + U^2 \chi(\mathbf{q}))$$

Spin susceptibility (RPA): peaked at momentum q_0

$$\Gamma^t(\mathbf{q}) \sim -\frac{U^2}{2}\chi(\mathbf{q}) = -\tilde{\Gamma}^s(\mathbf{q})/3$$

- Superconducting gap function from linearized gap equation

$$\Delta^{s/t}_{\mathbf{k}} = -\frac{1}{N}\sum_{\mathbf{k}'} V^{s/t}(\mathbf{k},\mathbf{k}')\frac{\Delta^{s/t}_{\mathbf{k}'}}{2E_{\mathbf{k}'}}\tanh\left(\frac{\beta E_{\mathbf{k}'}}{2}\right)$$

Linearize: on r.h.s. $E_{\mathbf{k}} = \epsilon_{\mathbf{k}}$

→ only contributions from FS

$$-\frac{1}{V_G}\int_{\mathrm{FS}} dS' \, \Gamma(\mathbf{k}-\mathbf{k}')\frac{g_i(\mathbf{k}')}{|v_{\mathrm{F}}(\mathbf{k}')|} = \lambda_i g_i(\mathbf{k})$$

Eigenvalue and gap symmetry function

Rømer, et al., PRB **92**, 104505 (2015)

Connection to critical temperature (in principle)

$$T_c = \omega_0 e^{-\frac{1}{\lambda_i}}$$

Singlet pairing

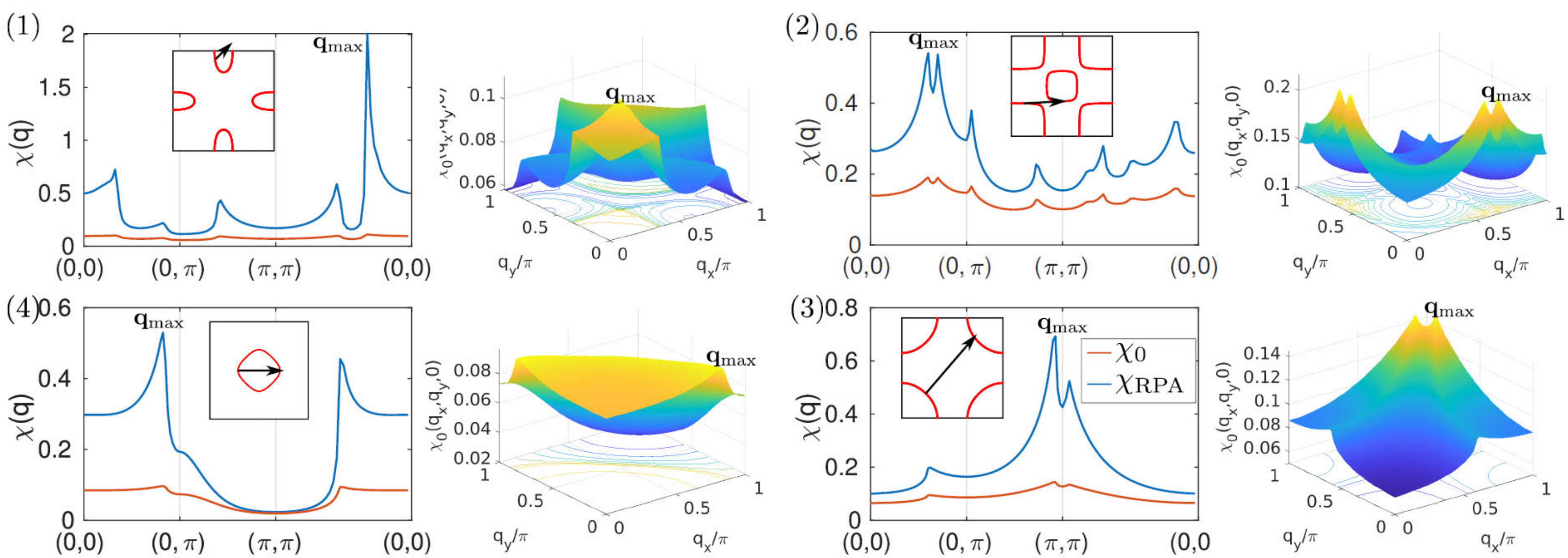
- Spin susceptibility in single band models

$$\chi_0^{+-}(\mathbf{q}, i\nu_n) = -\frac{1}{N}\sum_{\mathbf{k}} \frac{n_F(\epsilon_{\mathbf{k+q}}) - n_F(\epsilon_{\mathbf{k}})}{i\nu_n + \epsilon_{\mathbf{k+q}} - \epsilon_{\mathbf{k}}}$$

  - Onsite interaction: RPA enhancement

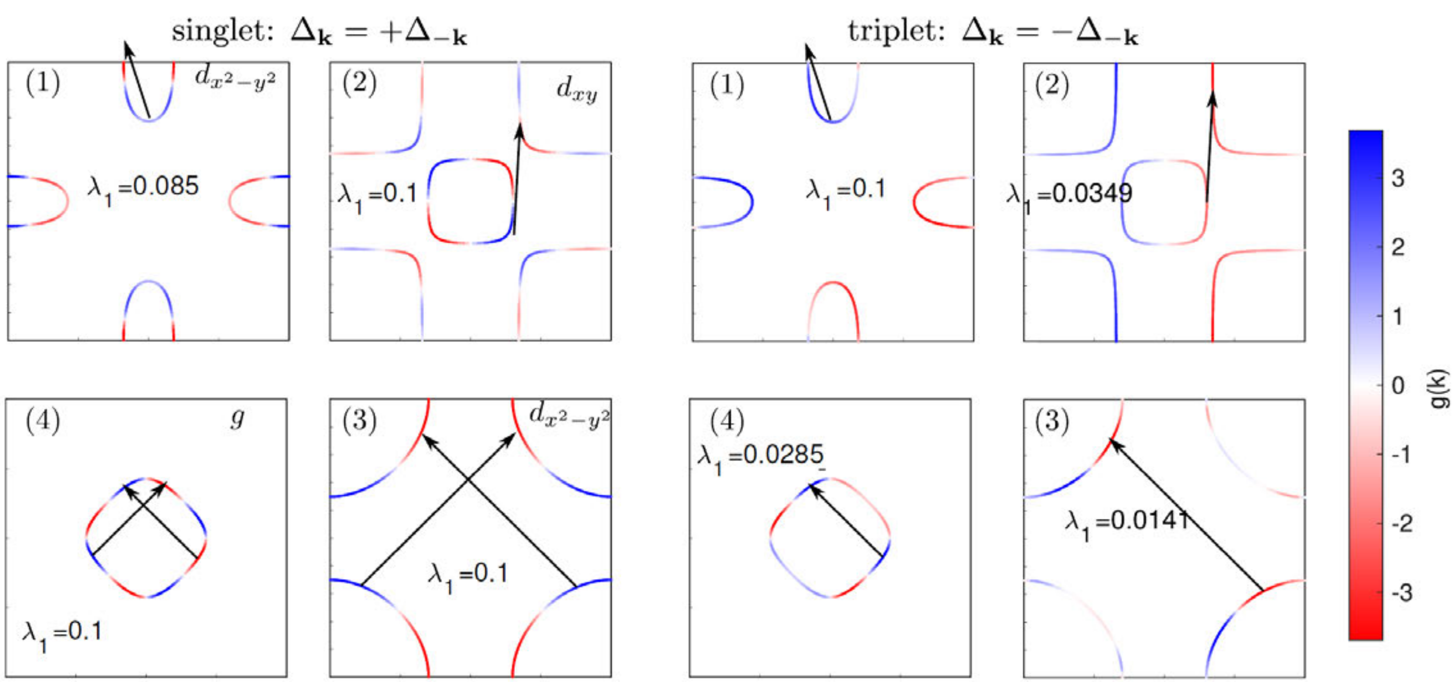
$$\chi_s(\mathbf{k}-\mathbf{k}') = \frac{\chi_0^{zz}(\mathbf{k}-\mathbf{k}')}{1 - U\chi_0^{zz}(\mathbf{k}-\mathbf{k}')}$$

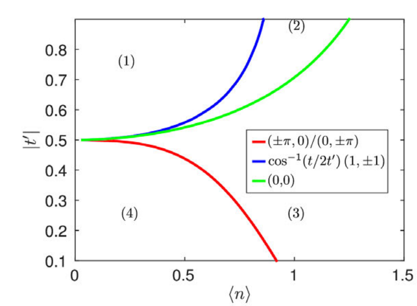
- Examples



Singlet pairing

- Ingredients:
  - Repulsive pairing interaction $$\Gamma^s(\mathbf{q}) \sim \frac{3}{2}(U + U^2\chi(\mathbf{q}))$$
  - Fermi surface geometry (nesting): solve for eigenvalues/eigenvectors to $$M^{s/t}_{\mathbf{k},\mathbf{k}'} = -\frac{1}{V_G}\frac{l_{\mathbf{k}'}}{|v_F(\mathbf{k}')|}V^{s/t}(\mathbf{k},\mathbf{k}')$$
- Examples





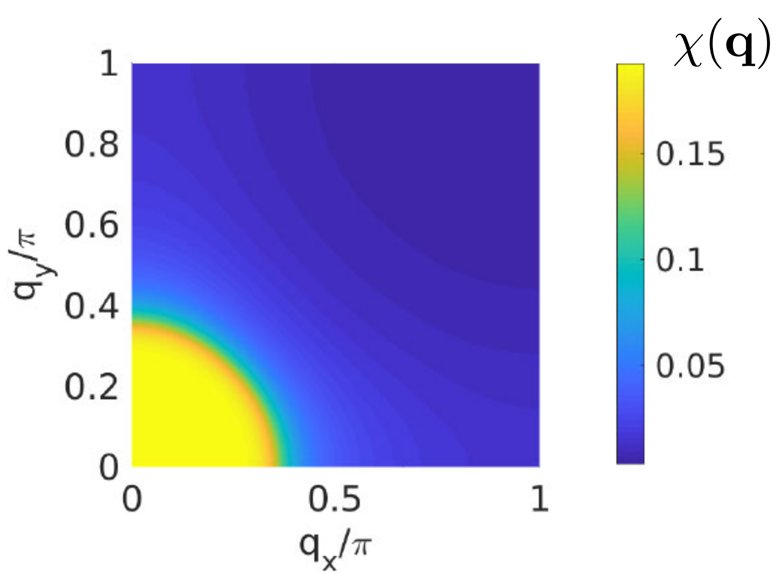
Triplet pairing

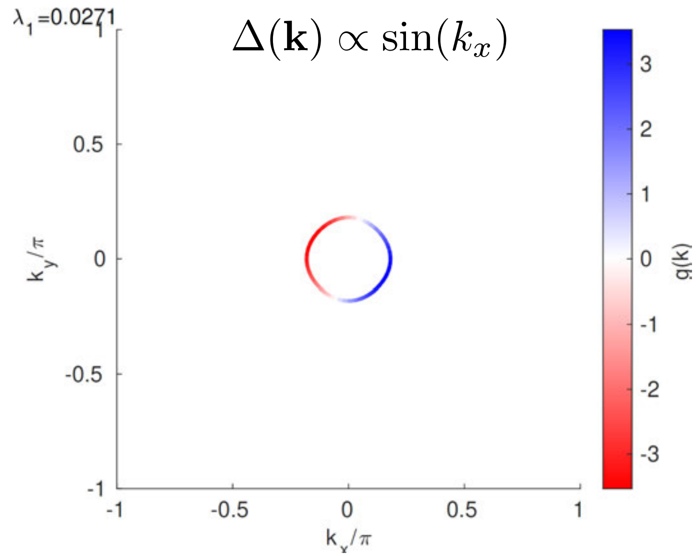
A. Layzer and D. Fay, Int. J. Magn. **1**, 135 (1971)

- Pairing dominated by small q_0 interactions
  - susceptibility peaked at q=0 (close to ferromagnetic instability)

$$\Gamma^t(\mathbf{q}) \sim -\frac{U^2}{2}\chi(\mathbf{q}) = -\tilde{\Gamma}^s(\mathbf{q})/3$$

- Odd parity state $\Delta^t_{\mathbf{k}} = -\Delta^t_{-\mathbf{k}}$





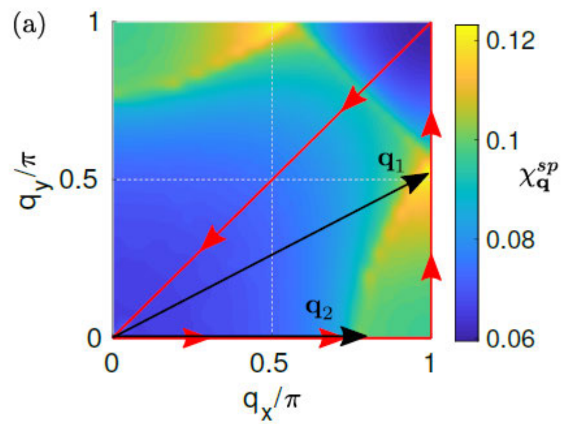
Beyond conventional picture: Kreisel, Quan, Hirschfeld, Phys. Rev. B **105**, 104507 (2022)

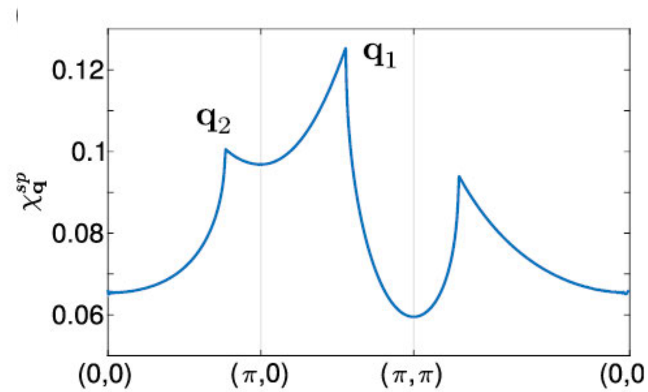
Triplet pairing: large momentum spin fluctuations

- Square lattice Hubbard model close to quarter filling

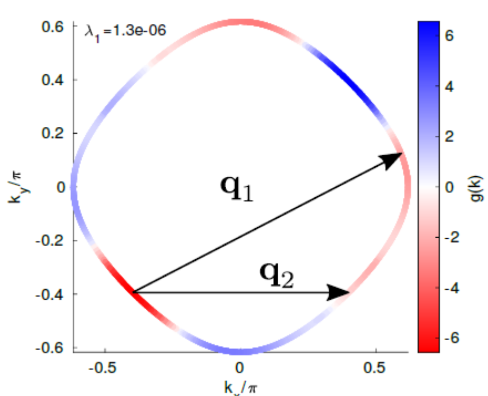
$$\xi_{\mathbf{k}} = -2t[\cos(k_x) + \cos(k_y)]$$

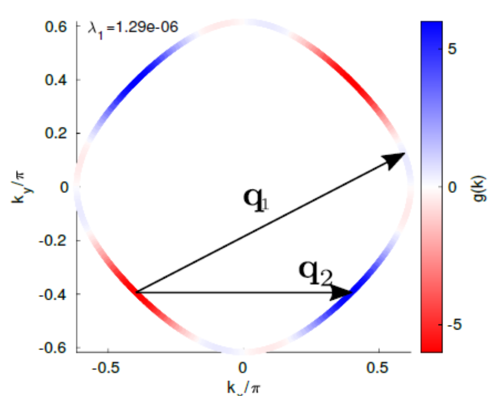
- Susceptibility

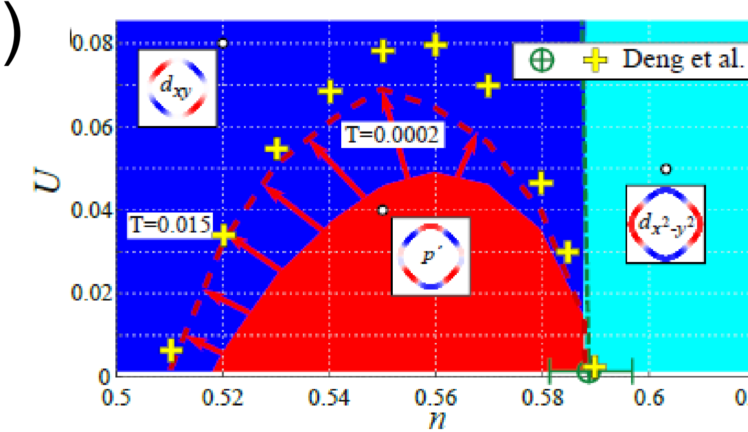




- Pairing (from linearized gap equation)







Deng, et al., EPL **110** 57001 (2015)
Rømer, et al., Phys. Rev. Research **2**, 013108 (2020)
Roig, et al., in preparation

Multiband generalization

- electronic structure $$H_0 = \sum_{\mathbf{k}\sigma\ell\ell'} t_{\mathbf{k}}^{\ell\ell'} c_{\ell\sigma}^\dagger(\mathbf{k}) c_{\ell'\sigma}(\mathbf{k}) \qquad t_{\mathbf{k}}^{\ell\ell'} = \sum_\delta t_\delta^{\ell\ell'} \exp(i\mathbf{k}\cdot\delta)$$
- Hubbard-Kanamouri interaction $$\begin{aligned} H_{\rm int} &= U \sum_{i,\ell} n_{i\ell\uparrow} n_{i\ell\downarrow} + U' \sum_{i,\ell'<\ell} n_{i\ell} n_{i\ell'} \\ &+ J \sum_{i,\ell'<\ell} \sum_{\sigma,\sigma'} c_{i\ell\sigma}^\dagger c_{i\ell'\sigma'}^\dagger c_{i\ell\sigma'} c_{i\ell'\sigma} + J' \sum_{i,\ell'\neq\ell} c_{i\ell\uparrow}^\dagger c_{i\ell\downarrow}^\dagger c_{i\ell'\downarrow} c_{i\ell'\uparrow} \end{aligned}$$
- Generalized Lindhard function

$$\chi^0_{\ell_1\ell_2\ell_3\ell_4}(\mathbf{q}, i\nu_n) = -\sum_{\mathbf{k},\mu,\nu} \frac{a_\nu^{\ell_4}(\mathbf{k}) a_\nu^{\ell_2,*}(\mathbf{k}) a_\mu^{\ell_1}(\mathbf{k}+\mathbf{q}) a_\mu^{\ell_3,*}(\mathbf{k}+\mathbf{q}) [n_F(\epsilon_{\mu,\mathbf{k}+\mathbf{q}}) - n_F(\epsilon_{\nu,\mathbf{k}})]}{i\nu_n + \epsilon_{\mu,\mathbf{k}+\mathbf{q}} - \epsilon_{\nu,\mathbf{k}}}$$

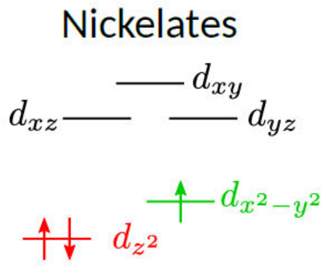
- Matrix equation for RPA

$$\chi^{\rm RPA}_{{\rm s/c}\,\ell_1\ell_2\ell_3\ell_4}(\mathbf{q}, \omega) = \left\{ \chi^0(\mathbf{q}, \omega) \left[1 \mp \bar{U}^{\rm s/c} \chi^0(\mathbf{q}, \omega)\right]^{-1} \right\}_{\ell_1\ell_2\ell_3\ell_4}$$

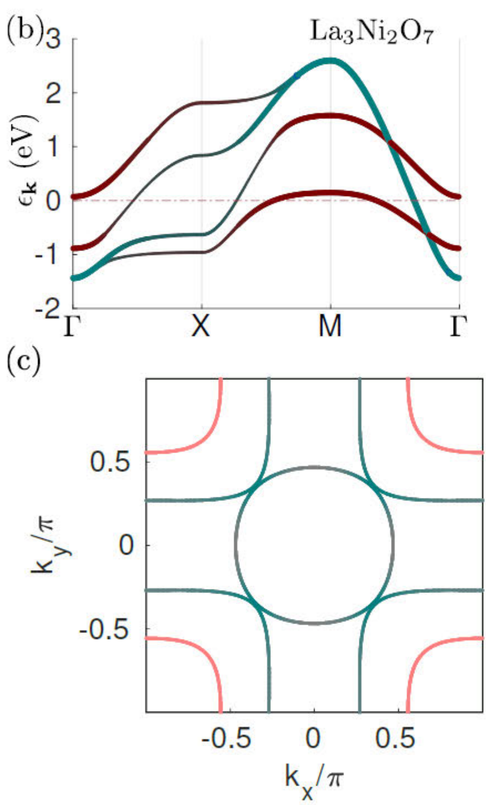
- Pairing interaction (orbital space)

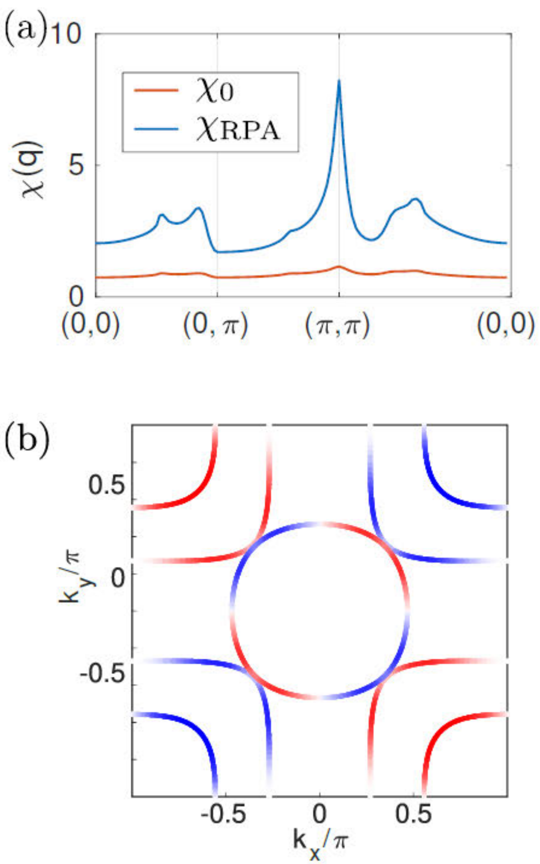
$$\Gamma_{\ell_1\ell_2\ell_3\ell_4}(\mathbf{k}, \mathbf{k}') = \frac{1}{2} \left[3\bar{U}^{\rm s} \chi_{\rm s}^{\rm RPA}(\mathbf{k}-\mathbf{k}') \bar{U}^{\rm s} + \bar{U}^{\rm s} - \bar{U}^{\rm c} \chi_{\rm c}^{\rm RPA}(\mathbf{k}-\mathbf{k}') \bar{U}^{\rm c} + \bar{U}^{\rm c}\right]_{\ell_1\ell_2\ell_3\ell_4}$$

More realistic examples

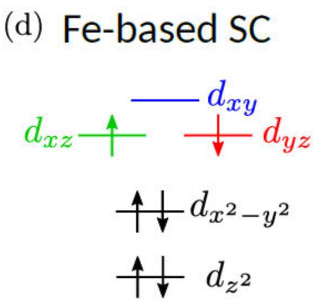


- (bilayer) Nickelates
  - Bands, Fermi surface → susceptibility, pairing

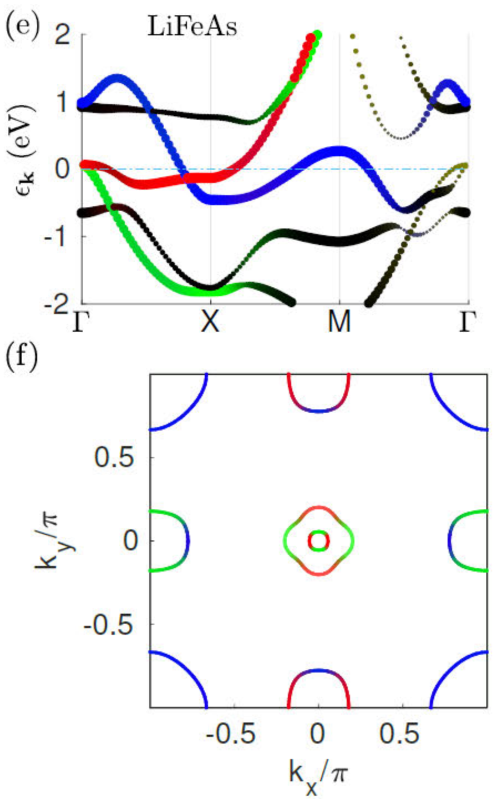


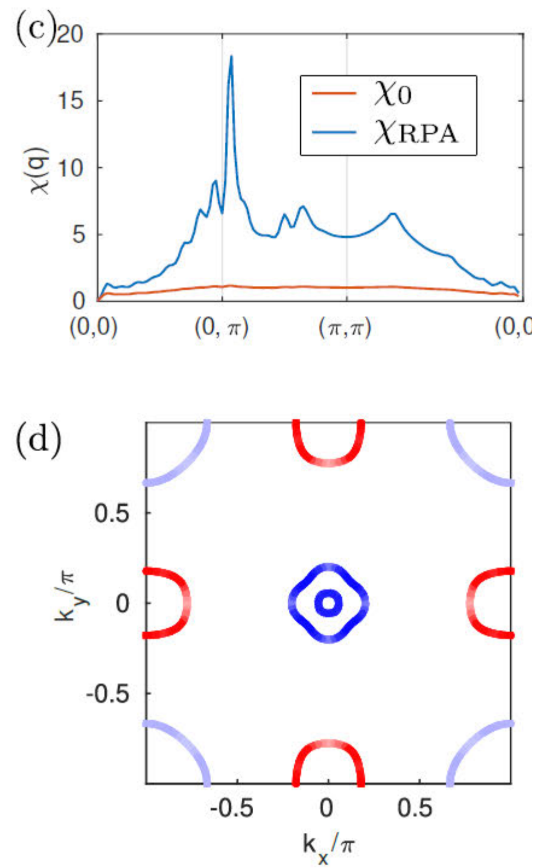


More realistic examples

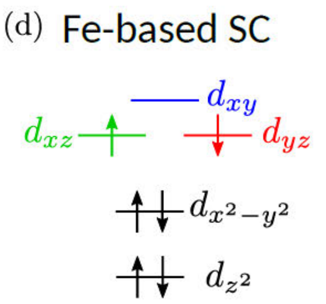


- Fe-based materials: 111
  - Bands, Fermi surface → susceptibility, pairing

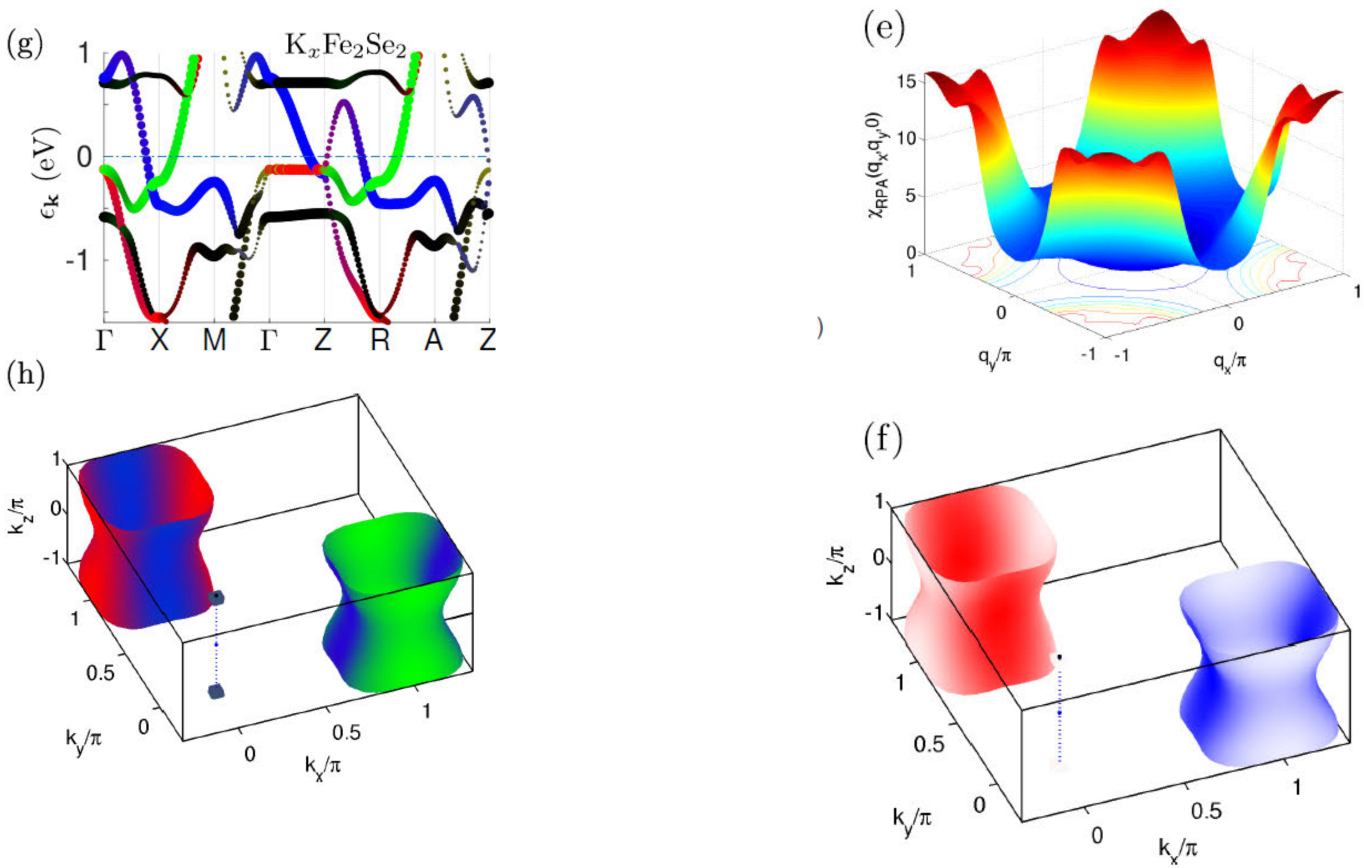




More realistic examples

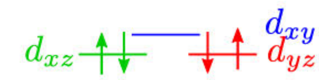


- Fe-based materials: 122
  - Bands, Fermi surface → susceptibility, pairing

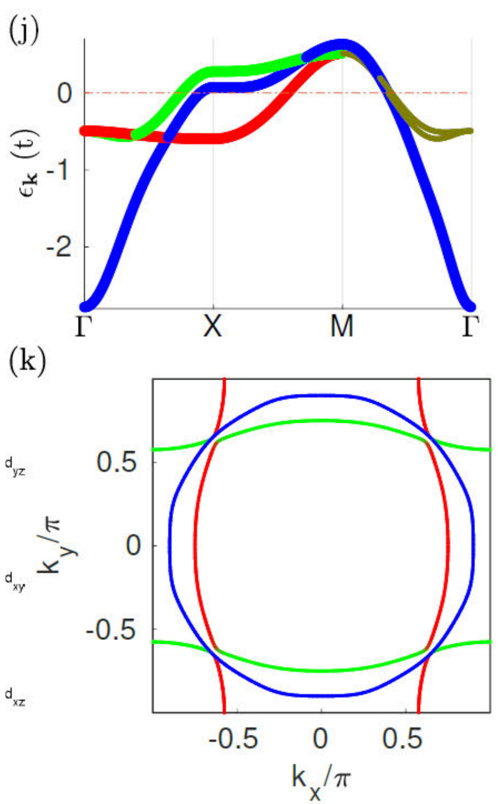


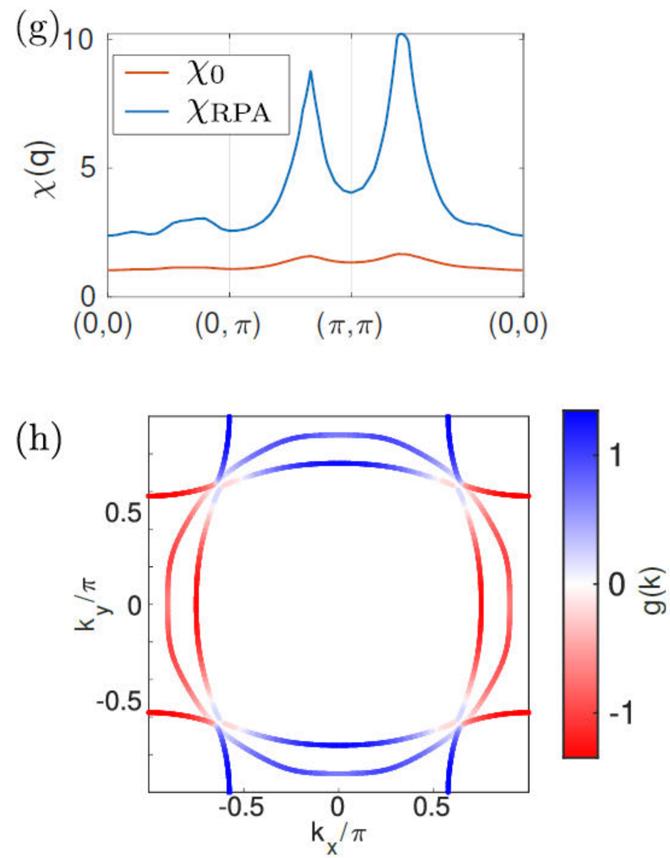
More realistic examples

(i) Sr_2RuO_4



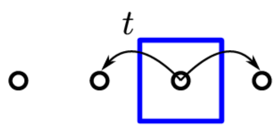
- Sr_2RuO_4
  - Bands, Fermi surface → susceptibility, pairing





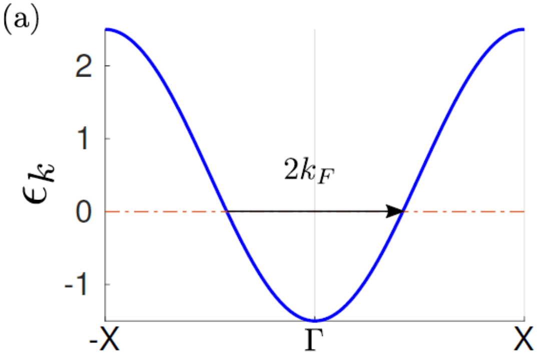
Sublattice degree of freedom

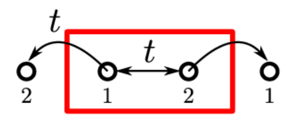
- Consider single band model in 1D



$$\epsilon_k = -2t\cos ka$$

(a)





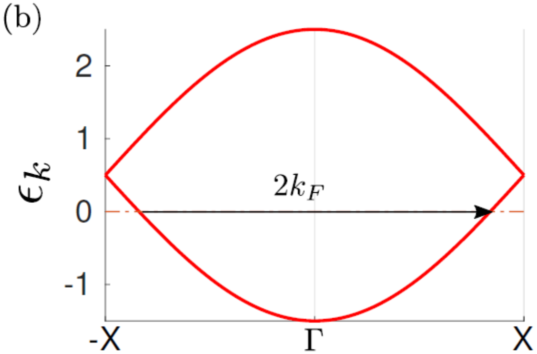
$$\tilde{H}(k) = -t\begin{pmatrix} 0 & 1+\exp(ika') \\ 1+\exp(-ika') & 0 \end{pmatrix} \qquad H(k) = -t\begin{pmatrix} 0 & 2\cos(ka'/2) \\ 2\cos(ka'/2) & 0 \end{pmatrix}$$

Periodic with Brillouin Zone

$$U = \begin{pmatrix} 1 & 0 \\ 0 & \exp(\frac{ika'}{2}) \end{pmatrix}$$

Knows about position of lattice points

(b)



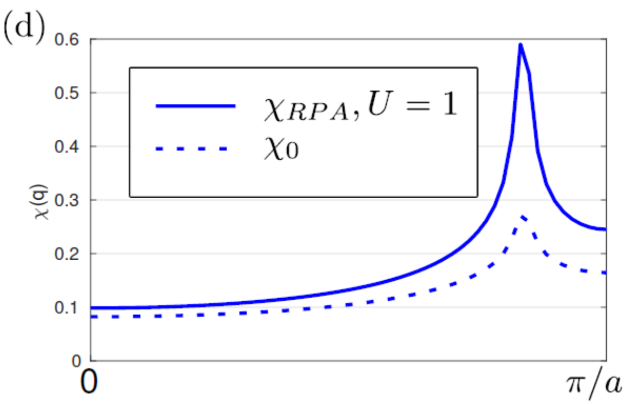
Same eigenenergies, but not same eigenstates

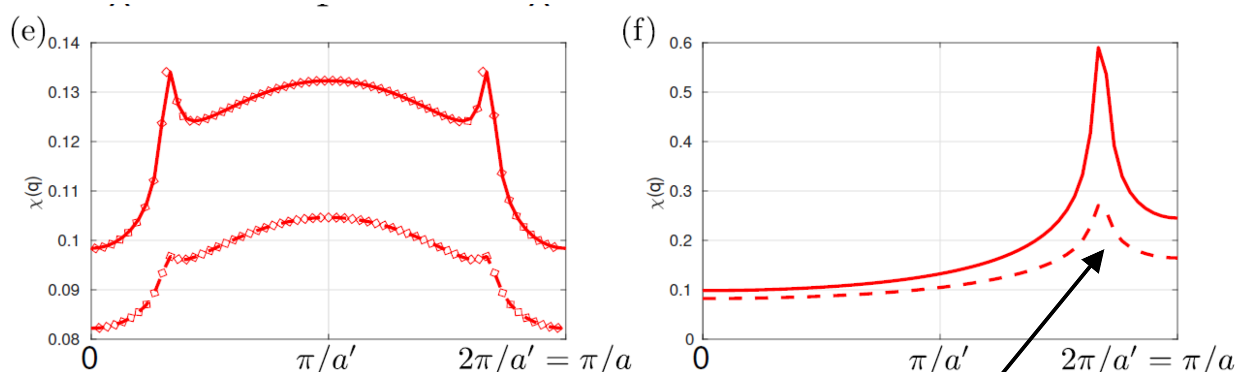
Susceptibility: Sublattice degree of freedom

$$\chi^0_{\ell_1\ell_2\ell_3\ell_4}(\mathbf{q}, i\nu_n) = -\sum_{\mathbf{k},\mu,\nu} \frac{a^{\ell_4}_{\nu}(\mathbf{k}) a^{\ell_2,*}_{\nu}(\mathbf{k}) a^{\ell_1}_{\mu}(\mathbf{k}+\mathbf{q}) a^{\ell_3,*}_{\mu}(\mathbf{k}+\mathbf{q}) [n_F(\epsilon_{\mu,\mathbf{k}+\mathbf{q}}) - n_F(\epsilon_{\nu,\mathbf{k}})]}{i\nu_n + \epsilon_{\mu,\mathbf{k}+\mathbf{q}} - \epsilon_{\nu,\mathbf{k}}}$$

$$\epsilon_k = -2t\cos ka$$

$$\tilde{H}(k) = -t \begin{pmatrix} 0 & 1+\exp(ika') \\ 1+\exp(-ika') & 0 \end{pmatrix} \qquad H(k) = -t \begin{pmatrix} 0 & 2\cos(ka'/2) \\ 2\cos(ka'/2) & 0 \end{pmatrix}$$





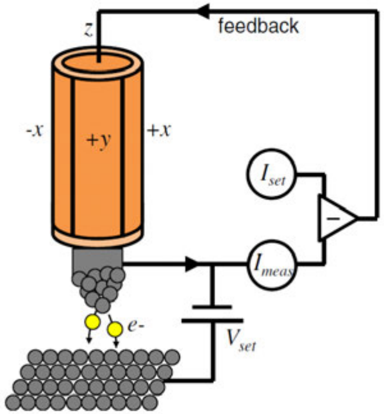
$$\chi_{\text{phys}}(\mathbf{q}) = \frac{1}{2}\sum_{\ell\ell'} \chi_{\ell\ell'\ell'\ell}(\mathbf{q})$$

Correct physical quantity!

$$\chi^{+-}_{\ell_1\ell_2\ell_3\ell_4}(q, i\nu_n) = \chi^{+-}_{\ell_1\ell_2\ell_3\ell_4}(q + 4\pi/a', i\nu_n) = \chi^{+-}_{\ell_1\ell_2\ell_3\ell_4}(q + 2\pi/a, i\nu_n)$$

(selected) spectroscopic probes

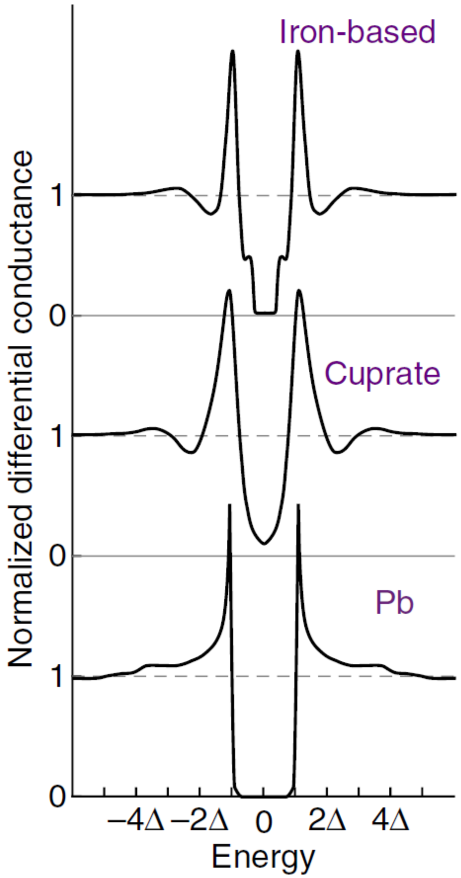
- Tunneling (density of states)



Tunnelling current:

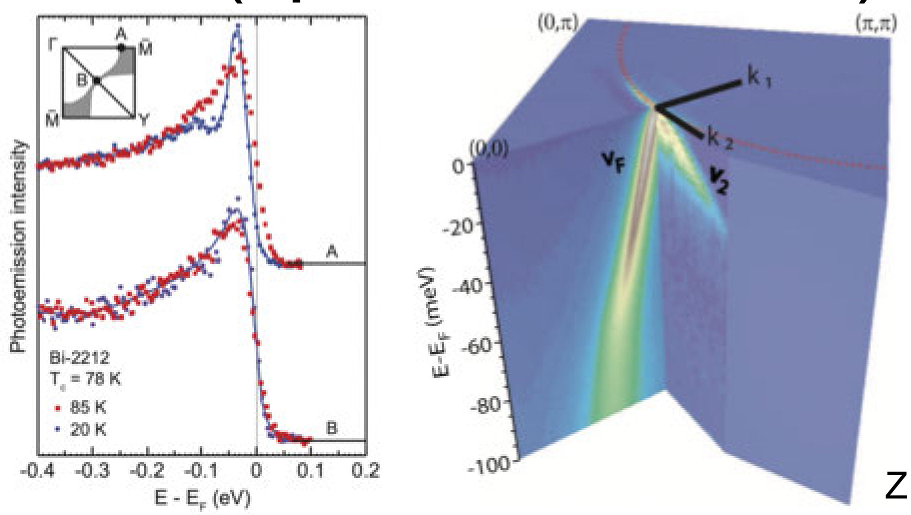
$$I(V,x,y,z) = -\frac{4\pi e}{\hbar}\rho_t(0)|M|^2\int_0^{eV}\rho(x,y,z,\epsilon)d\epsilon$$

Local Density Of States (LDOS)
of sample at given energy at the tip position



Chi, et al. Nat. Commun (2017)

- ARPES (spectral function)



Z. X. Shen et al., PRL **70**, 1553 (1993)

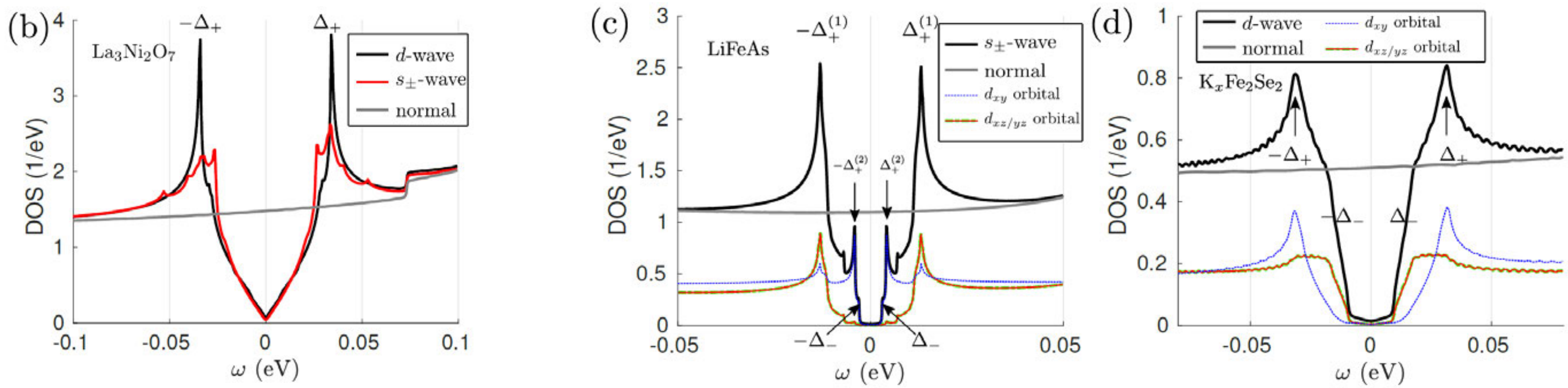
Density of states

- "electronic density of states"

$$\widehat{G}^{(0)}(\mathbf{k},\omega) = \left[(\omega + i\eta)\mathbb{1} - \widehat{H}_{\mathrm{BdG}}(\mathbf{k})\right]^{-1} \qquad \widehat{H}_{\mathrm{BdG}}(\mathbf{k}) = \begin{pmatrix} h(\mathbf{k}) & \Delta^{s/t}(\mathbf{k}) \\ \Delta(\mathbf{k})^{s/t\,\dagger} & -h(-\mathbf{k})^{T} \end{pmatrix}$$

$$\rho(\omega) = -\frac{1}{\pi}\,\mathrm{Tr}\,\mathrm{Im}\,G^{(0)}_{11}(\mathbf{k},\omega) = -\frac{1}{2\pi}\,\mathrm{Tr}\,\mathrm{Im}\left[G^{(0)}_{11}(\mathbf{k},\omega) + G^{(0)}_{\bar{1}\bar{1}}(\mathbf{k},-\omega)\right]$$

- Sensitive to saddle points and nodal behavior, but not to the sign of the order parameter

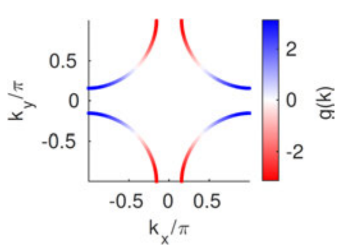


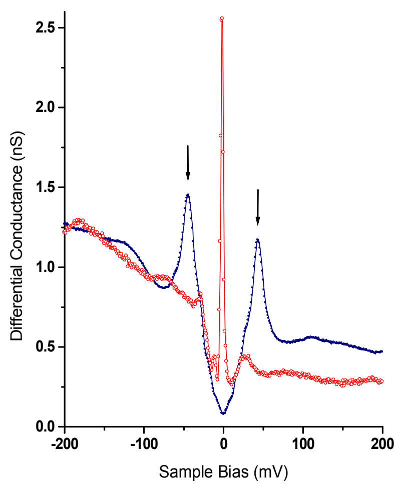
Disorder in superconductors

- Conventional superconductor $\Delta(\mathbf{k}) = \Delta_0$
  - Nonmagnetic impurity → no bound states
  - Magnetic impurity → Yu-Shiba-Rusinov bound states
- Unconventional superconductor, sign change: $\Delta(\mathbf{k}) = -\Delta(\tilde{\mathbf{k}})$
  - Nonmagnetic impurity → in-gap bound states

A. V. Balatsky, I. Vekhter, and Jian-Xin Zhu Rev. Mod. Phys. **78**, 373 (2006)

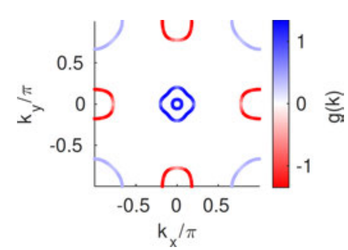
Cuprates: d-wave order parameter

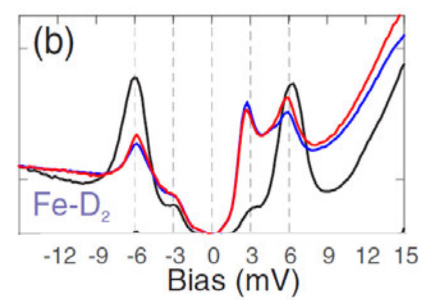




Pan et al., Nature **403**, 746 (2000)

Iron-based SC: s_{+-} order parameter



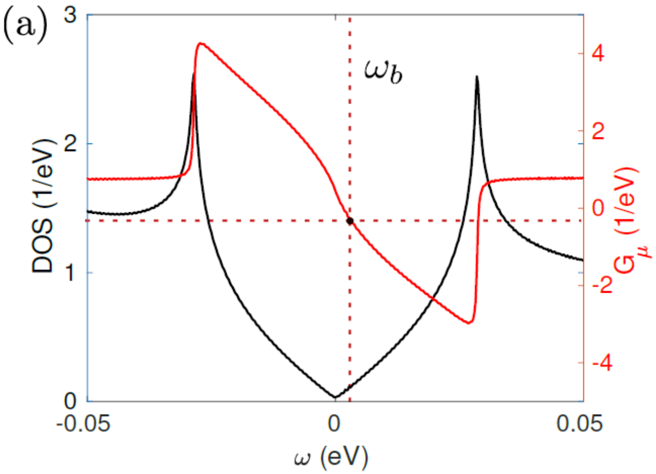


S. Chi, *et al.* Phys. Rev. B **94**, 134515 (2016)

Disorder in superconductors

- Non-magnetic impurity $\widehat{H}_{\rm imp} = S_z \tau^0 \otimes M$

$\sum_{\mathbf{k}} \Delta_{\mathbf{k}} = 0 \qquad G^{(0)}_{1\bar{1}}(\omega) = 0$



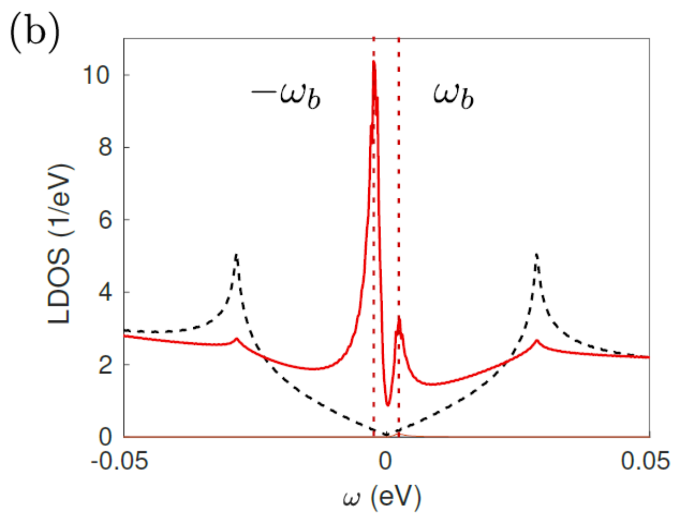
- T-matrix approximation

$$\widehat{G}(\mathbf{r},\mathbf{r}',\omega) = \widehat{G}^{(0)}(\mathbf{r}-\mathbf{r}',\omega) + \widehat{G}^{(0)}(\mathbf{r},\omega)\widehat{T}(\omega)\widehat{G}^{(0)}(-\mathbf{r}',\omega)$$

$$\widehat{T}(\omega) \equiv \left[\mathbb{1} - \widehat{H}_{\rm imp}\widehat{G}^{(0)}(\mathbf{0},\omega)\right]^{-1}(\omega)\widehat{H}_{\rm imp}$$

- Local density of states*

$$\rho_\alpha(\mathbf{r},\omega) = -\frac{1}{\pi}{\rm Im}\left[G_{\alpha\alpha}(\mathbf{r},\mathbf{r},\omega) + G_{\bar{\alpha}\bar{\alpha}}(\mathbf{r},\mathbf{r},-\omega)\right]$$



Resonances:
if Re(HG)=1, Im(G)~0

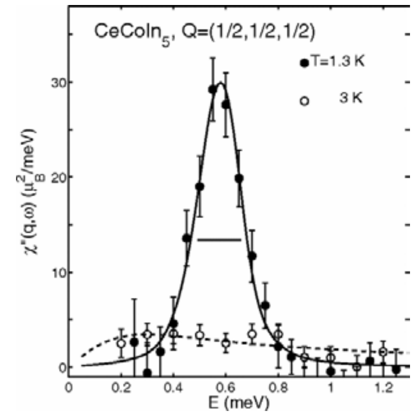
$$G^{(0)}_{1\bar{1}}(\omega) = 0$$

*here only lattice DOS is discussed, for STM tunneling “continuum” LDOS important
S. Chi, et al. Phys. Rev. B 94, 134515 (2016)

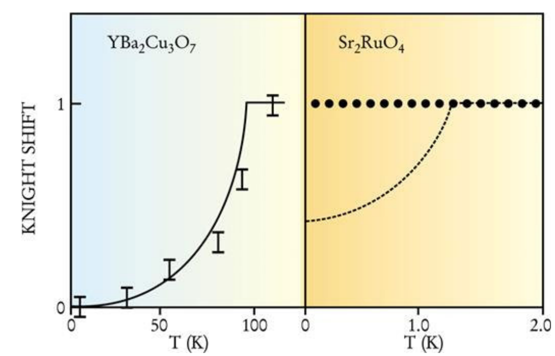
(selected) spectroscopic probes

- Neutron scattering (spin susceptibility)
  - Enhancement of scattering below 2Δ in the SC state

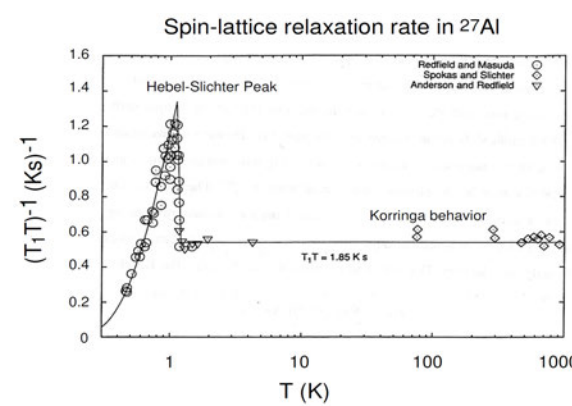
C. Stock, et al., Phys. Rev. Lett. 100, 087001 (2008)



- Nuclear spin resonance (NMR)
  - Screening from external field due to SC (Knight shift)
  - Low energy spin fluctuations (relaxation rate)



Maeno, Rice, Sigrist Physics Today **54**, 42 (2001)



L. C. Hebel, C. P. Slichter Phys. Rev. **107**, 901 (1957)

Spin susceptibility in the superconducting state

- Convolution of normal and anomalous GF

$$\chi_0^{+-}(\mathbf{q},\tau) = -\frac{1}{N}\sum_{\mathbf{k}}\left(G^0_{11}(\mathbf{k}+\mathbf{q},-\tau)G^0_{\bar{1}\bar{1}}(\mathbf{k},\tau) + G^0_{\bar{1}1}(\mathbf{k}+\mathbf{q},-\tau)G^0_{1\bar{1}}(\mathbf{k},\tau)\right)$$

- Contribution of magnitude and sign of order parameter

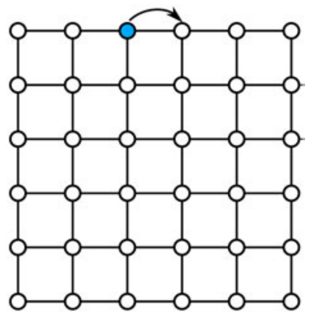
$$\begin{aligned}\chi_0^{+-}(\mathbf{q},\omega) = \frac{1}{N}\sum_{\mathbf{k},E>0}\Bigg[&\left(1-\frac{\epsilon_{\mathbf{k}}\epsilon_{\mathbf{k+q}}+\Delta^*_{\mathbf{k+q}}\Delta_{\mathbf{k}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\frac{1-f(E_{\mathbf{k}})-f(E_{\mathbf{k+q}})}{\omega+E_{\mathbf{k+q}}+E_{\mathbf{k}}+i\eta}\\ &+\left(1-\frac{\epsilon_{\mathbf{k}}\epsilon_{\mathbf{k+q}}+\Delta^*_{\mathbf{k+q}}\Delta_{\mathbf{k}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\frac{f(E_{\mathbf{k}})+f(E_{\mathbf{k+q}})-1}{\omega-E_{\mathbf{k+q}}-E_{\mathbf{k}}+i\eta}\\ &+\left(1+\frac{\epsilon_{\mathbf{k}}\epsilon_{\mathbf{k+q}}+\Delta^*_{\mathbf{k+q}}\Delta_{\mathbf{k}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\frac{f(E_{\mathbf{k}})-f(E_{\mathbf{k+q}})}{\omega+E_{\mathbf{k+q}}-E_{\mathbf{k}}+i\eta}\\ &+\left(1+\frac{\epsilon_{\mathbf{k}}\epsilon_{\mathbf{k+q}}+\Delta^*_{\mathbf{k+q}}\Delta_{\mathbf{k}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\frac{f(E_{\mathbf{k+q}})-f(E_{\mathbf{k}})}{\omega+E_{\mathbf{k}}-E_{\mathbf{k+q}}+i\eta}\Bigg]\end{aligned}$$

Important for imaginary part at ω>0 (neutron resonance)

$$\left(1-\frac{\Delta_{\mathbf{k+q}}\Delta_{\mathbf{k}}}{|\Delta_{\mathbf{k}}||\Delta_{\mathbf{k+q}}|}\right)\approx\begin{cases}0\\2\end{cases}$$

Hebel-Slichter peak

- Single band square lattice model



$$\mathcal{H}_0 = \sum_{\mathbf{k}\sigma} \left[-2t\left(\cos k_x + \cos k_y\right) - \mu\right] c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$$

- Spin relaxation rate $\alpha \equiv \frac{1}{T_1 T} \propto \lim_{\omega\to 0} \frac{1}{\mathcal{N}} \sum_{\mathbf{q}} \mathrm{Im} \frac{\chi_0^{+-}(\mathbf{q},\omega)}{\omega}$

$$\chi_0^{+-}(\mathbf{q},\omega) = \frac{1}{\mathcal{N}} \sum_{\mathbf{k},E>0} \Bigg[\left(1 - \frac{\xi_\mathbf{k}\xi_{\mathbf{k+q}} + \Delta^*_{\mathbf{k+q}}\Delta_\mathbf{k}}{E_\mathbf{k}E_{\mathbf{k+q}}}\right) \frac{1 - f(E_\mathbf{k}) - f(E_{\mathbf{k+q}})}{\omega + E_{\mathbf{k+q}} + E_\mathbf{k} + i\eta} + \left(1 - \frac{\xi_\mathbf{k}\xi_{\mathbf{k+q}} + \Delta^*_{\mathbf{k+q}}\Delta_\mathbf{k}}{E_\mathbf{k}E_{\mathbf{k+q}}}\right) \frac{f(E_\mathbf{k}) + f(E_{\mathbf{k+q}}) - 1}{\omega - E_{\mathbf{k+q}} - E_\mathbf{k} + i\eta}$$
$$+ \left(1 + \frac{\xi_\mathbf{k}\xi_{\mathbf{k+q}} + \Delta^*_{\mathbf{k+q}}\Delta_\mathbf{k}}{E_\mathbf{k}E_{\mathbf{k+q}}}\right) \frac{f(E_\mathbf{k}) - f(E_{\mathbf{k+q}})}{\omega + E_{\mathbf{k+q}} - E_\mathbf{k} + i\eta} + \left(1 + \frac{\xi_\mathbf{k}\xi_{\mathbf{k+q}} + \Delta^*_{\mathbf{k+q}}\Delta_\mathbf{k}}{E_\mathbf{k}E_{\mathbf{k+q}}}\right) \frac{f(E_{\mathbf{k+q}}) - f(E_k)}{\omega + E_\mathbf{k} - E_{\mathbf{k+q}} + i\eta} \Bigg],$$

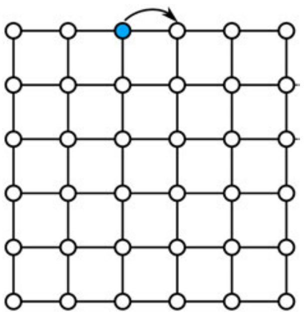
- Relevant quantity

$$\left(1 + \frac{\Delta_{\mathbf{k+q}}\Delta_\mathbf{k}}{|\Delta_\mathbf{k}||\Delta_{\mathbf{k+q}}|}\right) \approx \begin{cases} 2 & \text{same sign} \\ 0 & \text{different sign} \end{cases}$$

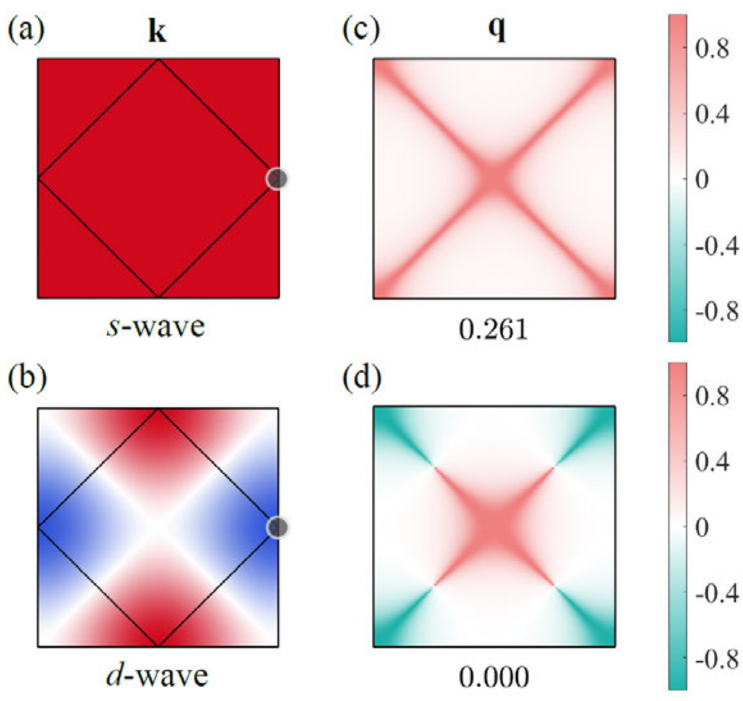
$$B(\mathbf{q},\mathbf{k}_n) = \frac{\Delta^*_{\mathbf{k}_n+\mathbf{q}}\Delta_{\mathbf{k}_n}}{E_{\mathbf{k}_n}E_{\mathbf{k}_n+\mathbf{q}}}$$

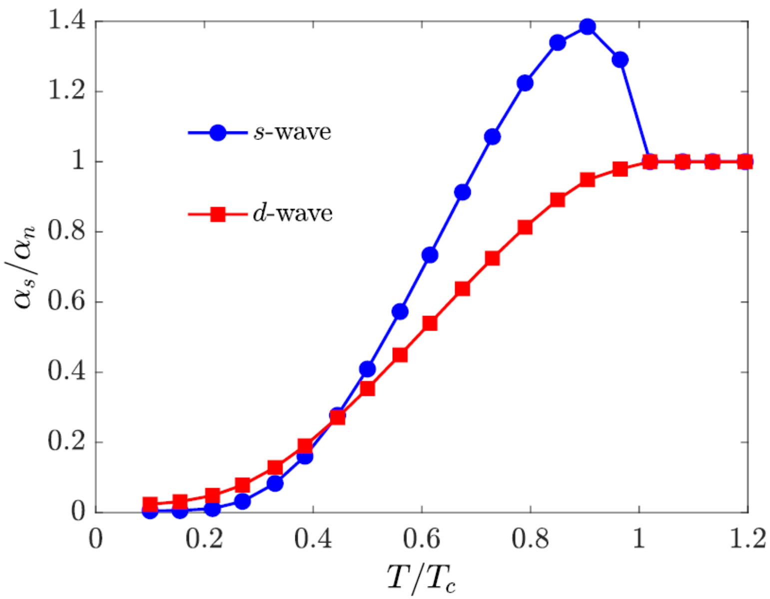
Hebel-Slichter peak

- Relevant quantity $$B(\mathbf{q}, \mathbf{k}_n) = \frac{\Delta^*_{\mathbf{k}_n+\mathbf{q}} \Delta_{\mathbf{k}_n}}{E_{\mathbf{k}_n} E_{\mathbf{k}_n+\mathbf{q}}}$$



- Fixed k_n

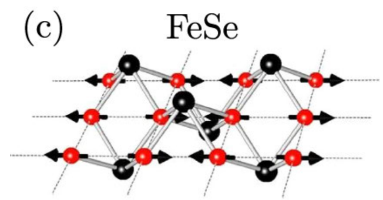




$$\left(1 + \frac{\Delta_{\mathbf{k}+\mathbf{q}} \Delta_{\mathbf{k}}}{|\Delta_{\mathbf{k}}||\Delta_{\mathbf{k}+\mathbf{q}}|}\right) \approx \begin{cases} 2 & \text{same sign} \\ 0 & \text{different sign} \end{cases}$$

Summary (1)

- A number of material candidates for unconventional superconductivity

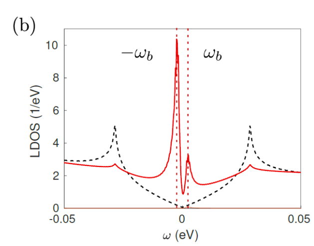


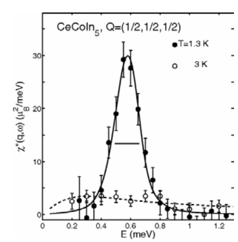
- Momentum dependent interaction allows for momentum dependent order parameter and different symmetries

$$\Delta_{\mathbf{k}} \sim \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$$

$$E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}^{s/t}|^2}.$$

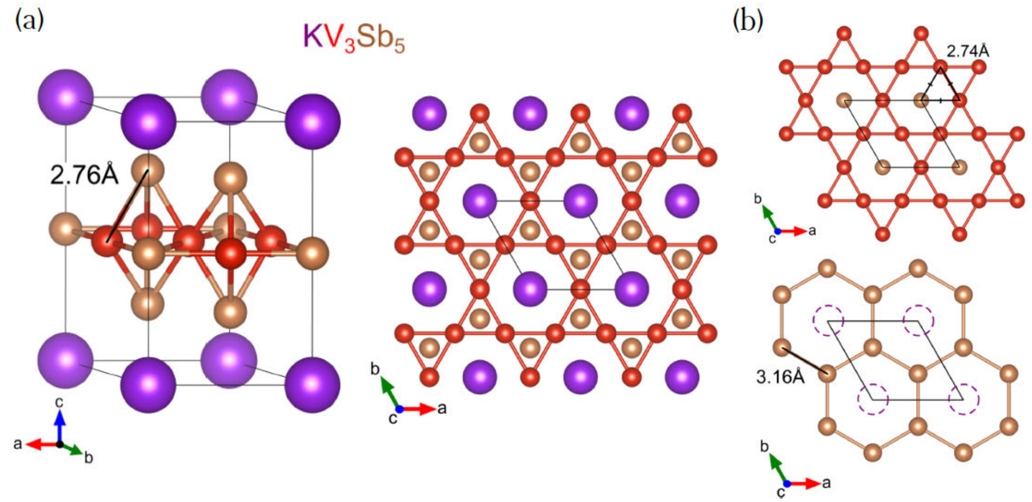
- Experimental probes sensitive to the sign of the order parameter are rare
  - Impurity resonances in LDOS
  - Neutron resonance
  - Hebel-Slichter peak in NMR relaxation rate



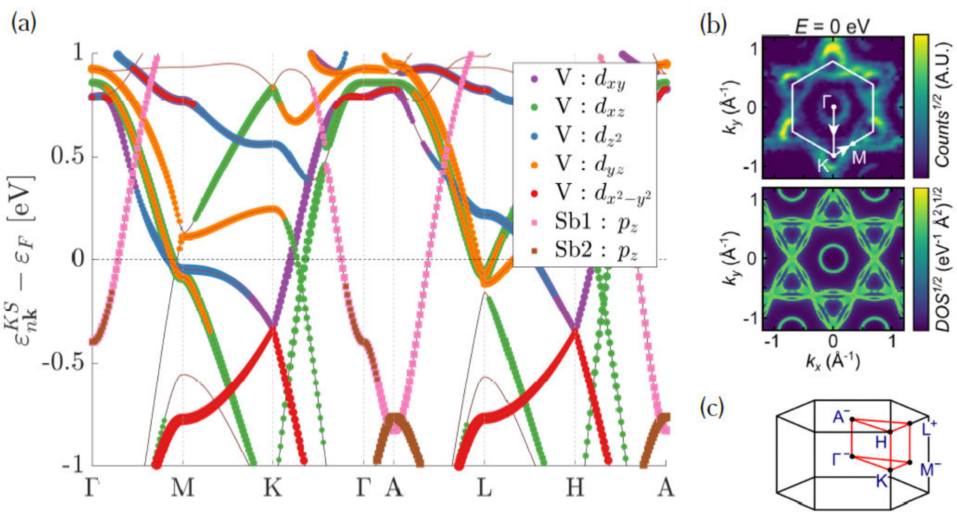


Frontiers in materials: Kagome superconductors

- Hexagonal system



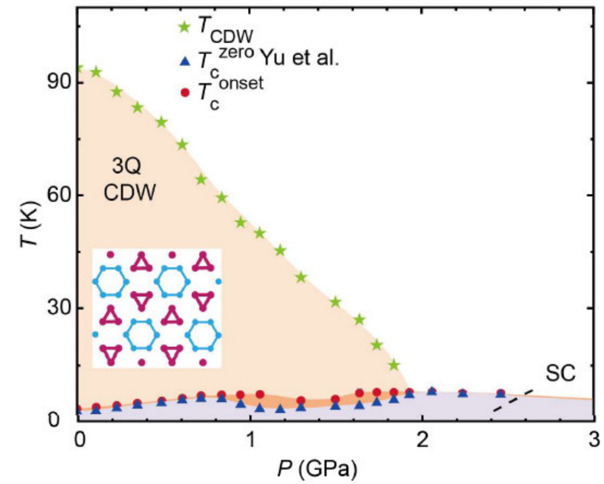
- Electronic structure dominated by V states



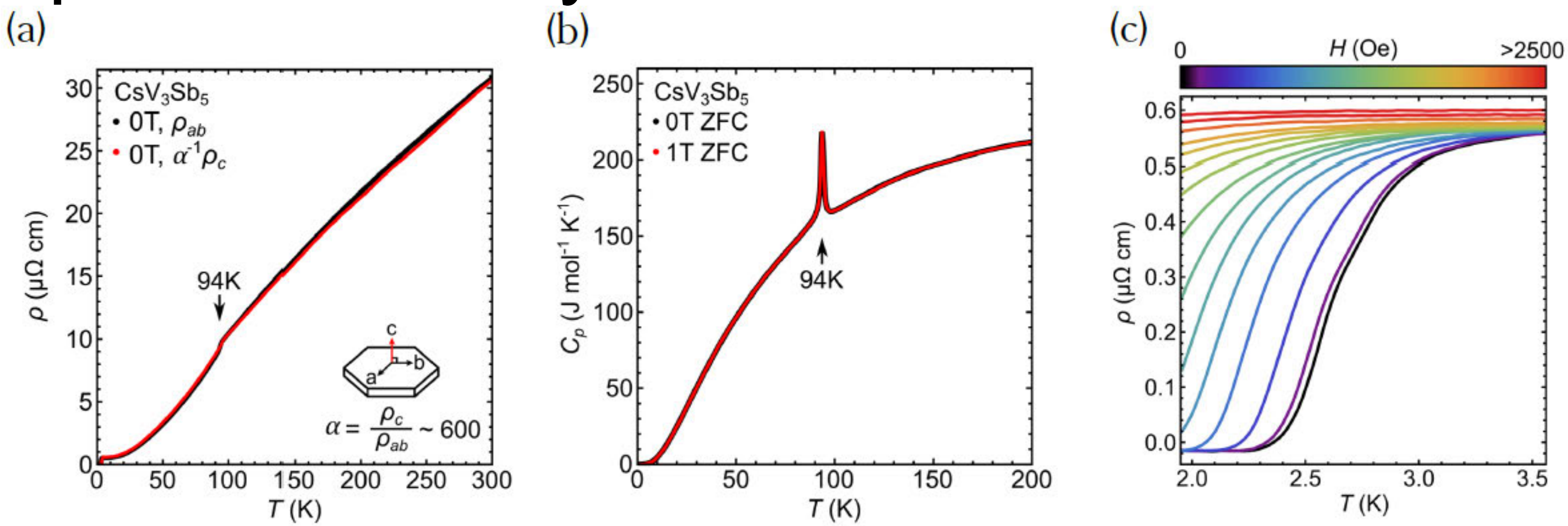
B. Ortiz *et al.*,
Phys. Rev. Lett **125**, 247002 (2020)

Kagome superconductors

- CDW at high temperature



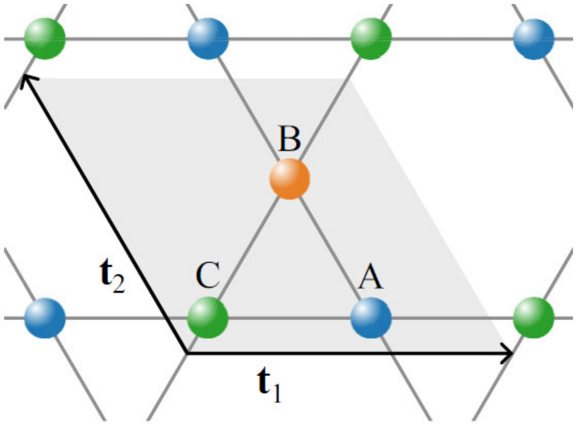
- superconductivity at ~3K



B. Ortiz *et al.*,
Phys. Rev. Lett **125**, 247002 (2020)

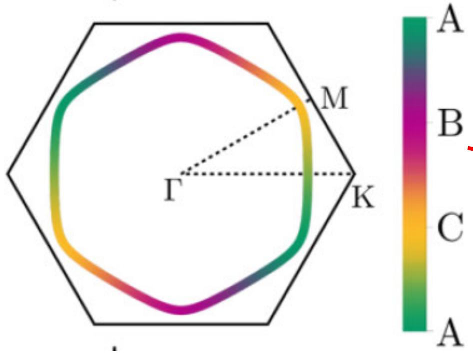
Kagome systems

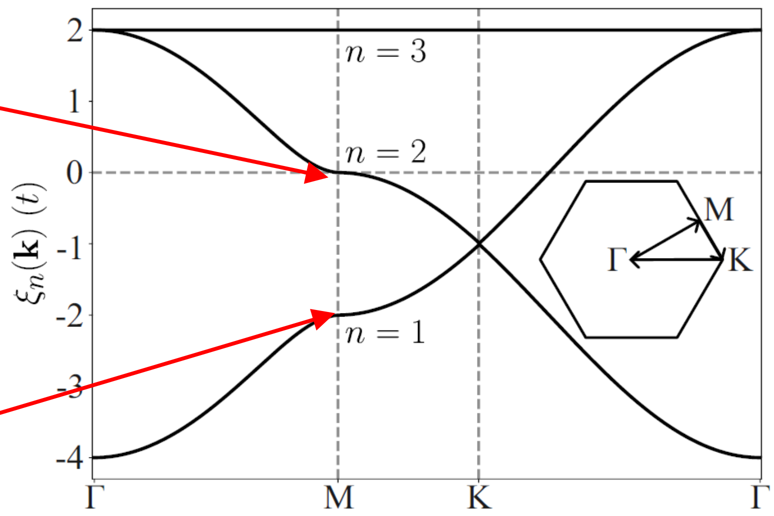
- Basics: electronic structure



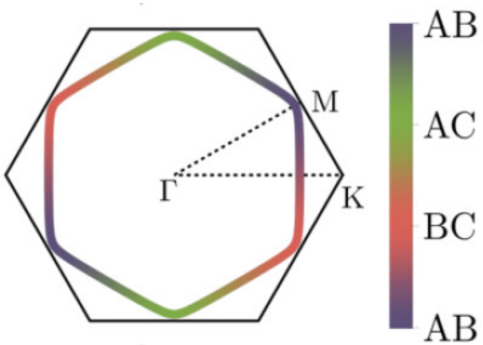
$$\mathcal{H}_0 = \sum_{\mathbf{k},\sigma} \psi^\dagger_{\mathbf{k}\sigma} H_0(\mathbf{k}) \psi_{\mathbf{k}\sigma} \qquad \psi_{\mathbf{k}\sigma} = \begin{pmatrix} c_{\mathbf{k}\sigma A} & c_{\mathbf{k}\sigma B} & c_{\mathbf{k}\sigma C} \end{pmatrix}^T$$

Upper v. Hove point





lower v. Hove point



Band structure and susceptibility

- Strategy: use periodic Hamiltonian to calculate

$$\tilde{H}_0(\mathbf{k}) = -\begin{pmatrix} \mu & t(1+e^{2ik_3}) & t(1+e^{-2ik_1}) \\ t(1+e^{-2ik_3}) & \mu & t(1+e^{-2ik_2}) \\ t(1+e^{2ik_1}) & t(1+e^{2ik_2}) & \mu \end{pmatrix}$$

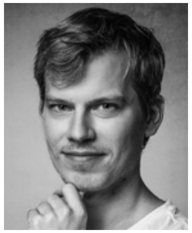
$$T(\mathbf{k}) = \begin{pmatrix} e^{-ik_1} & 0 & 0 \\ 0 & e^{-ik_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Then, unitary transformation to express in other basis

$$\chi^{+-}_{0,\alpha\beta}(\mathbf{q},\omega) = \begin{pmatrix} \tilde{\chi}^{+-}_{0,AA} & e^{i(q_2-q_1)}\tilde{\chi}^{+-}_{0,AB} & e^{-iq_1}\tilde{\chi}^{+-}_{0,AC} \\ e^{i(q_1-q_2)}\tilde{\chi}^{+-}_{0,BA} & \tilde{\chi}^{+-}_{0,BB} & e^{-iq_2}\tilde{\chi}^{+-}_{0,BC} \\ e^{iq_1}\tilde{\chi}^{+-}_{0,CA} & e^{iq_2}\tilde{\chi}^{+-}_{0,CB} & \tilde{\chi}^{+-}_{0,CC} \end{pmatrix}$$

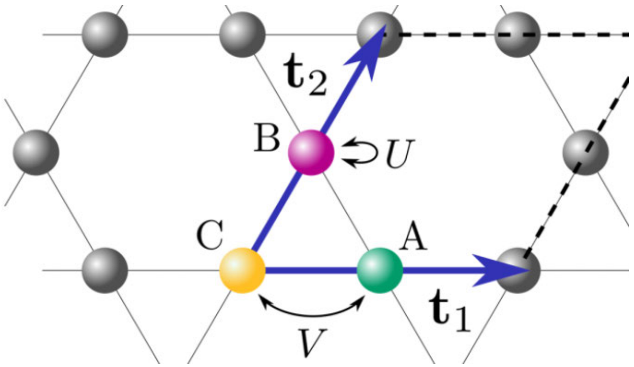
Spin-fluctuation pairing

Astrid Tranum Rømer



Morten Holm Christensen

- Band structure+onsite/NN interactions



$$\mathcal{H}_{\mathrm{U}} = U \sum_{i\mu s} n_{\mu,i,s} n_{\mu,i,\bar{s}}$$

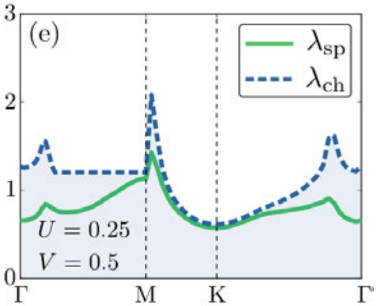
$$\mathcal{H}_{\mathrm{V}} = V \sum_{\substack{ij \\ \mu s s'}} n_{\mu,i,s} n_{\overline{\mu},j,s'}$$

- What are the leading superconducting instabilities?

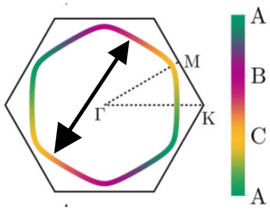
Main ingredients:

$$-\frac{\sqrt{3}}{2(2\pi)^2} \oint_{\mathrm{FS}} d\mathbf{k}'_f \frac{1}{|v(\mathbf{k}'_f)|} \Gamma_{s/t}(\mathbf{k}_f, \mathbf{k}'_f) \Delta(\mathbf{k}'_f) = \lambda \Delta(\mathbf{k}_f)$$

$$\Gamma_{s/t}(n_1\mathbf{k}, n_2\mathbf{k}') = \sum_{\{s\}} [\mathcal{V}(n_1\mathbf{k}, n_2\mathbf{k}')]^{s\bar{s}}_{\bar{s}s} \mp [\mathcal{V}(n_1\mathbf{k}, n_2\mathbf{k}')]^{ss}_{\bar{s}\bar{s}}$$



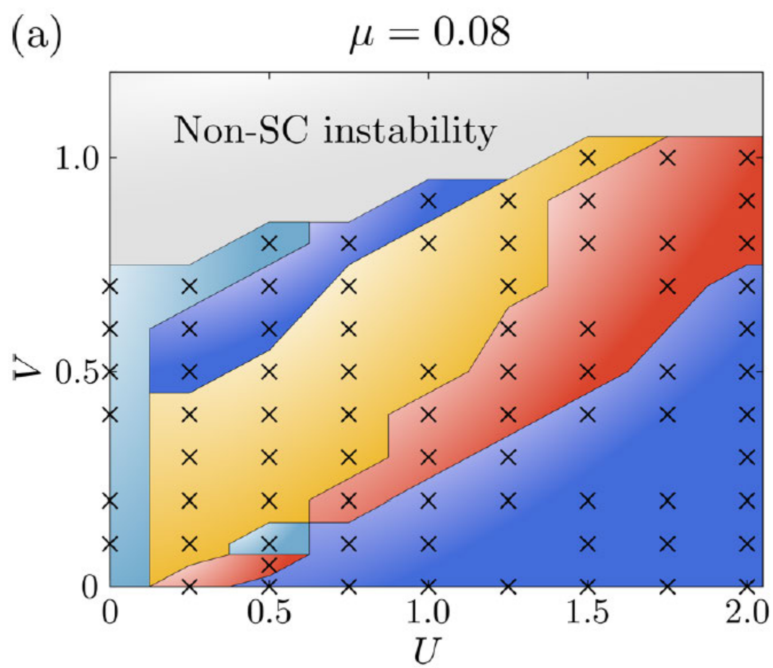
Spin and charge susceptibilities: dominated by v. Hove singularities and sublattice interference

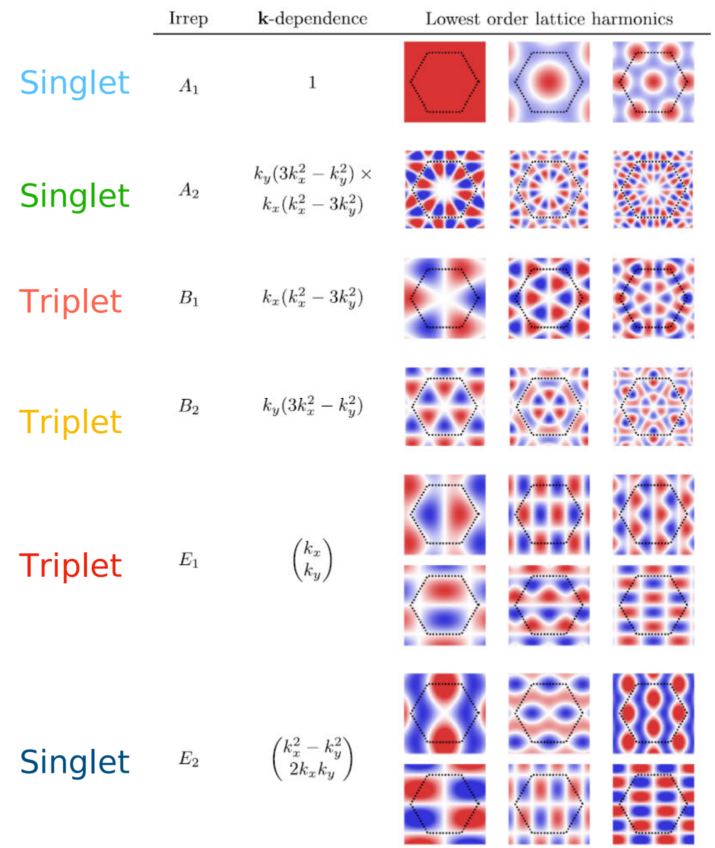


Perfect nesting, but sublattice mismatch

Phase diagram

- C_{6v}



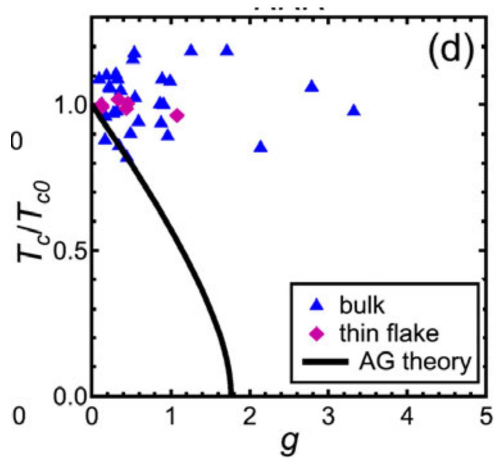


- Which state is realized?
- Experimental verifications?
  - TRSB in SC state (two component order parameter?)
  - Sign change from disorder effects?

Hanbin Deng, et al., arXiv:2408.02898 (2024)

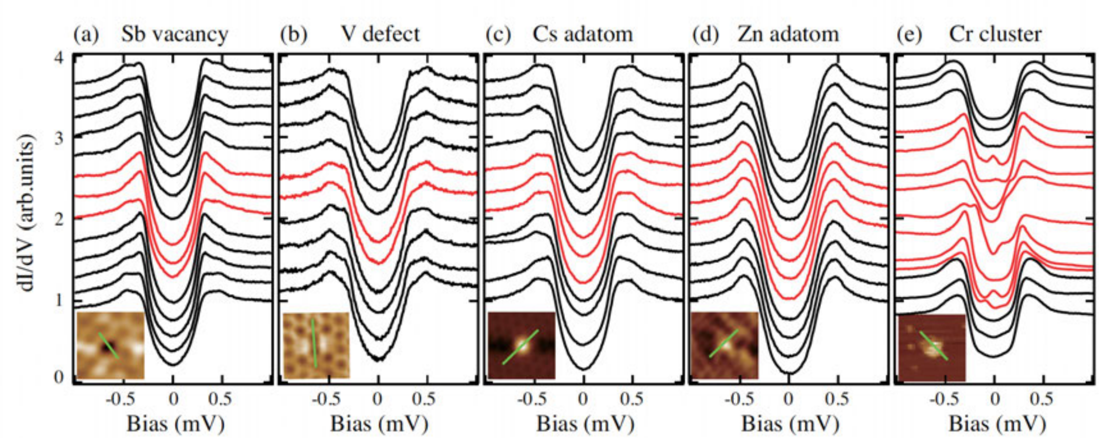
Kagome materials: Conventional or unconventional?

- T_c suppression due to disorder?



Zhang, et al., Nano Lett. **23**, 872 (2023)

- Bound states at nonmagnetic impurities?



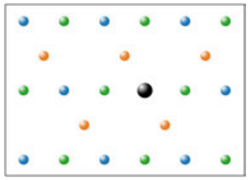
Are kagome superconductors conventional?

Xu, et al, PRL **127**, 187004 (2021)

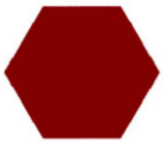
The case of the Kagome SC

- Open question: How does disorder affect SC in the Kagome system?
- Answer depends on
  - Nature of the defects (“symmetry”)
  - Superconducting phase

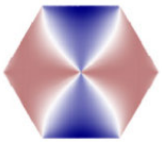
$$H_{\mathrm{imp}} = V\tau^z \otimes \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



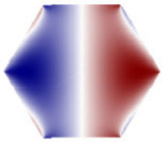
A_g



$E_{2g}^{(1)}$



$E_{2u}^{(1)}$



No subgap states if $\left\{\hat{h}_0^{\mathrm{BdG}} + \hat{h}_{\mathrm{imp}}^{\mathrm{BdG}}, \hat{h}_{\Delta}^{\mathrm{BdG}}\right\} = 0$

Generalized Anderson’s theorem:
M. Scheurer PhD thesis (2016)

A bit of theory details

Sofie Castro Holbæk

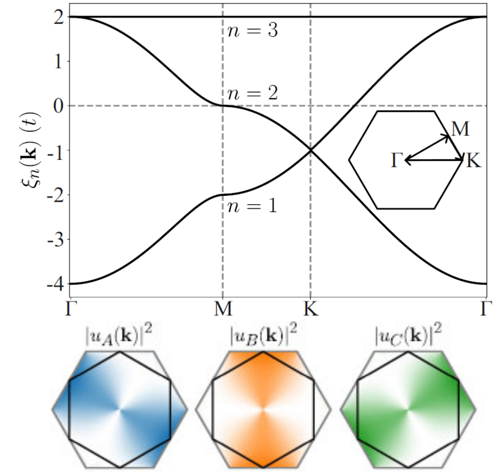
- T-matrix approach

$$\widehat{G}(\mathbf{r},\mathbf{r}',\omega) = \widehat{G}^{(0)}(\mathbf{r}-\mathbf{r}',\omega) + \widehat{G}^{(0)}(\mathbf{r},\omega)\widehat{T}(\omega)\widehat{G}^{(0)}(-\mathbf{r}',\omega)$$

$$\widehat{T}(\omega) \equiv \widehat{D}^{-1}(\omega)\widehat{H}_{\text{imp}}$$

$$\widehat{D}(\omega) \equiv [\mathbb{1} - \widehat{H}_{\text{imp}}\widehat{G}^{(0)}(\mathbf{0},\omega)].$$

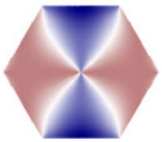
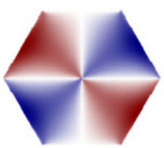
- Multiband electronic structure



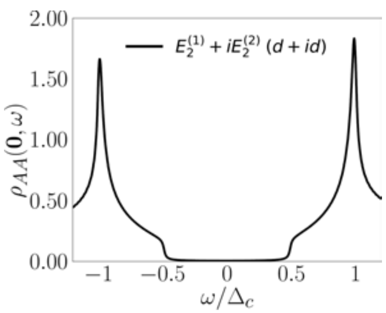
$$\widehat{H}_{\text{BdG}}(\mathbf{k}) = \begin{pmatrix} h(\mathbf{k}) - \mu\mathbb{1} & -\Delta(\mathbf{k}) \\ -\Delta(\mathbf{k})^{\dagger} & -h(-\mathbf{k})^{T} + \mu\mathbb{1} \end{pmatrix}$$

- Local anomalous Green function

$$\widehat{G}^{(0)}(\mathbf{k},\omega) = [(\omega + i\eta)\mathbb{1} - \widehat{H}_{\text{BdG}}(\mathbf{k})]^{-1}$$

 $+i$ 

$$\mathrm{d}_{xy} + id_{x^2-y^2}$$

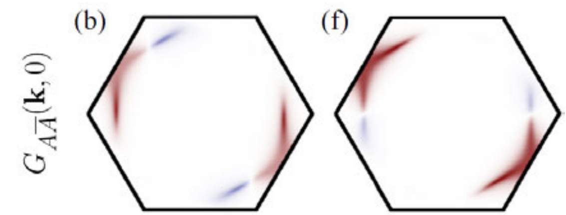


$$E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}^{s/t}|^2}.$$

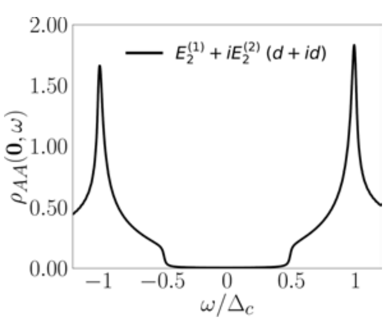
Results: d+id

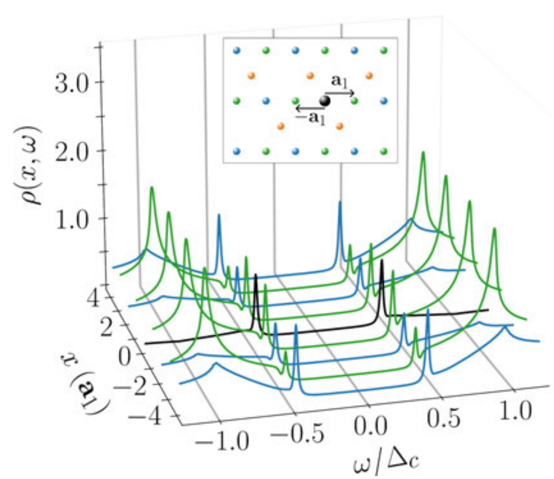
- Denominator of T-matrix $\det[\widehat{D}(\omega)] = 0$

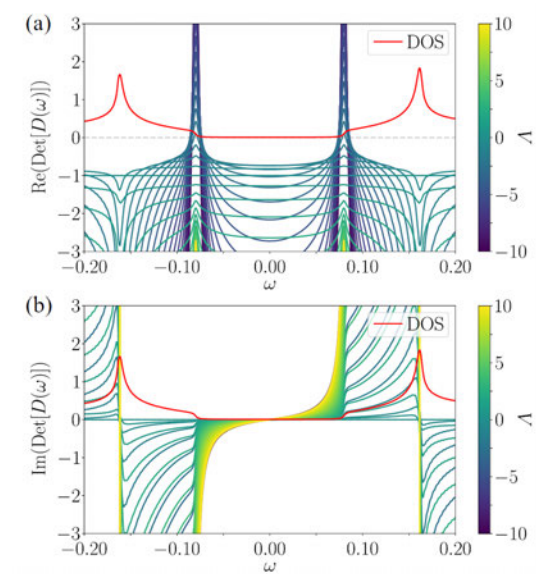
$$V^2[G^{(0)}_{AA}(\omega)G^{(0)}_{\bar{A}\bar{A}}(\omega) - G^{(0)}_{A\bar{A}}(\omega)G^{(0)}_{\bar{A}A}(\omega)] + V[G^{(0)}_{AA}(\omega) - G^{(0)}_{\bar{A}\bar{A}}(\omega)] - 1 = 0.$$



- Important quantity: $G^{(0)}_{A\bar{A}}(\omega) \neq 0$
- Absence of in-gap bound states!

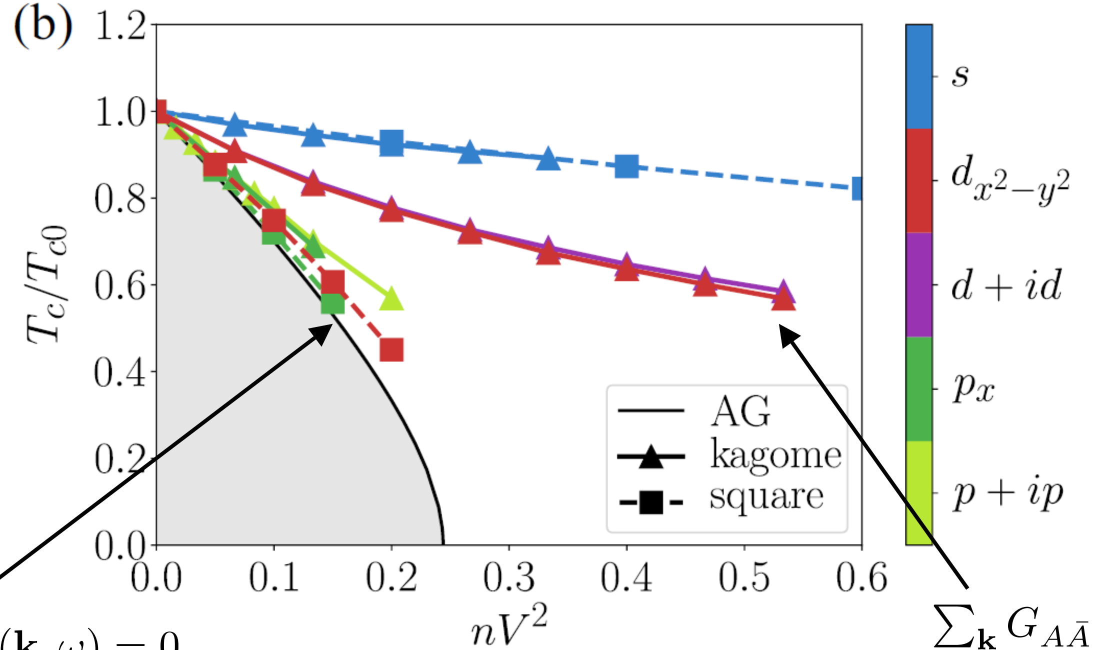


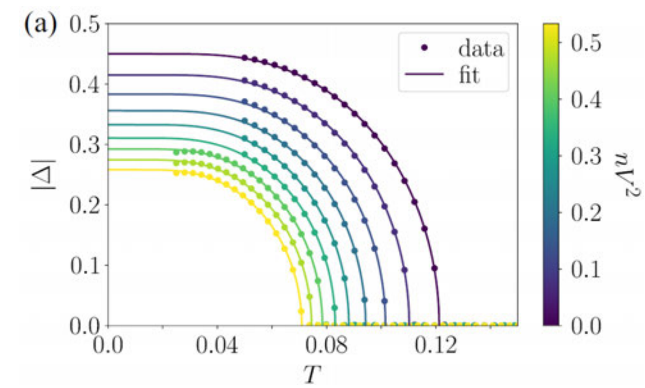




What about T_c suppression?

- Calculate T_c within Abrikosov-Gorkov theory





$$\widehat{\Sigma}(i\omega_n) = \frac{nV^2}{N} \sum_{\mathbf{k}} \widehat{h}_{\text{imp}} \widehat{G}^{(0)}(\mathbf{k}, i\omega_n) \widehat{h}_{\text{imp}}$$

$$\sum_{\mathbf{k}} G_{A\bar{A}}(\mathbf{k},\omega) = 0$$
$$\sum_{\mathbf{k}} \Delta(\mathbf{k}) = 0$$

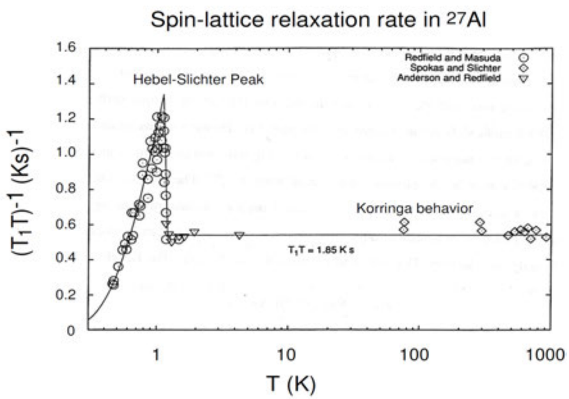
$$\sum_{\mathbf{k}} G_{A\bar{A}}(\mathbf{k},\omega) \neq 0$$
$$\sum_{\mathbf{k}} \Delta(\mathbf{k}) = 0$$

- sign-changing spin-singlet superconductivity is robust* to non-magnetic point-like disorder
  $\longrightarrow$ cannot exclude unconventional SC

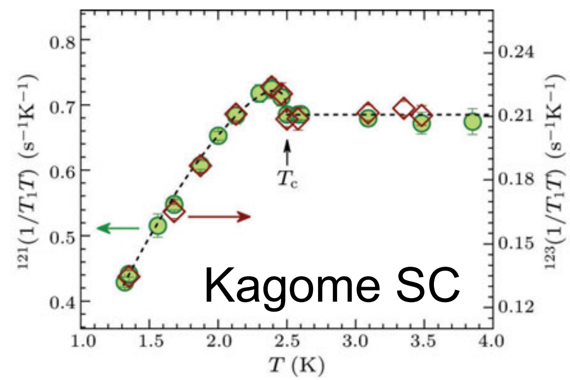
*Robust = no in-gap bound states + weak T_c suppression rate

NMR spin relaxation rate?

- Are kagome superconductors unconventional?



L. C. Hebel and C. P. Slichter Phys. Rev. **113**, 1504 (1959)

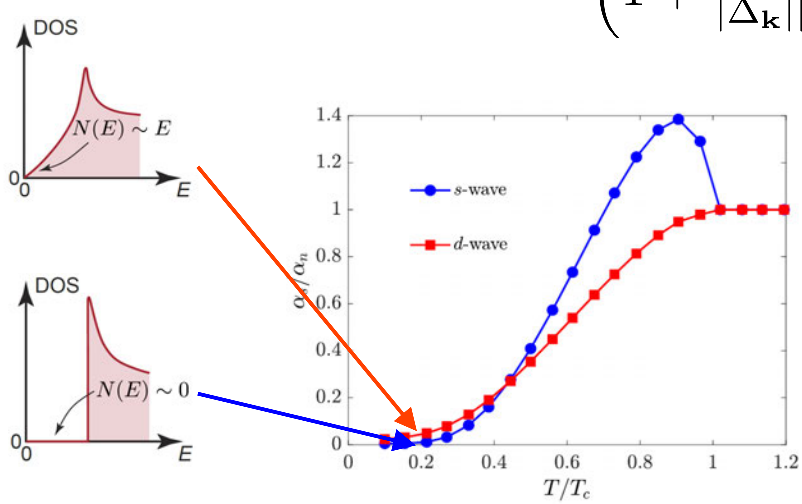


Chao Mu, et al., Chinese Phys. Lett. **38** 077402 (2021)

- What can we learn?

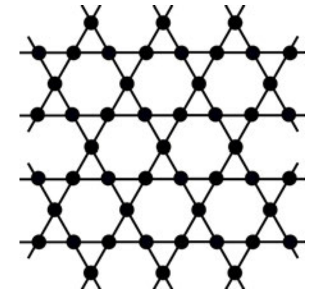
$$\left(1+\frac{\Delta_{\mathbf{k}+\mathbf{q}}\Delta_{\mathbf{k}}}{|\Delta_{\mathbf{k}}||\Delta_{\mathbf{k}+\mathbf{q}}|}\right)\approx\begin{cases}2 & \text{same sign}\\ 0 & \text{different sign}\end{cases}$$

1) Low energy quasiparticles for T→ limit



2) sign change in from Hebel Slichter peak below T_c

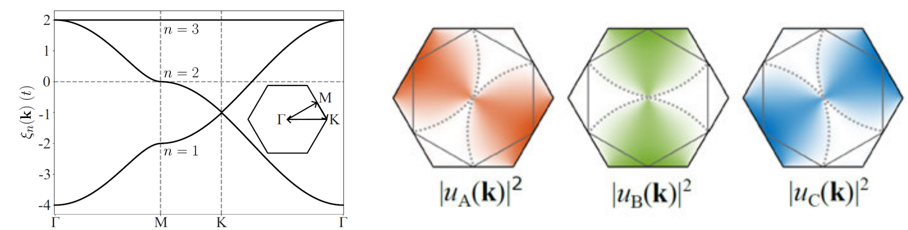
Hebel-Slichter peak



- Multiband system: sublattice interference

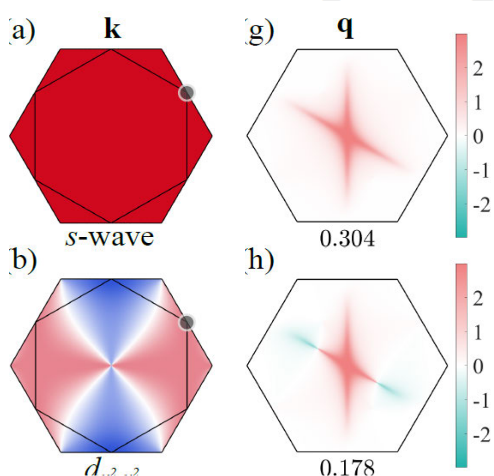
$$\chi_0^{+-}(\mathbf{q},\omega) = \frac{1}{\mathcal{N}}\sum_{\mathbf{k},E>0}\Bigg[\left(1-\frac{\xi_\mathbf{k}\xi_{\mathbf{k+q}}+\Delta^*_{\mathbf{k+q}}\Delta_\mathbf{k}}{E_\mathbf{k}E_{\mathbf{k+q}}}\right)\frac{1-f(E_\mathbf{k})-f(E_{\mathbf{k+q}})}{\omega+E_{\mathbf{k+q}}+E_\mathbf{k}+i\eta}+\left(1-\frac{\xi_\mathbf{k}\xi_{\mathbf{k+q}}+\Delta^*_{\mathbf{k+q}}\Delta_\mathbf{k}}{E_\mathbf{k}E_{\mathbf{k+q}}}\right)\frac{f(E_\mathbf{k})+f(E_{\mathbf{k+q}})-1}{\omega-E_{\mathbf{k+q}}-E_\mathbf{k}+i\eta}$$
$$+\left(1+\frac{\xi_\mathbf{k}\xi_{\mathbf{k+q}}+\Delta^*_{\mathbf{k+q}}\Delta_\mathbf{k}}{E_\mathbf{k}E_{\mathbf{k+q}}}\right)\frac{f(E_\mathbf{k})-f(E_{\mathbf{k+q}})}{\omega+E_{\mathbf{k+q}}-E_\mathbf{k}+i\eta}+\left(1+\frac{\xi_\mathbf{k}\xi_{\mathbf{k+q}}+\Delta^*_{\mathbf{k+q}}\Delta_\mathbf{k}}{E_\mathbf{k}E_{\mathbf{k+q}}}\right)\frac{f(E_{\mathbf{k+q}})-f(E_k)}{\omega+E_\mathbf{k}-E_{\mathbf{k+q}}+i\eta}\Bigg]\sum_{\alpha\beta}g_{\alpha\beta}(\mathbf{k},\mathbf{q}),$$

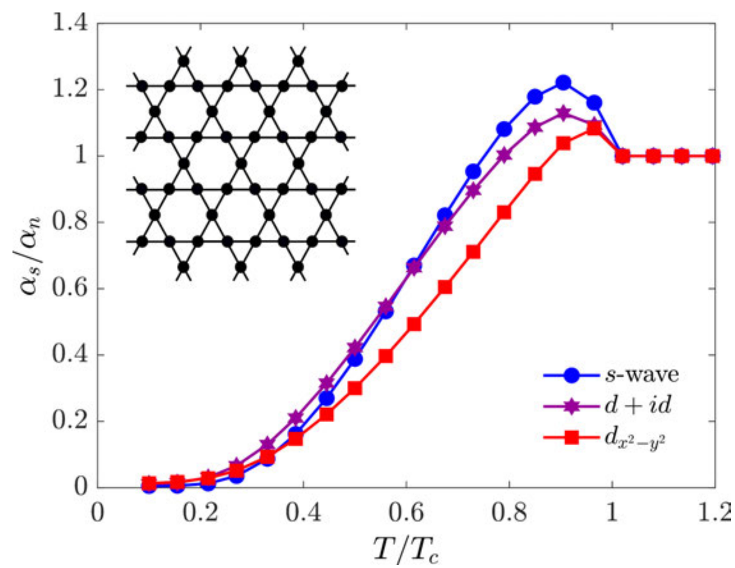
$$g_{\alpha\beta}(\mathbf{k},\mathbf{q}) = u_{2\alpha}(\mathbf{k}+\mathbf{q})u_{2\beta}(\mathbf{k}+\mathbf{q})u_{2\alpha}(\mathbf{k})u_{2\beta}(\mathbf{k})$$



- Relevant quantity

$$B_\mathrm{d}(\mathbf{q},\mathbf{k}_n) = \frac{\sum_{\alpha\beta} g_{\alpha\beta}(\mathbf{k}_n,\mathbf{q})}{Z}\,\frac{\Delta^*_{\mathbf{k}_n+\mathbf{q}}\Delta_{\mathbf{k}_n}}{E_{\mathbf{k}_n}E_{\mathbf{k}_n+\mathbf{q}}}$$

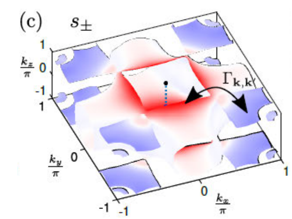




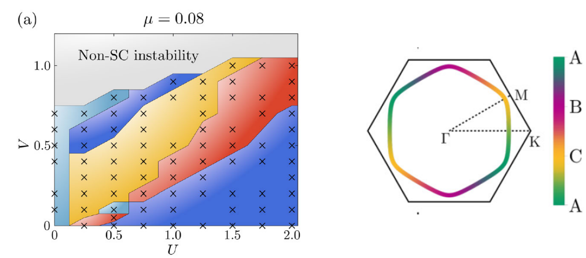
Unconventional superconductivity DOES exhibit Hebel-Slichter peak

Summary (2)

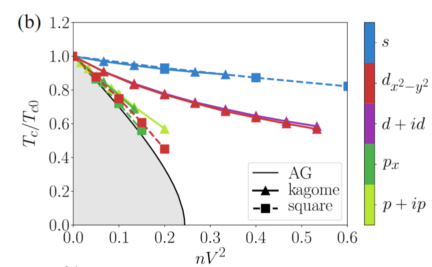
- Unconventional pairing from spin fluctuations: Interplay of Fermi surface and structure of susceptibility
  - Fe-based superconductors: multiband, dominating s+- state
  - Kagome superconductors: rich phase diagram
- Unconventional Kagome superconductors: two surprises
  - Robust against disorder
  - Features Hebel-Slichter peak



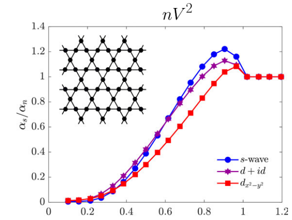
A. Kreisel, et al. Phys. Rev. Lett. **129**, 077002 (2022)



A. T. Rømer *et al.*, Phys. Rev. B **106**, 174514 (2022)



S. C. Holbæk, et al., Phys. Rev. B **108**, 144508 (2023)



Yi Dai, et al., arXiv:2404.10835 (2024)