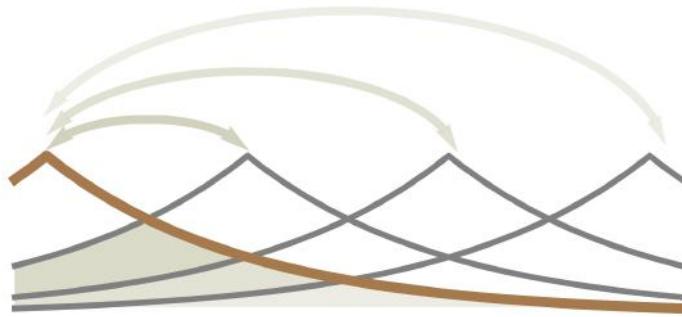


MANY-BODY LOCALIZATION



Julian Léonard
TU Wien



Jülich, 16th September 2024







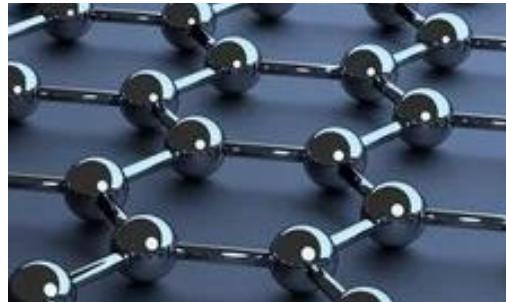
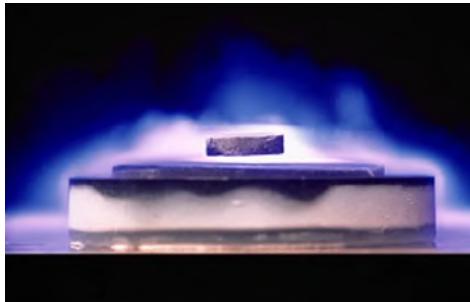
$$\begin{aligned}8\pi p_0 &= -\frac{1}{R^2} e^{-g} - \frac{d^2 g}{dt^2} - \frac{3}{4} \left(\frac{dg}{dt} \right)^2 + K \quad \left| \frac{ds^2 = e^g}{(1+g)^2} \right. \\8\pi g_{00} &= \frac{3}{R^2} e^{-g} - \frac{d^2 g}{dt^2} + \frac{3}{4} \left(\frac{dg}{dt} \right)^2 - K \quad \left. + d \right. \\8\pi (g_{00} + p_0) &= -\frac{d^2 g}{dt^2} \quad \left| \begin{array}{l} \\ \\ \end{array} \right. \\ \frac{8\theta}{RL} &= \text{count} \times \left(1 + \frac{8\lambda}{K} \right) \quad \left| \begin{array}{l} \\ \\ \end{array} \right.\end{aligned}$$



THE CHALLENGE OF MANY-BODY QUANTUM SYSTEMS

Understand and design many-body systems

One of the biggest challenges
of 21st century quantum physics



Technological relevance

- High-T_c superconductivity
- Magnetism
- Novel quantum sensors
- Quantum technologies

Fundamental interest

- Parameter changes
- Benchmark theories
- Many “simple” models not solvable
- Discern different effects

THE CHALLENGE OF MANY-BODY QUANTUM SYSTEMS

International Journal of Theoretical Physics, Vol. 21, No. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

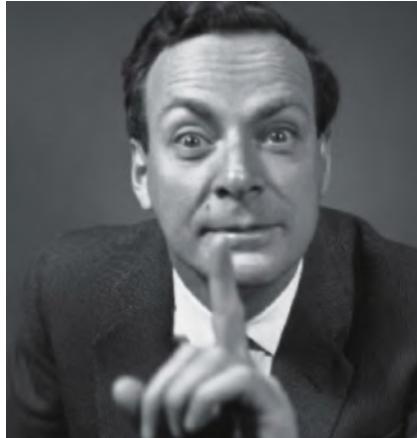
Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

I. INTRODUCTION

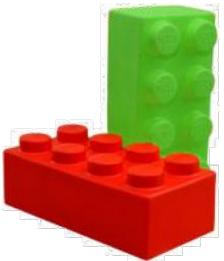
On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have my own things to say and to talk about and there's no implication that anybody needs to talk about the same thing or anything like it. So what I want to talk about is what Mike Dertouzos suggested that nobody would talk about. I want to talk about the problem of simulating physics with computers and I mean that in a specific way which I am going to explain. The reason for doing this is something that I learned about from Ed Fredkin, and my entire interest in the subject has been inspired by him. It has to do with learning something about the possibilities of computers, and also something about possibilities in physics. If we suppose that we know all the physical laws perfectly, of course we don't have to pay any attention to computers. It's interesting anyway to entertain oneself with the idea that we've got something to learn about physical laws; and if I take a relaxed view here (after all I'm here and not at home) I'll admit that we don't understand everything.

The first question is, What kind of computer are we going to use to simulate physics? Computer theory has been developed to a point where it realizes that it doesn't make any difference; when you get to a *universal computer*, it doesn't matter how it's manufactured, how it's actually made. Therefore my question is, Can physics be simulated by a universal computer? I would like to have the elements of this computer *locally interconnected*, and therefore sort of think about cellular automata as an example (but I don't want to force it). But I do want something involved with the



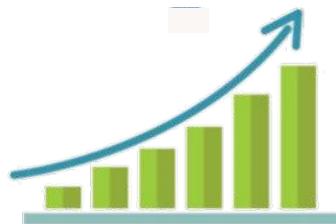
R. P. Feynman's vision
A quantum simulator to study
the properties of another
quantum system

WHICH PLATFORM?



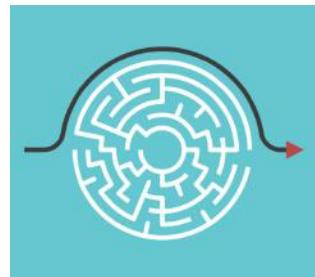
Building blocks

Identical, well understood



Scalable

Ability to connect many building blocks



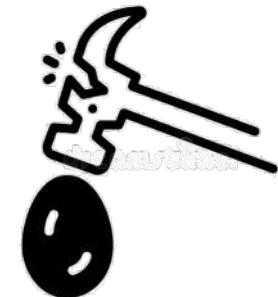
Simple

No behaviour beyond the computation



Controllable

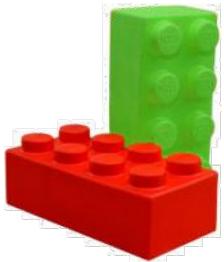
Many control parameters, precise, independent



Robust

Reproducible results, insensitive to noise

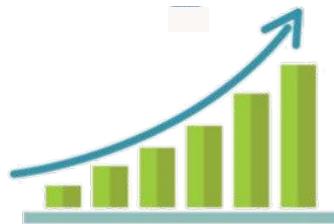
ATOMS ARE IDEAL PLATFORM



Building blocks

Identical, well understood

→ exactly identical, atomic physics highly precise ($\sim 10^{-20}$)



Scalable

Ability to connect many building blocks

→ possible to trap thousands of atoms



Simple

No behaviour beyond the computation

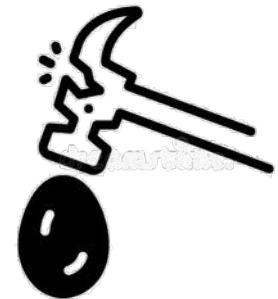
→ isolated in vacuum



Controllable

Many control parameters, precise, independent

→ optical control (motion, interactions,...)

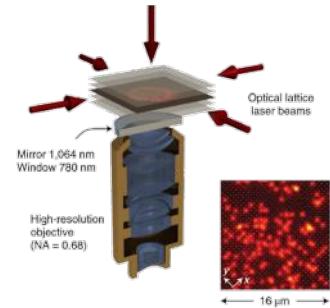
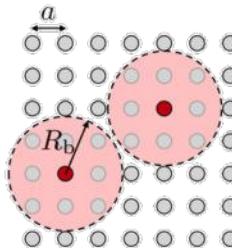
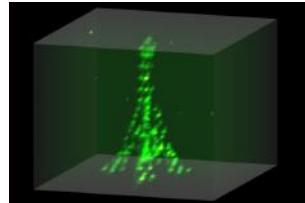
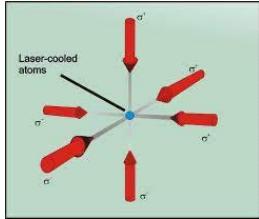


Robust

Reproducible results, insensitive to noise

→ even at microscopic level

QUANTUM SIMULATION TOOLBOX



State preparation

Laser cooling
Optical pumping
...
...

Potential control

Holographic beam shaping
Spin-dependent beams
Gauge fields

Interaction control

Feshbach resonance
Photon-mediated interactions
Dipoles, Rydberg,

State readout

Microscopy
Fluorescence imaging
Dispersive readout...

Key advantages

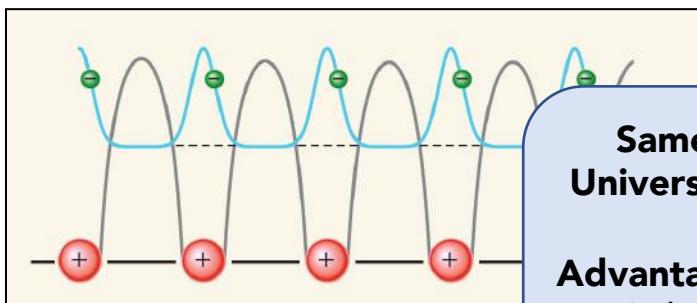
- Excellent coherence properties
- Easy to create large numbers of neutral atoms
- Easy to manipulate with light

Key challenges

- Bare atoms interact weakly
- Hard to obtain individual control over large numbers of atoms

REALIZING ELECTRONIC SYSTEMS

Electrons in a crystal

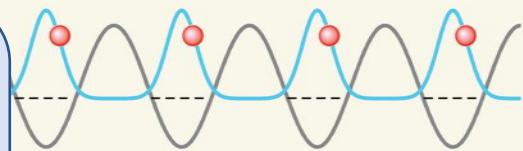


Lattice constant: $\sim \text{\AA}$

Densities: $\sim 10^{25}/\text{cm}^3$

Temperatures: $\sim \text{K}$

Atoms in an optical potential optical lattice



Lattice constant: $\sim \mu\text{m}$

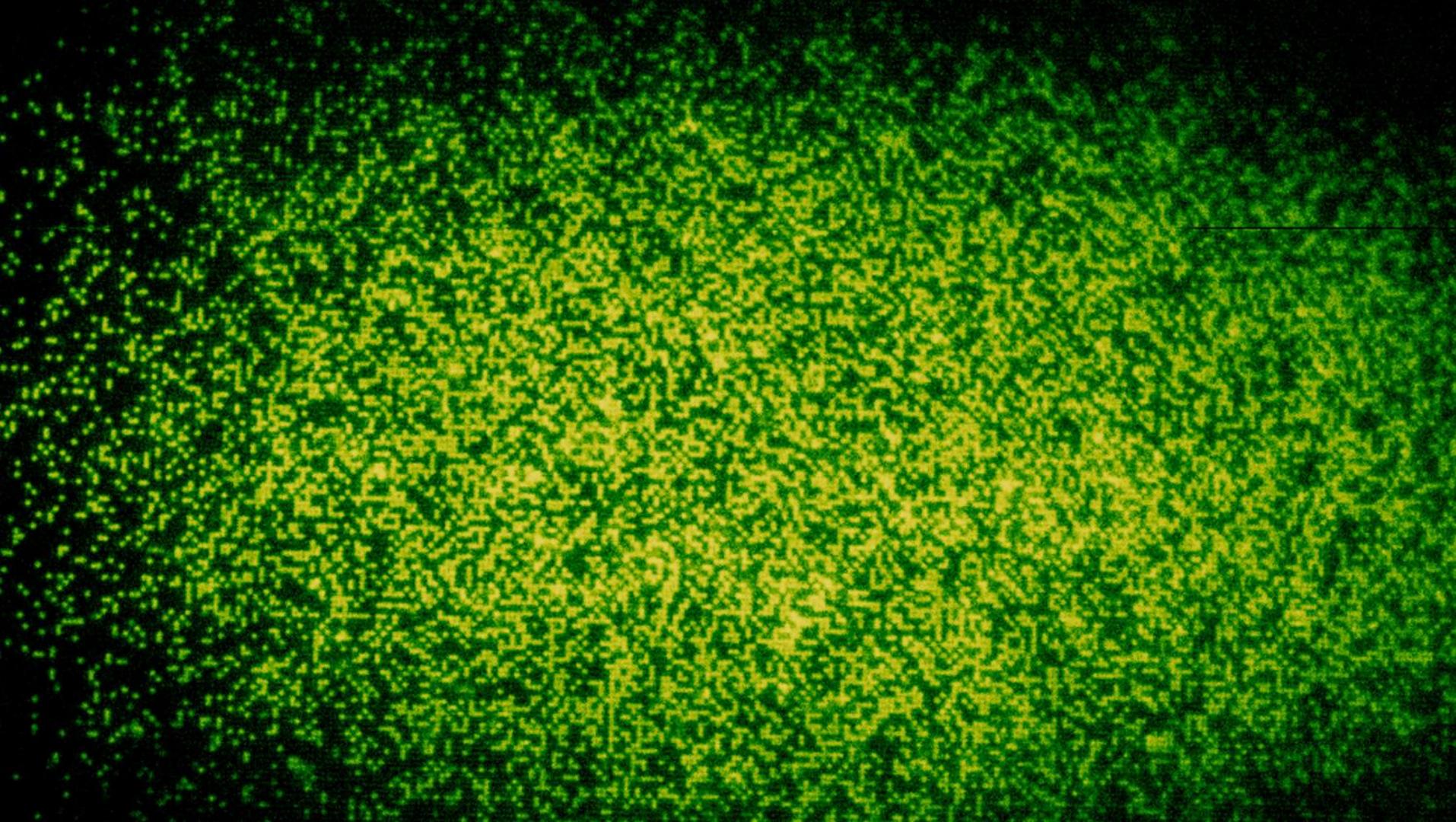
Densities: $\sim 10^{14}/\text{cm}^3$

Temperatures: $\sim \text{pK}$

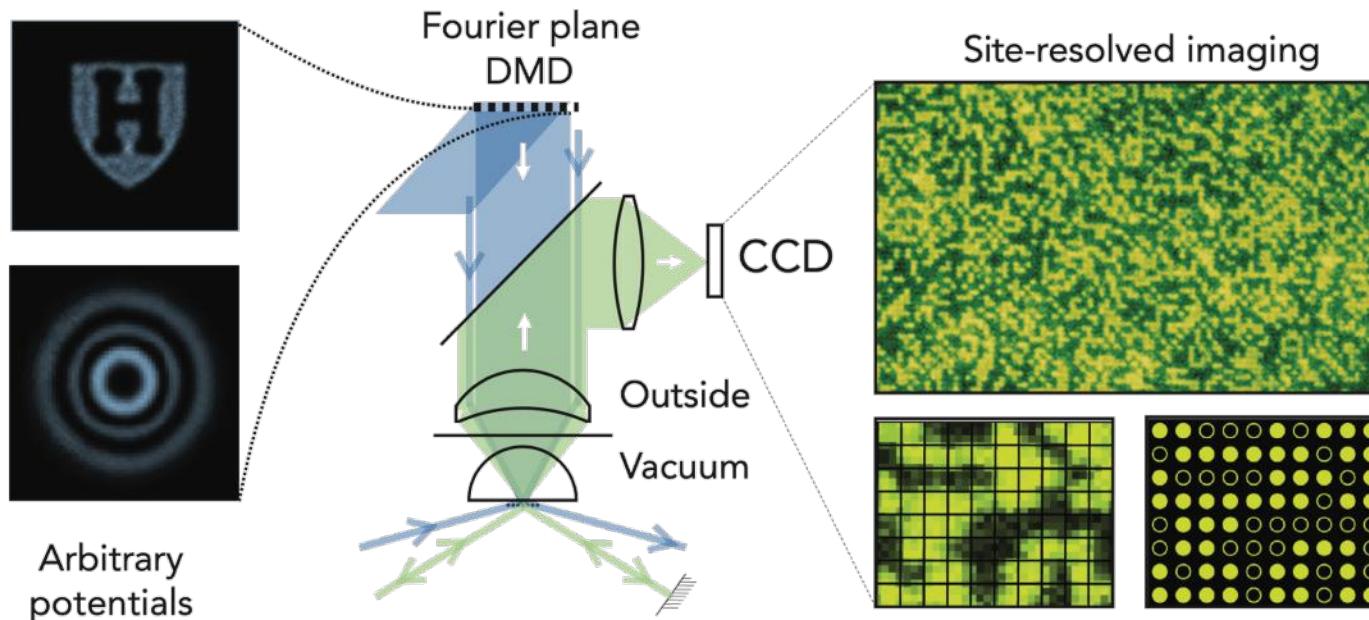
Same quantum regime: $\lambda/d > 1$
Universality of quantum mechanics!

Advantages

- Coherent: > 1000 tunneling times
- Scalable: > 1000 sites
- Fermionic and bosonic statistics
- Tunable interaction, temperature, doping, ...
- Microscopic control



QUANTUM GAS MICROSCOPY



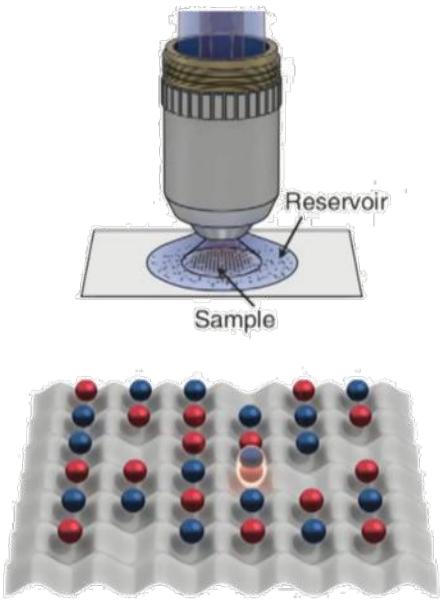
Arbitrary
potentials

State preparation:
Potential:
Interaction:
Readout:

Laser cooling, evaporative cooling
Optical standing waves
Collisional on-site interactions
Fluorescence microscopy

W. Bakr et al., *Science* **329**, 547 (2010)
J. Sherson et al., *Nature* **467**, 68 (2010)

MEASURING THE MANY-BODY STATE



Site-resolved occupation measurement

$$|\psi\rangle = \left| \begin{array}{cccc} \bullet & \circ & \circ & \\ \circ & \bullet & \circ & \\ \circ & \circ & \bullet & \\ \bullet & \circ & \circ & \end{array} \right\rangle + \left| \begin{array}{cccc} \circ & \bullet & \circ & \\ \circ & \circ & \bullet & \\ \bullet & \circ & \circ & \\ \circ & \circ & \circ & \end{array} \right\rangle + \left| \begin{array}{cccc} \bullet & \bullet & \bullet & \\ \bullet & \bullet & \bullet & \\ \bullet & \bullet & \bullet & \\ \bullet & \bullet & \bullet & \end{array} \right\rangle + \dots$$

Extendable to other observables

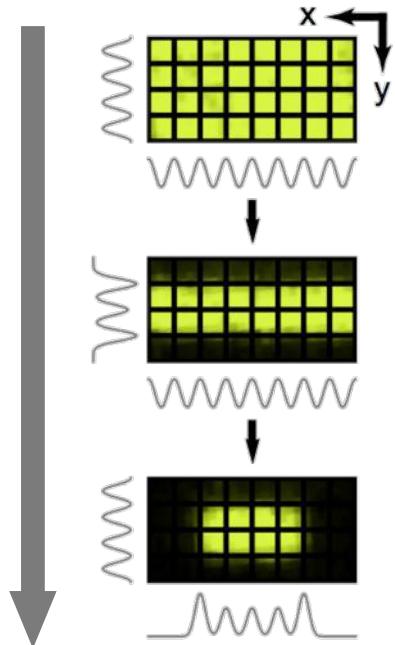
- All density correlations between sites/particles
- Local currents
- Entanglement entropy
- Quantum state purity
- ...

PURE QUANTUM STATES

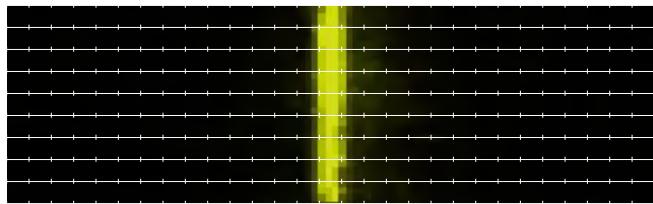
low-entropy
Mott insulator



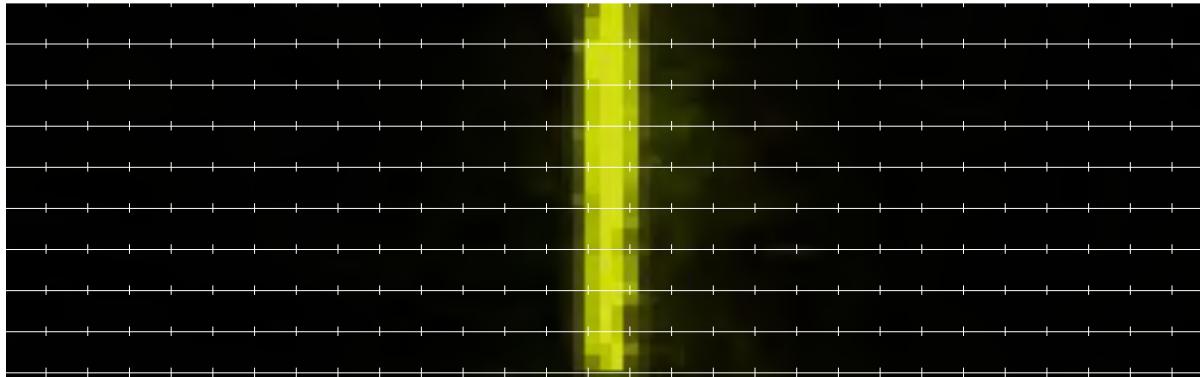
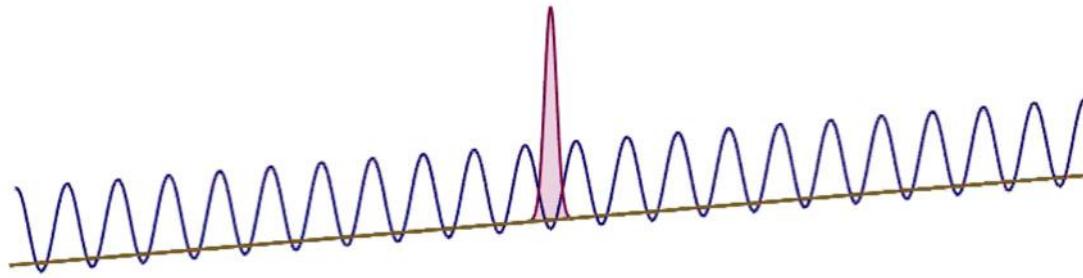
Local potentials

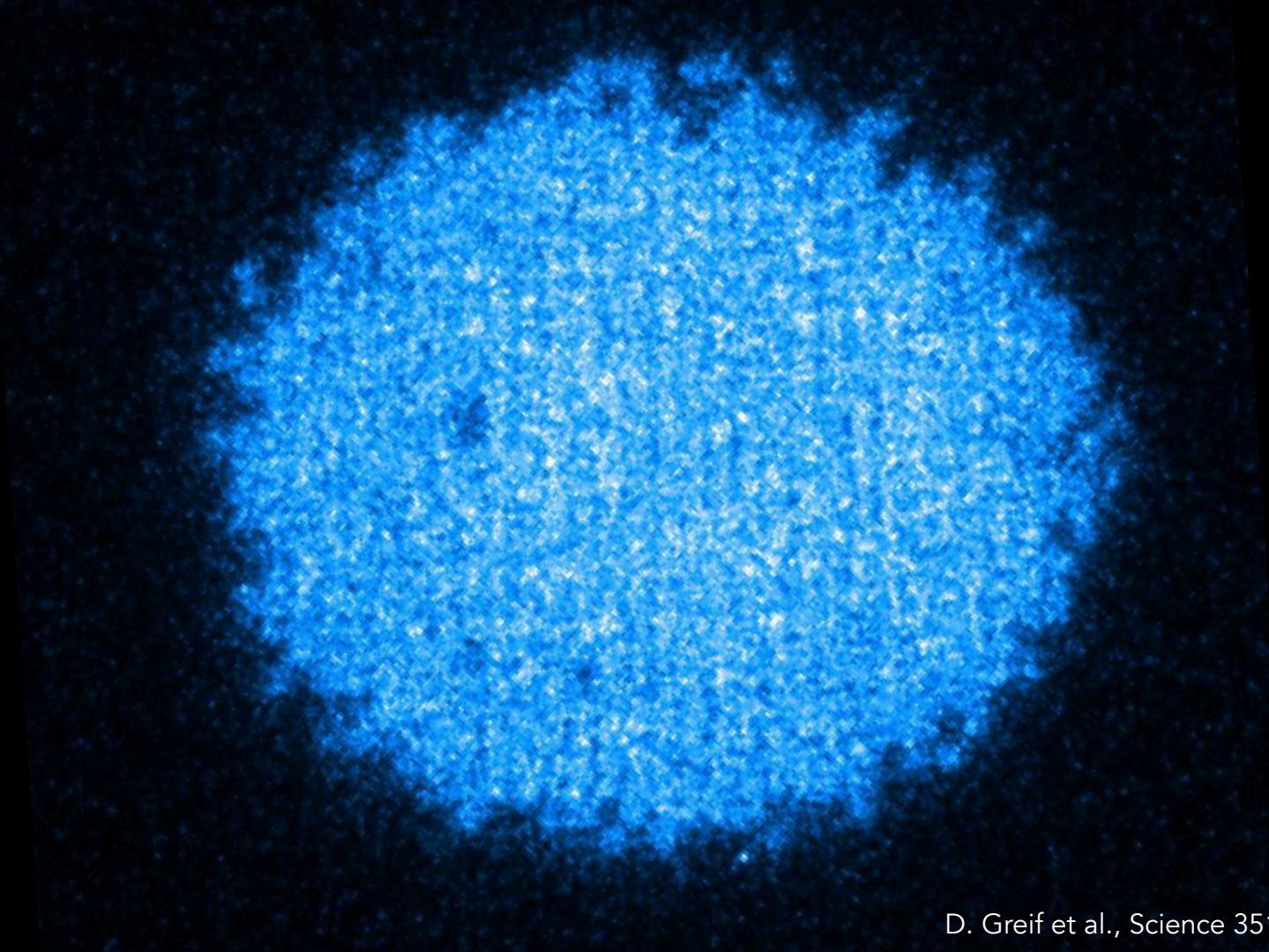


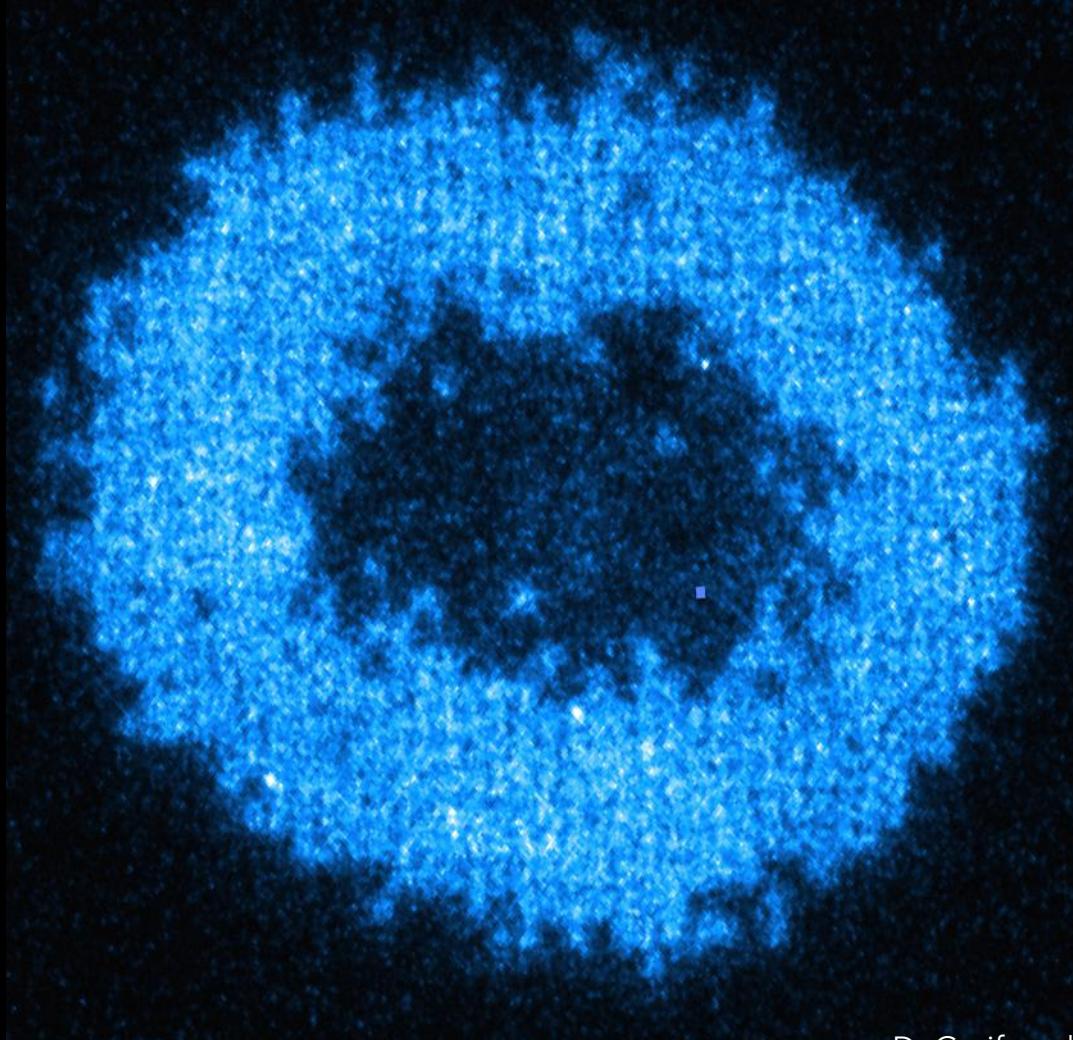
high fidelity
state preparation



SITE-RESOLVED ADDRESSING

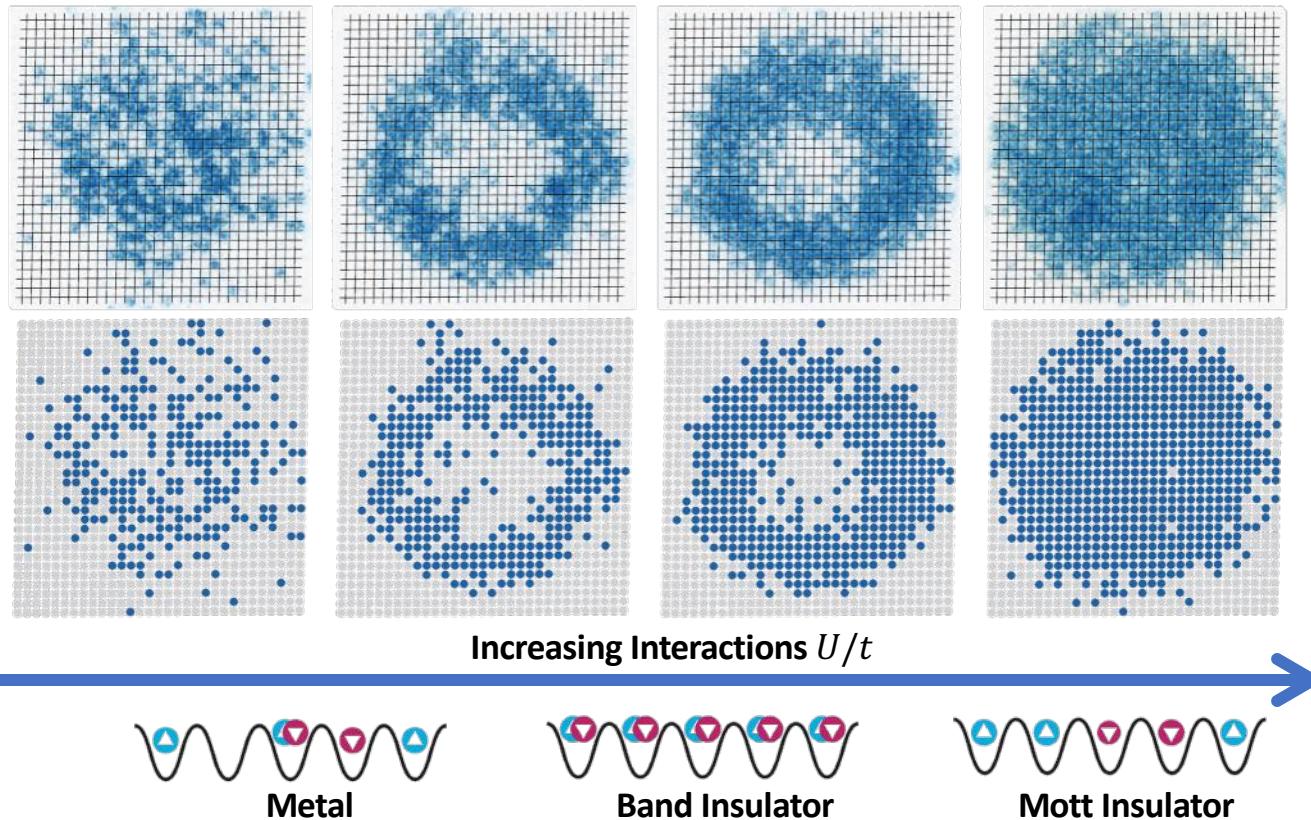




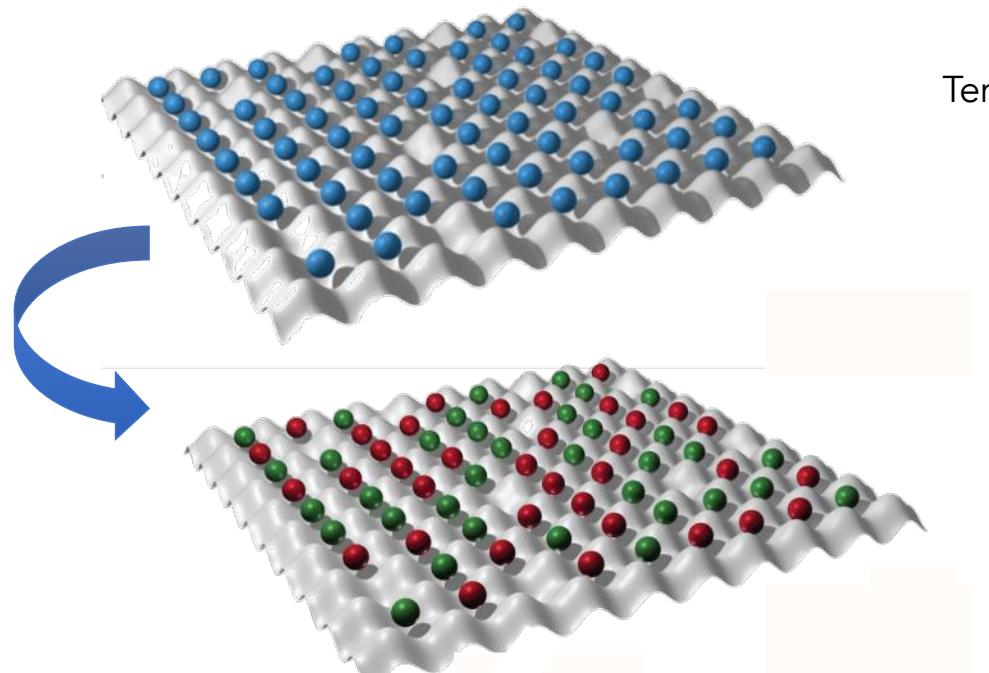


D. Greif et al., Science 351, 953 (2016)

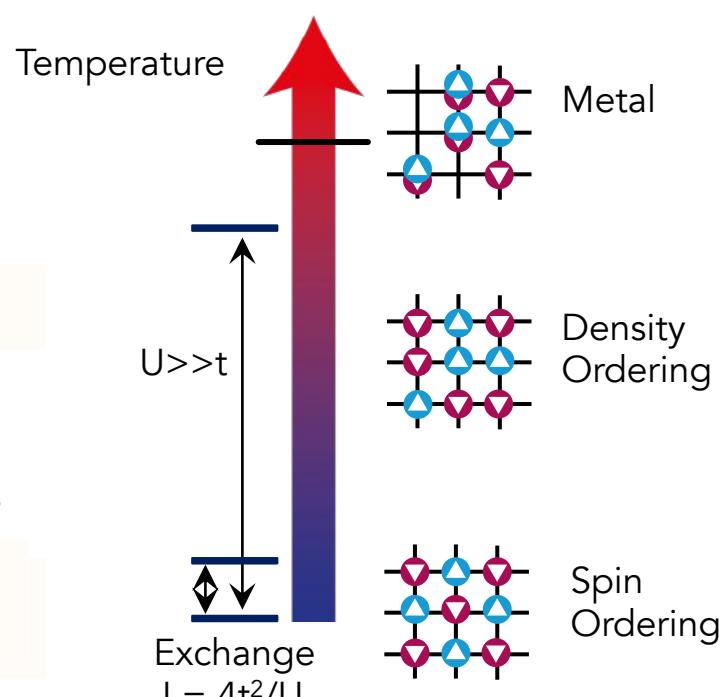
FERMIONIC HUBBARD MODEL



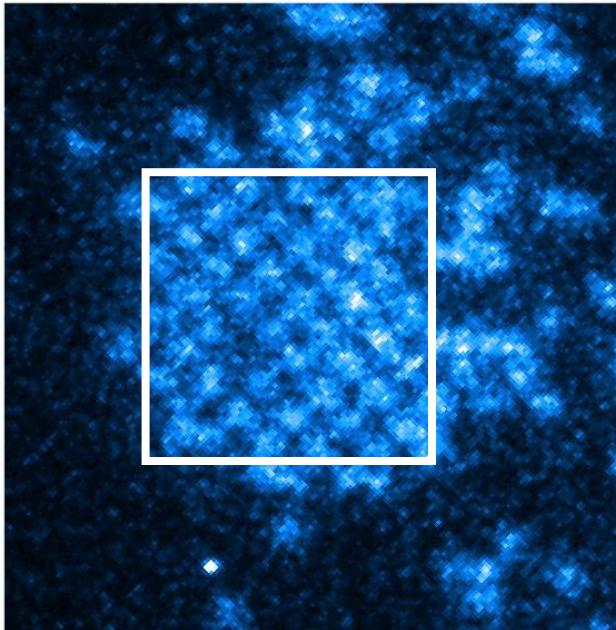
QUANTUM MAGNETISM



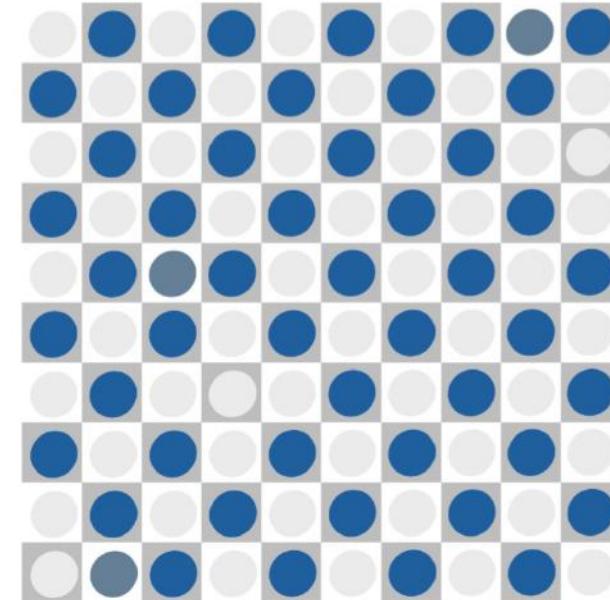
$$\hat{H} = \underbrace{-t \sum_{\langle i,j \rangle, \sigma} (c_{i,\sigma}^\dagger c_{j,\sigma} + \text{H.c.})}_{\text{tunneling}} + \underbrace{U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}}_{\text{On-site interaction}}$$



ANTIFERROMAGNETIC ORDER

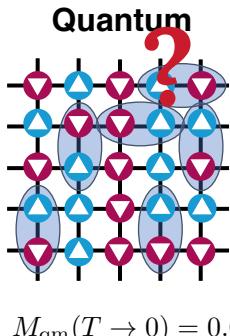
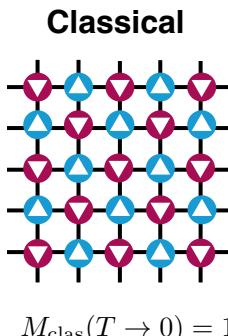
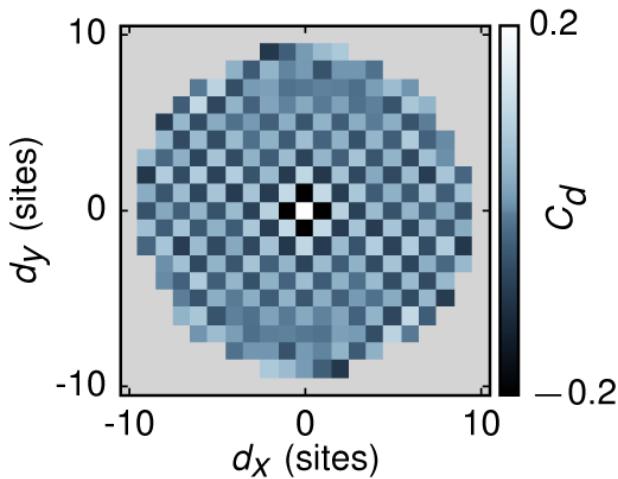


$|\uparrow\rangle$ only



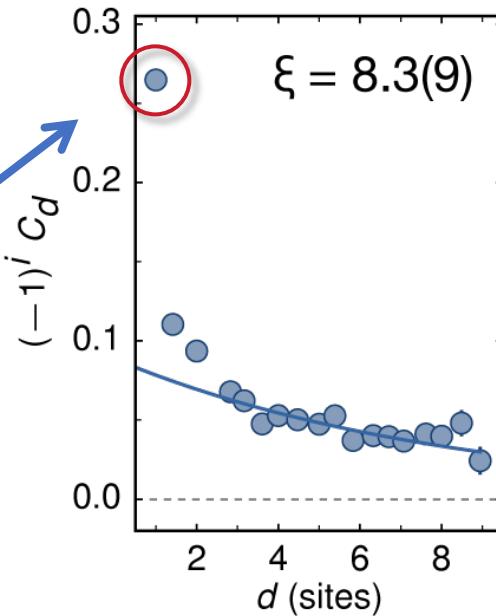
Temperature: $T/t = 0.25$
 $T/J = 0.5$

ANTIFERROMAGNETIC ORDER

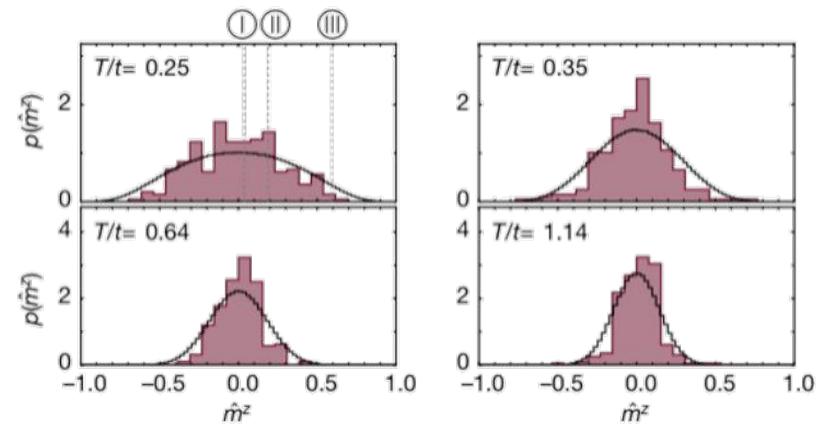
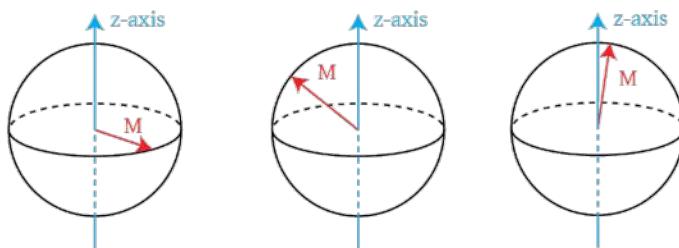
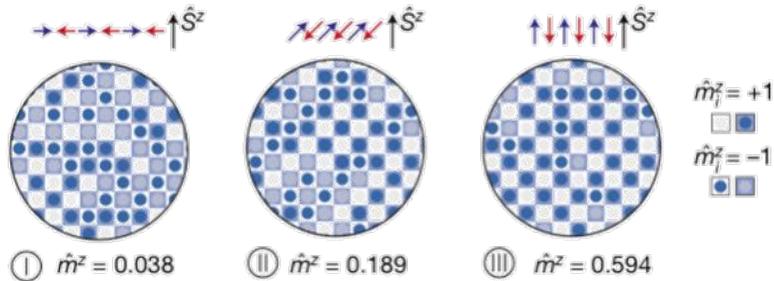


2D: exponential decay
of correlations expected

$$\langle S_i^z S_{i+d}^z \rangle \propto \exp(-d/\xi)$$

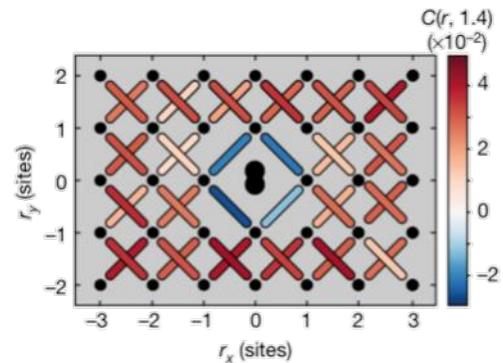
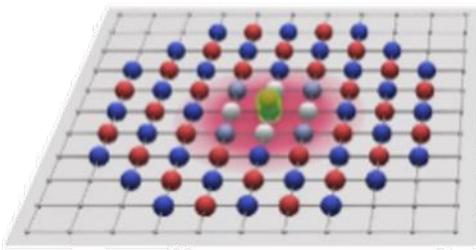
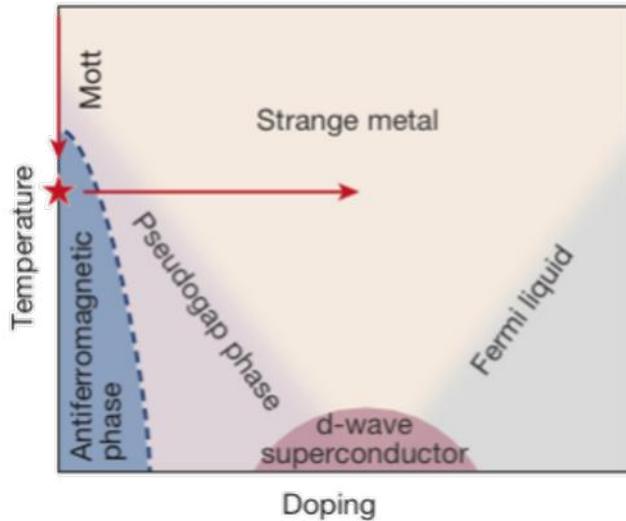


QUANTUM ANTIFERROMAGNET



Greiner group (Harvard)
A. Mazurenko et al., Nature 545, 462 (2017)

DOPING AN ANTIFERROMAGNETIC

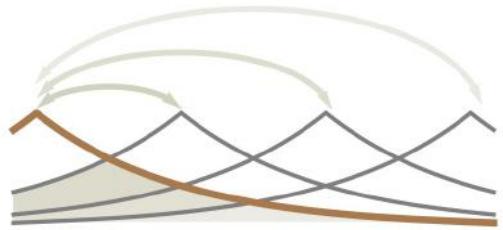


Controlled doping

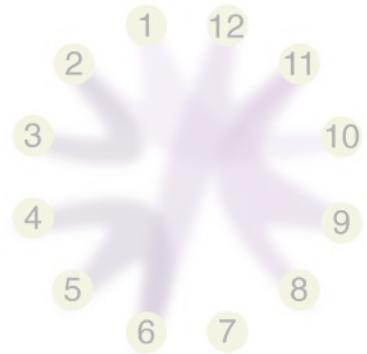
→ Gain microscopic understanding of high-T_c superconductivity

No numerics possible:
Many open questions
about the phase diagram

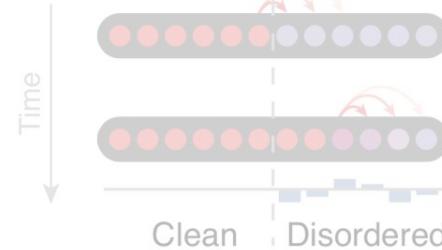
OUTLINE



**Localization and
entanglement**



MBL “transition”



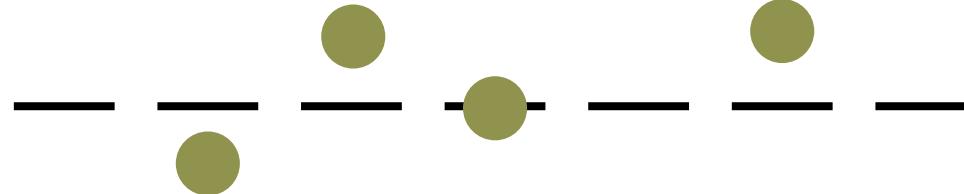
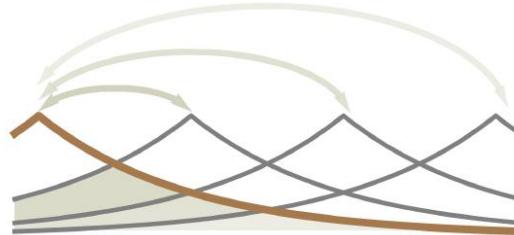
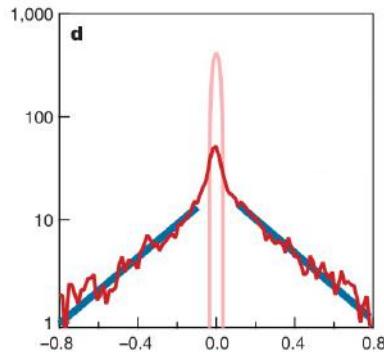
Quantum avalanches

MANY-BODY LOCALIZATION

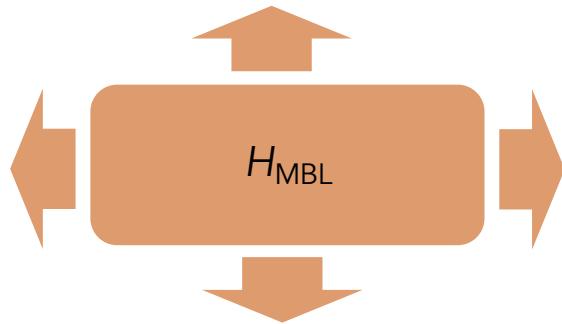
Disorder:
Anderson localization

+

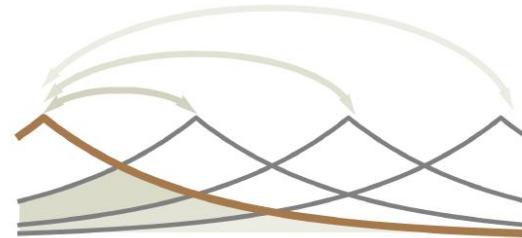
Interactions:
Many-body localization



TWO COMPETING EXPONENTIALS



Hilbert space
grows exponentially
→ thermalization,
ergodicity breaking



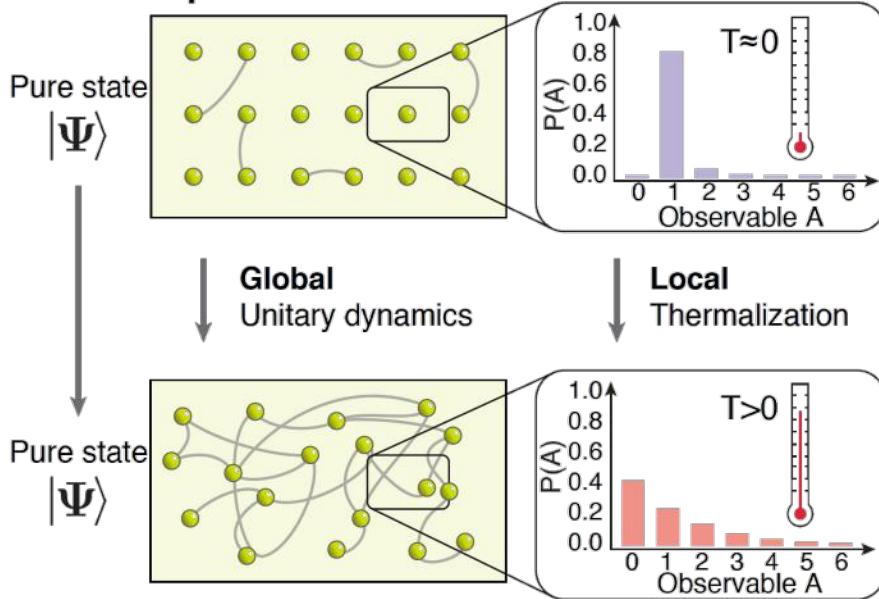
Localization
Integrals of motion,
reduces number of
degrees of freedom

Which one wins?

Basko, Aleiner, Altshuler 2006: localization wins
Also Anderson 1958, Imbrie 2014

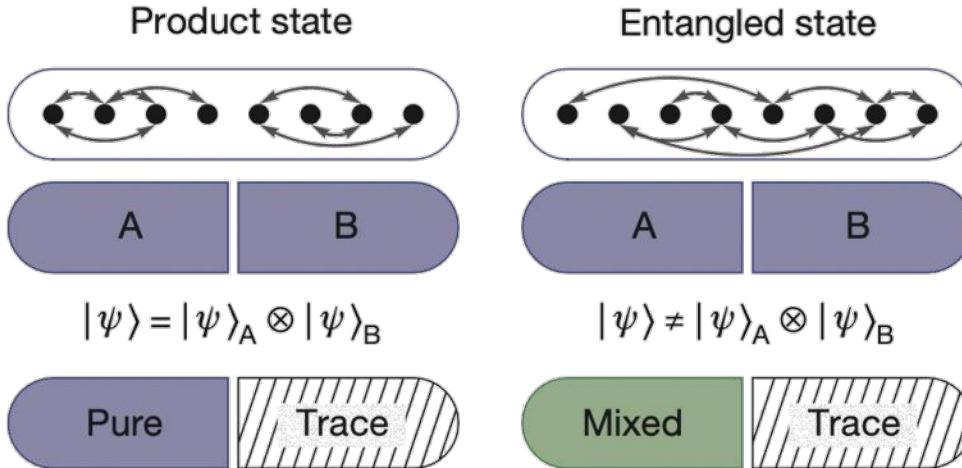
QUANTUM THERMALIZATION

Quantum quench



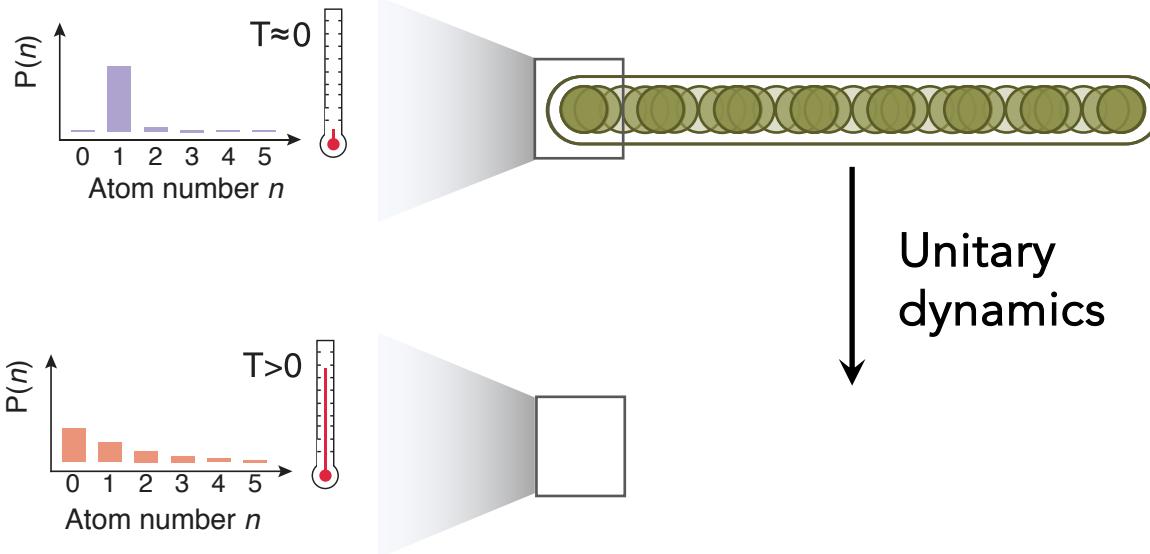
- Local quantum correlations get lost in global d.o.f
- Classical hydrodynamics of remaining slow modes
- Entanglement entropy and classical entropy become indistinguishable

MANY-BODY ENTANGLEMENT



Entanglement:
local **purity** of the quantum state

QUANTUM THERMALIZATION



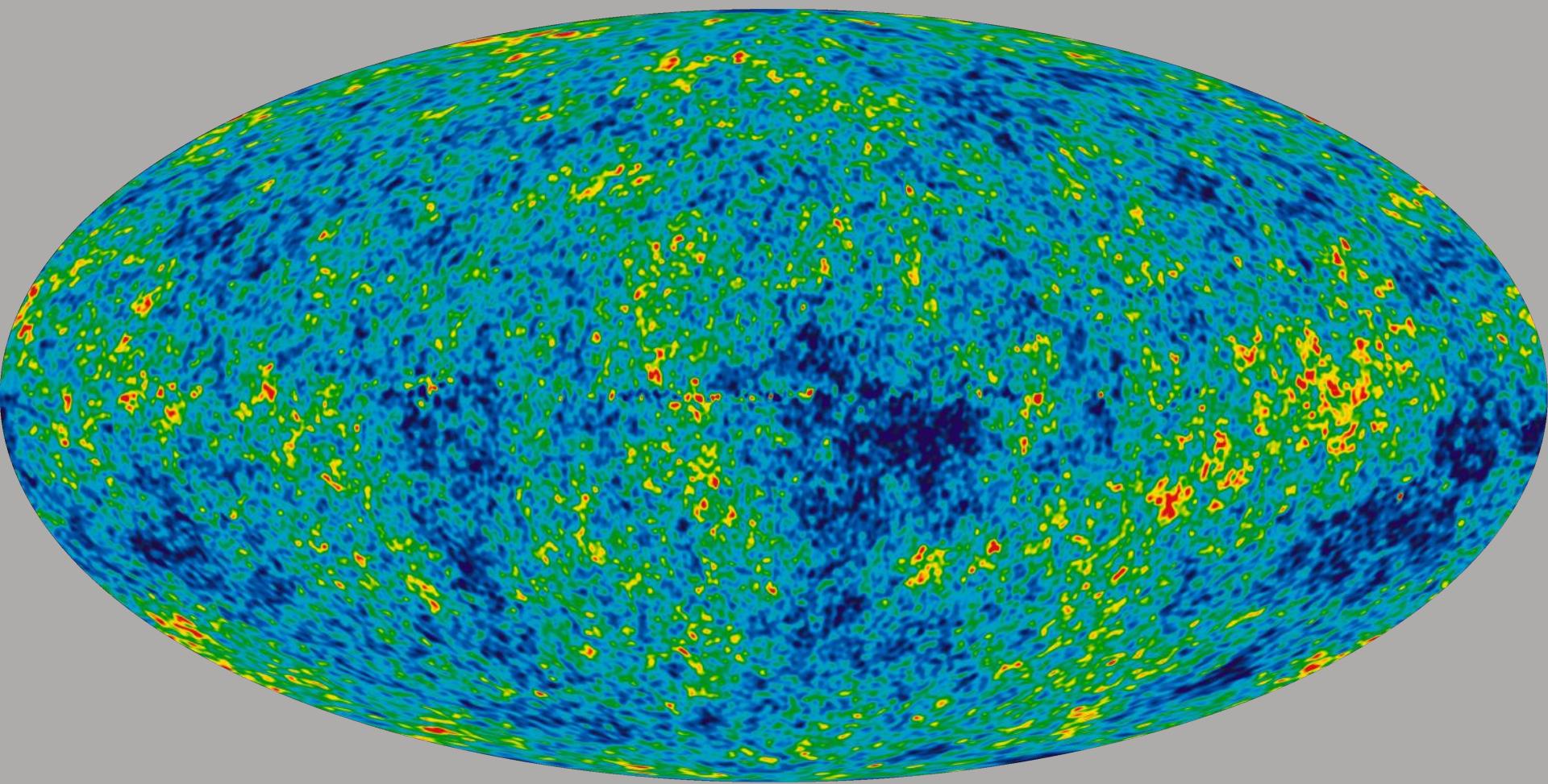
From local observables it is impossible to tell if the global state is pure.

Globally pure
Locally pure

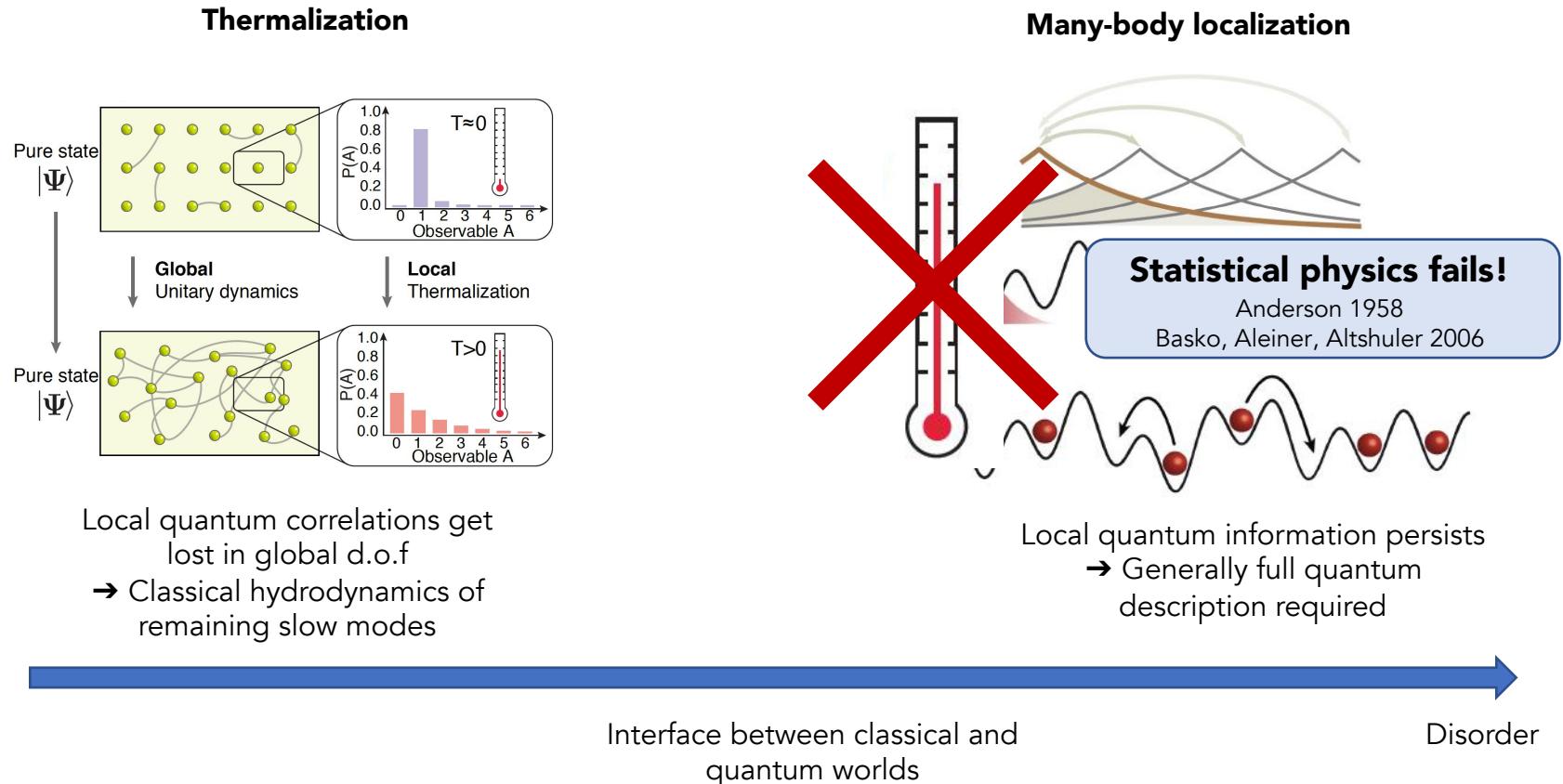
Unitary
dynamics

Globally pure
Locally thermal
(ETH)

A. Kaufman et al., Science 353, 794 (2016)
also: Schmiedmayer, Martinez, Schätz...



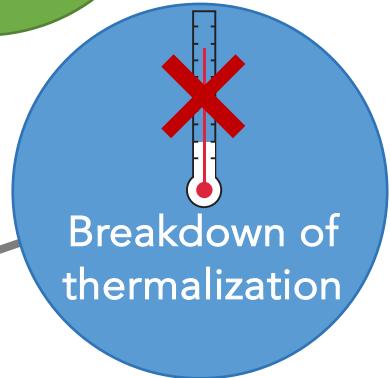
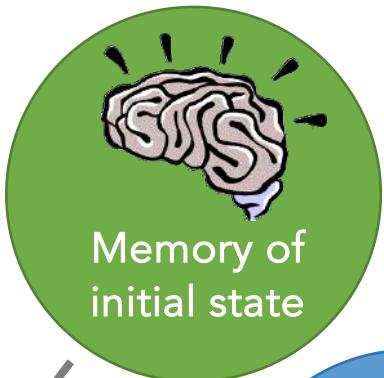
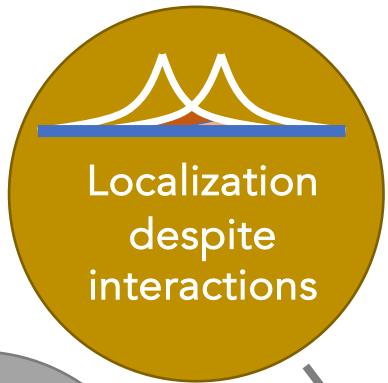
THERMALIZATION VS LOCALIZATION



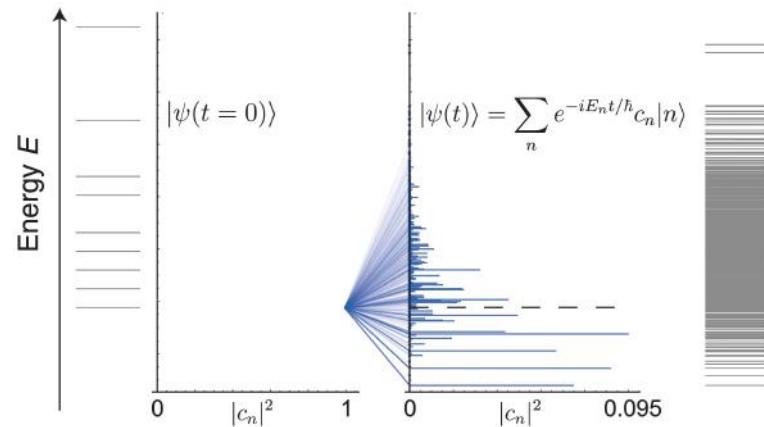
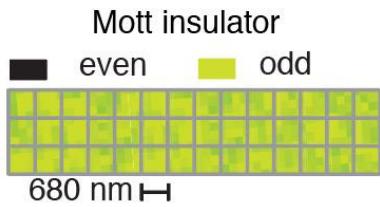
QUANTUM INFORMATION

CONDENSED MATTER

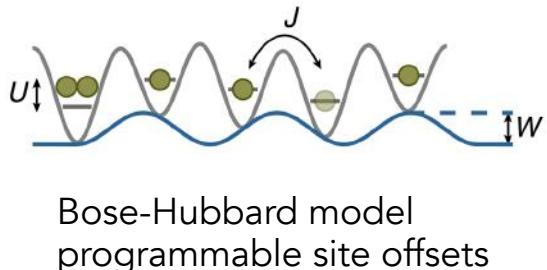
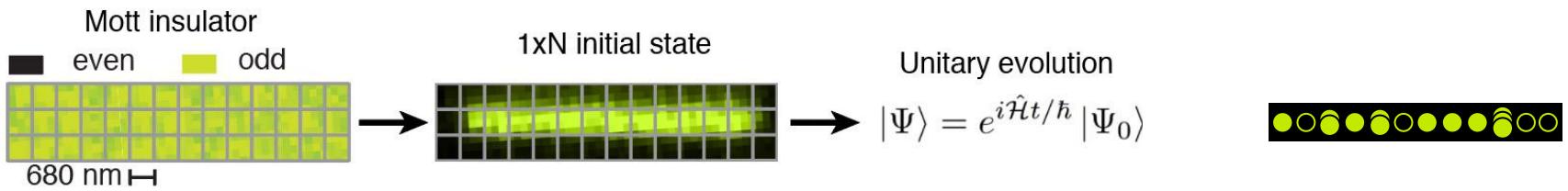
STATISTICAL PHYSICS



PROTOCOL



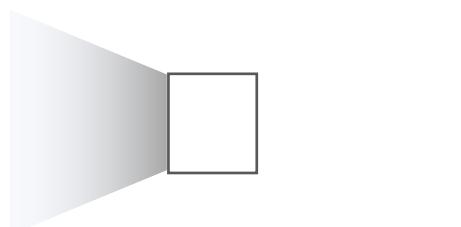
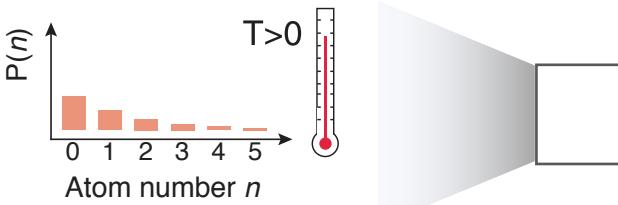
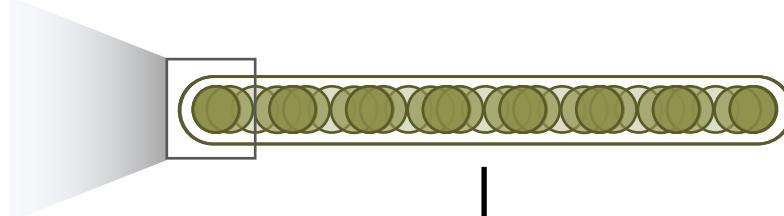
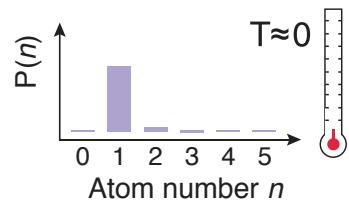
PROTOCOL



$$\begin{aligned}\hat{\mathcal{H}} = & -J \sum_i \left(\hat{a}_i^\dagger \hat{a}_{i+1} + h.c. \right) \\ & + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + W \sum_{i \in L_{\text{dis}}} h_i \hat{n}_i\end{aligned}$$

$U = 3J$
 h_i quasi-periodic with golden ratio 1.618

QUANTUM THERMALIZATION

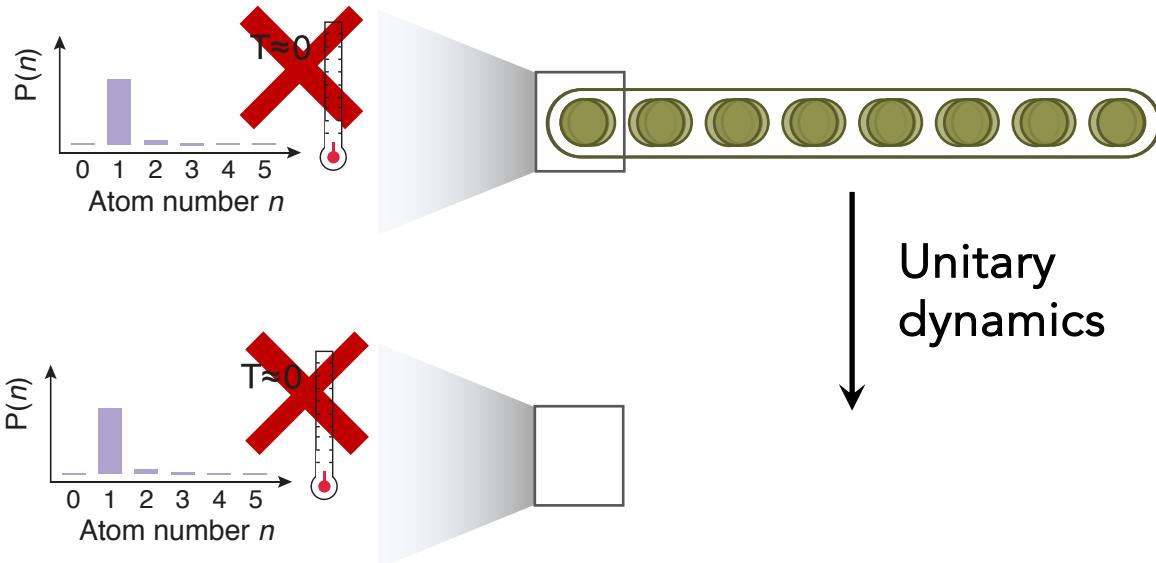


Unitary
dynamics

Globally pure
Locally pure

Globally pure
Locally thermal
(ETH)

BREAKDOWN OF THERMALIZATION



No thermalization!

QUANTUM THERMALIZATION

Product state (locally pure)

$$|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$$

Subsystem A



Subsystem B

Diagonal reduced density matrix:

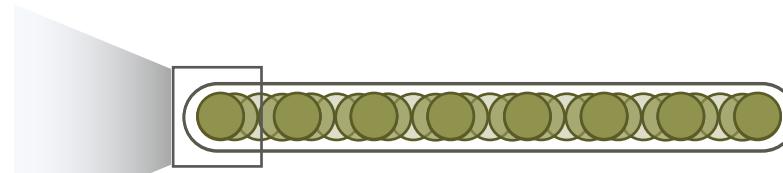
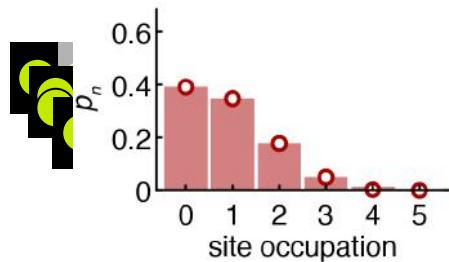
$$\hat{\rho}_A = \sum_n p_n |n\rangle \langle n|$$

Entangled state (locally mixed)

$$|\Psi\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$$

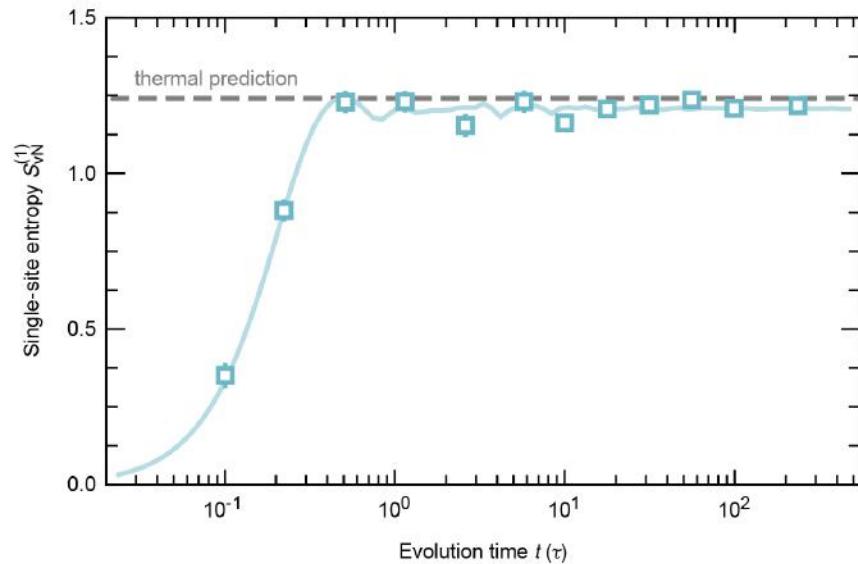
...

QUANTUM THERMALIZATION

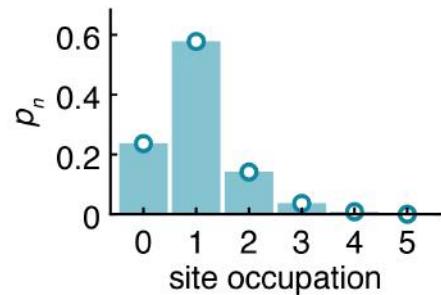


On-site von-Neumann entropy:

$$S_{\text{vN}}^{(1)} = - \sum_n p_n \log p_n$$

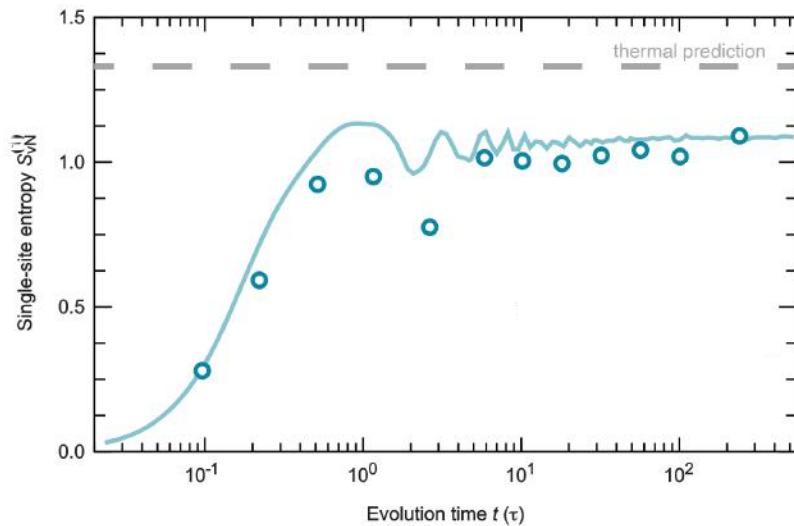


BREAKDOWN OF THERMALIZATION

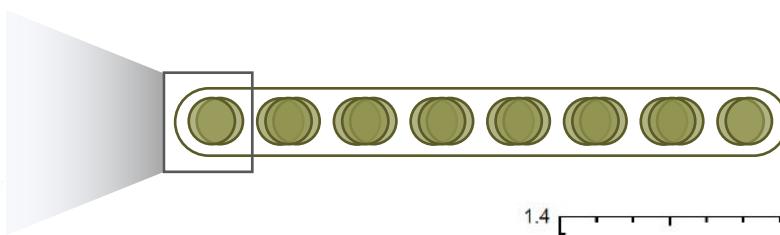
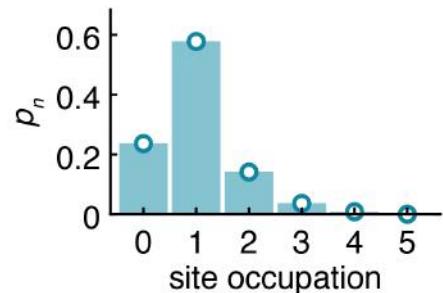


On-site von-Neumann entropy:

$$S_{\text{vN}}^{(1)} = - \sum_n p_n \log p_n$$

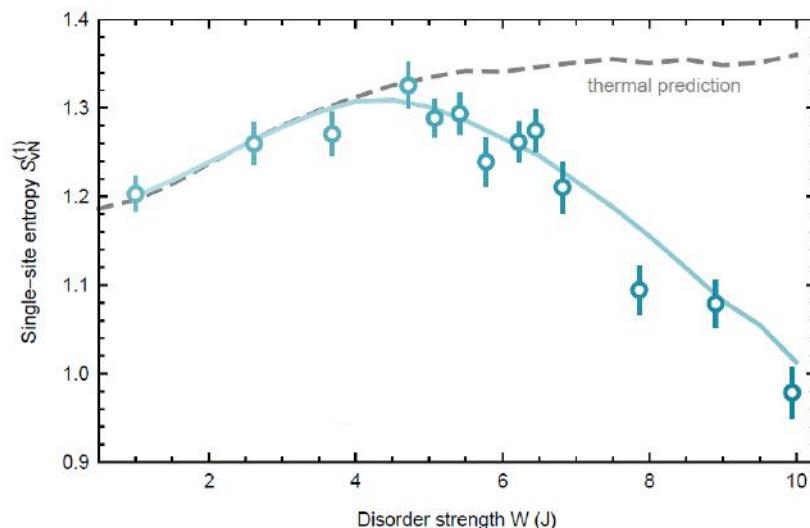


BREAKDOWN OF THERMALIZATION

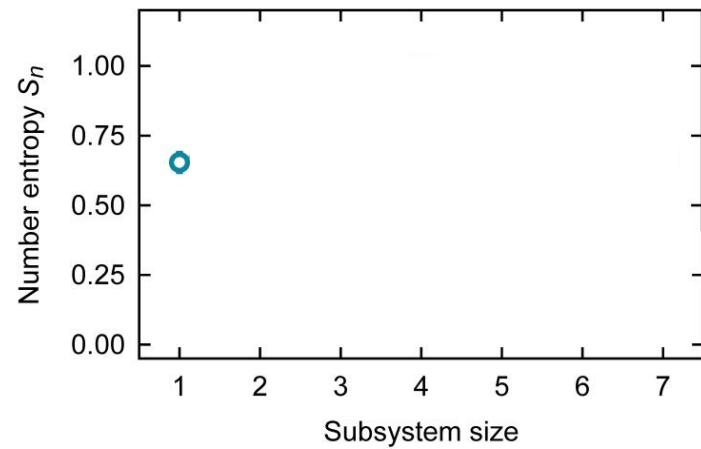
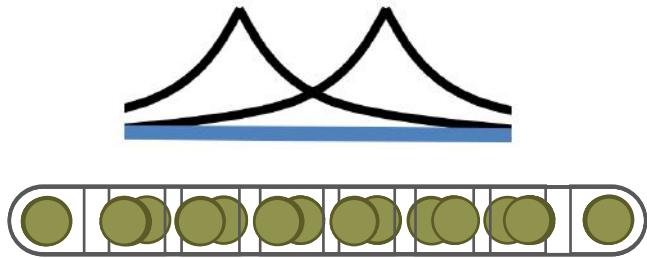


On-site von-Neumann entropy:

$$S_{\text{vN}}^{(1)} = - \sum_n p_n \log p_n$$

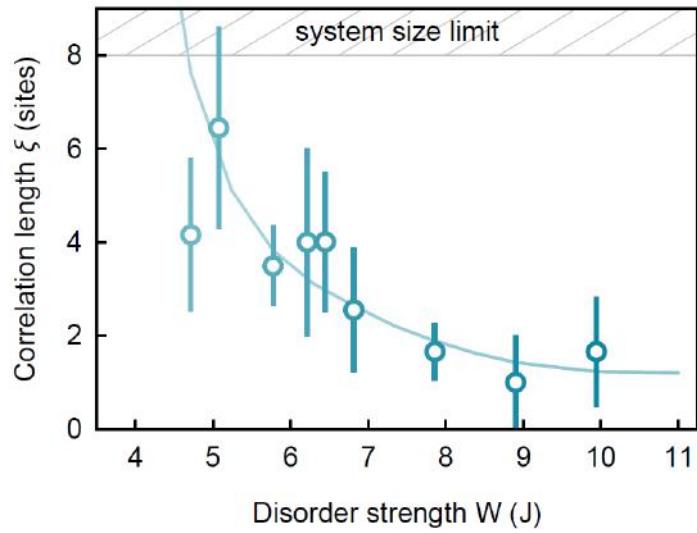
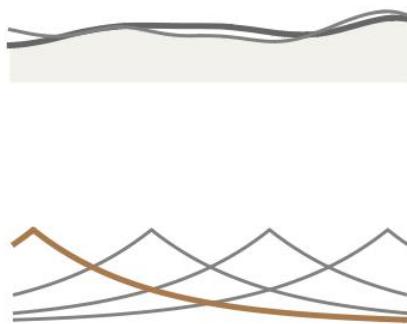
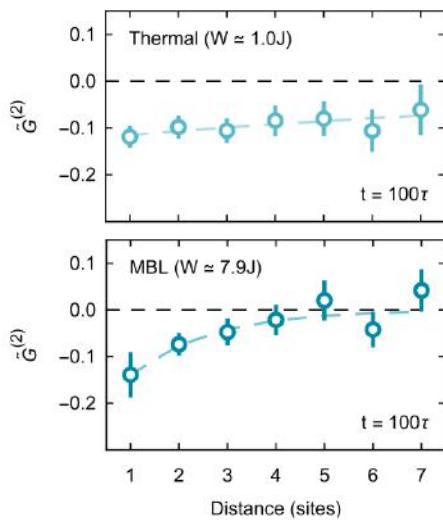


ABSENCE OF TRANSPORT

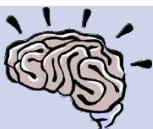


Independent of subsystem size
→ localized wavefunctions

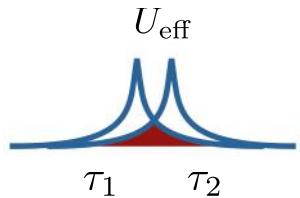
LOCALIZATION



SIGNATURES OF MBL

	Thermal	Anderson ($U=0$)	MBL ($U>0$)
	✗	✓	✓
	✗	✓	✓
	✗	✓	✓
	✗	✓	✓
	✗	✗	✓

LONG-RANGE ENTANGLEMENT



Slow growth of entanglement:

$$\tau_1 \otimes \tau_2 = (\dots + \dots) \otimes (\dots + \dots) e^{iU_{\text{eff}}t/\hbar}$$

$$= \dots + \dots + \dots + \dots e^{iU_{\text{eff}}t/\hbar}$$

$$\rightarrow \dots + \dots + \dots - \dots$$

local transformation $= \dots + \dots$

→ Entanglement from eigenstate superposition!

LOGARITHMIC GROWTH OF ENTANGLEMENT

Number entanglement



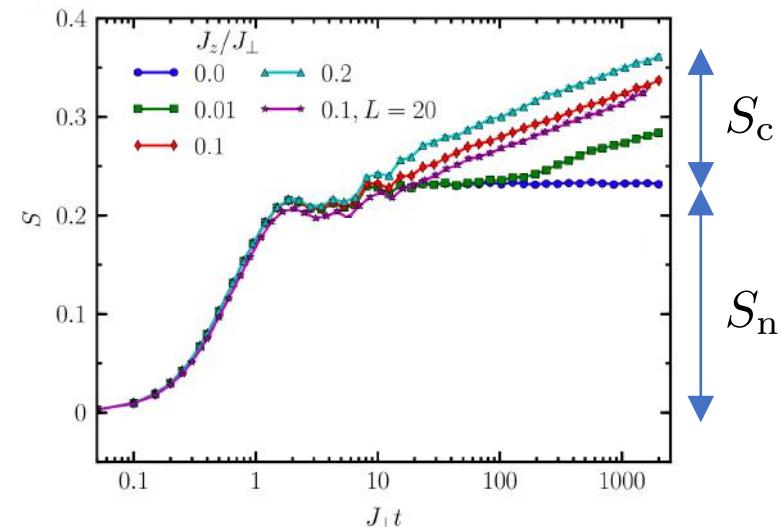
+



Configurational entanglement



+

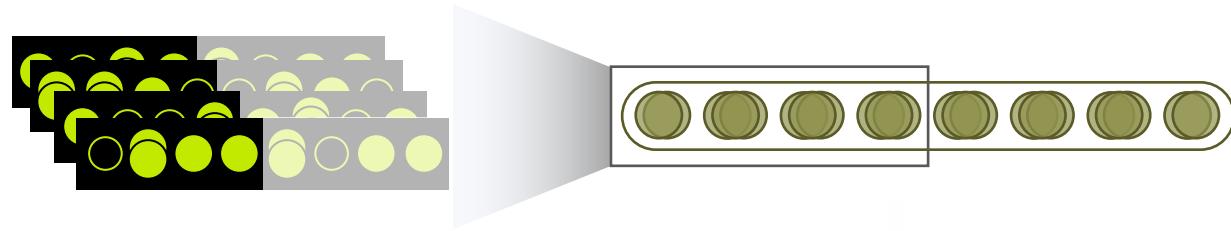


J. H. Bardarson et al., Phys. Rev. Lett. 109, 017202 (2012)

Separation of total entanglement entropy

$$S_{vN} = S_n + S_c$$

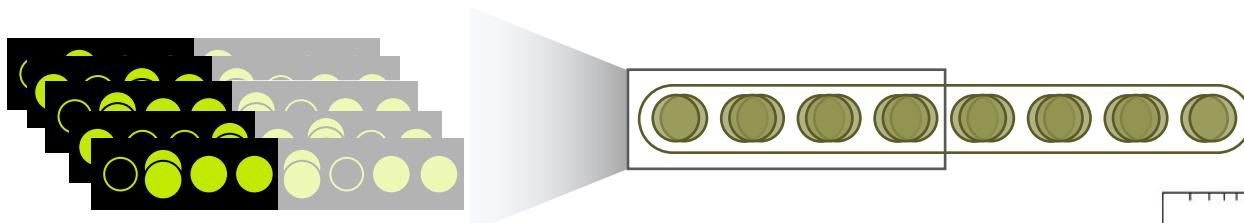
ABSENCE OF TRANSPORT



Number entropy:

$$S_n = - \sum_n p_n \log p_n$$

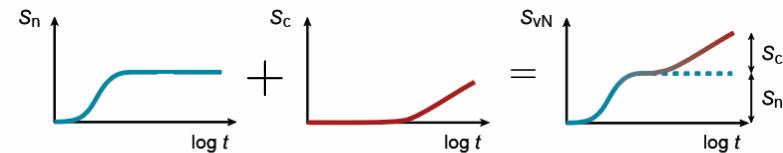
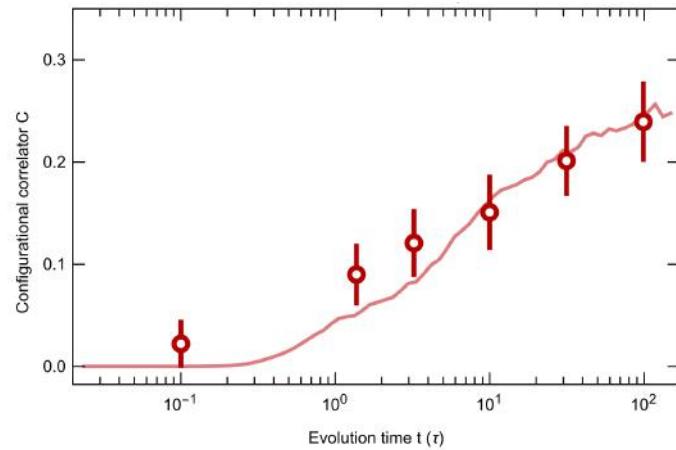
LOGARITHMIC GROWTH OF ENTANGLEMENT



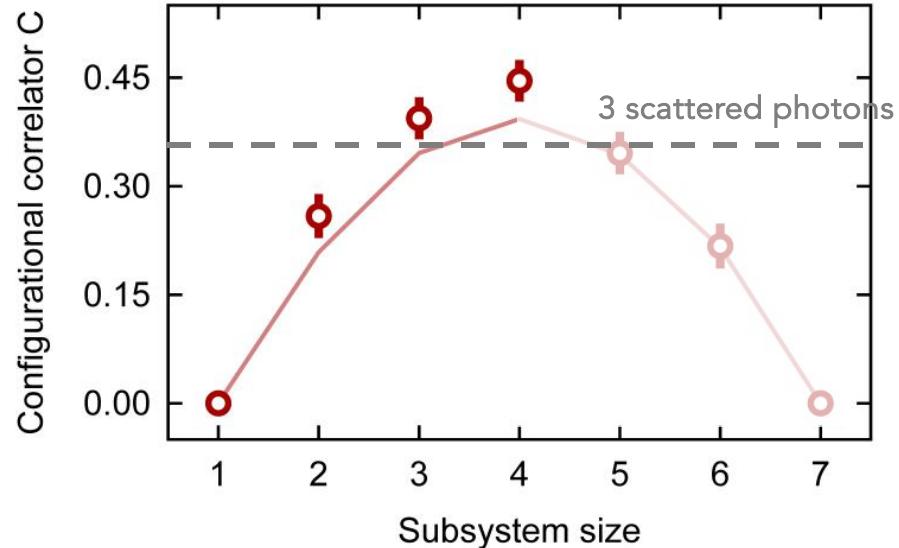
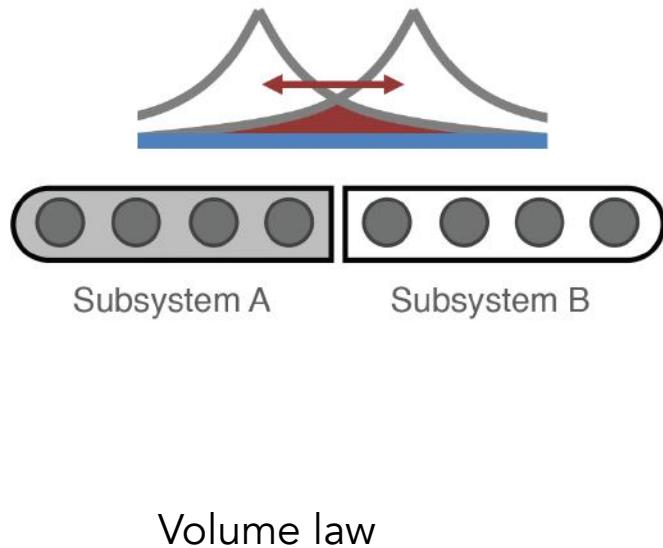
A B

$$C = \sum_{A,B} |p(A \otimes B) - p(A)p(B)|$$

$C \propto S_c \rightarrow$ probe for many-body entanglement



SUBSYSTEM SIZE

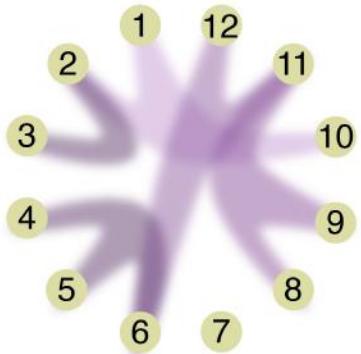


→ probe for quantum state purity

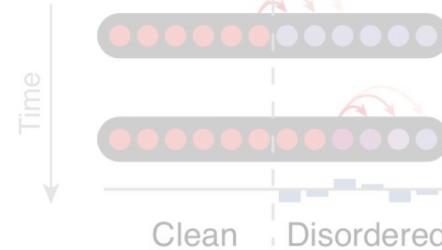
OUTLINE



Localization and entanglement

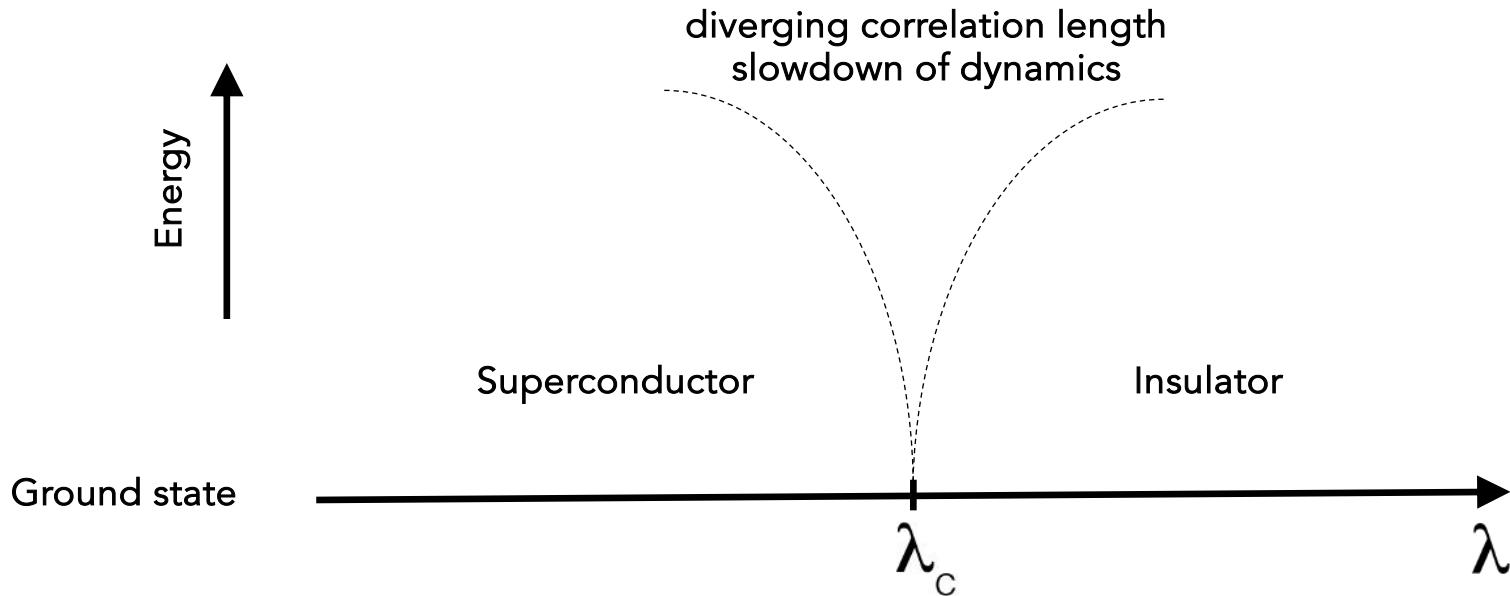


MBL “transition”

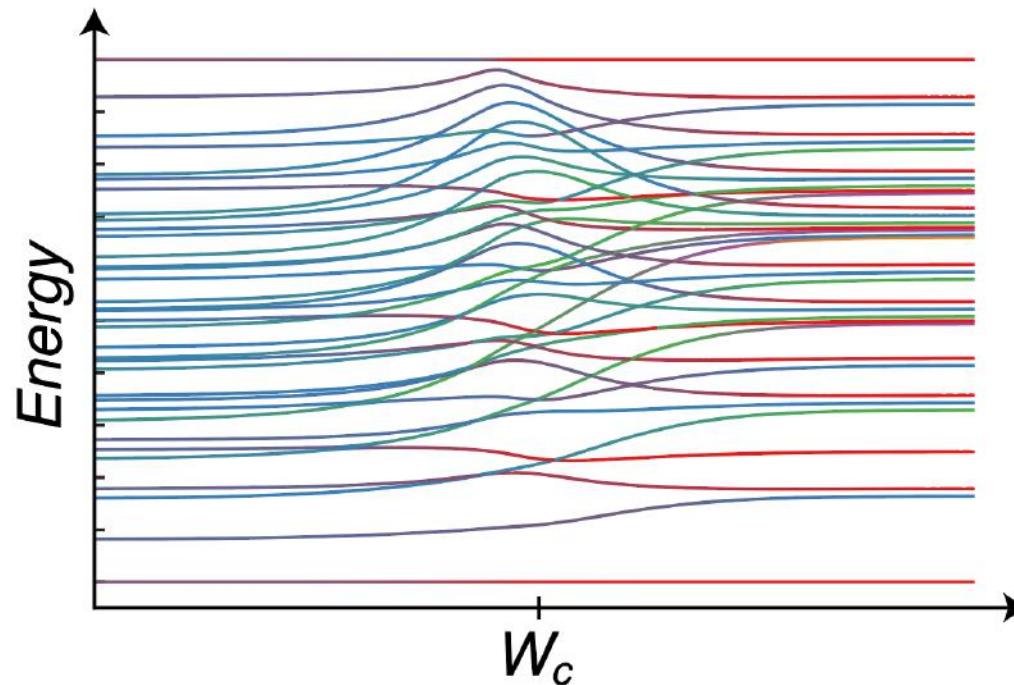


Quantum avalanches

EQUILIBRIUM CRITICAL BEHAVIOUR



MANY-BODY LOCALIZATION TRANSITION

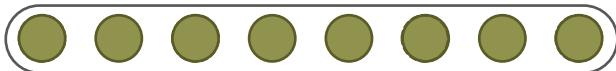


All eigenstates determine critical behaviour!

→ requires new theoretical and experimental concepts to understand

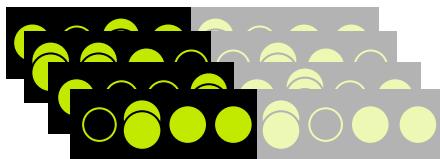
TWO-POINT DENSITY CORRELATIONS

$$G_c^{(2)}(d) = \langle \hat{n}_i \hat{n}_{i+d} \rangle - \langle \hat{n}_i \rangle \langle \hat{n}_{i+d} \rangle$$



Anti-correlations
from particle hopping

$$G_c^{(2)}(d) < 0$$

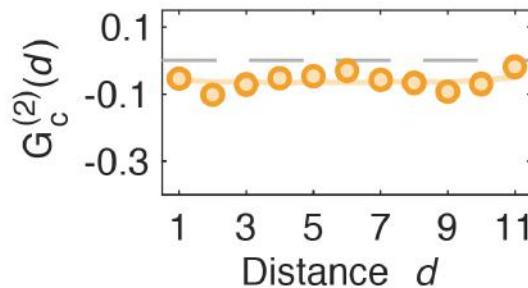
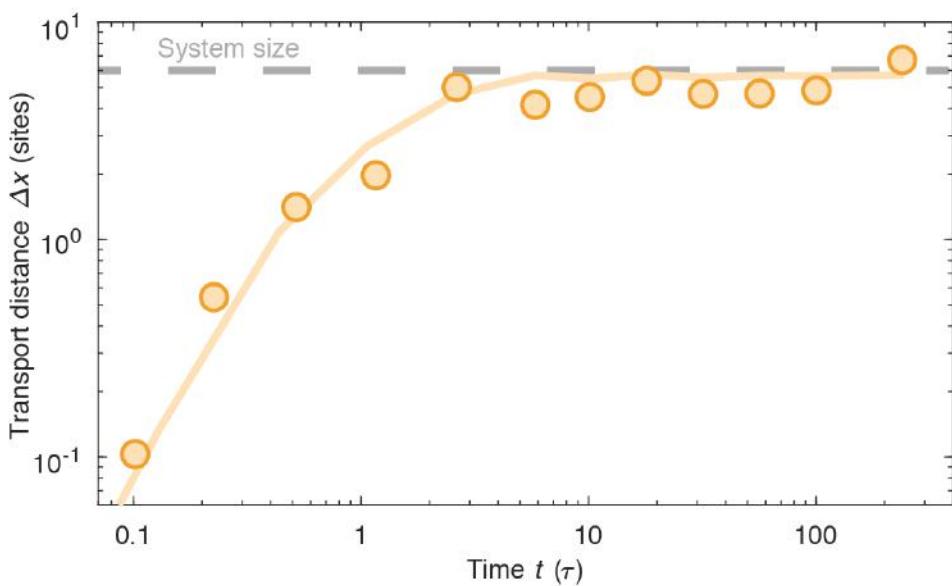


Define transport distance:

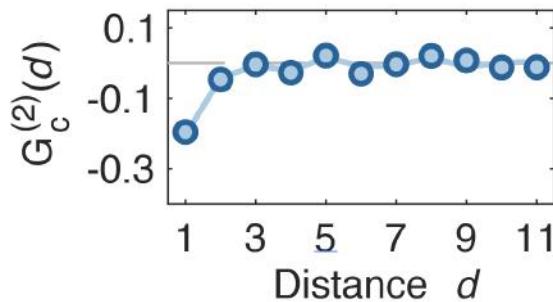
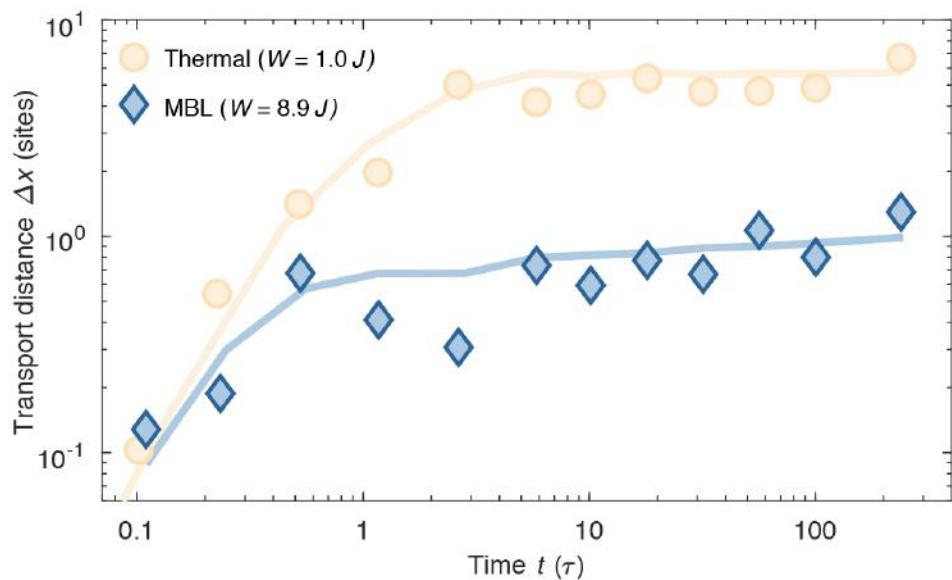
$$\Delta x \propto \sum_d d \times G_c^{(2)}(d)$$

(average hopping distance)

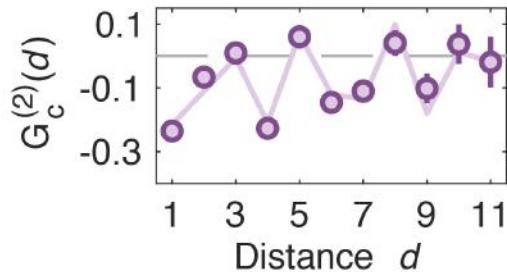
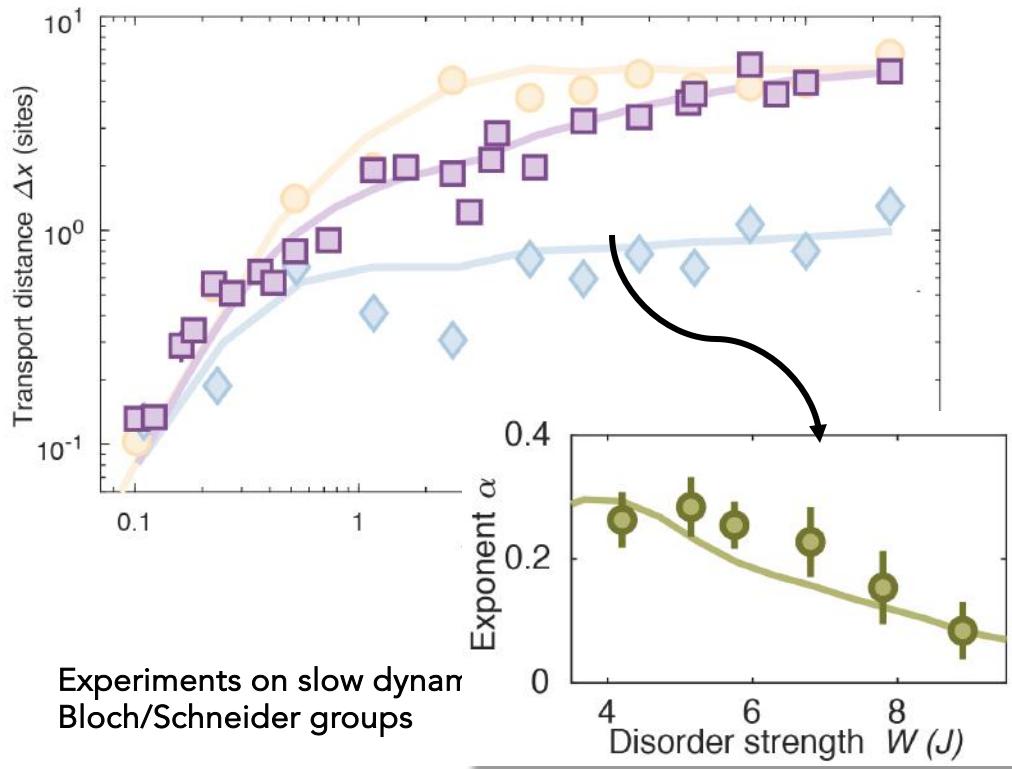
THERMAL REGIME



MBL REGIME

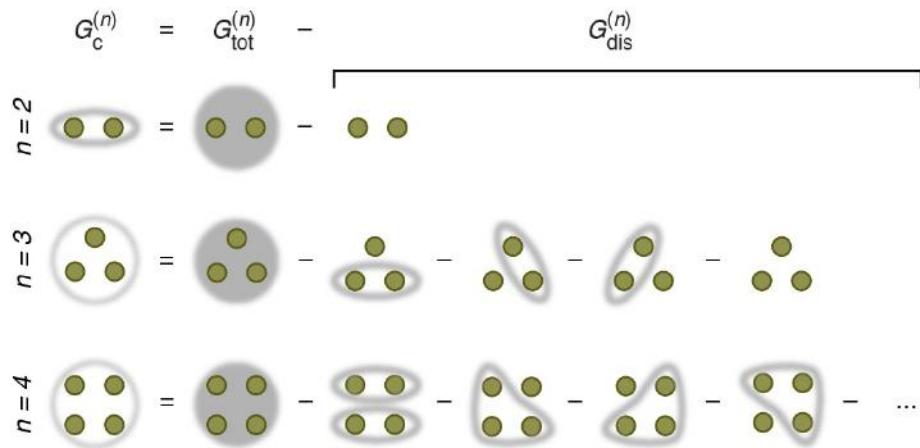


QUANTUM CRITICAL REGIME

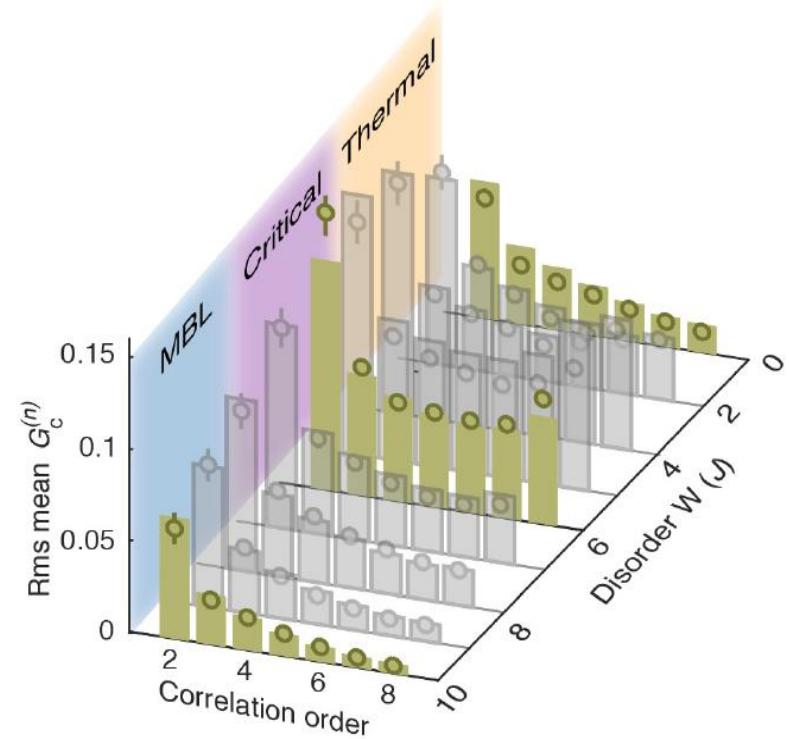


Subdiffusive transport
through complex network

MULTI-POINT QUANTUM CORRELATIONS



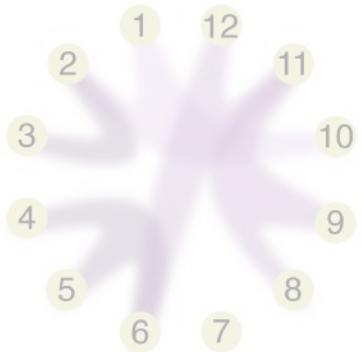
Hierarchy of correlations: non-separable at order n



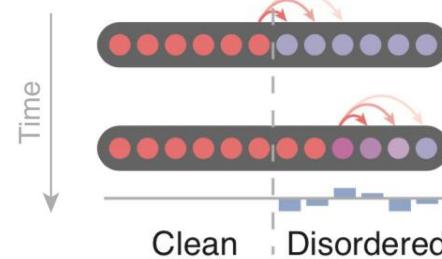
OUTLINE



Localization and
entanglement



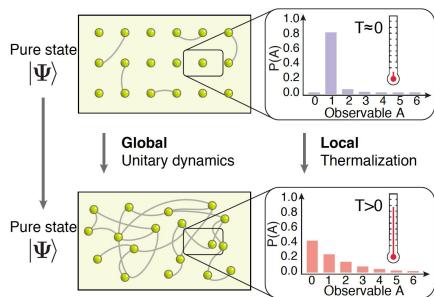
MBL "transition"



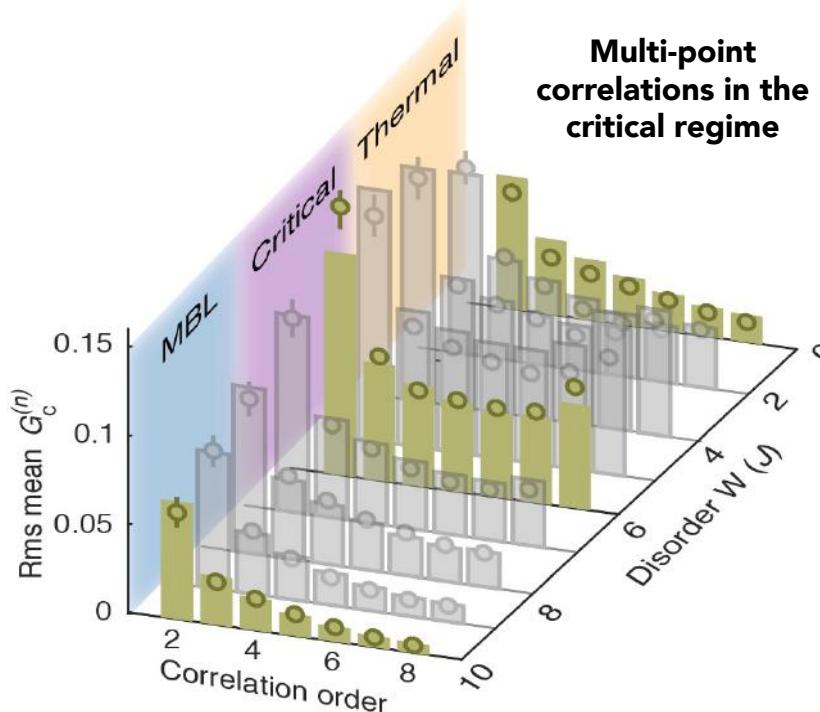
Quantum avalanches

THERMALIZATION VS LOCALIZATION

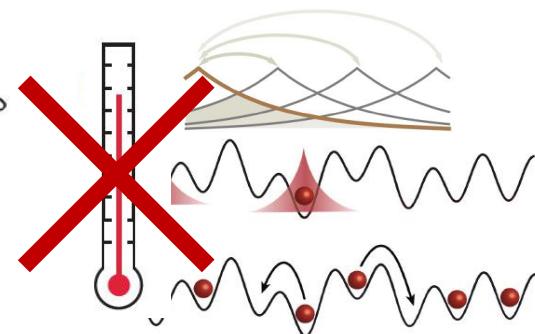
Thermalization



Multi-point correlations in the critical regime



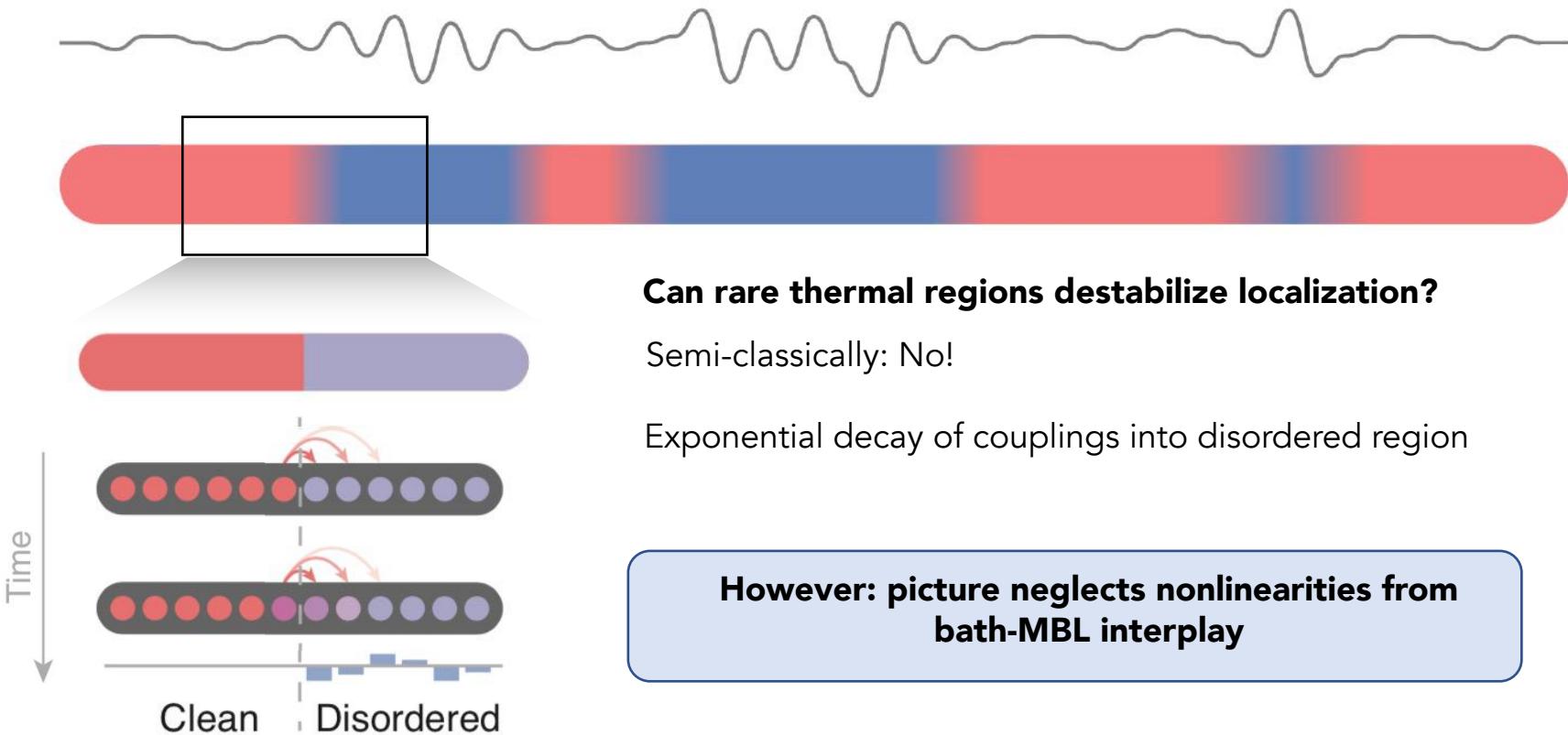
Many-body localization



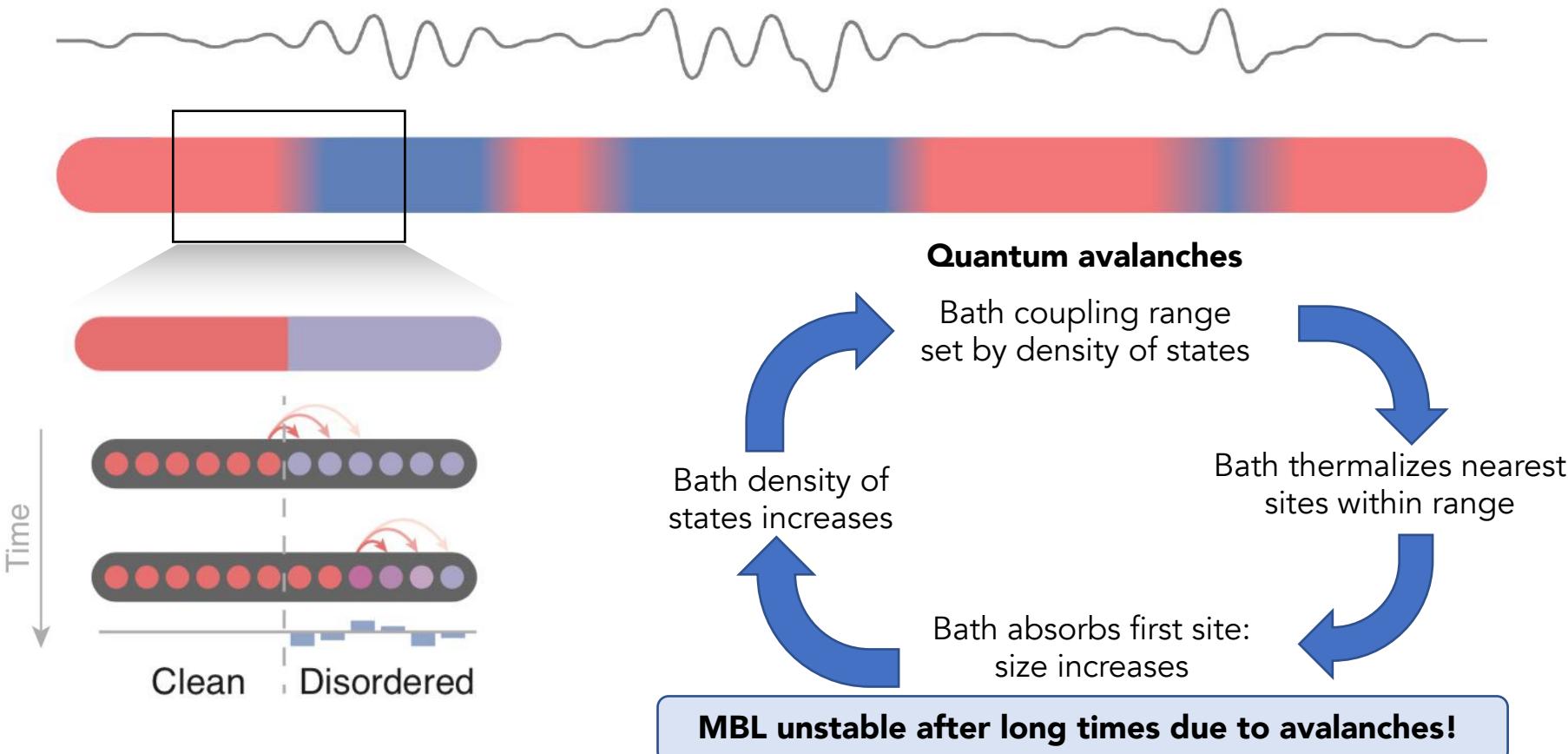
Interface between classical and quantum worlds

Disorder

THERMAL INCLUSIONS



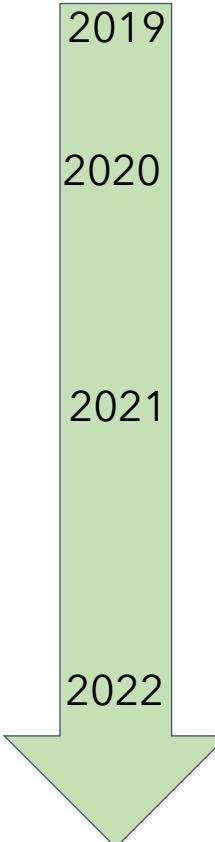
THERMAL INCLUSIONS



IS MBL STABLE?

CONTRA

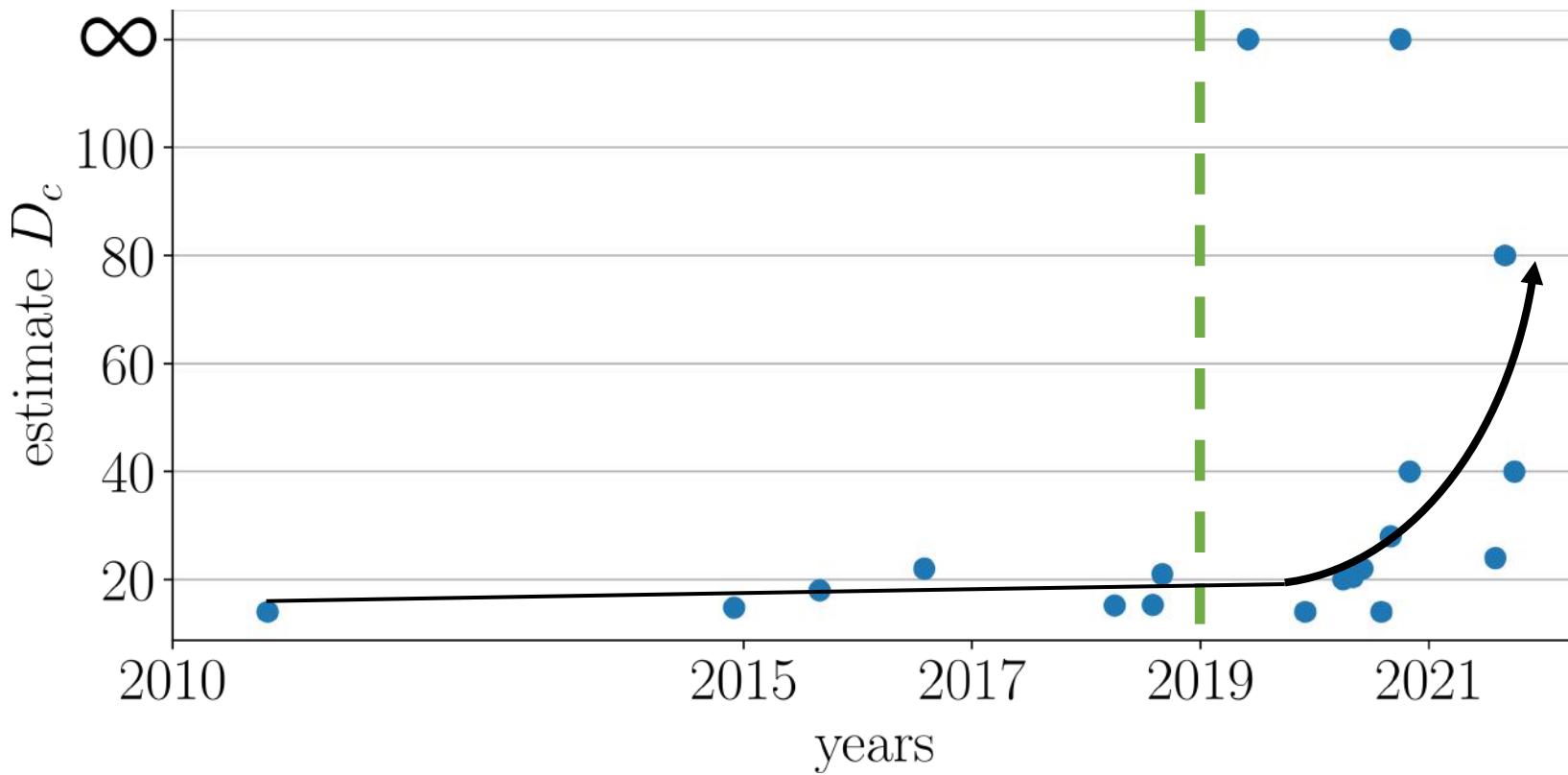
- Quantum chaos challenges many-body localization**
[Jan Šuntajs et al.](#)
- Evidence for unbounded growth of the number entropy in many-body localized phases**
[Maximilian Kiefer-Emmanouilidis et al.](#)
- Ergodicity Breaking Transition in finite disordered spin chains** [Jan Šuntajs et al.](#)
- Dynamical obstruction to localization in a disordered spin chain** [Dries Sels et al.](#)
- Slow delocalization of particles in many-body localized phases**
[Maximilian Kiefer-Emmanouilidis et al.](#)
- Unlimited growth of particle fluctuations in many-body localized phases**
[Maximilian Kiefer-Emmanouilidis et al.](#)
- Markovian baths and quantum avalanches**
[Dries Sels](#)
- Particle fluctuations and the failure of simple effective models for many-body localized phases**
[Maximilian Kiefer-Emmanouilidis et al.](#)



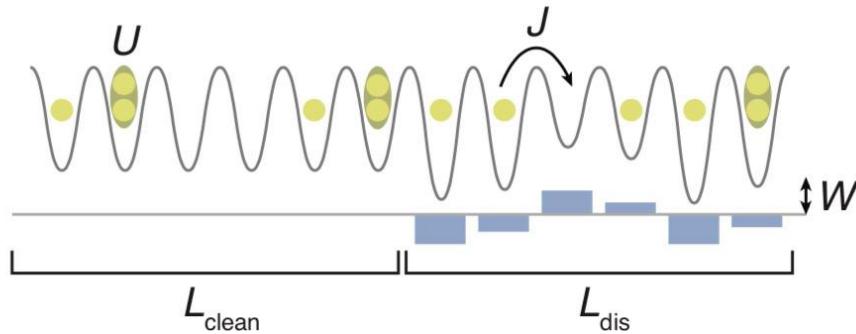
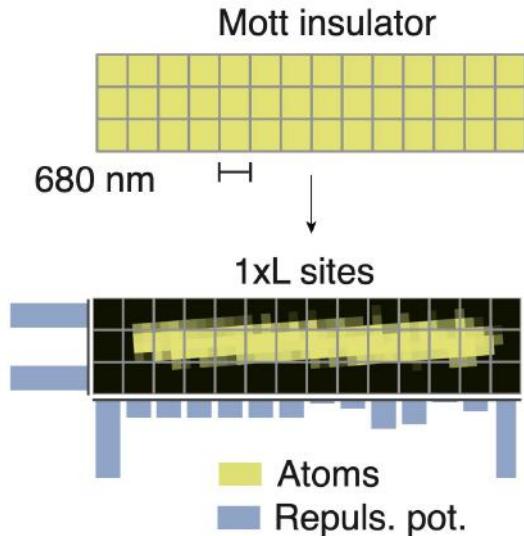
PRO

- Distinguishing localization from chaos: Challenges in finite-size systems**
[Dmitry Abanin et al.](#)
- Can we study the many-body localization transition**
[Rajat K. Panda et al.](#)
- Thouless time analysis of Anderson and many-body localization transitions** [Piotr Sierant et al.](#)
- Polynomially filtered exact diagonalization approach to manybody localization** [Piotr Sierant et al.](#)
- Is there slow particle transport in the MBL phase**
[David J. Luitz et al.](#)
- Avalanches and many-body resonances in many-body localized systems**
[Alan Morningstar et al.](#)
- Can we observe the many-body localization**
[Piotr Sierant et al.](#)
- Resonance-induced growth of number entropy in strongly disordered systems**
[Roopayan Ghosh et al.](#)

IS MBL STABLE?



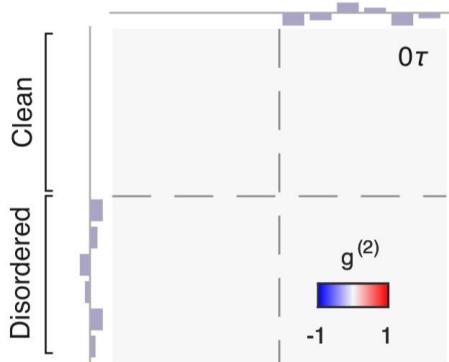
CLEAN-DISORDER INTERFACE



- L_{clean} sites without disorder
- L_{dis} sites with disorder (quasi-periodic)

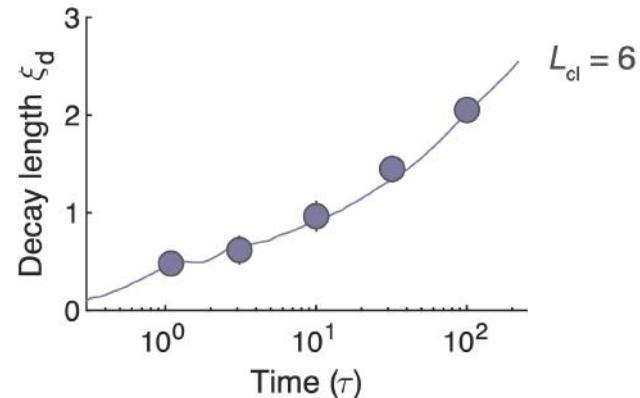
AVALANCHE DYNAMICS

Quenched initial state



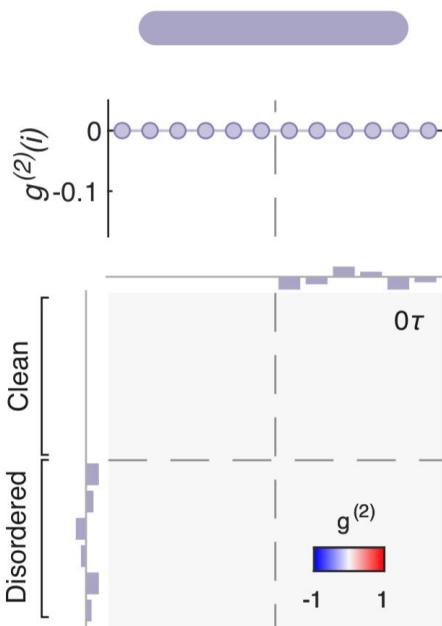
$$g^{(2)}(i, j) = \langle \hat{n}_i n_j \rangle - \langle \hat{n}_i \rangle \langle \hat{n}_j \rangle$$

Accelerated thermalization

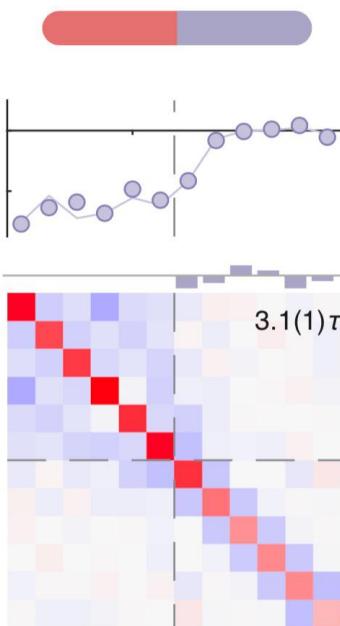


AVALANCHE DYNAMICS

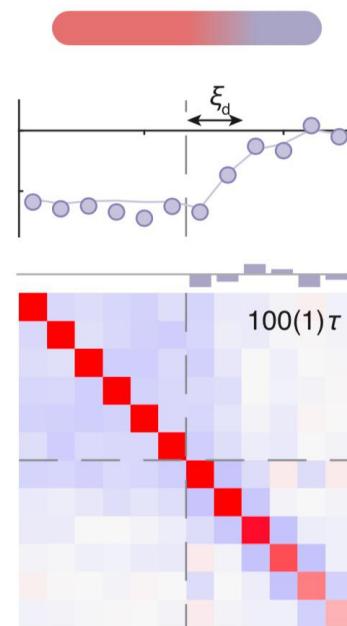
Quenched initial state



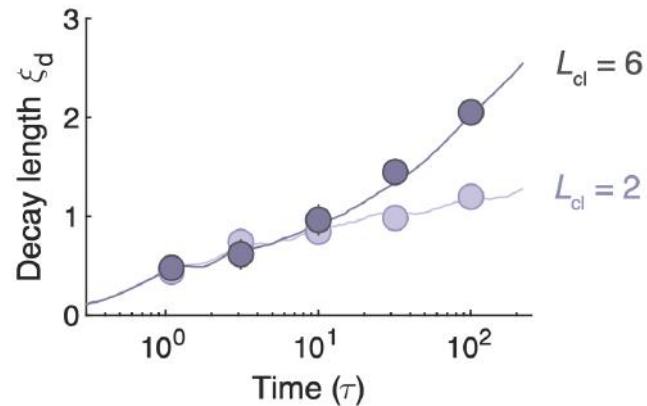
Separate dynamics



Transport across interface

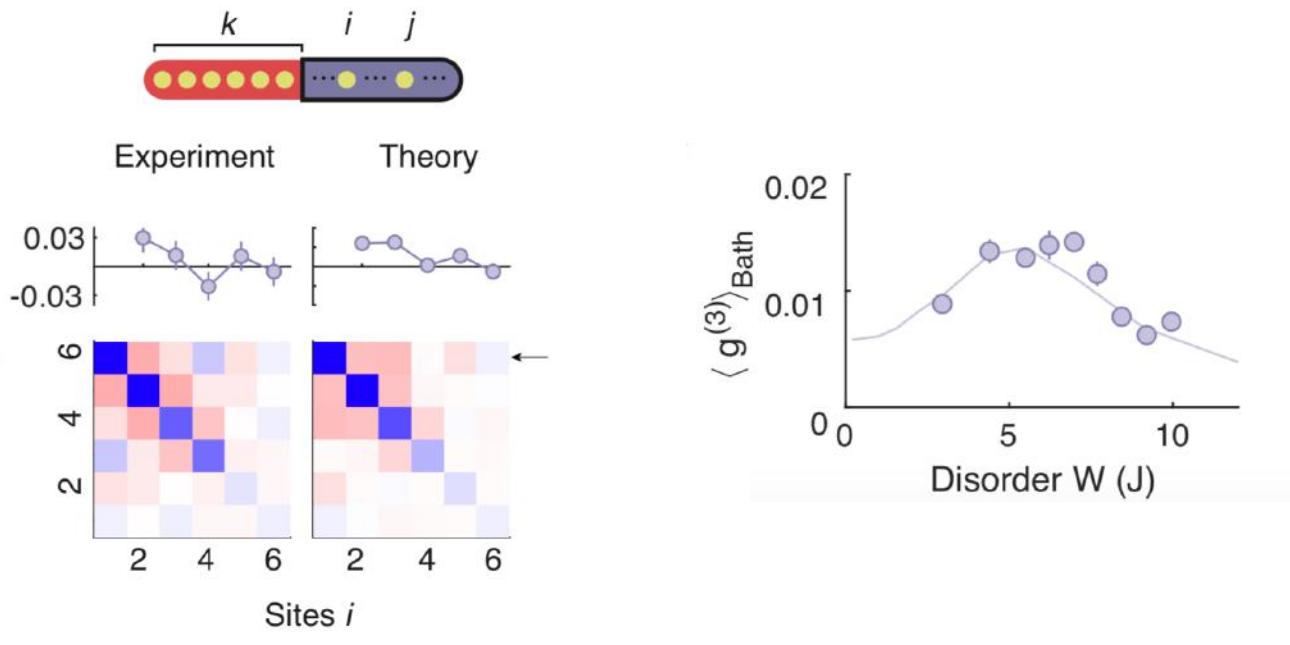


Accelerated thermalization



$$g^{(2)}(i, j) = \langle \hat{n}_i n_j \rangle - \langle \hat{n}_i \rangle \langle \hat{n}_j \rangle$$

THREE-BODY CORRELATIONS



→ driven by many-body processes

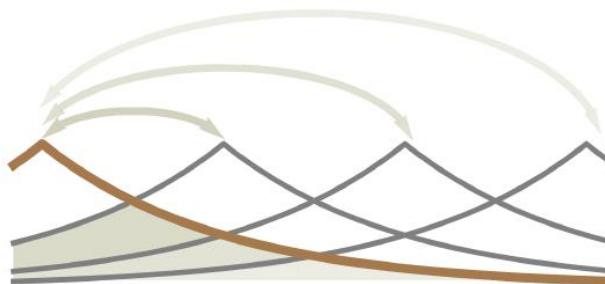
OUTLOOK

What we know

- Localization is possible over accessible time scales
- Long-range entanglement despite localization
- Avalanches destabilize MBL over disorder range
- MBL may be a prethermal phenomenon
→ more research required

What we don't know yet

- Is MBL stable at high disorder?
- If not, are there ways to stabilize MBL?
- Is there any strict exception to statistical physics thermalization?
- What about time crystals, many-body scars?







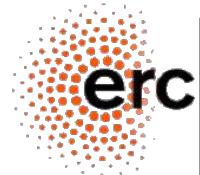
R. Rosa-Medina F. Silva-Tarouca S. Roschinski J. Schabbauer M. Michalek

J. L.

I. Safa

T. Schubert S. Waddington M. Stümmer

quantA



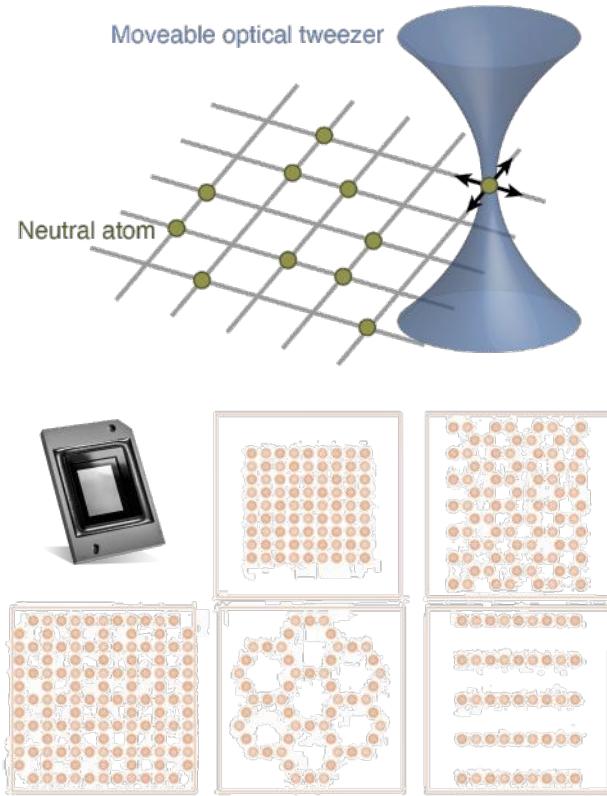
European
Research
Council

FWF

Der Wissenschaftsfonds.

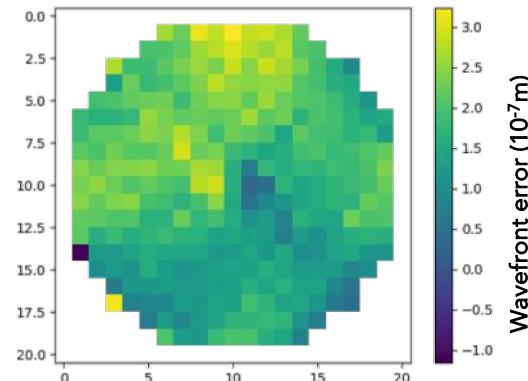


OUTLOOK: ASSEMBLING QUANTUM MATTER

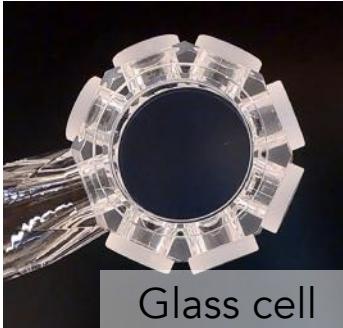


Programmable fermionic systems

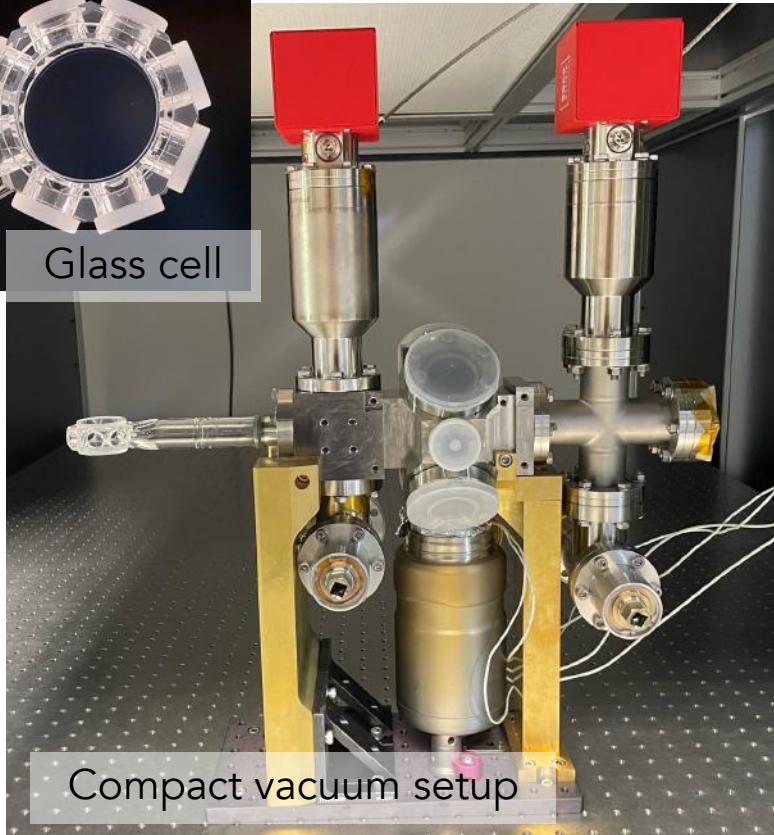
- Deterministic initial state by loading from a fermionic tweezer array
- Programmable potentials, beyond square lattices
- Reuse sample (\rightarrow 10-100Hz) to get statistics for quantitative results



ASSEMBLING QUANTUM MATTER



Glass cell



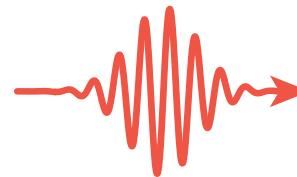
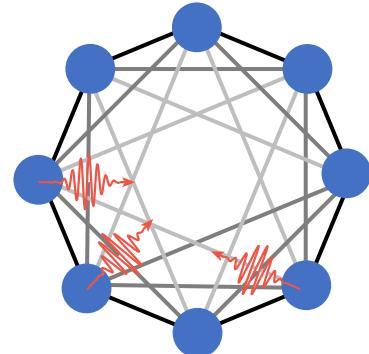
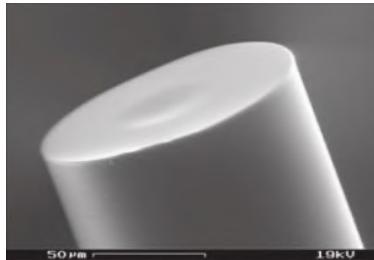
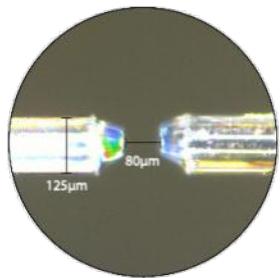
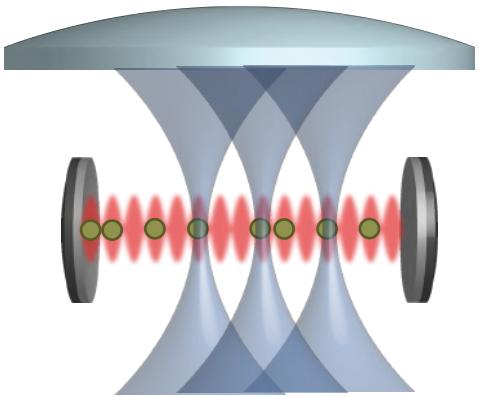
Compact vacuum setup

Key features

- Lattice constructed and filled site by site
- Fermionic Lithium to reach fast energy scales due to light mass
- Controllable interactions via Feshbach resonance
- Blue-detuned lattices for improved coherence time

OUTLOOK: PHOTON-COUPLED ATOMIC ARRAYS

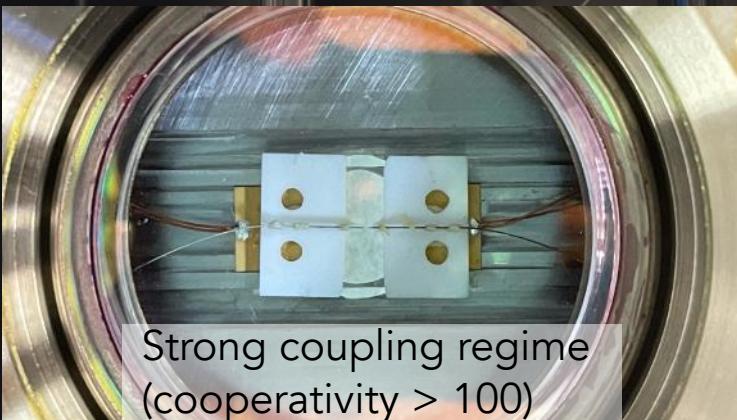
New frontier:
Local **and** non-local control of interactions



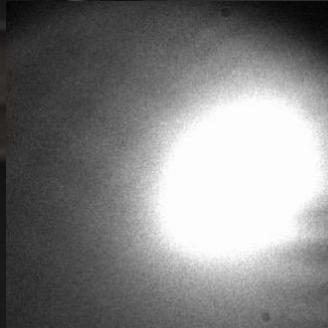
Light-induced coupling

- Spatial control: any distance, any groups of atoms
- Dynamical control
- New readout techniques

Programmable couplings among *all* atoms



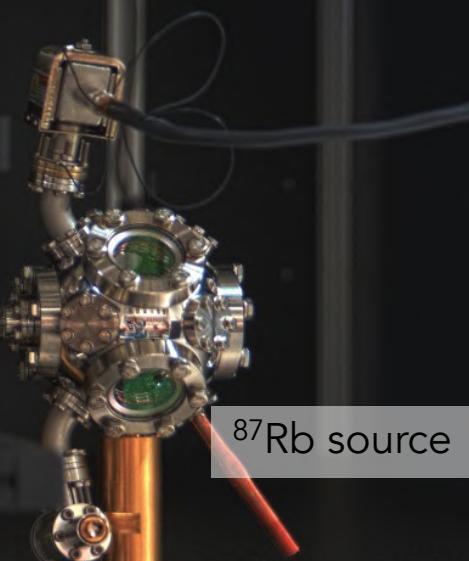
Strong coupling regime
(cooperativity > 100)



3DMOT



2DMOT



^{87}Rb source

3 μm



THANK YOU