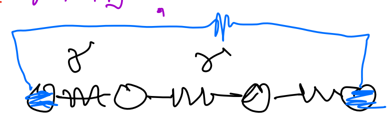
Hubbard Model

- Captures many essential qualitative phenomena of strongly correlated systems:
  - Mott Metal insulator transition
  - magnetism
  - CDW and SC (if $U<0$)
- Geometry of lattice: Flat bands, frustration (Manmana)
- Interplay of disorder/interactions (Leonard)
- Don't oversell: Real materials are complex
  - cuprates? (Hirsch)

Something so powerful must be HARD?

Classical Mechanics



N masses

$x_\ell(t)$

$$\boxed{m\frac{d^2 x_\ell}{dt^2} = -\gamma(x_\ell - x_{\ell-1}) - \gamma(x_\ell - x_{\ell+1})}$$

$$x_\ell(t) = a_\ell\, e^{i\omega t}$$

linear algebraic eqns

$$\boxed{m\omega^2 a_\ell = +\gamma(a_\ell - a_{\ell-1}) - \gamma(a_\ell - a_{\ell+1})}$$

$$a_\ell = a_0\, e^{-ik\ell}$$

$$\omega^2(k) = 2\gamma(1-\cos(k))$$

Any real k, ω

Dispersion reln.

Boundary conditions PBC

$$x_0 = x_N \qquad x_1 = x_{N+1}$$

$$a_\ell = a_0 e^{-ik\ell}$$

quantize k values

$$k = \frac{2\pi}{N} \{1, 2, 3, \dots N\}$$

as expected N solutions

Simplest description of lattice vibrations

$$\omega^2 = 2\gamma(1 - \cos k)$$

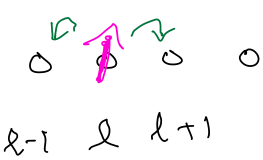
$$\cos k \approx 1 - \tfrac{1}{2}k^2$$

$$\boxed{\omega \sim k}$$ Phonons acoustic

Hubbard Hamiltonian $(U=0)$

$$\mathcal{H} = -t \sum_\ell \left(c_\ell^\dagger c_{\ell+1} + c_{\ell+1}^\dagger c_\ell \right) - \mu \sum n_\ell$$

Kinetic energy



$c_\ell^\dagger$ creates e^- on site ℓ

$\Downarrow$

$c_k^\dagger$ creates e^- with momentum k

$$c_k^\dagger = \frac{1}{\sqrt{N}} \sum_\ell e^{ik\ell} c_\ell^\dagger$$

$$c_\ell^\dagger = \frac{1}{\sqrt{N}} \sum_k e^{-ik\ell} c_k^\dagger$$

$$-\frac{t}{N}\sum_{l}\sum_{k}\sum_{k'} e^{-ik(l+1)} c_k^\dagger e^{+ik'l} c_{k'} \qquad c_{l+1}^\dagger c_l$$

$$\frac{1}{N}\sum_{l} e^{-ikl} e^{+ik'l} = \delta_{k,k'}$$

$$-t\sum_{k} e^{-ik} c_k^\dagger c_k$$

$$e^{+ik}$$

$$\boxed{H = \sum_k (-2t\cos k)\, c_k^\dagger c_k}$$

$U=0$ 1D Hubbard Hamiltonian

classical mechanics : different masses push each other and are coupled

$\longrightarrow$ normal modes which are independent

Real space Hubbard

$l-1$ $\quad l \quad$ $l+1$

$c_{l+1}^\dagger c_l$

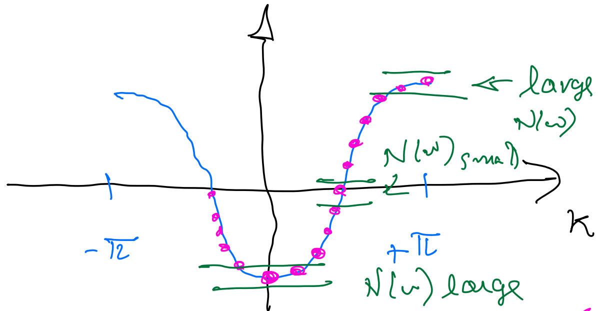
mixes different sites

but in k space

$c_k^\dagger c_k$

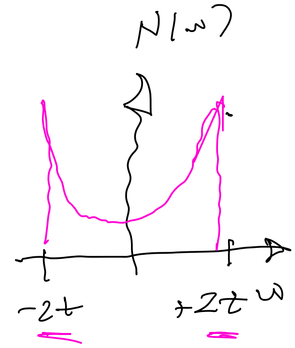
different k are not mixed

$$E(k) = -2t\cos k$$

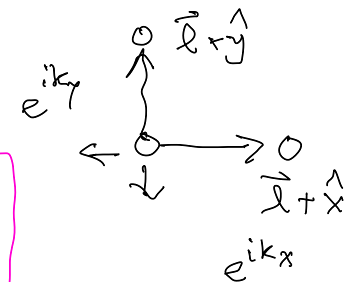


"Band structure" of 1D Hubbard model

$$N(\omega) = \frac{1}{N}\sum_k \underbrace{\delta(\omega - E(k))}_{\text{counts the \# of Energies which have the value } \omega} = \frac{1}{\sqrt{4t^2-\omega^2}}$$



2D square lattice



$$\boxed{E(k_x k_y) = -2t(\cos k_x + \cos k_y)}$$

what does "Fermi surface look" like ?

what is the geometrical contour of k_x, k_y for constant E ?

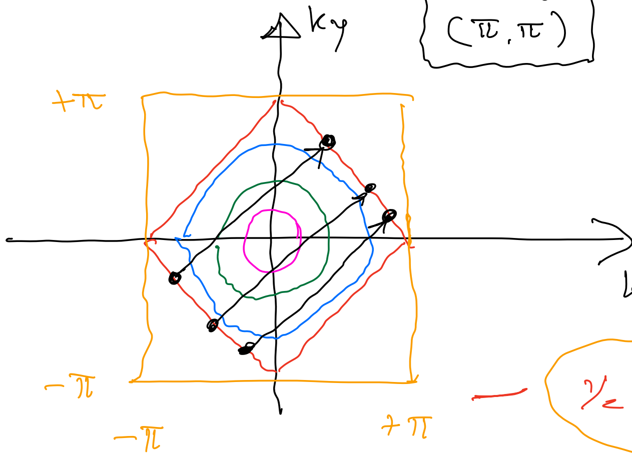
smallest possible E is at $k_x = k_y = 0$

$$E = -4t$$

$$-2t\left(1 - \frac{k_x^2}{2} + 1 - \frac{k_y^2}{2}\right)$$

$$E = -4t + \underbrace{t(k_x^2 + k_y^2)}_{\text{circle}}$$

Nesting (π, π)



Fermi surface : trajectory of constant E

— or —

as you add fermions to system fill up energy levels out to some value

$\frac{1}{2}$ filling

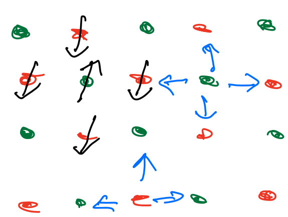
$\boxed{\frac{1}{2}\text{ filled 2D Hubbard}}$ model is very sensitive to things happening at (π, π)

$\Longrightarrow$ Antiferromagnetic order

Any $U \neq 0$

$U=0$ another geometry : Lieb lattice

flat bands $\Leftarrow$ E indep of $\vec{k}$!



Square lattice

Bipartite

AF

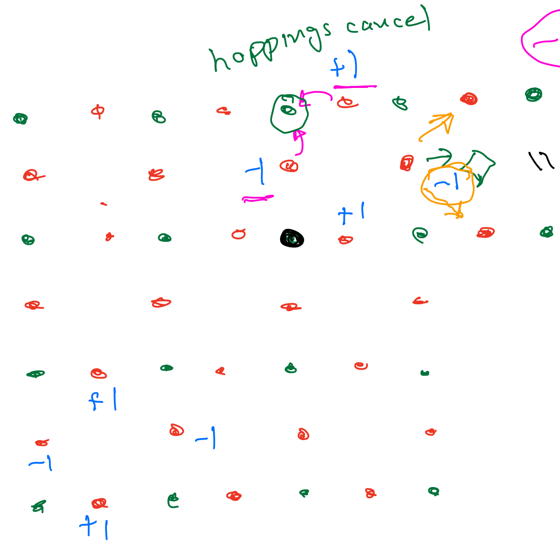
X

$N_{red} = N_{green}$

real space

$$\hat{H}\begin{pmatrix}0\\0\\0\\+1\\-1\\+1\\-1\\0\\0\\0\end{pmatrix} \overset{E=0}{=} 0\begin{pmatrix}0\\0\\0\\+1\\-1\\+1\\-1\\0\\0\\0\end{pmatrix}$$

eigenvector of $E=0$



$-t+t$

"Lieb lattice"

Bipartite

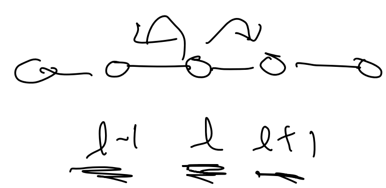
$\boxed{N_{red} = 2N_{green}}$

Lieb

There are eigenstates of H which are localized? $\leftarrow$ $c_k^\dagger$

Remember up to now $\boxed{e^{ikl}}$ spread out throughout lattice

$$H = \begin{array}{c|cccc} & & l-1 & & l+1 \\ \hline & 0 & -t & 0 & 0 \\ & -t & 0 & -t & 0 \\ l & 0 & \boxed{-t} & 0 & \boxed{-t} \\ & 0 & 0 & -t & 0 \end{array}$$



There are $\frac{N}{3}$ vectors of eigenvalue $E=0$!

Many states all with identical energy $\Rightarrow$ "flat band"

Better way is to go to k space just as for 1D chain or 2D square lattice

$$\begin{pmatrix} 0 & -t(1+e^{ik_x}) & -t(1+e^{ik_y}) \\ -t(1+e^{-ik_x}) & 0 & 0 \\ -t(1+e^{-ik_y}) & 0 & 0 \end{pmatrix}$$

$\longrightarrow$ 3 bands

one has $\underline{E(k_x k_y) = 0}$

$U = 0$ Hubbard model:

By going to k space can get at "band structure" for different geometries can in itself be quite important

Square lattice
perfect nesting

Lieb lattice
flat bands

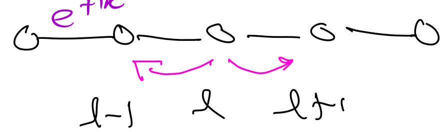
Non Hermitian Hamiltonian ?!?!

$$\boxed{-t \sum_\ell \left(e^{h} c^\dagger_{\ell+1} c_\ell + e^{-h} c^\dagger_\ell c_{\ell+1} \right)} \; \boxed{- \sum \mu_\ell n_\ell}$$

1D chain

$h \neq 0$

e^{-ik} e^{+ik}



!

$\neq$ if $h \neq 0$

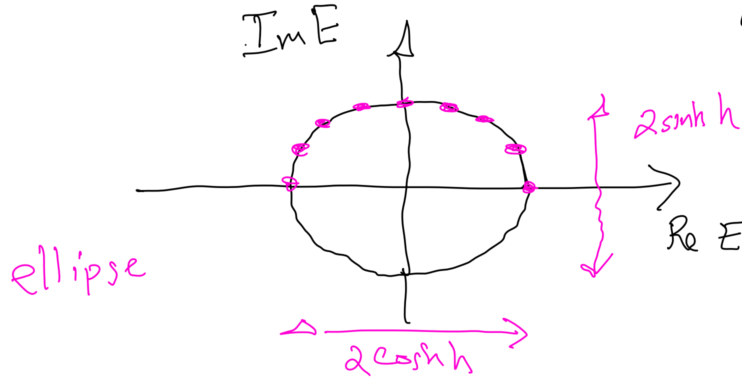
$$c_\ell = \frac{1}{\sqrt{N}} \sum_k e^{ik\ell} c_k$$

$$E(k) = -t\left(e^{h}e^{-ik} + e^{-h}e^{+ik}\right)$$

$$\boxed{-2t\cos k}$$

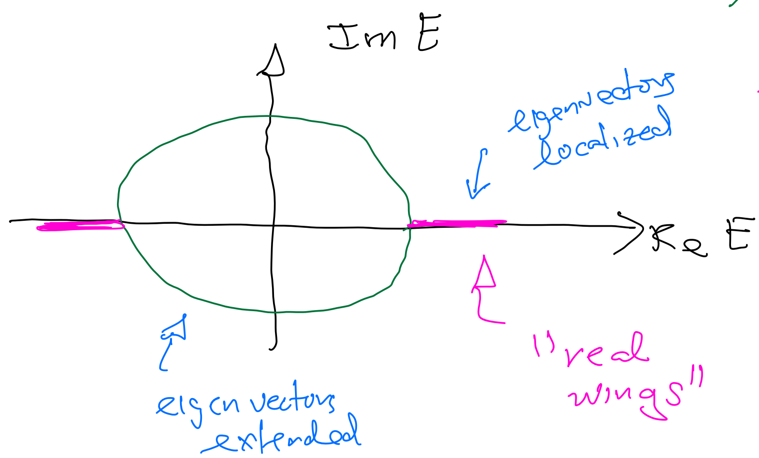
$$= -2t(\cosh h\cos k - i\sinh h\sin k)$$

$k = \frac{2\pi}{N}\{1,2,3,\ldots\}$



$\mu_e = 0$

$\mu_e \neq 0$



$\boxed{\text{Hatano - Nelson model}}$

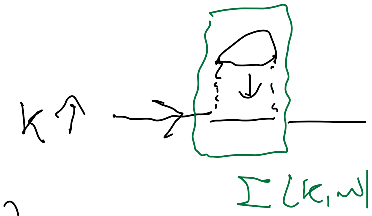
describes motion of flux lines in type II sc

What do you do if E is complex ?!

Thermodynamics ...

flow current pushes flux lines preferentially in 1 direction

! → Jonah Huang



$$G = \frac{1}{\omega - E_k - \Sigma(k,\omega)}$$

from interactions has real/imag parts

$U=0$

e^{+iEt}

$E_r + iE_i$

$\Rightarrow e^{-E_i t}$ decay

sort of like origin $E(k)$ being complex

$c_k^\dagger \backslash c_k$ $\overset{U=0}{=}$

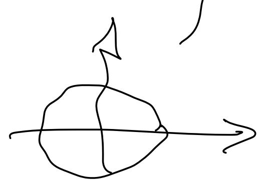
Analogies between Im part from non-Hermitian and from U ?

Boundaries are crucial

OBC → spectrum is real

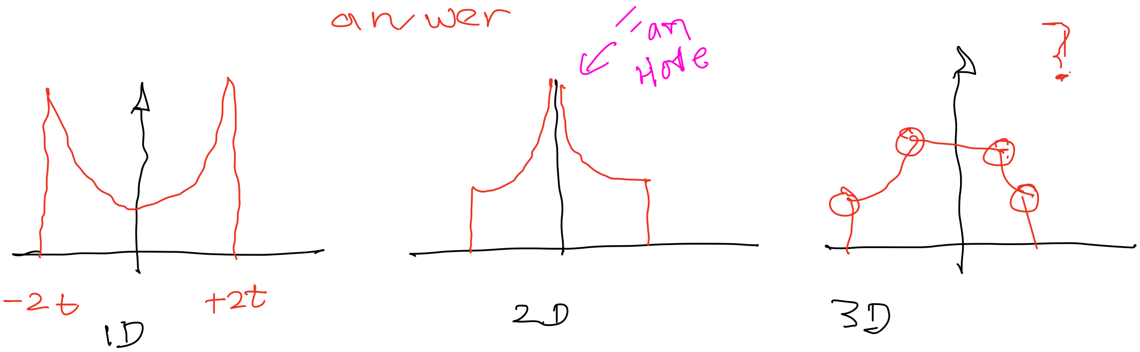
NO if OBC

PBC



* great answer

I am Here



CuO_2 planes

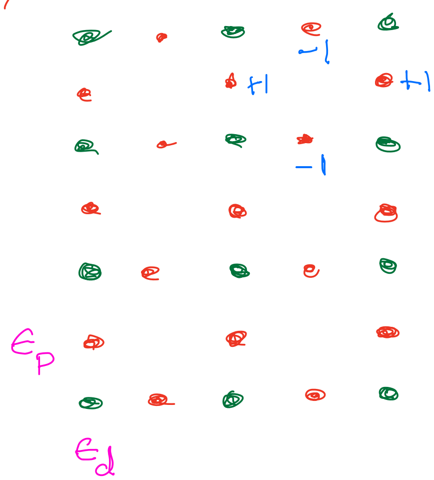
$Y_1Ba_2Cu_3O_{7+\delta}$

$LaSrCuO$

⋮

etc etc.

$\boxed{\text{Lieb}}$



$\boxed{\epsilon_d \neq \epsilon_p}$

D.

kill the flat bands?

Eva ~ flat bands are ~~p~~ p character