

Mean-Field Theory of Spin and Charge Order in the Two-Dimensional Hubbard Model

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Outline

- Introduction
- Unrestricted Hartree-Fock theory
- Mean-field solutions in the thermodynamic limit
- Renormalized mean-field theory



Robin Scholle



Pietro Bonetti



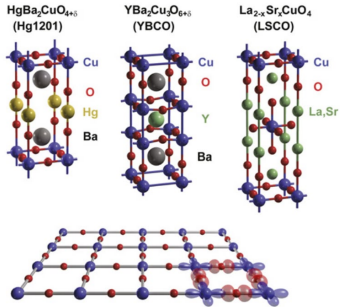
Demetrio Vilardi

Scholle, Bonetti, Vilardi, *wm*, PRB **108**, 035139 (2023)

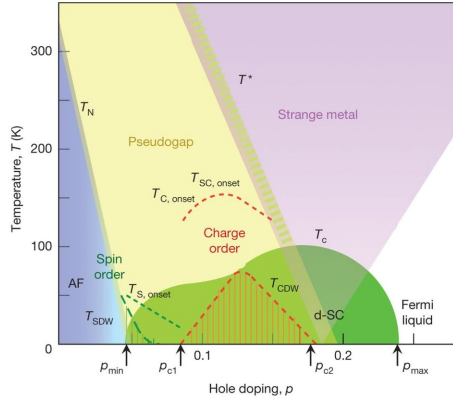
Scholle, *wm*, Vilardi, Bonetti, PRB **109**, 235149 (2024)

Scholle, Bonetti, *wm*, Vilardi, PRB **114**, 185107 (2026)

Cuprate superconductors



Barišić *et al.*, PNAS (2013)

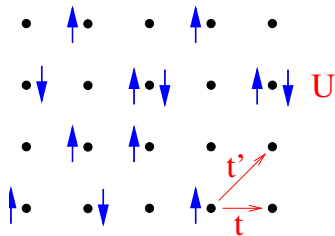


Keimer *et al.*, Nature (2015)

Two-dimensional Hubbard model

Effective single-band model for CuO_2 -planes in HTSC:

(Anderson '87, Zhang & Rice '88)



Hamiltonian $H = H_{\text{kin}} + H_{\text{int}}$

$$H_{\text{kin}} = \sum_{i,j} \sum_{\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} n_{\mathbf{k}\sigma}$$

$$H_{\text{int}} = U \sum_j n_{j\uparrow} n_{j\downarrow}$$

Bare band structure: $\epsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y$

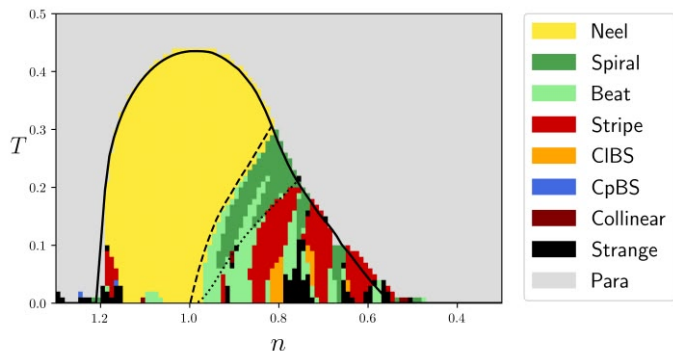
Phase diagram and other properties extremely **hard to compute** !

Short history

- There are tons of theoretical papers on the Hubbard model, with dozens of different methods applied
- Recent progress with **exact** numerical methods on finite lattices: **spin-charge stripe** states and **d-wave superconductivity**
- Dozens of **mean-field** studies, mostly restricted to **special magnetic states**
- **Unrestricted** Hartree-Fock for special parameter sets (no phase diagram), most notably discovery of stripe order by **Zaanen & Gunnarsson 1989**

Unrestricted Hartree-Fock theory

- Hartree-Fock equations in **real space**
- **Classification** of magnetic states
- Results from unrestricted real space calculations



Hartree-Fock equations in real space

Mean-field decoupling of H_{int} :

$$H_{\text{int}}^{\text{mf}} = \sum_j \left(\overbrace{\Delta_{j\downarrow} n_{j\uparrow} + \Delta_{j\uparrow} n_{j\downarrow}}^{\text{Hartree}} \overbrace{-\Delta_{j-} c_{j\uparrow}^\dagger c_{j\downarrow} - \Delta_{j+} c_{j\downarrow}^\dagger c_{j\uparrow}}^{\text{Fock}} \right) - U^{-1} \sum_j (\Delta_{j\uparrow} \Delta_{j\downarrow} - \Delta_{j-} \Delta_{j+})$$

Gap parameters self-consistently determined by

$$\Delta_{j\sigma} = U \langle n_{j\sigma} \rangle \quad \Delta_{j+} = \Delta_{j-}^* = U \langle c_{j\uparrow}^\dagger c_{j\downarrow} \rangle$$

Allows for arbitrary spin and charge patterns

$$\Delta_{j\uparrow} + \Delta_{j\downarrow} = U \langle n_j \rangle \quad \Delta_{j\uparrow} - \Delta_{j\downarrow} = 2U \langle S_j^z \rangle$$

$$\Delta_{j+} + \Delta_{j-} = 2U \langle S_j^x \rangle \quad \Delta_{j+} - \Delta_{j-} = 2iU \langle S_j^y \rangle$$

$$n_j = n_{j\uparrow} + n_{j\downarrow} \text{ charge,} \quad \vec{S}_j = (S_j^x, S_j^y, S_j^z) \text{ spin}$$

Hartree-Fock solution in real space

Finite lattice with $L = L_x L_y$ sites

$$H^{\text{mf}} = H_0 + H_{\text{int}}^{\text{mf}} = \sum_{j,j'} \sum_{\sigma,\sigma'} c_{j\sigma}^\dagger \mathcal{H}_{jj'}^{\sigma\sigma'} c_{j'\sigma'} + \text{const}$$

$\mathcal{H}_{jj'}^{\sigma\sigma'}$ hermitian $2L \times 2L$ matrix

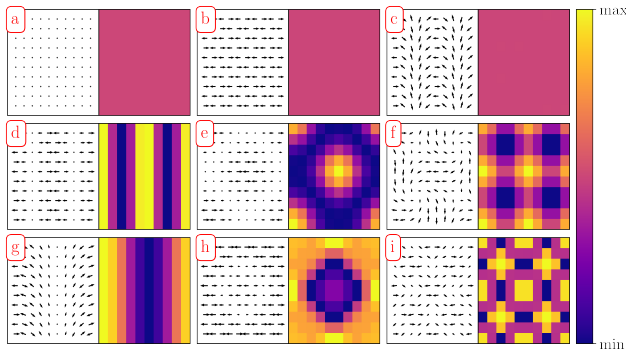
$2L$ eigenvalues ϵ_ℓ , $2L$ eigenvectors $v_{j\sigma}^\ell$, with $\ell = 1, \dots, 2L$

Gap parameters from $\langle c_{j\sigma}^\dagger c_{j'\sigma'} \rangle = \sum_{\ell=1}^{2L} (v_{j\sigma}^\ell)^* v_{j'\sigma'}^\ell f(\epsilon_\ell - \mu)$

Solve **iteratively**, starting from an **arbitrary** spin and charge configuration
It almost always converges to a **unique** (modulo symmetry) solution!

Classification of magnetic states

A zoo of spin (left) and charge (right) patterns:



- a) no order b) Néel antiferromagnet c) planar circular spiral
 d) collinear unidirectional stripe e) collinear bidirectional stripe
 f) coplanar bidirectional stripe g) beat order h) other collinear orders
 i) "strange" order

Fourier decomposition

Charge modulation follows spin amplitude modulation

$$\langle n_j \rangle \approx \text{const} \langle \vec{S}_j^2 \rangle + \text{const}$$

Fourier representation of spin pattern $\langle \vec{S}_j \rangle = \sum_{\mathbf{q}} \vec{\mathcal{S}}_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}_j}$

In most cases only **one or two modes** contribute significantly:

$$\vec{\mathcal{S}}_{\mathbf{q}} = \frac{1}{2} \left(\vec{S} \delta_{\mathbf{q}, \mathbf{Q}} + \vec{S}^* \delta_{\mathbf{q}, -\mathbf{Q}} \right) \text{ or}$$

$$\vec{\mathcal{S}}_{\mathbf{q}} = \frac{1}{2} \left(\vec{S}_x \delta_{\mathbf{q}, \mathbf{Q}_x} + \vec{S}_x^* \delta_{\mathbf{q}, -\mathbf{Q}_x} \right) + \frac{1}{2} \left(\vec{S}_y \delta_{\mathbf{q}, \mathbf{Q}_y} + \vec{S}_y^* \delta_{\mathbf{q}, -\mathbf{Q}_y} \right)$$

Usually $\mathbf{Q}_x = (\pi - 2\pi\eta, \pi)$, $\mathbf{Q}_y = (\pi, \pi - 2\pi\eta)$, and $\mathbf{Q} \in \{\mathbf{Q}_x, \mathbf{Q}_y\}$

Diagonal $\mathbf{Q} = (\pi - 2\pi\eta, \pi - 2\pi\eta)$ are rare for Hubbard with t, t'

Specific magnetic states

- *Circular planar spiral*, oriented in xy plane:

$$\langle \vec{S}_j \rangle = S_0 \begin{pmatrix} \cos(\mathbf{Q} \cdot \mathbf{r}_j) \\ \sin(\mathbf{Q} \cdot \mathbf{r}_j) \\ 0 \end{pmatrix} \quad \vec{S} = S_0 \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$$

- *Collinear unidirectional stripe*, oriented in x direction:

$$\langle \vec{S}_j \rangle = S_0 \begin{pmatrix} \cos(\mathbf{Q} \cdot \mathbf{r}_j) \\ 0 \\ 0 \end{pmatrix} \quad \vec{S} = S_0 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

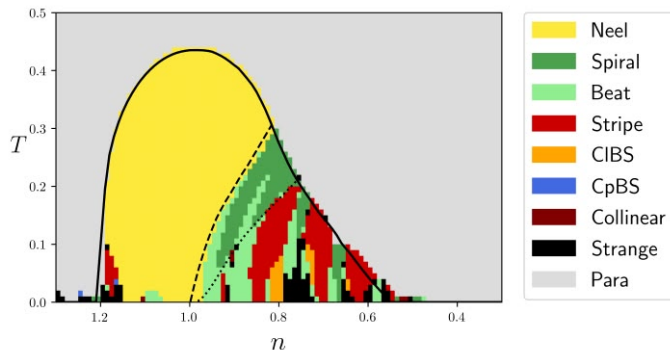
- *Coplanar bidirectional stripe*, oriented in xy plane:

$$\langle \vec{S}_j \rangle = S_0 \begin{pmatrix} \cos(\mathbf{Q}_x \cdot \mathbf{r}_j) \\ \cos(\mathbf{Q}_y \cdot \mathbf{r}_j) \\ 0 \end{pmatrix} \quad \vec{S}_x = S_0 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \vec{S}_y = S_0 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

- *Beat states*: superposition of two close wave vectors $\mathbf{Q} \approx \mathbf{Q}'$
(finite size artifact)

Finite size phase diagram

Hartree-Fock phase diagram for $t' = -0.15t$ and $U = 3t$ on 20×20 lattice (lines from thermodynamic limit)



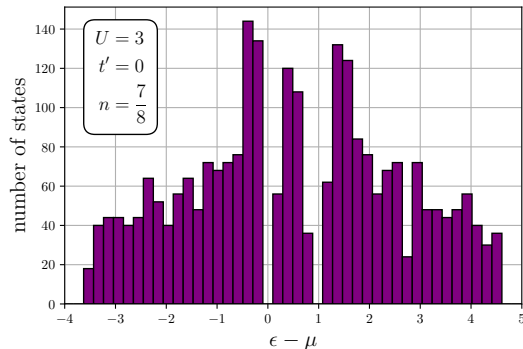
- Néel state at half-filling and for electron doping ($n \geq 1$)
- Spiral, stripe, beat, and strange states for hole doping ($n < 1$)

Insulating phases away from half-filling?

Exact numerical calculations on small finite lattices find special points with **insulating stripe** ground states, e.g., $t' = 0$, $n = 7/8$

Zheng et al., Science (2017); Jiang & Devereaux, Science (2019);

Qin et al., Phys. Rev. X (2020)



Hartree-Fock
density of states for

$$t' = 0, U = 3t, \\ n = \frac{7}{8}, T = 0$$

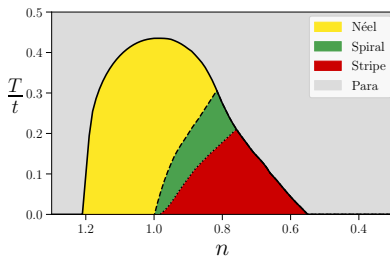
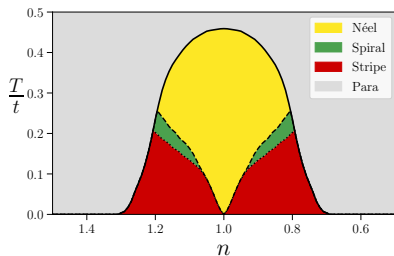
on 32×40 lattice

Gap at Fermi level

Consistent with Hartree-Fock approximation? **Yes!**

Mean-field solutions in the thermodynamic limit

- Néel and spiral states
- Commensurate coplanar states
- Landau theory
- Spiral to stripe transition and complete phase diagram



Néel and spiral states

- Néel special case of spiral with $\mathbf{Q} = (\pi, \pi)$
- Special **symmetry** of circular spiral state: translation $\otimes$ spin rotation
- For spiral oriented in **xy** plane, symmetry-breaking expectation values $\langle c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}+\mathbf{Q}\downarrow} \rangle$ (nothing else)

$\Rightarrow$ Mean-field Hamiltonian

$$H_{\text{int}}^{\text{mf}} = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger \begin{pmatrix} \epsilon_{\mathbf{k}} & -\Delta^* \\ -\Delta & \epsilon_{\mathbf{k}+\mathbf{Q}} \end{pmatrix} \Psi_{\mathbf{k}} + L|\Delta|^2/U$$

with $\Psi_{\mathbf{k}} = (c_{\mathbf{k}\uparrow}, c_{\mathbf{k}+\mathbf{Q}\downarrow})$ and $\Delta = UL^{-1} \sum_{\mathbf{k}} \langle c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}+\mathbf{Q}\downarrow} \rangle$

Diagonalization $\Rightarrow$ two quasi-particle bands

$$E_{\mathbf{k}}^\pm = \epsilon_{\mathbf{k}}^\pm \pm \sqrt{(\epsilon_{\mathbf{k}}^-)^2 + |\Delta|^2} \quad \text{with } \epsilon_{\mathbf{k}}^\pm = \epsilon_{\mathbf{k}} \pm \epsilon_{\mathbf{k}+\mathbf{Q}}$$

Free energy $F = -T \sum_{\mathbf{k}} \sum_{\ell=\pm} \ln \left(1 + e^{-\beta(E_{\mathbf{k}}^\ell - \mu)} \right) + L|\Delta|^2/U + \mu N$

Magnetic gap $|\Delta|$ and $\mathbf{Q}$ determined by minimum of F

Stability of spiral state

Charge-spin susceptibility $\chi^{ab}(\mathbf{q}, \mathbf{q}')$,

where $a, b = 0, 1, 2, 3$ label charge (0) and spin (1, 2, 3) components,
 $\mathbf{q}' - \mathbf{q} \in \{\mathbf{0}, \pm\mathbf{Q}, \pm 2\mathbf{Q}\}$

Conserving approximation for $\chi^{ab}(\mathbf{q}, \mathbf{q}')$:

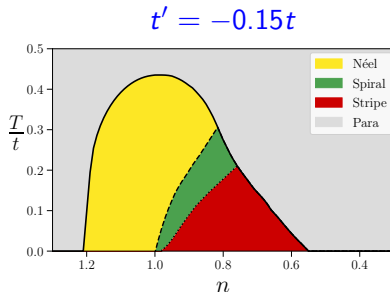
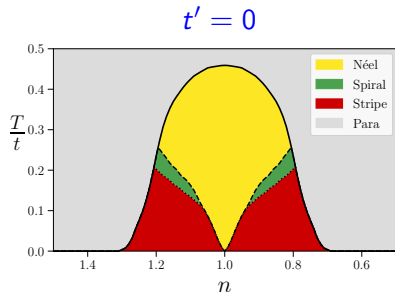
random phase approximation (RPA), most convenient in rotated spin frame with all spins aligned in same direction

Kampf 1996, Bonetti & wm 2022

- Divergence at $\mathbf{q}, \mathbf{q}' = \pm\mathbf{Q}$: Goldstone modes
- Divergence at other wave vectors:
instability toward other magnetic/charge order

Preliminary phase diagram in thermodynamic limit

- Overall critical temperature T^* from divergence of spin susceptibility in paramagnetic (no order) state
- Transition Néel-spiral from spiral solution with generic $\mathbf{Q}$
- Transition to magnetic state with charge order (“stripe”) from divergence of charge-spin susceptibility in spiral state



Nature of “stripe” regime remains to be clarified!

Commensurate coplanar states: ansatz

Unrestricted real-space calculations indicate that all mean-field states are

- **Coplanar**: all spins aligned in a common plane
- **Unidirectional**: spins are Néel ordered in (at least) one direction

Assuming **commensurate** order with a **periodicity** P in x direction (and Néel order in y direction, the wave vectors contributing to the Fourier expansion of $\vec{S}_j$ and ρ_j have the form

$$\mathbf{Q}_n = (2\pi n/P, \pi n) \text{ with } n = 0, 1, \dots, \bar{P} - 1; \bar{P} = \begin{cases} P & \text{for } P \text{ even} \\ 2P & \text{for } P \text{ odd} \end{cases}$$

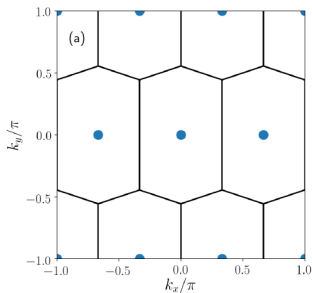
Choosing spins to be aligned in xy plane one then has

$$\begin{aligned} \vec{S}_j &= \sum_{n \text{ odd}} (M_n^x \vec{e}_x + M_n^y \vec{e}_y) e^{i\mathbf{Q}_n \cdot \mathbf{r}_j} \\ \rho_j &= \sum_{n \text{ even}} \rho_n e^{i\mathbf{Q}_n \cdot \mathbf{r}_j} \end{aligned}$$

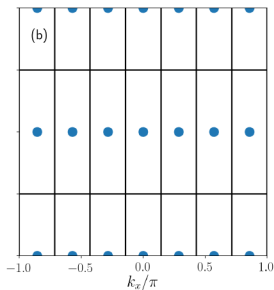
Reduced Brillouin zones

Reduced Brillouin zones of commensurate coplanar unidirectional states:

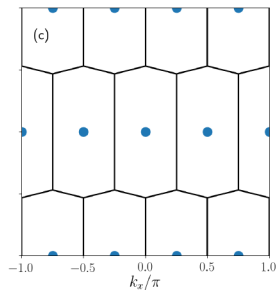
$$P = 6$$



$$P = 7, \bar{P} = 14$$



$$P = 8$$



Mean-field Hamiltonian in momentum representation

Mean-field interaction in SU(2) symmetric form:

$$H_{\text{int}}^{\text{MF}} = \sum_j \sum_{a=0}^3 s_a \left[\Delta_j^a c_j^\dagger \sigma^a c_j - (\Delta_j^a)^2 / U \right]$$

where $c_j = (c_{j\uparrow}, c_{j\downarrow})$, $s_0 = 1$, $s_{a \neq 0} = -1$, and $\Delta_j^a = \frac{1}{2} U \langle c_j^\dagger \sigma^a c_j \rangle$

Insert Fourier decomposition $\Rightarrow$

$$H_{\text{int}}^{\text{MF}} = \sum_{\mathbf{k}} \sum_{a=0}^3 \sum_{n=0}^{\bar{P}-1} s_a \Delta_n^a c_{\mathbf{k}}^\dagger \sigma^a c_{\mathbf{k}+\mathbf{Q}_n} - L \sum_{a=0}^3 \sum_{n=0}^{\bar{P}-1} s_a \frac{|\Delta_n^a|^2}{U}$$

$$\text{with } \Delta_n^a = \frac{U}{2L} \sum_{\mathbf{k}} \langle c_{\mathbf{k}}^\dagger \sigma^a c_{\mathbf{k}+\mathbf{Q}_n} \rangle$$

$$\Delta_n^0 = \begin{cases} \frac{1}{2} U \rho_n & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{cases}, \quad \Delta_n^{1,2} = \begin{cases} 0 & \text{for } n \text{ even} \\ U M_n^{x,y} & \text{for } n \text{ odd} \end{cases}$$

$\Rightarrow H^{\text{MF}} \equiv \bar{P} \times \bar{P}$ matrix

Landau theory

The **Landau theory** of phase transitions is based on a **polynomial expansion** of the **effective action** for the order parameter, applicable close to the transition temperature where the order parameter is small

The **structure** of the effective action is usually determined by **symmetries**

The **coefficients** can be derived, in principle, from a **microscopic theory** (e.g., Ginzburg-Landau from BCS)

In our case, the Landau theory can then be used to determine the type of magnetic order close to T^*

Hubbard action

Hubbard model action in path integral formalism

$$\begin{aligned}\mathcal{S}[c, c^*] &= \int_0^\beta d\tau \sum_{j,j'} c_j^*(\tau) [(\partial_\tau - \mu)\delta_{jj'} + t_{jj'}] c_{j'}(\tau) \\ &+ \int_0^\beta d\tau \sum_j n_{j\uparrow}(\tau) n_{j\downarrow}(\tau)\end{aligned}$$

$c_j = (c_{j\uparrow}, c_{j\downarrow})$ anticommuting fields, depending on imaginary time τ

See, for example, textbook by [Negele & Orland](#)

Effective action

Interaction in SU(2) symmetric form:

$$Un_{j\uparrow}n_{j\downarrow} = \frac{U}{4}(c_j^*c_j)^2 - \frac{U}{4}(c_j^*\vec{\sigma} \cdot \vec{n}_j c_j)^2$$

Decouple with auxiliary **charge** and **spin** fields (Hubbard-Stratonovich)

$$e^{-U \int_0^\beta d\tau \sum_j n_{j\uparrow} n_{j\downarrow}} = \int \mathcal{D}\rho \mathcal{D}\vec{S} e^{\int_0^\beta \sum_j U^{-1}(\rho_j^2 + \vec{S}_j^2) + c_j^*(i\rho_j + \vec{\sigma} \cdot \vec{S}_j) c_j}$$

Integrate out fermion fields c, c^* (Gaussian integral)

$\Rightarrow$ **Effective action**

$$\mathcal{S}^{\text{eff}}[\rho, \vec{S}] = \frac{1}{U} \int_0^\beta d\tau \sum_j (\rho_j^2 + |\vec{S}_j|^2) - \text{tr} \ln [1 - G_0(i\rho + \vec{\sigma} \cdot \vec{S})]$$

G_0 bare electron propagator

Taylor expansion $\text{tr} \ln [1 - G_0(i\rho + \vec{\sigma} \cdot \vec{S})] = - \sum_{n=1}^{\infty} \frac{1}{n} \text{tr} [G_0(i\rho + \vec{\sigma} \cdot \vec{S})]^n$

Quadratic term

Quadratic term in $\vec{S}$

$$\mathcal{S}_2^{\text{eff}}[\vec{S}, 0] = T \sum_{q_0} \int_{\mathbf{q}} \left[U^{-1} - \Pi_0(\mathbf{q}, iq_0) \right] \vec{S}(\mathbf{q}, iq_0) \cdot \vec{S}(-\mathbf{q}, -iq_0)$$

$$\Pi_0(\mathbf{q}, iq_0) = -T \sum_{k_0} \int_{\mathbf{k}} G_0(\mathbf{k}, ik_0) G_0(\mathbf{k} + \mathbf{q}, ik_0 + iq_0)$$

Critical temperature T^* determined by

$$U^{-1} - \lim_{\mathbf{q} \rightarrow 0} \Pi_0(\mathbf{q}, 0) = 0$$

Cubic and quartic terms

Mean-field theory: Only $q_0 = 0$ and $\mathbf{q} = \pm \mathbf{Q}_x, \pm \mathbf{Q}_y$ close to T^*

$\Rightarrow \vec{S}(\mathbf{q}, iq_0) = \vec{S}(\mathbf{q})\delta_{q_0,0}$ with

$$\vec{S}(\mathbf{q}) = \vec{M}_x \delta_{\mathbf{q}, \mathbf{Q}_x} + \vec{M}_x^* \delta_{\mathbf{q}, -\mathbf{Q}_x} + \vec{M}_y \delta_{\mathbf{q}, \mathbf{Q}_y} + \vec{M}_y^* \delta_{\mathbf{q}, -\mathbf{Q}_y}$$

Quartic spin potential

$$\begin{aligned} V_4(\vec{M}_x, \vec{M}_y) = & u_0(|\vec{M}_x|^2 + |\vec{M}_y|^2)^2 + u_1(|\vec{M}_x|^2 - |\vec{M}_y|^2)^2 \\ & + u_2(|\vec{M}_x \cdot \vec{M}_x|^2 + |\vec{M}_y \cdot \vec{M}_y|^2) + u_3(|\vec{M}_x \cdot \vec{M}_y|^2 + |\vec{M}_x \cdot \vec{M}_y^*|^2) \end{aligned}$$

Leading charge-spin coupling $\int_{\mathbf{q}, \mathbf{q}'} i\lambda(\mathbf{q}, \mathbf{q}') \vec{S}(\mathbf{q}) \cdot \vec{S}(-\mathbf{q}') \rho(\mathbf{q}' - \mathbf{q})$

$$\begin{aligned} \Rightarrow i\rho(\mathbf{q}) = & \phi_x \delta_{\mathbf{q}, 2\mathbf{Q}_x} + \phi_x^* \delta_{\mathbf{q}, -2\mathbf{Q}_x} + \phi_y \delta_{\mathbf{q}, 2\mathbf{Q}_y} + \phi_y^* \delta_{\mathbf{q}, -2\mathbf{Q}_y} \\ & + \phi_+ \delta_{\mathbf{q}, \mathbf{Q}_+} + \phi_+^* \delta_{\mathbf{q}, -\mathbf{Q}_+} + \phi_- \delta_{\mathbf{q}, \mathbf{Q}_-} + \phi_-^* \delta_{\mathbf{q}, -\mathbf{Q}_-} \end{aligned}$$

with $\mathbf{Q}_{\pm} = \mathbf{Q}_x \pm \mathbf{Q}_y$

Landau potential

Optimize $\rho(\mathbf{q})$ for fixed $\vec{S}(\mathbf{q}) \Rightarrow$ Landau potential

$$\begin{aligned} V(\vec{M}_x, \vec{M}_y) = & s(|\vec{M}_x|^2 + |\vec{M}_y|^2) \\ & + u_0(|\vec{M}_x|^2 + |\vec{M}_y|^2)^2 + u_1(|\vec{M}_x|^2 - |\vec{M}_y|^2)^2 \\ & + \tilde{u}_2(|\vec{M}_x \cdot \vec{M}_x|^2 + |\vec{M}_y \cdot \vec{M}_y|^2) + \tilde{u}_3(|\vec{M}_x \cdot \vec{M}_y|^2 + |\vec{M}_x \cdot \vec{M}_y^*|^2) \end{aligned}$$

Minimizing with respect to $\vec{M}_x$ and $\vec{M}_y$ one can determine the magnetic state right below T^*

Examples:

- $\vec{M}_x = 0$ or $\vec{M}_y = 0 \Rightarrow$ unidirectional order
- $\vec{M}_x = m(1, 0, 0)$ or rotation thereof $\Rightarrow$ unidirectional stripe
- $\vec{M}_x = m(1, i, 0)$ or rotation thereof $\Rightarrow$ spiral

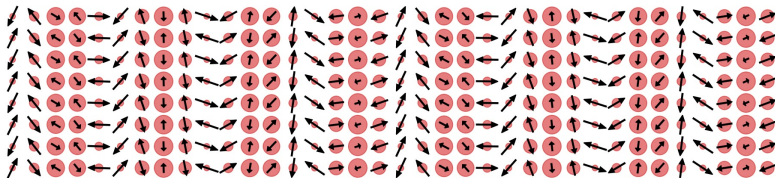
Spiral to stripe transition

Increasing hole doping in spiral regime leads to **divergence** of spin susceptibility at special **nesting** vector $\mathbf{Q}'$ of the hole pockets

$\Rightarrow$ Continuous transition to a **multi-spiral** state

$$\vec{S}_j = \begin{pmatrix} M \cos(\mathbf{Q} \cdot \mathbf{r}_j) + M'_+ \cos(\mathbf{Q}_+ \cdot \mathbf{r}_j - \phi') + M'_- \cos(\mathbf{Q}_- \cdot \mathbf{r}_j - \phi') \\ M \sin(\mathbf{Q} \cdot \mathbf{r}_j) - M'_+ \sin(\mathbf{Q}_+ \cdot \mathbf{r}_j - \phi') + M'_- \sin(\mathbf{Q}_- \cdot \mathbf{r}_j - \phi') \\ 0 \end{pmatrix}$$

with $\mathbf{Q}'_{\pm} = -(\mathbf{Q}' \pm \mathbf{Q})$

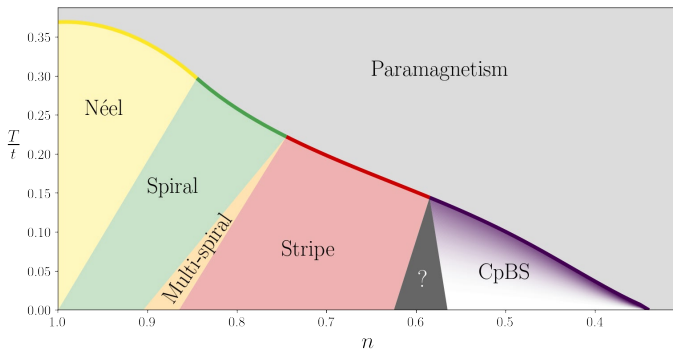


Coplanar (not collinear), unidirectional, with stripe charge order

Complete Hartree-Fock phase diagram

Further increasing hole-doping, transition to **collinear** stripe order (probably weakly first order)

Complete HF phase diagram for $U = 3t$ and $t' = -0.3t$:



Ground state transitions, T^* , and states near T^* computed;
 Transition lines for $0 < T < T^*$ only schematic

Renormalized mean-field theory

Renormalized mean-field theory:

Hartree-Fock theory with bare Hubbard interaction replaced by Hartree-Fock + BCS with effective interaction obtained from a renormalization group flow

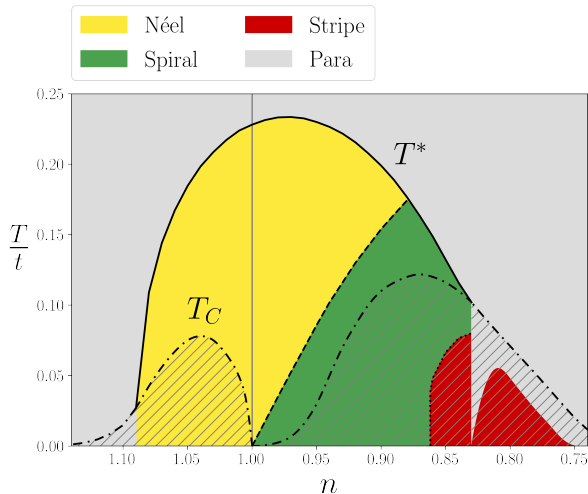
Reiss, Rohe, wm 2007; Wang, Eberlein, wm 2014

- Magnetic interaction reduced by fluctuations (mostly pairing)
- d-wave pairing interaction generated by fluctuations (mostly magnetic)

See Jülich Lectures on functional RG 2025

Renormalized mean-field phase diagram

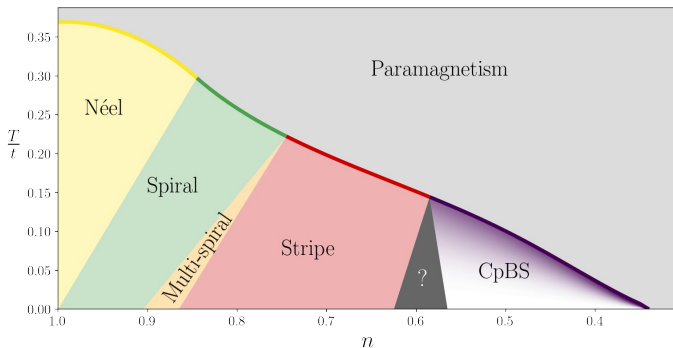
Renormalized mean-field phase diagram for $U = 4t$ and $t' = -0.2t$:



- Néel, spiral, and stripe as in plain HF
- Pairing coexisting with magnetic order
- Pairing reduces stripe order in favor of spiral
- No magnetic order at van Hove filling

Conclusion

- Comprehensive Hartree-Fock phase diagram of 2D Hubbard model: Néel, spiral, and stripe phases
- Pairing (superconductivity) can be obtained by renormalized MFT



Outlook:

Dynamical mean-field (DMFT) phase diagram for strong interaction