Exercise Sheet 4 due 14 November

1. *Airy functions*

Let $w(z) = \alpha\text{Ai}(z) + \beta\text{Bi}(z)$ be a general solution of the Airy differential equation $w''(z) = z\, w(z)$.

i. Show that

$$\varphi(x) = w\left(\sqrt[3]{\frac{2me\mathcal{E}}{\hbar^2}}\left(x - \frac{E}{e\mathcal{E}}\right)\right)$$

solves the time in-dependent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2} + e\mathcal{E}x\,\varphi(x) = E\varphi(x).$$

What happens when $\mathcal{E} < 0$. Sketch the solution.

ii. Express the normalization integral $\int_{x_1}^{x_2} |\varphi(x)|^2\, dx$ in terms of the integral over the Airy function $\int_{z_1}^{z_2} |w(z)|^2\, dz =: c$, where $z_i = \sqrt[3]{2me\mathcal{E}/\hbar^2}(x_i - E/e\mathcal{E})$.

2. *infinite potential well in electric field*

Consider an infinite potential well of width $L = 8\,\text{Å}$ with potential $V(x) = e\mathcal{E}x$ for $|x| < L/2$. Find the lowest three eigenenergies

i. for zero electric field, $\mathcal{E} = 0$

ii. for $\mathcal{E} = 100\,\text{V}/\mu\text{m}$