spherically symmetric potential

$$\left[-\frac{\hbar^2}{2m}\Delta_r + V(|\vec{r}|)\right]\psi(\vec{r}) = E\psi(\vec{r})$$

separation ansatz

$$\psi(\vec{r}) = \frac{u(r)}{r} Y_{l,m}(\theta,\phi)$$

$$-i\frac{\partial}{\partial\vartheta} Y_{l,m}(\vartheta,\varphi) = m\,Y_{l,m}(\vartheta,\varphi)$$

$$-\left(\frac{1}{\sin(\vartheta)}\frac{\partial}{\partial\vartheta}\sin(\vartheta)\frac{\partial}{\partial\vartheta} + \frac{1}{\sin^2(\vartheta)}\frac{\partial^2}{\partial\varphi^2}\right) Y_{l,m}(\vartheta,\varphi) = l(l+1)\,Y_{l,m}(\vartheta,\varphi)$$

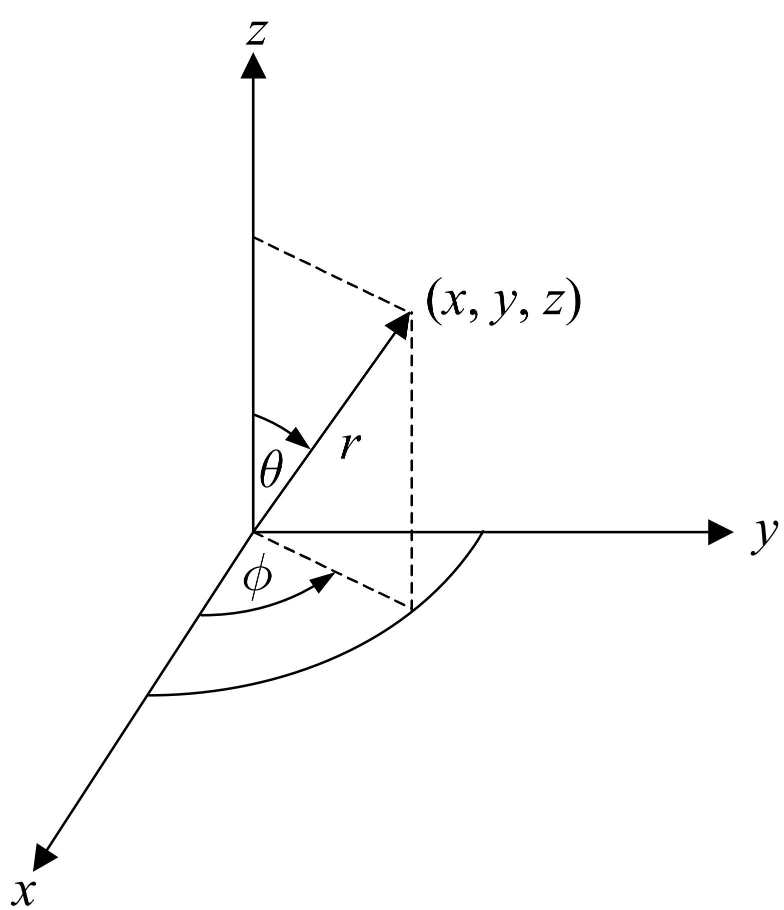
$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2} - V(r)\right) u(r) = E\,u(r)$$

Spherical Harmonics

spherical harmonics (Condon-Shortley phase-convention)

$$Y_{l,m}(\theta,\phi) = (-1)^{\frac{m+|m|}{2}} \sqrt{\frac{2l+1}{4\pi}} \sqrt{\frac{(l-|m|)!}{(l+|m|)!}} P_{l,m}(\cos\theta) e^{im\phi}$$

where $P_{l,m}(\cos\theta) = \frac{(-1)^l}{2^l l!} \sin^m\theta \frac{d^{l+m}\sin^{2l}\theta}{d\cos^{l+m}\theta}$ associated Legendre functions



properties

$$\int d\Omega \overline{Y_{l,m}(\theta,\phi)} Y_{l',m'}(\theta,\phi) = \delta_{l,l'}\delta_{m,m'}$$ orthonormality

$$\sum_{l=0}^{\infty}\sum_{m=-l}^{l} Y_{l,m}(\theta,\phi) Y_{l,m}(\theta',\phi') = \frac{\delta(\theta-\theta')\delta(\phi-\phi')}{\sin\theta}$$ completeness

$$Y_{l,m}(\pi-\theta,\pi+\phi) = (-1)^l Y_{l,m}(\theta,\phi)$$ parity

$$Y_{l,-m}(\theta,\phi) = (-1)^m \overline{Y_{l,m}(\theta,\phi)}$$